Basic Method for projecting 3D points in space onto 2D planes (X, Y and Z axes)

Project Title: 3D Geometry HULL Calculations

SEP Group: HULL2

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Research Scope:

Part A (Hull Calculation – 3D projection) requires the 3D geometry to be 'flattened'. The set of points in $\mathbb{R}^3$ be projected onto 3 planes (X, Y and Z axes). This short summary paper looks into projecting a set of points onto the Z axis.

Basic Concept:

The main idea for the proposed projection is to use a projection matrix and multiply it by the points in 3D space. Depending on the plane of projection, we can change the projection matrix accordingly.

The mathematics behind the basic idea:

If we have a point in 3D space ($\mathbb{R}^3$) let this be $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

If we would like to project this to the Z plane then we multiply this point by the projection matrix, we set the last value to be 0 as we would like to project it to the Z plane.

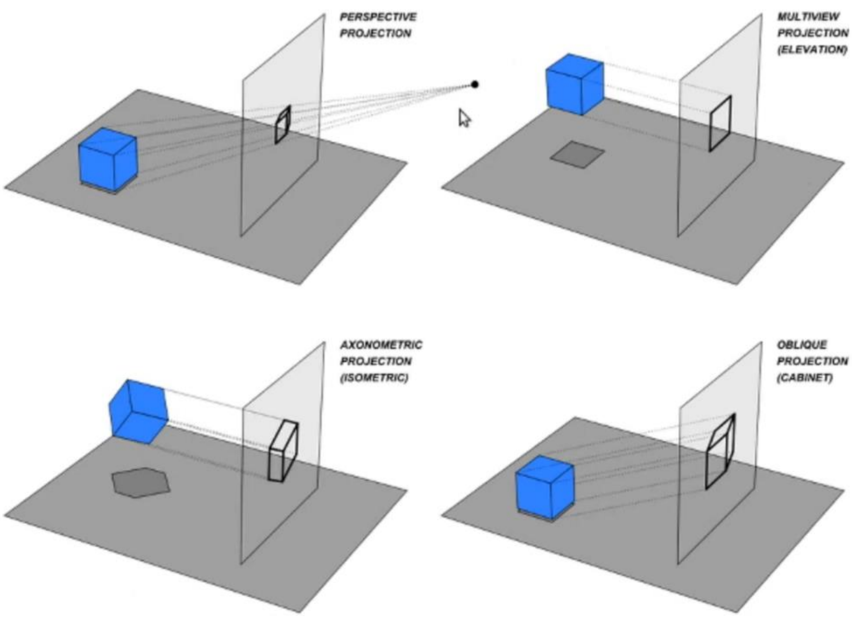
$$Projection\ Matrix = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$Projection\ of\ point\ onto\ the\ Z\ plane = Projection\ Matrix \times point$$

$$\therefore Projection\ of\ point\ onto\ the\ Z\ plane = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\therefore Projection\ of\ point\ onto\ the\ Z\ plane = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

There are various projection methods to use, however, Multiview projection would be the aim of the algorithm (as shown below).



The projection matrix will then be changed to $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ if the projection is onto the Y plane and $\begin{vmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ if onto the X plane, forcing the resultant points to be $\begin{bmatrix} x \\ 0 \\ z \end{bmatrix}$ and $\begin{bmatrix} 0 \\ y \\ z \end{bmatrix}$ respectively.

Algorithm Overview and Explanation:

Steps:

> **Point Construction**: Input the points in 3-dimensional space and multiply it by the corresponding projection matrix and save the output to a file. The multiplication of the points can be completed by multiplying it with all 3 axes consecutively eg. multiplying it by projection matrix to project onto X, Y and Z planes.
>
> **Save Points:** The output from the calculations can be output to a file and saved in it corresponding plane (X, Y and Z files).

Source:

Pythonista (2021*) How to make a 3D projection in Python* | *Rendering a cube in 2D! (No OpenGL),* 20 March. Available at: https://www.youtube.com/watch?v=qw0oY6Ld-L0