



# Youla parameterized adaptive vibration control against deterministic and band-limited random signals



Fanfan Qian, Zhizheng Wu<sup>\*</sup>, Maotong Zhang, Tao Wang, Yuanyuan Wang, Tengfei Yue

Department of Precision Mechanical Engineering, Shanghai University, Shanghai, China

## ARTICLE INFO

### Article history:

Received 26 April 2019  
Received in revised form 1 August 2019  
Accepted 6 September 2019

### Keywords:

Active vibration control  
Disturbance rejection  
Youla parameterized  
 $H_\infty$  controller

## ABSTRACT

Multiple narrow band deterministic and band-limited random disturbance signals will occur during machining process, which will cause vibration and have negative effect on the cutting accuracy. For instance, in a lathe system, spindle rotation can produce narrow band deterministic vibration disturbance and turning process can lead to band-limited random vibration disturbance. In this paper, a  $H_\infty$  controller with augmented Youla ( $Q$ ) parameterized adaptive regulation approach is proposed to reject both random and deterministic disturbances. The parameter  $Q$  is converged to the desired value by online tune of a recursive least squares (RLS) algorithm. An experimental evaluation on a lathe system has been carried out to indicate the effectiveness of this approach. The result illustrates that the proposed adaptive control method can effectively attenuate the lathe vibrations against the unknown deterministic and band-limited random disturbances.

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## 1. Introduction

Vibration is one of the key factors affecting the machining quality which can produce noise and decrease system life or even cause damage [1,2]. In the machining process, due to the effect of a variety of forces and the impact of the mechanical property of the elastic system, the workpiece and the tool often have a strongly relative vibration, so that the process components of various forming motion can be interfered and destroyed. It will not only make the processed surfaces have streaks which can seriously affect the processing accuracy and surface quality, but also make the connecting parts between the lathe and fixture loose, even shorten the service life of the mechanical system. In addition, the vibration noise could cause the uncomfortably and affect the health of the operator. Therefore, the vibration problem makes disadvantageous impact on the profile accuracy and surface roughness of machined parts, which is supposed to be eliminated.

Forced vibration and self-excited vibration are the main components of vibration in mechanical systems [3]. Forced vibration is caused by large cutting resistance or resonance phenomenon and self-excited vibration is caused by regenerative effect, which relates to the variation in cutting thickness [4,5]. For instance, in a lathe system, spindle rotation can lead to narrow band deterministic disturbance which is called self-excited vibration and turning process results in random interference which is called forced vibration. Normally, the feature and bandwidth of the forced vibration can be represented as band-limited random signals [6,7]. Self-excited vibration can be considered as narrow band deterministic signals which depend on the speed of spindle. Therefore, the vibration signals in the mechanical systems can normally be divided into band-limited random disturbance and unknown deterministic disturbance.

<sup>\*</sup> Corresponding author.

E-mail address: [zhizhengwu@shu.edu.cn](mailto:zhizhengwu@shu.edu.cn) (Z. Wu).

The main target of the active vibration control system is to reject the vibration signals in mechanical systems. In the literatures, many control methods such as  $H_\infty$  robust controller [8–11], PID controller [12,13], LQG controller [14,15] and model predictive control [16] have been extensively studied and proposed to correct the disturbances which are mainly featured with the random properties. However, the aforementioned feedback control approaches are usually not so effective for the unknown deterministic vibration, but are supposed to be perfectly eliminated based on the Internal Model Principle. Since excessively boosting the wide-range frequency gain of the controller could weaken the phase property and decrease the robustness of the closed-loop system. If the reference input signal is exactly known or relevant vibration of the disturbance signals is measurable, then the feedforward compensator techniques can be used to perfectly eliminate the disturbances. However, this approach requires an additional transducer and, in some situations, it may be difficult to obtain a measurement that is highly correlated with the disturbances. These control limitations of traditional control approaches could limit the rejection of the unknown deterministic disturbances [17].

The  $H_\infty$  controller can be designed to effectively reject the random disturbance within the known bandwidth. However, it is not so effective for the deterministic disturbance signals which are normally unknown and varied with the speed of the spindle. To completely eliminate the unknown deterministic disturbance, one way is to estimate the disturbance model online and then calculate the controller based on the Internal Model Principle [18]. Using the standard parameter estimation algorithm or the frequency estimation algorithm, the unknown model of the disturbance can be explicitly obtained. By incorporating Youla parameterization (also called the  $Q$  parameterized) scheme into the controller, the internal model of the disturbances may implicitly be contained in the controller with the appropriately designed  $Q$  parameters [19–22]. The parameter  $Q$  is converged to the desired value by online tune of a recursive least squares (RLS) algorithm, so that the unknown disturbances can be eliminated. In this paper, a Youla parameterized adaptive controller is proposed for the mechanical systems to deal with the unknown vibration caused both by the deterministic disturbance and the random disturbance, simultaneously.

In order to indicate the effectiveness of this approach, an experiment testing has been carried out on a lathe system using the inertial actuator [23,24]. The result illustrates that the proposed adaptive control method can effectively control the lathe vibration for the unknown disturbance rejection, especially to the deterministic disturbances.

The rest of the paper is organized as follows: In Section 2, the control problem of vibration in the mechanical systems is formulated. In Section 3, the design of the adaptive vibration control regulator is presented, which mainly consists of the constructions of the base controller and the parameter  $Q$ , respectively. The proposed adaptive regulation approach is applied in a lathe system for the vibration attenuation and the experiment results are evaluated in Section 4. Section 5 concludes the discussions.

## 2. Vibration control problem formulation

In mechanical systems, the vibration can be generalized as band-limited random and unknown deterministic signals. The objective of the active controller is to eliminate these signals. For a lathe system, the band-limited random dynamic vibration is produced during the cutting and therefore is time-varying in a range. The unknown deterministic vibration is caused by the spindle and also varies with the speed of the spindle. The nonlinear factors caused by the different cutter angles, nonlinear damping and varying spindle speed, etc. could further introduce the time-varying and random properties to the vibration signals. Therefore, the overall vibration attenuation in the lathe system can be formulated as an equivalent regulation problem against the unknown exogenous disturbances located at the output side of the plant. The vibration control block diagram is shown in Fig. 1, where  $e$  is the vibration error performance which is supposed to be zero,  $u$  is the control input,  $d$  represent the vibration signals. The feedback loop including the plant is controlled in order to reduce, as much as possible, the effect of the disturbance  $d$  on the controlled output. It is assumed that the characteristics of the compensator systems remain almost unchanged and linear during operation.

In the following, a Youla ( $Q$ ) parameterized adaptive regulation approach is proposed to attenuate the random and unknown deterministic dynamic signals, simultaneously. The constructing of the proposed adaptive controller mainly consists of two steps. Firstly, a robust  $H_\infty$  base controller is designed to increase the gain of the open loop system in the overall frequency range where the random disturbance locates, so that the random disturbance and the uncertainties can be properly dealt with. Secondly, the base controller is augmented with  $Q$  parameter to form a Youla-Kucera ( $Q$ ) parameterized all stabilizing controllers, which satisfies the interpolation condition of Internal Model Principle (IMP) to perfectly reject the deterministic disturbance. If the property of the deterministic disturbance is assumed to be unknown and possibly time

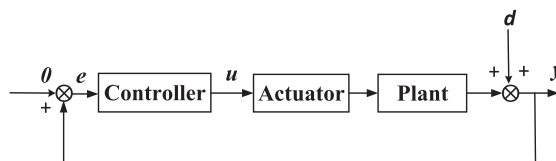


Fig. 1. Block diagram of the equivalent active vibration control system.

varying, then the adaptive algorithm is developed to online tune the  $Q$  parameter to the desired values so that the unknown deterministic disturbance can be perfectly rejected.

In the following, the discrete-time representation of the generalized plant is denoted as  $P(z^{-1}) = \frac{P_A(z^{-1})}{P_B(z^{-1})}$ . For example, in a lathe system  $P(z^{-1})$  represents the lathe dynamics with the actuator.  $d$  represents the equivalent vibration signals as:

$$d(k) = \sum_{\ell=1}^n A_{\ell}(k) \sin[d_{\ell}(k)k + \varphi_{\ell}(k)] + R(k) \quad (1)$$

where  $\sum_{\ell=1}^n A_{\ell}(k) \sin[d_{\ell}(k)k + \varphi_{\ell}(k)]$  represents the unknown deterministic dynamic signal. The amplitudes  $A_{\ell}$ , frequencies  $d_{\ell}(k)$ , phases  $\varphi_{\ell}$  are unknown and time-varying.  $R(k)$  represents the random signal. The object of the control problem is to construct an output feedback controller to regulate the performance variable  $e$  against the unknown external signal  $d$  such that the performance variable  $e$  is minimized.

### 3. Adaptive vibration control regulator

A Youla parameterized adaptive regulation approach is proposed to attenuate the unknown disturbances, which consists of two parts, the optimal inner-loop base controller and the augmented  $Q$  parameter. The  $Q$  parameter will adaptively converge to the desired value by online tune of a recursive least squares (RLS) algorithm so that the unknown disturbances can be attenuated.

#### 3.1. The inner-loop base controller design

The theory of  $H_{\infty}$  control has been well studied and developed in the literature [10]. The  $H_{\infty}$  standard design problem can be formulated as finding the controller  $K_0 = \frac{U_0}{V_0}$  to minimize the  $H_{\infty}$ -norm of the transfer matrix from  $d$  to  $z = \begin{bmatrix} Z_e \\ Z_u \end{bmatrix}$  as shown in Fig. 2. The terms  $W_e$  and  $W_u$  are weighting functions, which shape the transfer functions  $S = (I + PK_0)^{-1}$  and  $T = K_0(I + PK_0)^{-1}$ , respectively.

In the paper, the  $H_{\infty}$  controller is first designed as the inner-loop base controller to eliminate the random disturbance. However,  $H_{\infty}$  controllers can effectively suppress band-limited random dynamic signals, but cannot eliminate unknown deterministic signals perfectly. In the following, a Youla ( $Q$ ) parameterized adaptive regulation approach is further proposed to attenuate the unknown deterministic signals.

#### 3.2. $Q$ parameterized adaptive controller design

The proposed approach is to augment the base  $H_{\infty}$  controller with the  $J$  block and the  $Q$  parameters as shown in Fig. 3. Then the adaptive algorithm is developed to tune the  $Q$  parameters online to achieve regulation for the unknown deterministic signals.

The formulated  $J$  block with the  $H_{\infty}$  controller is given by:

$$J = \begin{bmatrix} K_0 & V^{-1} \\ V^{-1} & -V^{-1}P_A \end{bmatrix} \quad (2)$$

where  $K_0 = \frac{U_0}{V_0}$  is the base controller designed as an  $H_{\infty}$  controller.  $U_0$  and  $V_0$  are in  $RH_{\infty}$ . Define

$$\begin{aligned} V &= V_0/Z, \quad U = U_0/Z, \\ Z &= P_B * V_0 - U_0 * P_A \end{aligned} \quad (3)$$

The set of all stabilizing controllers  $K$  then can be formulated with the base stabilizing controller  $K_0$  and the  $Q$  parameters as [25]

$$K = (U + P_B Q)(V + P_A Q)^{-1} \quad (4)$$

If the  $Q \in RH_{\infty}$  is selected as a stable function, then the set of all stabilizing controllers  $K$  formulated as in (4) is internal stable. Moreover, every rational stabilizing controllers  $K$  has the form (4) for some  $Q \in RH_{\infty}$ .

The plant  $P$  and the block  $J$  can be combined into blocks  $T_{11}$ ,  $T_{12}$  and  $T_{21}$  as:

$$\begin{bmatrix} e \\ r \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & 0 \end{bmatrix} \begin{bmatrix} d \\ s \end{bmatrix} \quad (5)$$

Therefore, the block diagram of the closed-loop system with lumped  $P - J$  blocks can be given as Fig. 4.

Let  $D(z^{-1})$  and  $E(z^{-1})$  denote the  $Z$  transform of the disturbance input  $d$  and the performance variable  $e$ , respectively. The performance variable of the closed-loop system is then given by

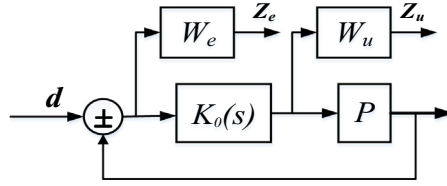


Fig. 2. Block diagram of  $H_\infty$  control with weighting functions.

$$E(z^{-1}) = [T_{11}(z^{-1}) + T_{12}(z^{-1})Q(z^{-1})T_{21}(z^{-1})]D(z^{-1}) \quad (6)$$

The  $Q(z^{-1})$  is considered as:

$$Q(z^{-1}) = \left( \sum_{i=1}^{n_q} \theta_i z^{1-i} \right) F(z^{-1}); i = 1, \dots, n_q, \quad (7)$$

where the stable weight function  $F(z^{-1}) = \frac{b_1 z^{s-1} + \dots + b_s}{z^s + a_1 z^{s-1} + \dots + a_s}$  is used to adjust the frequency response of  $Q(z^{-1})$  and thereby optimally shape the sensitivity functions of the closed-loop system. By properly choosing  $F(z^{-1})$  the robustness of the closed-loop system can be improved. Normally  $F(z^{-1})$  is practically selected as a weight function with low pass or multiple narrow band pass features corresponding to the frequency range of the deterministic dynamic signals.  $\theta = [\theta_1, \dots, \theta_{n_q}]^T$  denote the parameter vectors which can be tuned online so that it converges to the desired parameter vector, needed to achieve perfect tracking of the deterministic dynamic signals. Based on the IMP, the interpolation condition for achieving regulation against the disturbance  $D(z^{-1})$  in the closed-loop system can be written as [19]

$$A_\theta \theta + B_\theta = 0 \quad (8)$$

where  $A_\theta \in R^{n_p \times n_q}$ ,  $B_\theta \in R^{n_p \times 1}$  are the corresponding interpolation matrices, and  $n_p$  is the total number of the different poles in the expression for  $D(z^{-1})$ . Let  $q^{-l}$  denote the  $l$  time step delay operator, define  $\bar{Q}_k = \sum_{i=1}^{n_q} \theta_i (k-1) q^{1-i}$  and  $Q_k = \bar{Q}_k F(q^{-1})$ . The performance variable  $E(z^{-1})$  is then given by

$$E(z^{-1}) = T_{11}(z^{-1})D(z^{-1}) + T_{12}(z^{-1})\bar{Q}_k F(z^{-1})r(k) \quad (9)$$

where  $r(k)$  is one of the outputs of block  $J$  and thus can be obtained at each step  $k$ .

Denote  $\theta^0$  be a parameter vector satisfying the IMP and  $\bar{Q}^0$  be the Youla parameter that results from using parameter  $\theta^0$ . The corresponding performance error resulting from  $\bar{Q}^0$  can be then written as

$$e^0(k) = T_{11}(z^{-1})d(k) + \bar{Q}^0 T_{12}(z^{-1})F(z^{-1})r(k) \quad (10)$$

The expression for the performance variable can be rewritten as

$$\begin{aligned} e(k) &= T_{11}(z^{-1})d(k) + T_{12}(z^{-1})Q_k(z^{-1})r(k) - e^0(k) + e^0(k) \\ &= T_{12}(z^{-1})\bar{Q}_k F(z^{-1})r(k) - T_{12}(z^{-1})\bar{Q}_k^0 F(z^{-1})r(k) + e^0(k) \end{aligned} \quad (11)$$

Define the modified performance error  $\hat{e}(k) = e(k) - [T_{12}(q^{-1})\bar{Q}_k - \bar{Q}_k T_{12}(q^{-1})]F(q^{-1})r(k)$  and  $v(k) = T_{12}(q^{-1})F(q^{-1})r(k)$ , then the corresponding posteriori error of  $\hat{e}(k)$  is:

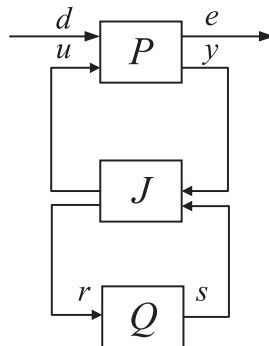


Fig. 3. Closed loop system with a  $Q$  parameterized controller.

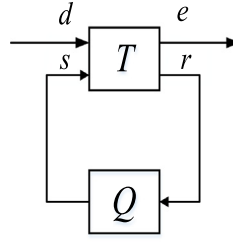


Fig. 4. Block diagram of the closed loop system with lumped  $P-J$  blocks.

$$\hat{e}(k+1) = \phi^T(k+1) \tilde{\theta}(k+1) + e^0(k+1) \quad (12)$$

where

$$\hat{\theta}(k) = \left[ \left( \theta_1^0 - \hat{\theta}_1(k) \right), \dots, \left( \theta_{n_q}^0 - \hat{\theta}_{n_q}(k) \right) \right]^T \quad (13)$$

$$\phi(k) = [-v(k), \dots, -v(k - n_q + 1)]^T \quad (14)$$

Therefore, the estimation of the unknown parameter vector  $\hat{\theta}(\cdot)$  can be performed with

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \frac{P(k)\phi(k+1)}{1 + \phi^T(k+1)P(k)\phi(k+1)} \hat{e}(k+1) \quad (15)$$

$$P(k) = \frac{1}{\lambda(k)} \left[ P(k-1) - \frac{P(k-1)\phi(k)\phi^T(k)P(k-1)}{1 + \phi^T(k)P(k-1)\phi(k)} \right] \quad (16)$$

where the adaptation gain  $P(0) > 0$ .  $\lambda(k)$  could be a time varying forgetting factor normally satisfying  $0 < \lambda(k) \leq 1$ . The proper selection of  $\lambda(k)$  is based on the disturbance properties. For example, for the fast time varying disturbance,  $\lambda(k)$  can be selected in each step to make the trace of the gain matrix  $P(k)$  be a constant. For the slow time varying or stationary disturbance,  $\lambda(k)$  can be simply a constant value typically located in  $\lambda \in (0.95, 1)$  or gradually tends toward 1 as  $t \rightarrow \infty$ , respectively.

Since  $F(z^{-1})$  is selected as a filter with band pass or low pass features w.r.t. the bandwidth of the deterministic and random disturbances and the posteriori errors are used in the adaptation, the above algorithms maintain good performance when the adaptation input  $d(k)$  contains random disturbance.  $T_{12}(q^{-1})$ ,  $F(q^{-1})$  and  $T_{21}(q^{-1})$  are all stable transfer functions, then if the disturbance  $d(k)$  is bounded, based on (14) it is easy to obtain that the regressive vector  $\phi(k)$  is bounded. Thereby, based on Strict Positive Real (SPR) condition, if it is assumed that there exists a parameter vector  $\theta_0$  satisfying the regulation condition (8) corresponding to the deterministic disturbance properties, then the algorithm given by (15) and (16) yields  $\lim_{k \rightarrow \infty} \hat{\theta}(k) = \theta_0$  almost sure and the regulation against the deterministic disturbances in the adaptive closed-loop system can be achieved [17].



Fig. 5. Experimental setup system.

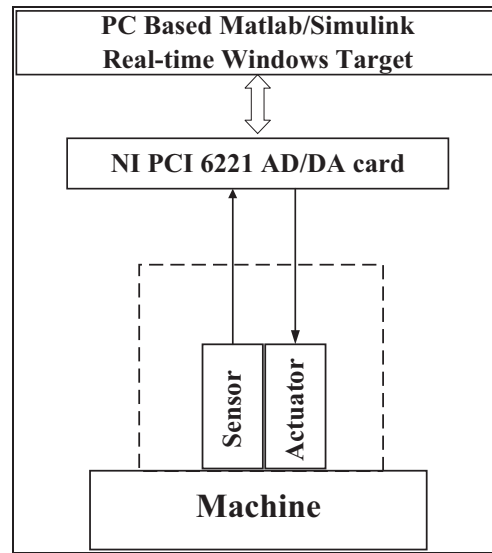


Fig. 6. Schematic block diagram of the experimental setup configuration.

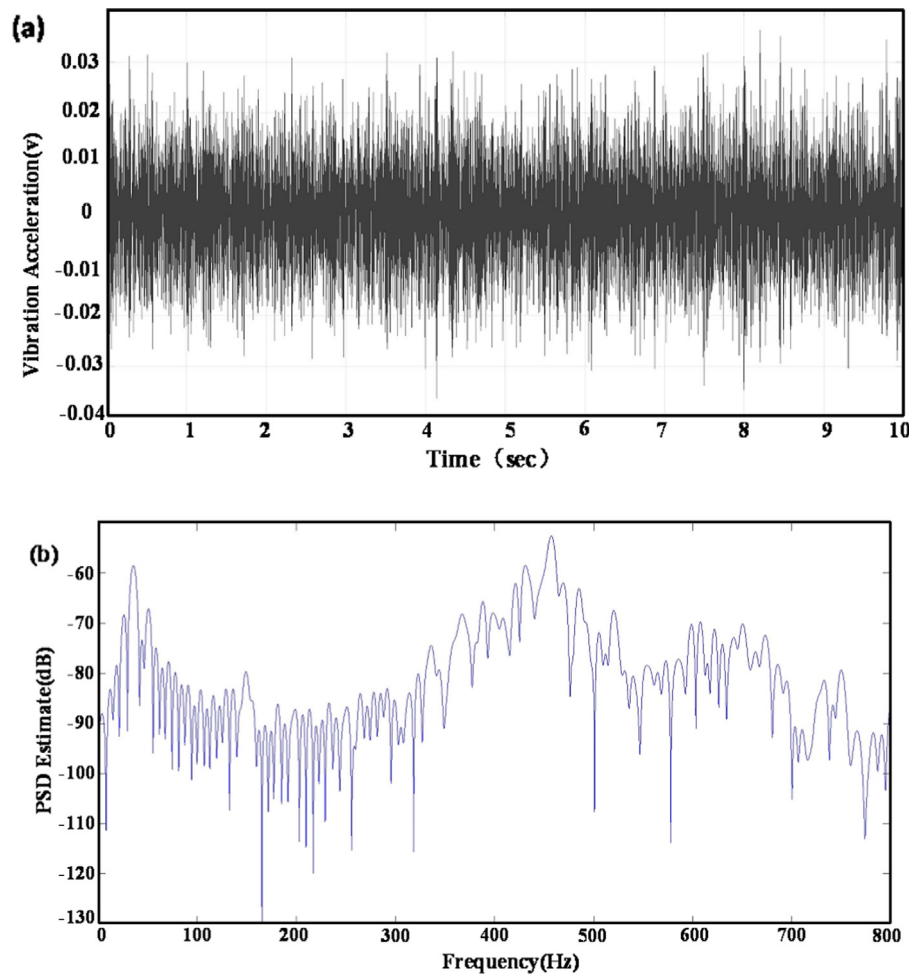


Fig. 7. (a) Experimental results of the disturbance signal (b) power spectral density estimate of the disturbance signal.

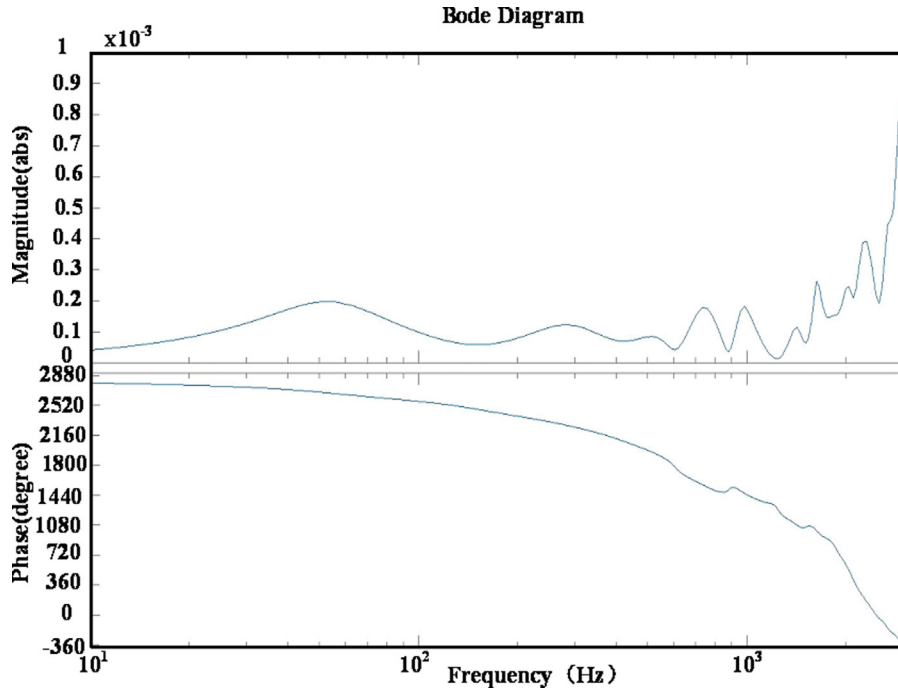


Fig. 8. Frequency response of the identified model.

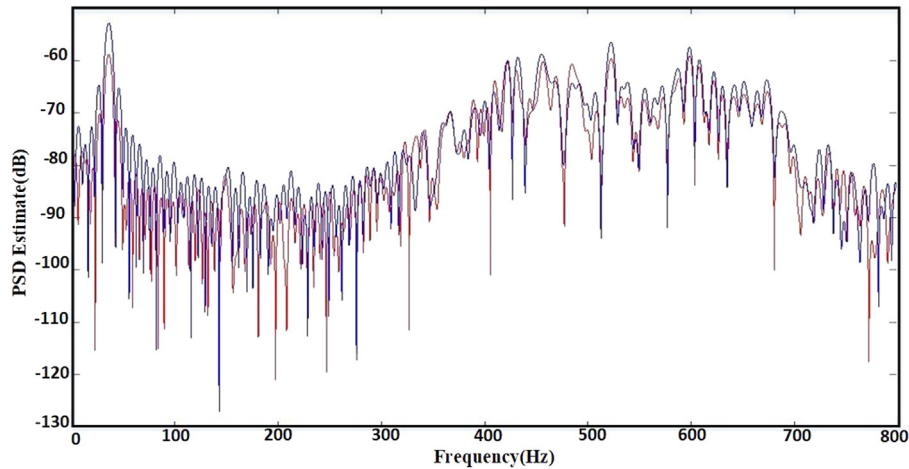


Fig. 9. Experimental results of the PID control system.

#### 4. Experimental verification

This section will verify the effectiveness of the proposed adaptive regulation on a lathe to actively attenuate the vibrations. The experimental equipment photograph is shown in Fig. 5, and the corresponding schematic diagram is shown in Fig. 6. The sampling period of the closed-loop experimental system is 6000 Hz. The Active Damping Devices (ADD) produced by Micromega company internally consists of an inertial actuator and a collocated acceleration sensor, which is used as the force actuator and the vibration sensor for the experiment setup system. The vibration of the lathe is measured in real-time by the collocated vibration sensor with a measuring range of  $\pm 20m/s^2$ . A band pass filter has been designed in the signal processing circuit of ADD to deal with the noise and drift. In order to eliminate the measurement noise produced in the path from the ADD to the controller, the measured vibration signal is further filtered with a Butterworth low pass filter with a cutting frequency of 800 Hz before it is fed back to the controller. During the experimental testing, the inertial actuator is attached to the outer metallic wall of the lathe spindle by 3 magnet pots, which are available by Micromega Dynamics. In

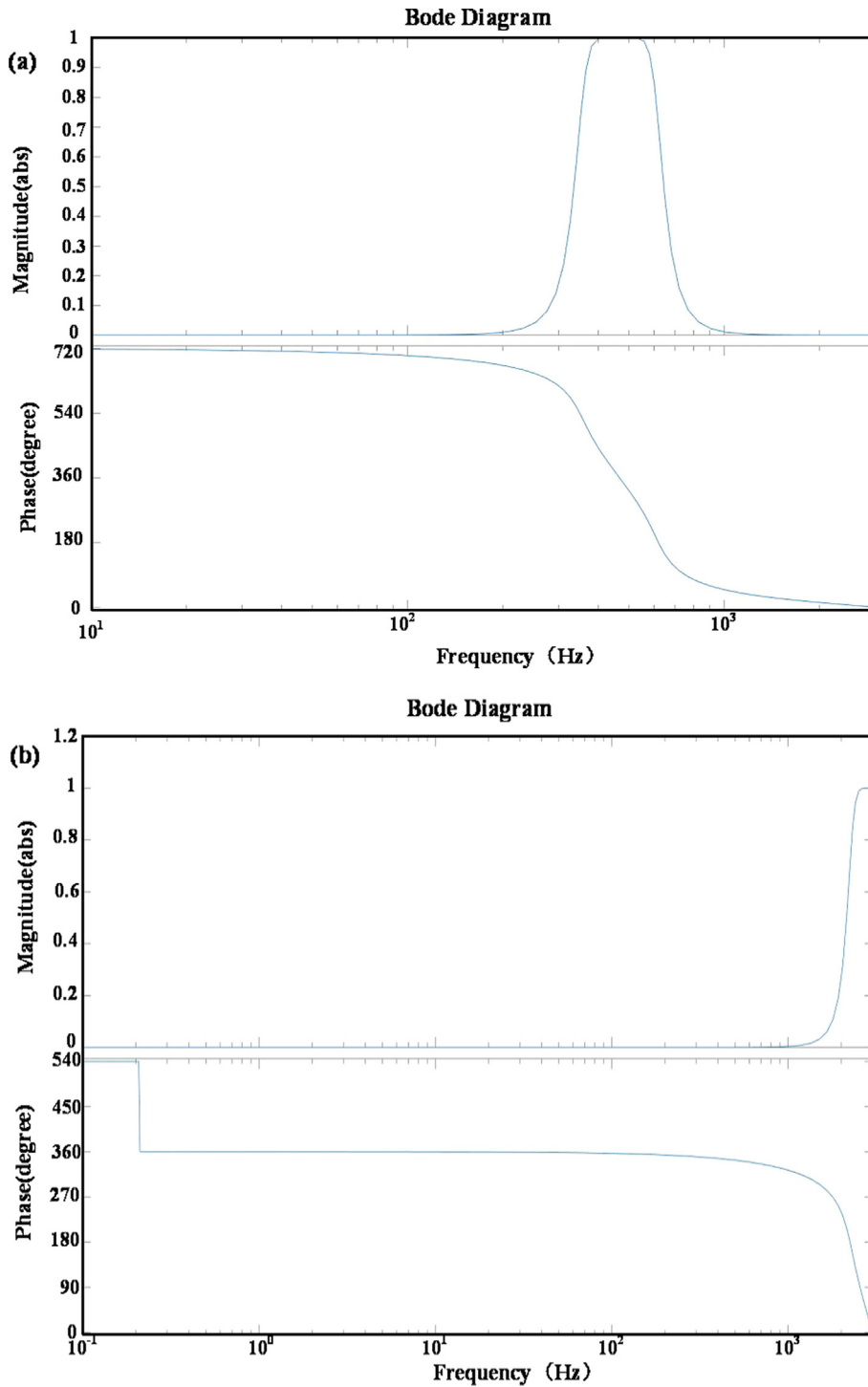


Fig. 10. (a) Frequency response of  $W_e$  (b) Frequency response of  $W_u$

order to verify the validity of the algorithm, the experiment testing was carried out in the horizontal dimension. In the practical case, the rotation speed of the spindle could be unknown and even be time-varying, therefore, and the vibration signals are normally not exactly known due to unknown vibration excitation. The experimental testing results based on the proposed adaptive controller with the inertial actuator are evaluated to verify the effectiveness of the proposed adaptive vibration suppression strategy.



The magnitude of the vibration measured by the sensor is shown in Fig. 7(a) and the corresponding power spectral density estimate of the vibration is presented in Fig. 7(b). It can be seen in Fig. 7(b) that the deterministic disturbances locate around 24 Hz, 36 Hz and 48 Hz and the random disturbances is located around 350–620 Hz. The model of the actuator attached to the lathe is identified using the system identification method. Based on the sweeping input signal and measured output signal, the model is identified with input-output data filtering as a thirty-order system. The frequency response of the identified model is presented Fig. 8.

In order to verify the advantages of this algorithm, the conventional PID controller is first evaluated in the experiment. The PID control algorithm is the most widely used in control engineering because of its adaptability and robustness. It has also been successfully applied to actively attenuate the vibration in the mechanical systems [12,13]. The key to effectively control the vibration with a PID-based controller is how to obtain the three controller parameters. The PID parameter tuning can normally be achieved by the theoretical approaches or engineering approaches. In this paper, an iteration engineering tuning method as shown in [12] is adopted to tune the three parameters.

The resulting experimental evaluation result is illustrated in Fig. 9, which shows that only the disturbances in the low frequency range has been successfully attenuated, however the random disturbances in the high frequency range from 350 to 620 Hz cannot be effectively attenuated and the deterministic disturbances located around 24 Hz, 36 Hz and 48 Hz are not perfectly eliminated too. Though the control performance of the PID controller may be further improved by optimally tuning the parameters using other approaches, the space for the performance improvement should be limited. This is mainly due to the fact that only three parameters in the PID controller can be tuned and thus are not enough for tuning to obtain a high open-loop gain in the whole disturbance frequency range and to satisfy the Internal Model Principle (IMP) for the perfect deterministic disturbance elimination. However, the robust  $H_\infty$  controller with augmented Youla ( $Q$ ) parameterized adaptive regulation approach proposed in this paper can solve these difficulties simultaneously.

In the following, the base  $H_\infty$  controller in the proposed adaptive controller is first designed. The  $H_\infty$  controller is designed by choosing the appropriate  $W_e$  and  $W_u$  weighting functions using the *augw* and *hinfsyn* functions in MATLAB based on the identified model. In order to effectively attenuate the band-limited vibration disturbance, the weighting functions are selected as:

$$W_e = \frac{0.0002831z^8 - 0.001133z^6 + 0.001699z^4 - 0.001133z^2 + 0.0002831}{z^8 - 6.41z^7 + 18.71z^6 - 32.38z^5 + 36.25z^4 - 26.89z^3 + 12.91z^2 - 3.673z + 0.4763} \quad (17)$$

$$W_u = \frac{0.01067z^4 - 0.04267z^3 + 0.064z^2 - 0.04267z + 0.01067}{z^4 + 1.942z^3 + 1.702z^2 + 0.706z + 0.1168} \quad (18)$$

The corresponding frequency response of the weighting functions are presented in Fig. 10. Firstly, the designed  $H_\infty$  controller is evaluated and applied in the experimental system to attenuate the vibration. It can be seen from the Fig. 11 that the disturbance in the range from 350–620 Hz has been dramatically attenuated with around –20db subduction. But the deterministic disturbances located around 24 Hz, 36 Hz and 48 Hz cannot be eliminated effectively.

The  $Q$  parameter in the controller is selected as  $Q(z^{-1}) = (\sum_{i=1}^6 \theta_i z^{1-i})F(z^{-1})$ , where  $F(z^{-1})$  is a Butterworth low pass filter with cutoff frequency at 50 Hz. The main idea of the adaptive regulation scheme is to tune the parameter vector  $\theta$  to converge to the desired value that achieves the regulation against deterministic signals. The forgetting factor in the adaptive algorithm is selected with respect to the error. By testing, the forgetting factor is set as 0.996. In the adaption algorithm, the  $U - D$  fac-

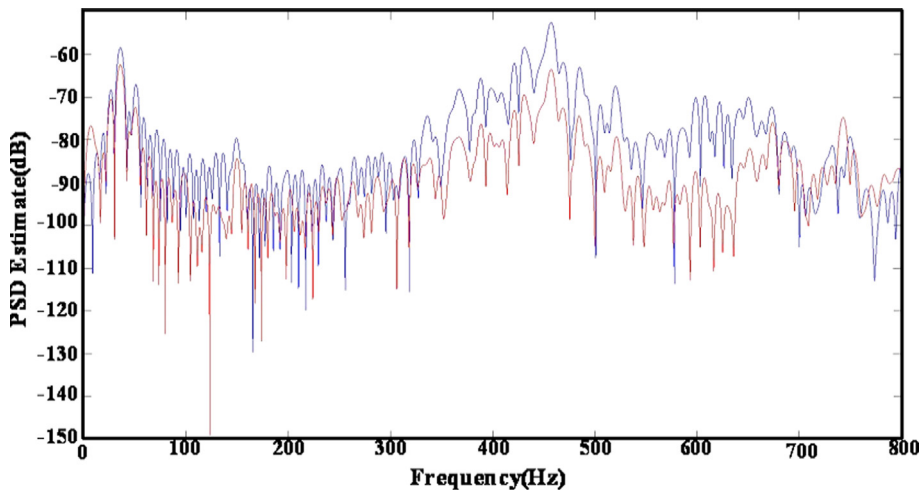


Fig. 11. Experimental results of the  $H_\infty$  control system.

torization algorithm is used to improve the numerical properties of the algorithm.  $U - D$  factorization decomposes the  $P(k)$  into the form of  $UDU^T$ , where  $U$  is an upper triangular matrix whose main diagonal element is 1 and  $D$  is a diagonal matrix [17]. This allows  $P(k)$  to remain positive definite despite the presence of rounding errors.  $U$  and  $D$  are updated at each step  $k$ .

Fig. 12(a) and (b) show the PSD of the error signal and the value of  $\theta$  in the  $Q$  parameters, respectively, with the designed adaptive controller. It can be seen from the Fig. 12 that the deterministic disturbances located around 24 Hz, 36 Hz and 48 Hz have been further eliminated. Therefore, the experimental results clearly indicated that the deterministic disturbances and the random disturbances have been both effectively attenuated with the proposed  $Q$  parameterized adaptive controller.

In the above experimental evaluation, the spindle speed is assume to be time varying or piecewise adjustable, therefore the  $\lambda(k)$  is set as a constant of 0.996, which could make the estimated parameters sensitive to the random disturbance signals and not converge as shown in Fig. 12(b). However, since the criterion of the proposed adaptive algorithm is to minimize the performance index  $J(k) = \sum_{i=1}^k \left[ \prod_{j=1}^{k-1} \lambda(j-i) \right] \left[ \tilde{\theta}(k)^T \phi(i-1) \right]^2$  for each step [17], the desired control performance is still achieved as shown in Fig. 12(a).

In order to show the stability and convergence of the estimated parameters with the adaptive algorithm, in the following, the deterministic disturbance signal is considered to be stationary, then the time varying  $\lambda(k)$  is selected as  $\lambda(t) = 0.9999 * \lambda(t-1) + 0.0001$  with  $\lambda(0) = 0.99$  and  $\lim_{t \rightarrow \infty} \lambda(t) = 1$ . The corresponding experimental results are shown in

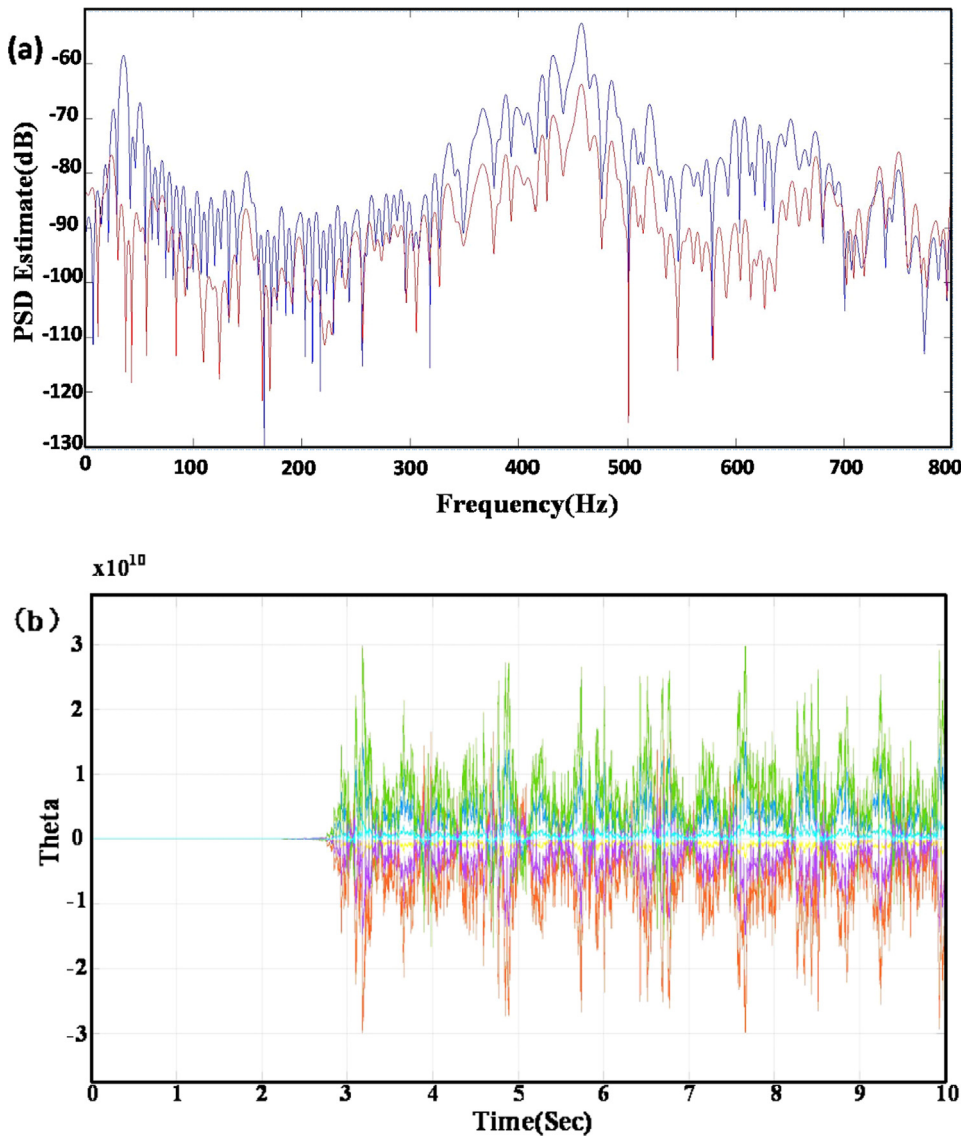
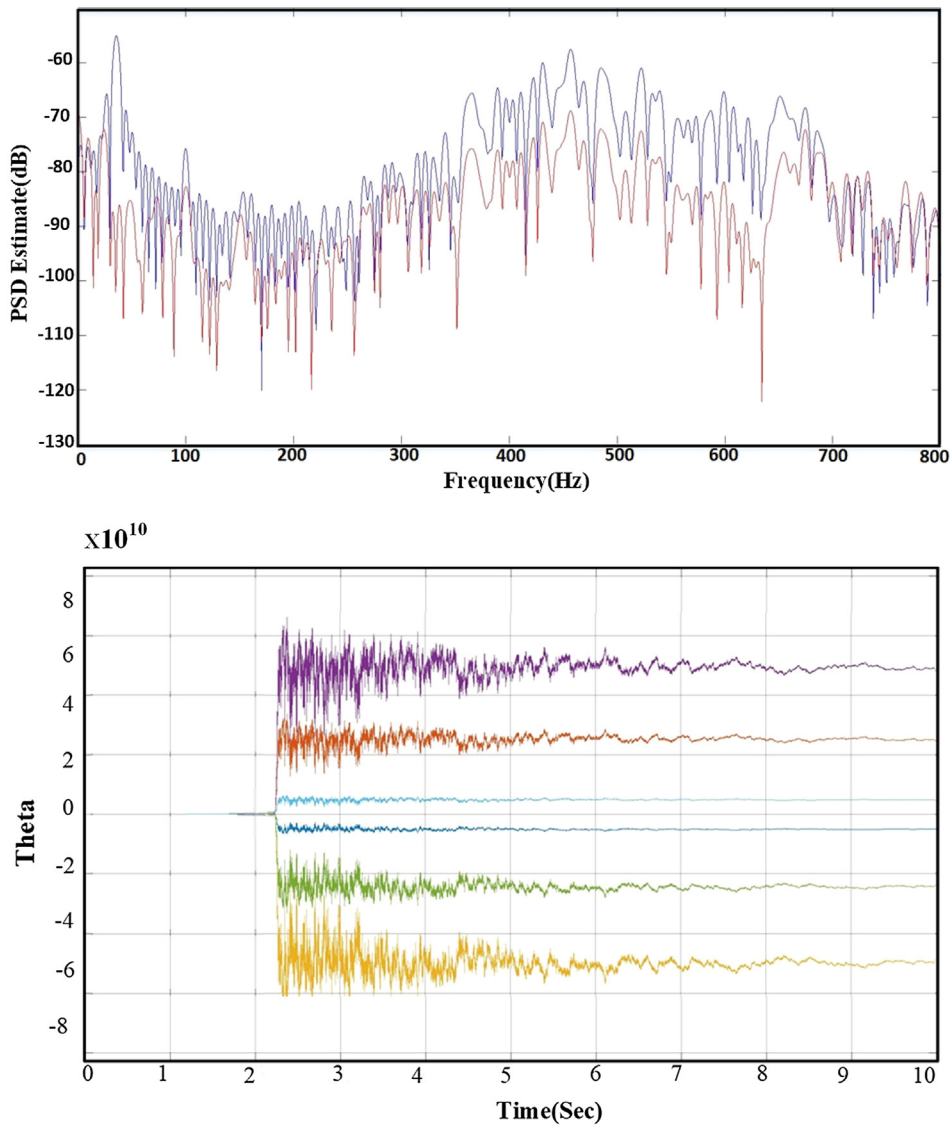


Fig. 12. Experimental results of the adaptive closed-loop control system with  $\lambda = 0.996$  (a) power spectral density estimate of the error (b) estimated parameter  $\theta$ .



**Fig. 13.** Experimental results of the adaptive closed-loop control system with time varying  $\lambda$  (a) power spectral density estimate of the error (b) estimated parameter  $\theta$ .

Fig. 13, which shows the estimated parameter  $\hat{\theta}(\cdot)$  has converged and the similar control performance against the unknown vibration disturbance is achieved.

## 5. Conclusion

An Youla ( $Q$ ) parameterized adaptive regulation approach is proposed to reject both deterministic and random vibration disturbances in this paper. The design of the adaptive controller mainly involves two steps. The first step is to design the base  $H_\infty$  controller for the random disturbance attenuation, and then the adaptive  $Q$  block is augmented in the second step to achieve the elimination for unknown deterministic vibration signals. A recursive least squares (RLS) algorithm with forgetting factor is used to tune the parameters online to the desired values that can achieve regulation against the deterministic signals. The experimental results on a lathe system indicate that the proposed adaptive controller can effectively deal with the unknown random and deterministic dynamic vibrations in the mechanical systems.

## Funding

This work was supported by the National Natural Science Foundation of China (51675321); Shanghai Municipal Natural Science Foundation (15ZR1415800); The Innovation Program of Shanghai Municipal Education Commission (14ZZ092).

## References

- [1] S. H. Crandall, W. D. Mark, Random vibration in mechanical systems, Michigan, 2014.
- [2] L.X. Guo, R. Li, Influence of vibration frequency variation on poroelastic response of intervertebral disc of lumbar spine, *J. Mech. Sci. Technol.* 33 (2019) 973–979, <https://doi.org/10.1007/s12206-019-0154-z>.
- [3] T. Ishibashi, H. Fujimoto, et al, High frequency variation speed control of spindle motor for chatter vibration suppression in NC machine tools, *IEEE Am. Control Conf. (ACC)* 4 (6) (2014), <https://doi.org/10.1109/ACC.2014.6858869>.
- [4] D. Wu, K. Chen, Chatter suppression in fast tool servo-assisted turning by spindle speed variation, *Int. J. Mach. Tools Manuf* 50 (2010) 1038–1047, <https://doi.org/10.1016/j.ijmachtools.2010.09.001>.
- [5] Y. Altintas, Z.M. Kilic, Generalized dynamic model of metal cutting operations, *CIRP Ann.* 62 (1) (2013) 47–50, <https://doi.org/10.1016/j.cirp.2013.03.034>.
- [6] J. Zhou, X.Y. Mao, et al, Prediction of cutting force in milling process using vibration signals of machine tool, *Int. J. Adv. Manuf. Technol.* 99 (2018) 965–984, <https://doi.org/10.1007/s00170-0182464-1>.
- [7] X.W. Zhang, C.X. Wang, et al, Robust active control based milling chatter suppression with perturbation model via piezoelectric stack actuators, *Mech. Syst. Sig. Process.* 120 (2019) 808–835, <https://doi.org/10.1016/j.ymssp.2018.10.043>.
- [8] S. Elliott, *Signal Processing for Active Control*, Academic Press, San Diego, California, 2001.
- [9] Y. He, X.A. Chen, et al, Active vibration control of motorized spindle based on mixed /Kalman filter robust state feedback control, *J. Vib. Control* (2019), <https://doi.org/10.1177/1077546318820935>.
- [10] G. James, Feedback and optimal sensitivity: Model reference transformations, multiplicative seminorms and approximate inverses, *IEEE Trans. Autom. Control* 26 (2) (1981) 301–320, <https://doi.org/10.1109/TAC.1981.1102603>.
- [11] H.D. Choi, C.K. Ahn, M.T. Lim, et al, Dynamic output-feedback control for active half-vehicle suspension systems with time-varying input delay, *Int. J. Control Autom. Syst.* 14 (1) (2016) 59–68, <https://doi.org/10.1007/s12555-015-2005-8>.
- [12] Y.L. Li, X.J. Wang, et al, Active vibration and noise control of vibro-acoustic system by using PID controller, *Sound Vib.* 348 (2015) 57–70, <https://doi.org/10.1016/j.jsv.2015.03.017>.
- [13] S.M. Khot, N.P. Yelve, R. Tomar, et al, Active vibration control of cantilever beam by using PID based output feedback controller, *J. Vib. Control* 18 (3) (2012) 366–372, <https://doi.org/10.1177/1077546311406307>.
- [14] A. Montazer, J. Poshtan, A. Yousefikhoma, Design and analysis of robust minimax LQG controller for an experimental beam considering spill-over effect, *IEEE Trans. Control System Technol.* 19 (5) (2011) 1251–1259, <https://doi.org/10.1109/TCST.2010.2071873>.
- [15] P.S. Ogun, M.R. Jackson, Active vibration control and real-time cutter path modification in rotary wood planning, *Mechatronics* 46 (1) (2017) 21–31, <https://doi.org/10.1016/j.mechatronics.2017.06.007>.
- [16] D.H. Li, H.G. Cao, et al, Model predictive control based active chatter control in milling process, *Mech. Syst. Sig. Process.* 128 (2019) 266–281, <https://doi.org/10.1016/j.ymssp.2019.03.047>.
- [17] I.D. Landau, R. Lozano, M. M'Saad, A. Karimi, *Adaptive Control: Algorithm, Analysis and Applications*, Springer-Verlag, London, 2011.
- [18] B.A. Francis, W.M. Wonham, The internal model principle of control theory, *Automatica*. 12 (5) (1976) 457–465, [https://doi.org/10.1016/0005-1098\(76\)90006-6](https://doi.org/10.1016/0005-1098(76)90006-6).
- [19] Z.Z. Wu, F. Ben Amara, Regulator synthesis for bimodal linear systems, *IEEE Trans. Autom. Control* 56 (2) (2011) 390–394, <https://doi.org/10.1109/TAC.2010.2090790>.
- [20] Z.Z. Wu, F. Ben Amara, Youla parameterized adaptive regulation against sinusoidal exogenous inputs applied to a benchmark problem, *Eur. J. Control.* 19 (4) (2013) 289–299, <https://doi.org/10.1016/j.ejcon.2013.04.001>.
- [21] Z.Z. Wu, M.T. Zhang, et al, Youla parameterized adaptive vibration suppression with adaptive notch filter for unknown multiple narrow band disturbances, *J. Vib. Control* 25 (3) (2019), <https://doi.org/10.1177/1077546318794539>.
- [22] I.D. Landau, On the use of Youla–Kucera parameterization in adaptive active noise and vibration control – a review, *Int. J. Control* 1 (13) (2018), <https://doi.org/10.1080/00207179.2018.1548773>.
- [23] J. Coey, Permanent magnet applications, *J. Magn. Magn. Mater.* 248 (3) (2002) 441–456, [https://doi.org/10.1016/S0304-8853\(02\)00335-9](https://doi.org/10.1016/S0304-8853(02)00335-9).
- [24] M. Lallart, C. Richard, et al, Electrostrictive bending actuators: modeling and experimental investigation, *Sens. Actuators, A* 179 (4) (2012) 169–177, <https://doi.org/10.1016/j.sna.2012.02.023>.
- [25] V. Kucera, *Discrete Linear Control: The Polynomial Equation Approach*, John Wiley & Sons, New York, London, Sydney, 1997.