Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		2221799

Using a series of logical comparisons to determine the best trading strategy of Bitcoin and gold

Predicting the unpredictable is perhaps every person's dream and the one thing, no matter how hard people have worked for it, that is impossible to do with 100% accuracy. Such is the case with asset trading. Sudden bad news can send an asset's value pummeling within a very short period of time. However, in the absence of such significant outside factors, there are ways to predict general trends or close estimates to future asset values.

The main purpose behind our model is not to predict future asset price. However, we will employ a popular, but simple, **log-return model** to predict the **future price** of Bitcoin and gold based on price data from the last 26 days. 26 is perhaps a good balance between too long ago that the data becomes irrelevant and too recent that short term fluctuations play too big of a role. Importantly, however, is that price prediction is simply a tool employed by the model and not the main purpose. Any reasonably accurate price-prediction model can be substituted for the one used in this report.

The strategy employed by the trader will consist of a **series of comparisons** the trader must carry out every day. There will be a slight difference in the comparisons done on the weekends, when gold markets are closed but bitcoin markets are open, and weekdays, when both are open.

The units of comparisons will be the ratios of expected price tomorrow over current price today. The basic idea is that of the expected change tomorrow is greater than a certain threshold, the trader should buy. If it is less than another threshold, the trader should sell. Otherwise, the trader should hold. On weekdays, the trader should compare the expected change in gold value and bitcoin value and choose whichever is higher. Then, they should compare the expected change in value to the thresholds they have set.

Key words: log-return model, future price, series of comparisons

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1 Overview

1.1 Introduction

Investments are an important part of a country's economy. They allow companies to raise money and allow people to place their money in an asset that, ideally, will rise in value. With the development of modern society, more people started to move their assets to those products that can be value-preserved. Rare metals are a popular choice with people because they are generally not highly risky investments. Since gold was discovered thousands of years ago, it was used for trading and its value persevered even in the 21st century.[1] Its price fluctuates as the actual currency's inflation or appreciation.[1] Throughout history, governments had much sway, sometimes all of it, in what kind of currency is used. However, the development of the internet in recent years had allowed a new type of currency to emerge; with the urgent need for an innovative design and private platform to trade without the monitor of officials and big companies, Satoshi Nakamoto created Bitcoin in 2009 as a digital asset and a cryptocurrency.[2] Unlike traditional currencies or value-preserve products like Gold and Silver, Bitcoin was famous for its substantial risk and payoff at the same time.[2] It was not officially recognized when it was first created and was even banned in some countries like China.[2] However, it only became more popular as time went by. In the mid-late 2010s, more and more people invested in this area, giving a sort of legitimacy to the whole cryptocurrency industry. Some celebrities also promoted cryptocurrencies, which helped in increasing their fame dramatically. Where there is a need, there is a market. Investors saw the success of Bitcoin and tried to imitate it; "Dogecoin" and other cryptocurrencies came one after another. The prosperity of this industry is attracting millions of investors around the world.

1.2 Problem restatement

Market traders need to predict what will happen to the value of assets to a reasonable extent to decide whether to buy, sell, or hold. As perhaps the most popular form of cryptocurrency, Bitcoin is an asset of particular interest to many traders. In addition, metals like gold provide reasonably safe investments and are also of interest to traders. The main goal of traders is to maximize their returns. Mathematical models may provide reasonable predictions that can help traders navigate the unpredictability of the markets. To maximize trader returns, this paper will:

- Produce a model to predict a trading strategy on each day based on previous price data.
- Analyze the strategy's sensitivity to changes in transaction costs.

1.3 Data tidying

There were some dates in the LBMA-GOLD data that did not correspond to any values (empty cells). Moreover, there are dates that are "skipped." We assume these dates are either weekends or the market is closed, and we assign them NA values. Finally, we merged the Bitcoin data and the gold data into one dataset.

1.4 The basic principle behind the model

The model employs predictions about future Bitcoin and gold prices to determine whether the trader should buy, sell, or hold. However, we would like to stress that the main idea behind our model is **not** to predict future asset prices. This is simply a tool that will be helpful in deciding whether the trader should buy, sell or trade based on a series of logical comparisons that are the foundation of the model. The trader can use any model to predict asset price, but ideally a reasonably accurate one. We will use a few simple models here to predict asset price, but we would like to note that there are sophisticated asset prediction models that have been produced with millions in investments for large companies. Again, the trader is free to use any of those models, but the principle of logical comparisons provided here is the main idea of the report.

2 Assumption and nomenclature

2.1 Assumptions

Assumption 1: the trader is buying and selling at the closing price. In reality, prices may fluctuate significantly during the day.

Assumption 2: Bitcoin and gold prices fluctuate normally as do other assets in general. In other words, they are not affected by sudden news or internet posts that encourage large scale buying or selling.

Assumption 3: we cannot predict the price of gold on days the market is closed or that any trading occurs on those days. Gold value prediction models will assume that the closing price at the last day before market close is the last data point to use to predict the closing price at the next date the market opens.

Assumption 4: data older than 26 days is not relevant in predicting future asset price. Times change and people's opinions on assets do as well. We feel the last 26 days of data are the most indicative of the trend the price will follow. Less than that, the prediction may be highly influenced by short-term fluctuations. More than that, the prediction may be highly influenced by irrelevant factors.

Assumption 5: because of our need of 26 days of data, the trader can begin trading bitcoin 26 days after 9/11/2016 and begin trading gold 26 **open-market** days after 9/12/2016.

Assumption 6: Bitcoins and gold can be bought in any non-negative amount. For example, the trader can buy 1.456 of a Bitcoin or 2.82 troy ounces of gold.

2.2 Nomenclature

Symbol	Meaning
$\boldsymbol{t}$	A day in a period at which the market is open; $t = 1,2,3 \ldots$
$\boldsymbol{P_t}$	Closing price at the end of day t

$\widehat{\boldsymbol{P}}_{\boldsymbol{g},\boldsymbol{t+1}}, \widehat{\boldsymbol{P}}_{\boldsymbol{b},\boldsymbol{t+1}}$	Predicted closing price of **g**old or **b**itcoin at the end of day $t+1$
$\boldsymbol{r_{t+1}}$	Log return at day
$\boldsymbol{A}, \boldsymbol{C}$	$1+$ the minimum percentage of revenue the trader would like from Bitcoin or gold, $A, C > 1$
$\boldsymbol{B}, \boldsymbol{D}$	$1-$ the minimum percentage of loss the trader would tolerate from Bitcoin or gold, $B, D < 1$

3 Prediction of asset prices

3.1 Predicting Asset Price: The Log Return Model

We will adapt the one period log return model to fit this situation. Let r_{t+1} be the return at day t+1, and P_t be the price at the end of day t. The one period log return model is defined as:[3,4]

$$r_{t+1} = \ln \frac{P_{t+1}}{P_t}$$

We are interested in price, so we rewrite this as:

$$P_{t+1} = P_t e^{r_{t+1}}$$

This pattern follows until P_0, so we can rewrite it as:

$$P_{t+1} = P_0 e^{\sum_{i=1}^{t+1} r_i}$$

We note that this requires the log-return at r_{t+1}. Our model will need to predict the price P_{t+1} before r_{t+1} is available, so we will estimate r_{t+1} as the average of r_i. Finally, we rewrite the formula as:

$$P_{t+1} = P_0 e^{\frac{1}{t}\sum_{i=1}^{t} r_i + \sum_{i=1}^{t} r_i}$$

Finally, we will not begin "period" at day 1 of five years. Instead, we will only use the last 26 points of data for any given prediction. This is because

4 A series of logical comparisons

4.1 The Tools of Comparisons

In this report, the main tools of comparisons used will be ratios. Specifically, the ratio of an asset' s predicted price at day $t+1$, assuming we are currently at day t, divided by the closing value at day t. For example,

$$\frac{\hat{P}_{g,t+1}}{P_{g,t}} \geq 1.05$$

means that the predicted value tomorrow of a troy ounce of **g**old (or at the next open trading period), $\hat{P}_{g,t+1}$, is at least 5% higher than the current value, $P_{g,t}$.

4.2 The First Logical Comparison: Are the Gold Markets Closed?

Gold markets are generally open on weekdays and closed on weekends. Further, there may be other times of the year on which the gold markets are closed. On the other hand, Bitcoin can be traded every day. The first step in determining what strategy the trader should take is to check whether the gold markets are closed or open. Using R, we have assigned dates in which there is no price value or that appear in the Bitcoin data but not the gold data as NA values. They are assumed to be days in which the market is closed.

4.3 If Gold markets are closed, should the trader buy, sell, or hold Bitcoin?

The trader should buy Bitcoin if:

$$\frac{\hat{P}_{b,t+1}}{P_{b,t}} \geq A$$

where A represents $1 +$ the minimum percentage of revenue the trader would like, $A > 1$.

$$[Cash_t, Gold_t, Bitcoin_t] = \left[0, Gold_{t-1}, \frac{Cash_{t-1}}{1.02 \times P_{b,t}} + Bitcoin_{t-1}\right]$$

(Assuming the trader buys Bitcoin with all the cash, and $\alpha_{bitcoin} = 2\%$. Note: we use $Cash_{t-1}$ which is really the cash at the start of day t. After buying Bitcoin, $Cash_t = 0$, so we used $Cash_{t-1}$ to avoid confusion)

What happens in this case is gold stays the same, the trader spends all of the cash on Bitcoin, and the trader's total amount of Bitcoin becomes yesterday's amount ($Bitcoin_{t-1}$) plus how much Bitcoin they can afford:

$$\frac{Cash_{t-1}}{1.02 \times P_{b,t}}$$

The trader should sell Bitcoin if:

$$\frac{\hat{P}_{b,t+1}}{P_{b,t}} \leq B$$

where B represents $1 -$ the minimum percentage of loss the trader would tolerate, $B < 1$.

$$[Cash_t, Gold_t, Bitcoin_t] = \left[0.98 Bitcoin_{t-1} \times P_{b,t} + Cash_{t-1}, Gold_{t-1}, 0\right]$$

(Assuming the trader sells all Bitcoin for cash, and $\alpha_{bitcoin} = 2\%$)

What happens in this case is gold stays the same, the trader sells all Bitcoin for cash so the trader's total amount of Bitcoin becomes 0, and their cash is 98% of the value of the Bitcoin (after the transaction fee is taken out):

$$0.98 Bitcoin_{t-1} \times P_{b,t}$$

The trader should hold Bitcoin if:

$$B < \frac{\hat{P}_{b,t+1}}{P_{b,t}} < A$$

$$[Cash_t, Gold_t, Bitcoin_t] = [Cash_{t-1}, Gold_{t-1}, Bitcoin_{t-1}]$$

A and B will depend on, among other things,

- The uncertainty in $\hat{P}$; if the asset price prediction model produces wildly inaccurate predictions, choosing a higher A or lower B value might be a safer bet.
- Buying and selling Bitcoin involves a transaction cost, $\alpha_{bitcoin}$, equal to 2% of the amount traded. This should be taken into account (e.g. B should not be more than 0.98)
- The optimism/risk factor of the trader. If the trader is optimistic for large increases in price (or does not want to take big risks for small gains), they can set A high. If the trader is willing to take risks (or realistically optimistic about small but steady gains), they can set A relatively low.

We note that the model in this report produces a **daily** trading strategy and is meant to be used over a relatively long term (say, at least a year). We recommend setting A low and B high, specifically $A = 1.03$ and $B = 0.98$. Practically speaking, large increases in price over a single day seldom occur. A realistic threshold should be somewhat low; "slow and steady," so to speak. As for the risk factor, we reiterate the long-term timescale this model is meant for; the price prediction models will produce inaccurate estimates at times, which might lead to the trader buying or selling at inappropriate times. However, on average, the differences between the predicted and actual values are centered around 0. In other words, even though the trader will lose money at times, the predictions are accurate enough that the trader will make a profit in the long term. Moreover, setting B high is somewhat of a safe strategy. At $B = 0.98$, the trader will sell if the predicted price at $t + 1$ is at least 2% lower than the price at t!

The profit from selling is $(1 - \alpha_{bitcoin})(Bitcoin_{sold})(P_{b,t})$
The cost of buying is $(1 + \alpha_{bitcoin})(Bitcoin_{bought})(P_{b,t})$

The strategy in this report is a high-risk high-reward strategy. Therefore, we recommend that if the condition for buy is met, all the cash available goes to buying. If the condition for sell is met, all the bitcoin is sold. Of course, there is the risk that the value of Bitcoin goes to 0. However, this is unlikely since Bitcoin is the most popular form of cryptocurrency and this strategy is a daily trading strategy (it is highly unlikely Bitcoin will go to 0 in one day).

4.4 If Gold markets are open

If the gold markets are open, the trader must decide whether to buy, sell, or hold Bitcoin, gold, or both.

Given that the value of gold generally does not fluctuate much, we recommend buying Bitcoin instead of gold if the predicted change in price for Bitcoin is higher than the predicted change in price for gold – 5%. This way, the trader does risk losing the transaction fees on a low-reward investment (gold) and keeps the money to buy Bitcoin.

If

$$\frac{\hat{P}_{b,t+1}}{P_{b,t}} \geq \frac{\hat{P}_{g,t+1}}{P_{g,t}} - 0.05$$

the trader should continue with the steps in 4.3.

If

$$\frac{\hat{P}_{b,t+1}}{P_{b,t}} < \frac{\hat{P}_{g,t+1}}{P_{g,t}} - 0.005$$

the trader should do the following:

We note that $\alpha_{gold} = 1\%$.

The trader should buy gold if:

$$\frac{\hat{P}_{g,t+1}}{P_{g,t}} \geq C$$

where C represents $1 +$ the minimum percentage of revenue the trader would like, $C > 1$.

$$[Cash_t, Gold_t, Bitcoin_t] = \left[0, \frac{Cash_{t-1}}{1.01 \times P_{g,t}} + Gold_{t-1}, Bitcoin_{t-1}\right]$$

The trader should sell gold if:

$$\frac{\hat{P}_{g,t+1}}{P_{g,t}} \leq D$$

where D represents $1 -$ the minimum percentage of loss the trader would tolerate, $B < 1$.

$$[Cash_t, Gold_t, Bitcoin_t] = \left(0.99Gold_{t-1} \times P_{b,t} + Cash_{t-1}, 0, Bitcoin_{t-1}\right)$$

The trader should hold gold if:

$$D < \frac{\hat{P}_{g,t+1}}{P_{g,t}} < C$$

$$[Cash_t, Gold_t, Bitcoin_t] = [Cash_{t-1}, Gold_{t-1}, Bitcoin_{t-1}]$$

Given that the value of gold generally does not fluctuate much, we recommend setting C and D high, specifically $C = 1.05$ and $D \leq 0.99$.

5 Results

5.1 Parameters $\alpha_{gold} = 0.01, \alpha_{bitcoin} = 0.02, A = 1.03, B = 0.98, C = 1.05, D = 0.99$

After tidying up the data in R, we made a dataset called "assets" with several columns:

- "Date" "Pricebitcoin" "Predictedbitcoin" "Pricegold" "Predictedgold"
  "Cash" "Bitcoin" "Valuebitcoin" "Gold" "Valuegold"

"Predictedbitcoin" and "Predictedgold" are the predicted prices for each asset on each day. "Bitcoin" and "Gold" are the amounts of Bitcoin and Gold. "Pricebitcoin" and "Pricegold" are the closing prices. "Valuebitcoin" and "Valuegold" are the values of Bitcoin and gold in USD.

Then, we wrote many nested IF functions in R to carry out the strategy. The code can be found in the appendix at the end of the report.

Using our parameters $\alpha_{gold} = 0.01, \alpha_{bitcoin} = 0.02, A = 1.03, B = 0.98, C = 1.05, D = 0.99$, we obtain the following on 9/10/2021:

$$[Cash, Gold, Bitcoin] = [0, 112.71358, 0]$$

$$Value_{bitcoin} = 112.71358 \times 46368.69 = \$5{,}226{,}381.05$$

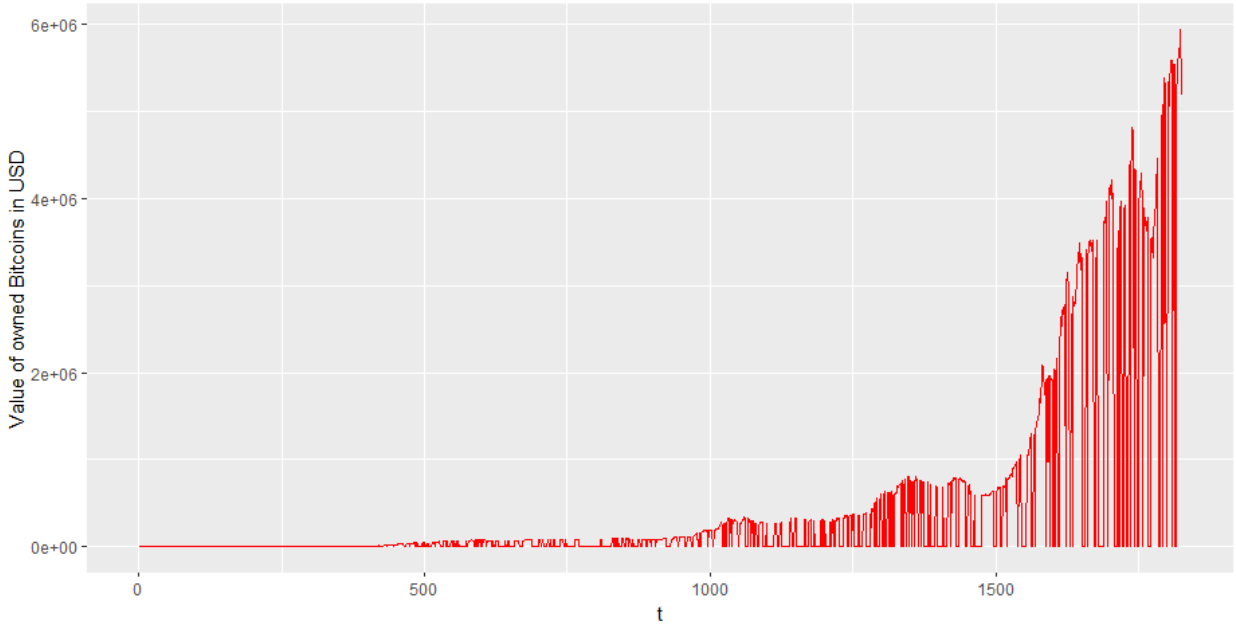


Figure 1: Plot of the value in USD of Bitcoins owned against time if our strategy was used

We note in the figure above that value goes to zero sometimes. This is when all the bitcoins are sold.

5.2 Sensitivity analysis

We will try out different values of $\alpha_{gold}, \alpha_{bitcoin}$ below and report Cash, Gold, Bitcoin on 9/10/2021:

$\alpha_{gold} = 0.02, \alpha_{bitcoin} = 0.02$

$$[Cash, Gold, Bitcoin] = [0,112.71358,0]$$

$\alpha_{gold} = 0.03, \alpha_{bitcoin} = 0.02$

$$[Cash, Gold, Bitcoin] = [0,112.71358,0]$$

It appears there were no gold transactions at all. This explains why the transaction cost of the gold has no effect.

$\alpha_{gold} = 0.01, \alpha_{bitcoin} = 0.01$

$$[Cash, Gold, Bitcoin] = [0,4991.963,0]$$

$\alpha_{gold} = 0.01, \alpha_{bitcoin} = 0.03$

$$[Cash, Gold, Bitcoin] = [0,2.541357,0]$$

It appears the model is extremely sensitive to changes in transaction costs of Bitcoin.
If the transaction cost was decreased by 1 percent, the trader would have about 44 times the number of bitcoins at the end of the five-year period.
If the transaction cost was increased by 1 percent, the trader would have about 0.023 times the number of bitcoins at the end of the five-year period.

We note that transaction costs have such a large effect because, as can be seen figure 1, many transactions occur using this strategy. If we lower "B" such that $B = 0.95$, the trader will reduce the number of transactions:

$$\alpha_{gold} = 0.01, \alpha_{bitcoin} = 0.03, B = 0.95, everything\ else\ unchanged$$

$$[Cash, Gold, Bitcoin] = [0,11.53275,0]$$

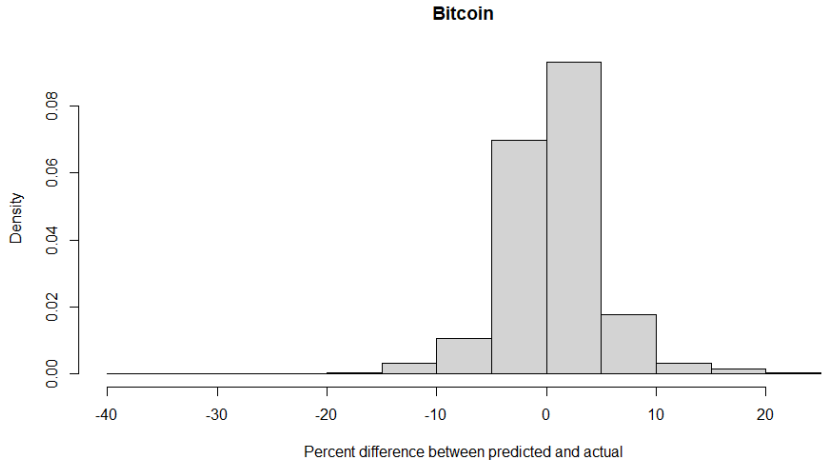
We notice that this is a better outcome than if we left $B = 0.98$.

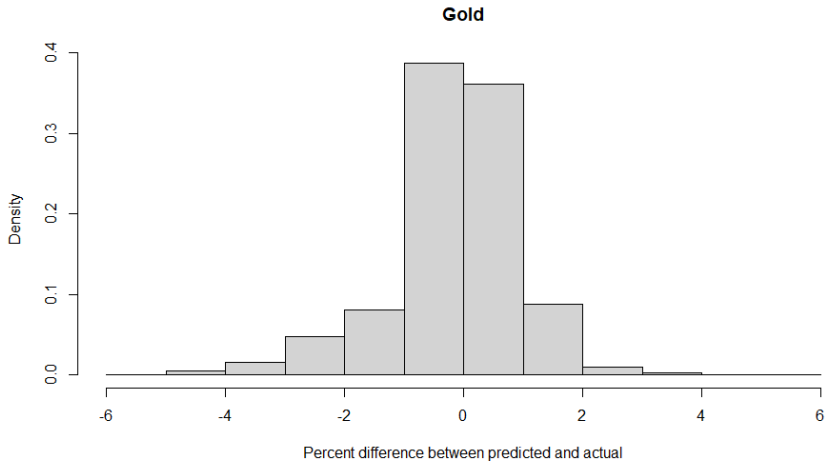
6 Validity of methodology

6.1 Prediction of future asset prices

The main objective of this report is to provide a way for the trader to decide when to buy, sell, or hold based on ratio comparisons, and not to predict the future price of the stock. However, it is important for the price prediction model, which we have produced here as the average estimates from two (SMA and log-return) models, to be somewhat accurate. Whether or not the trader chooses to use another predictive model, we feel we should show that this model is at least somewhat reasonable at predicting future stock prices. In other words, we want to show that the results we will obtain are based on reasonably accurate predictions, as opposed to random predictions that somehow got us good results.

By looking at the graphs, the predictions look reasonably accurate. Below are histograms of the differences between the predicted values and the actual values as percentages of actual:





6.2 Strengths of the model

- The asset price predictions models are reasonably accurate and are based on existing models used by traders
- The thresholds to buy and sell lead to an approximately %522,637 over a five-year period
    - The thresholds are designed to minimize risk and to find optimal times to buy

6.3 Weaknesses of the model and considerations for future work

- The model does not take into account selling part of the gold or part of bitcoin to buy the other if it may be profitable. This can be done using more if, then conditions in the future.
- The strategies produced by the model never suggest gold given the parameters. While this was perhaps the reason we saw such a large increase in profit, it could be that the strategies only works well for assets with large fluctuations like Bitcoin.
- Large, short-term fluctuations may lead to inaccurate price predictions. However, using another price prediction model that accounts for this can solve this issue

7 Memorandum

TO: Market trader

FROM: MCM Team 2221799

DATE: February 21, 2022

SUBJECT: Daily trading strategy using a series of logical comparisons

We believe we have found an effective daily trading strategy. We recognize the difficulty in determining whether to buy, sell, or hold an asset due to the somewhat unpredictable nature of the market. However, we believe we have found a model that, on average, will produce favorable results. We would like to mention that this model is meant for long term trading, ideally over one year, and produces a daily trading strategy.

Our model

The model uses future price predictions as a tool to determine whether to buy, sell, or hold bitcoin and gold. We would like to iterate that you are is free to use any model to predict price, but a reasonably accurate one is preferrable. The main idea behind the model is a series of comparisons. We have used a reasonably accurate, log-return price prediction model for our simulations.

First, given that bitcoin markets are always open, but gold markets are not, you should check whether gold markets are closed. If they are closed, then you should check whether the ratio of the expected price of bitcoin tomorrow over the current price of bitcoin today is greater than or equal to "A." For example, A = 1.05 means that you would like for the predicted price tomorrow to be at least 5% higher than the price today. If the ratio is greater than or equal to A, you should buy: you can make profit should you choose to sell tomorrow, and if you hold, your assets will have increased in value anyways. You should also check whether the ratio of the expected price of bitcoin tomorrow over the current price of bitcoin today is less than or equal to "B." For example, B = 0.96 means that you would tolerate a maximum of a 4% loss should you sell tomorrow. If this is satisfied, then you should sell today: you will avoid losing should you sell tomorrow, and, if you hold the cash, you can avoid the decrease in value. If none of these conditions are satisfied, you should hold. The next day, you will get another trading strategy.

From here on, we will refer to the ratio of the expected price tomorrow over the current price today as the "expected increase," "expected decrease," or "expected change" in price.

If the gold markets are open, the first thing you should do is to check whether it is better to buy gold or bitcoin. Gold is an interesting investment. It is a valuable metal, but it is also a somewhat "boring" investment. It does not see huge price fluctuations, which is safe, but it also does not increase wildly. For that reason, it may not be as good of an investment for day trading. We recommend comparing the expected change in price of bitcoin and gold. If the expected

change in price for bitcoin is greater than or equal to the expected price change in gold – 5%, we recommend holding gold and using the strategy in the previous paragraph for bitcoin. If, however, the expected change in gold – 5% is higher than bitcoin, we recommend the following: if the expected change in gold price is higher than C, buy. If the expected change in gold price is lower than D, sell. Otherwise, hold. We added the "– 5%" because we believe Bitcoin should be preferred over gold unless the change in price in gold is significant enough to warrant investment in it.

Our recommendations and results

We recommend setting $A = 1.03, B = 0.98, C = 1.05, D = 0.99$. You can change these parameters as needed, but we recommend setting A, C relatively low because this is a daily trading strategy and relatively high increases in price rarely occur over a day. Moreover, we recommend setting B, D high, which minimizes risk. B should not be higher than 0.98 and D should not be higher than 0.99 because there is a 2% transaction cost for Bitcoin and 1% transaction cost for gold. In other words, you should not sell if you will lose more money due to the transaction cost than the expected loss tomorrow.

We have simulated our models using only the above parameters and the past gold and bitcoin prices from 9/11/2016 to 9/10/2021. If you start on 9/11/2016 with $1000, you will end up with 112.7136 Bitcoin worth $5,226,381 by 9/10/2021!

Other recommendations we have are to exercise caution at times of large fluctuations. If there is a period in which news largely affect the price of the stock, you should be wary of the recommendations given by the model. You can use other price-prediction models if that is the case. We would also like to note that this model will, at times, recommend relatively many transactions during a short period of time. This makes it especially sensitive to transaction costs. If transaction costs are raised, we recommend lowering $B \; and \; D$ to reduce the number of transactions. This might be more risky, but it is worth it in the long term.

Sincerely,

Team 2221799

References

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[2] Wikimedia Foundation. (2022, February 20). *Bitcoin*. Wikipedia. Retrieved from https://en.wikipedia.org/wiki/Bitcoin?fbclid=IwAR2smNPKV172VzAer0PZuah0jiOVoos2GPHbUw1nCd2xpadoNCg0Q_4ky3k

[3] E.g. (2020, May 17). *Magic of log returns: Concept - part 1*. Investment Cache. Retrieved from https://investmentcache.com/magic-of-log-returns-concept-part-1/

[4] https://brian-ling.github.io/programming-in-r.html#another-simulation-example

Software and packages used: RStudio; tidyverse, readr,

Appendix

```
#Importing data
library(readr)
gold <- read_csv("LBMA-GOLD.csv")
bitcoin <- read_csv("BCHAIN-MKPRU.csv")
#

#Loading plugins
library(tidyverse)
library(rmarkdown)
library(knitr)
#

#Calculating log-return
#Bitcoin
logreturnpredictedbitcoin <- data.frame(logreturn = rep(NA, nrow(bitcoin)), PredictedLog =
rep(NA, nrow(bitcoin)))
for(i in 1:nrow(bitcoin)) {
  logreturnpredictedbitcoin$logreturn[i+1] <- log(bitcoin$Value[i+1]/bitcoin$Value[i])
}

#gold
logreturnpredictedgold <- data.frame(logreturn = rep(NA, nrow(gold)),PredictedLog = rep(NA,
nrow(gold)))
for(i in 1:nrow(gold)) {
  logreturnpredictedgold$logreturn[i+1] <- log(gold$`USD (PM)`[i+1]/gold$`USD (PM)`[i])
}

#Predicted prices using log-return based on the last n days of data, n = 26

n = 26

#Bitcoin
prediction_bitcoin_logreturn <- function(i,n) {
  bitcoin$Value[i-n] * exp(sum(c(logreturnpredictedbitcoin$logreturn[(i-n):i],
mean(logreturnpredictedbitcoin$logreturn[(i-n):i], na.rm = T)), na.rm = T))
}

for(i in (n+1):nrow(logreturnpredictedbitcoin)) {
  logreturnpredictedbitcoin$PredictedLog[i] <- prediction_bitcoin_logreturn(i, n)
}
#Gold
prediction_gold_logreturn <- function(i,n) {
  gold$`USD (PM)`[i-n] * exp(sum(c(logreturnpredictedgold$logreturn[(i-n):i],
mean(logreturnpredictedgold$logreturn[(i-n):i], na.rm = T)), na.rm = T))
```

```
}

for(i in (n+1):nrow(logreturnpredictedgold)) {
  logreturnpredictedgold$PredictedLog[i] <- prediction_gold_logreturn(i, n)
}

#Adding the predicted prices
gold[,"PredictedGold"] <- logreturnpredictedgold$PredictedLog
bitcoin[,"PredictedBitcoin"] <- logreturnpredictedbitcoin$PredictedLog
#

#Merging datasets
bitcoin$id <- 1:nrow(bitcoin) #id is used to fix the order of the dates, technically it corresponds
to "t" if the period starts at the first day
assets <- merge(bitcoin, gold, by = "Date" , all = TRUE)
assets <- assets[order(assets$id),] #fixing the order of the dates
assets <- assets[,-4]
colnames(assets) <- c("Date", "Pricebitcoin", "Predictedbitcoin", "Pricegold", "Predictedgold")
row.names(assets) <- 1:1826
#

#Adding columns for cash, Bitcoin, gold, Bitcoin value, gold value
assets[,c("Cash","Bitcoin","Valuebitcoin","Gold", "Valuegold")] <- NA
assets$Cash[1:n] <- 1000
assets$Bitcoin[1:n] <- 0
assets$Gold[1:n] <- 0
assets$Valuegold[1:n] <- 0
assets$Valuebitcoin[1:n] <- 0



my_function <- function(ag, ab, A,B,C,D, shift) {
  for(i in (n+1):1825) {
    if(is.na(assets$Predictedgold[i]) | is.na(assets$Predictedgold[i+1])) { #If gold market is closed
today or tomorrow
      if(assets$Predictedbitcoin[i+1]/assets$Pricebitcoin[i] > B &
assets$Predictedbitcoin[i+1]/assets$Pricebitcoin[i] < A) {
        assets$Cash[i] = assets$Cash[i-1]
        assets$Gold[i] = assets$Gold[i-1]
        assets$Bitcoin[i] = assets$Bitcoin[i-1]
      }
      if(assets$Predictedbitcoin[i+1]/assets$Pricebitcoin[i] >= A) {
        if(assets$Cash[i-1] > 0) {
          assets$Cash[i] = 0
          assets$Gold[i] = assets$Gold[i-1]
          assets$Bitcoin[i] = assets$Cash[i-1]/((1+ab)*assets$Pricebitcoin[i]) + assets$Bitcoin[i-1]
```

```
      } else {
        assets$Cash[i] = 0
        assets$Gold[i] = assets$Gold[i-1]
        assets$Bitcoin[i] = assets$Bitcoin[i-1]
      }
    }
    if(assets$Predictedbitcoin[i+1]/assets$Pricebitcoin[i] <= B) {
      if(assets$Bitcoin[i-1] > 0) {
        assets$Cash[i] = (1-ab) * assets$Bitcoin[i-1] * assets$Pricebitcoin[i] + assets$Cash[i-1]
        assets$Gold[i] = assets$Gold[i-1]
        assets$Bitcoin[i] = 0
      } else {
        assets$Cash[i] = assets$Cash[i-1]
        assets$Gold[i] = assets$Gold[i-1]
        assets$Bitcoin[i] = 0
      }
    }
  }
  if(!is.na(assets$Predictedgold[i]) & !is.na(assets$Predictedgold[i+1])) {
    if((assets$Predictedbitcoin[i+1]/assets$Pricebitcoin[i]) >
(assets$Predictedgold[i+1]/assets$Pricegold[i]) - shift) {
      if(assets$Predictedbitcoin[i+1]/assets$Pricebitcoin[i] > B &
assets$Predictedbitcoin[i+1]/assets$Pricebitcoin[i] < A) {
        assets$Cash[i] = assets$Cash[i-1]
        assets$Gold[i] = assets$Gold[i-1]
        assets$Bitcoin[i] = assets$Bitcoin[i-1]
      }
      if(assets$Predictedbitcoin[i+1]/assets$Pricebitcoin[i] >= A) {
        if(assets$Cash[i-1] > 0) {
          assets$Cash[i] = 0
          assets$Gold[i] = assets$Gold[i-1]
          assets$Bitcoin[i] = assets$Cash[i-1]/((1+ab)*assets$Pricebitcoin[i]) + assets$Bitcoin[i-1]
        } else {
          assets$Cash[i] = 0
          assets$Gold[i] = assets$Gold[i-1]
          assets$Bitcoin[i] = assets$Bitcoin[i-1]
        }
      }
      if(assets$Predictedbitcoin[i+1]/assets$Pricebitcoin[i] <= B) {
        if(assets$Bitcoin[i-1] > 0) {
          assets$Cash[i] = (1-ab) * assets$Bitcoin[i-1] * assets$Pricebitcoin[i] + assets$Cash[i-1]
          assets$Gold[i] = assets$Gold[i-1]
          assets$Bitcoin[i] = 0
        } else {
          assets$Cash[i] = assets$Cash[i-1]
          assets$Gold[i] = assets$Gold[i-1]
```

```
          assets$Bitcoin[i] = 0
        }
       }
      }
     if(((assets$Predictedbitcoin[i+1]/assets$Pricebitcoin[i]) <=
(assets$Predictedgold[i+1]/assets$Pricegold[i])-shift)) {
       if(assets$Predictedgold[i+1]/assets$Pricegold[i] >= C) {
        if(assets$Cash[i-1] > 0) {
          assets$Cash[i] = 0
          assets$Gold[i] = assets$Cash[i-1]/((1+ag)*assets$Pricegold[i]) + assets$Gold[i-1]
          assets$Bitcoin[i] = assets$Bitcoin[i-1]
        } else {
          assets$Cash[i] = 0
          assets$Gold[i] = assets$Gold[i-1]
          assets$Bitcoin[i] = assets$Bitcoin[i-1]
        }
       }
       if(assets$Predictedgold[i+1]/assets$Pricegold[i] <= D) {
        if(assets$Gold[i-1] > 0) {
          assets$Cash[i] = (1-ag) * assets$Gold[i-1] * assets$Pricegold[i] + assets$Cash[i-1]
          assets$Gold[i] = 0
          assets$Bitcoin[i] = assets$Bitcoin[i-1]
        } else {
          assets$Cash[i] = assets$Cash[i-1]
          assets$Gold[i] = 0
          assets$Bitcoin[i] = assets$Bitcoin[i-1]
        }
       }
       if(assets$Predictedgold[i+1]/assets$Pricegold[i] > D &
assets$Predictedgold[i+1]/assets$Pricegold[i] < C) {
         assets$Cash[i] = assets$Cash[i-1]
         assets$Gold[i] = assets$Gold[i-1]
         assets$Bitcoin[i] = assets$Bitcoin[i-1]
       }
      }
    }
    assets$Cash[1826] = assets$Cash[1825]
    assets$Gold[1826] = assets$Gold[1825]
    assets$Bitcoin[1826] = assets$Bitcoin[1825]
    assets$Valuebitcoin[1826] = assets$Pricebitcoin[1826] * assets$Bitcoin[1826]
    assets$Valuegold[1826] = assets$Pricegold[1826] * assets$Gold[1826]
    assets$Valuebitcoin[i] = assets$Pricebitcoin[i] * assets$Bitcoin[i]
    assets$Valuegold[i] = assets$Pricegold[i] * assets$Gold[i]
  }
  return(tail(assets))
}
```

```
my_function(ag = 0.01, ab = 0.02, A = 1.03, B = 0.98, C = 1.05, D = 0.95, shift =0.05)

ggplot(data = assets1, aes(x = 1:1826, y = Valuebitcoin)) +
  geom_line(col = "red") +
  labs(x = "t", y = "Value of owned Bitcoins in USD")


c1 <- c("Predicted" = "blue", "Actual" = "red")
c2 <- c("Predicted" = "violet", "Actual" = "orange")

ggplot(data = assets, aes(x = 1:1826)) +
  geom_line(aes(y = Pricebitcoin, col = "Actual")) +
  geom_line(aes(y = Predictedbitcoin, col = "Predicted")) +
  labs(x = "t", y = "Price of Bitcoin (USD)", color = "Legend") +
  scale_color_manual(values = c1)

hist(100*(assets$Predictedbitcoin-assets$Pricebitcoin)/assets$Pricebitcoin, freq = F, main =
"Bitcoin",
     xlab = "Percent difference between predicted and actual")

ggplot(data = assets, aes(x = 1:1826)) +
  geom_line(aes(y = Pricegold, col = "Actual")) +
  geom_line(aes(y = Predictedgold, col = "Predicted")) +
  labs(x = "t", y = "Price of Gold (USD)", color = "Legend") +
  scale_color_manual(values = c2)

hist(100*(assets$Predictedgold-assets$Pricegold)/assets$Pricegold, freq = F, main = "Gold",
     xlab = "Percent difference between predicted and actual")
```