PAI – Solutions Theoretical Test 2

PL1

Consider a vocabulary with only three propositions: A, B, C. How many models are there for the following sentences? (i.e. in how many models are the following sentences satisfied?)

$\neg A \lor \neg B \lor \neg C$ 7 $(A \Rightarrow B) \land A \land \neg B \land C$ 0 $B \lor C$ 6

PL2

Determine if the sentence

$$(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$$

is:

- ○ valid
- ◉ satisfiable
- ○ unsatisfiable

PL3

Which of the following are model checking types of proofs for propositional sentences?

- [x] Using a truth table to enumerate all possibilities
- [ ] Using a search algorithms on a tree generated by the application of inference rules
- [x] Using a heuristic search algorithms in the model space

PL4

We have the following rules:

$$A \land B \land C \Rightarrow D$$
$$C \Rightarrow G$$
$$F \Rightarrow G$$
$$D \land F \Rightarrow E$$
$$G \land A \Rightarrow B$$
$$E \land G \Rightarrow H$$

If the knowledge base is composed by the facts: A, B, C and we are using Forward Chaining, in how many steps of applying the above rules will we get fact E into the knowledge base?

- ○ 1
- ○ 2
- ○ 3
- ◉ It is not possible to add E to the knowledge base.

PL5

We have the same rules as in the previous question:

$$A \land B \land C \Rightarrow D$$
$$C \Rightarrow G$$
$$F \Rightarrow G$$
$$D \land F \Rightarrow E$$
$$G \land A \Rightarrow B$$
$$E \land G \Rightarrow H$$

Which of the following sets of facts is/are enough to be in the initial knowledge base so that fact H can be proven through either backward or forward chaining?

- [ ] A, B, C
- [x] A, C, F
- [ ] B, C, F
- [ ] A, D, G
- [x] D, F

PL6

You have the following sentence:

$$\neg(\neg A \land B \land C) \Leftrightarrow A \land \neg(B \land C)$$

Which of the following statements are logically equivalent?
Hint: Use the rules on slide 14 of *P2_Logic.pdf*.

- [x] $((A \lor \neg(B \land C)) \Rightarrow (A \land (\neg B \lor \neg C))) \land (A \lor \neg B \lor \neg C \Rightarrow \neg(\neg A \land B \land C))$
- [ ] $A \land \neg B \land \neg C \Leftrightarrow A \land \neg B \land \neg C$
- [x] $(A \land \neg B) \lor (A \land \neg C) \Leftrightarrow A \lor \neg B \lor \neg C$

FOL1

What can we say about the following sentence?

$$\forall x, y(x = y)$$

- ○ It is valid (necessarily true in all models)
- ○ It is true if the model has only 2 variables
- ◉ It is true if the model has only one variable

FOL2

What can you say about the following sentence?

$$(\exists x, x = x) \Rightarrow (\forall y, \exists z, y = z)$$

- ◉ It is valid
- ○ It is satisfiable
- ○ It is unsatisfiable

FOL3

Take the sentence:

There is a country that borders both Greece and Romania.

Say if the following logical expressions:

- correctly express the English sentence (**Y**)
- are syntactically valid but do not correctly express the English sentence (**N**) or
- are syntactically invalid (**I**)

Use only the capital letters: **Y**, **N**, **I** for your answers.

$\exists c, Country(c) \Rightarrow [Border(c, Romania) \land Border(c, Greece)]$ N

$[\exists c, Country(c)] \Rightarrow [Border(c, Romania) \land Border(c, Greece)]$ I

$\exists c, Country(c) \land Border(c, Romania) \land Border(c, Greece)$ Y

$\exists c, Border(Country(c), Romania \land Greece)$ I

FOL4

We have the following sentences (A) and (B) in First Order Logic:

$(A) \forall x, \exists y, (x \geq y)$

$(B) \exists y, \forall x, (x \geq y)$

We assume that the variable srange over all natural numbers: 0, 1, 2,... and that the present predicate means "greater than or equal to". Under this interpretation, state which of the following are true:

- [x] (A) is true under this interpretation
- [x] (B) is true under this interpretation