Study of persistence behaviour in Ising models and the similarity in financial markets

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Abstract

Persistence probability in Ising models were numerically reviewed, coarsening and blocking phenomena were demonstrated. Studying shares of companies regularly listed in NASDAQ 100 shown power law decay in persistence probability that was also observed in Ising models. Both close price and adjusted close price were studied to examine some corporate actions which represent market forces.

Introduction

The Hamiltonian of an Ising system

$$H = -\sum_{\langle ij \rangle} J_{ij} s_i s_j,$$

$< ij >$ demonstrating that the summation takes over the nearest neighbours of i only, $i = \overline{1, N}$.

The pure ferromagnetic Ising model has $J_{ij} = J = 1$. For a disordered Ising model, J_{ij} follows the probability distribution $P(J_{ij}) = (1-p)\,\delta(J_{ij} + \lambda J) + p\delta(J_{ij} - J), p \in [0,1]$, the ferromagnetic bond concentration. $\lambda = 0$ for the bond diluted model and $\lambda = 1$ gives the random bond Ising model.

Glauber dynamics at zero temperature was used to decide whether a spin flips from s_i to $-s_i$ or not, by considering the change in energy ΔE, if $\Delta E < 0$, flip the spin, if $\Delta E = 0$, flip the spin with probability 0.5, and spin flip is forbidden for positive energy change.

Persistence probability P(t) is defined for the fraction of spin sites haven't changed sign upto some time, and time is measured in Monte Carlo update sweeps (MCS).

Pure Ising

The typical length by which two spins are correlated is $L(t) \sim t^{1/2}$. The dynamics in finite systems will be slow down at late time and will eventually stop after the number of $MCS \sim L^2$ (L the lattice size). Therefore, a fraction of spins that won't flip, these are not like blocked spins in disordered models.

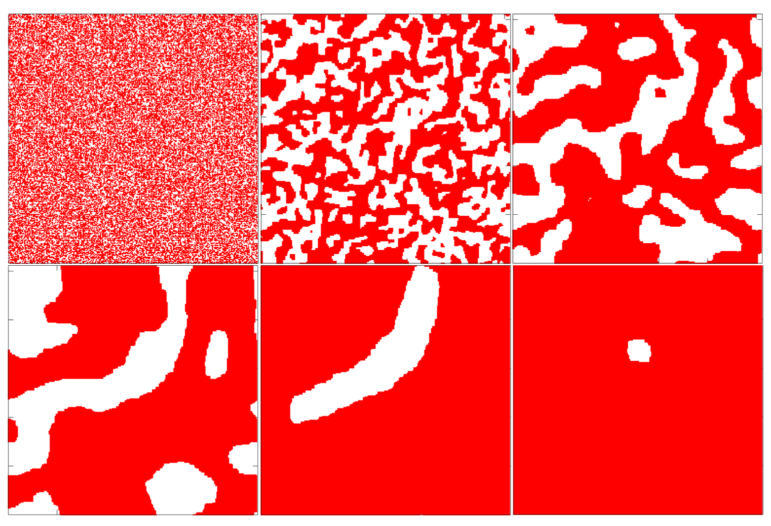


Figure 1: *Spin systems, Top: initially with random configuration, 10, 100, Bottom: at 400, 2000, 3900 MCS, respectively. Spin -1's are red, +1's are white.*

In pure ferromagnetic Ising model, persistence probability decays algebraically as $P(t) \sim t^{-\theta}$, the persistence exponent is $\theta \sim 0.2110(1)$ and no blocking is present.

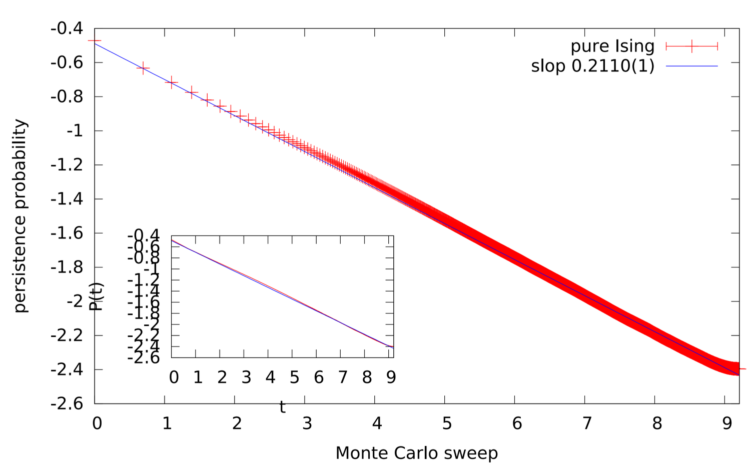


Figure 2: *Log-log plots of P(t) vs t for pure Ising model. The tail doesn't follow the linear trend is resulted from coarsening.*

Bond-dilute Ising

In disordered Ising models, there are three regimes in persistence behaviour, initially with power law decay, an intermediate regime with non-algebraic decay and the last regime with the present of blocking - fraction of non-flipping spins.

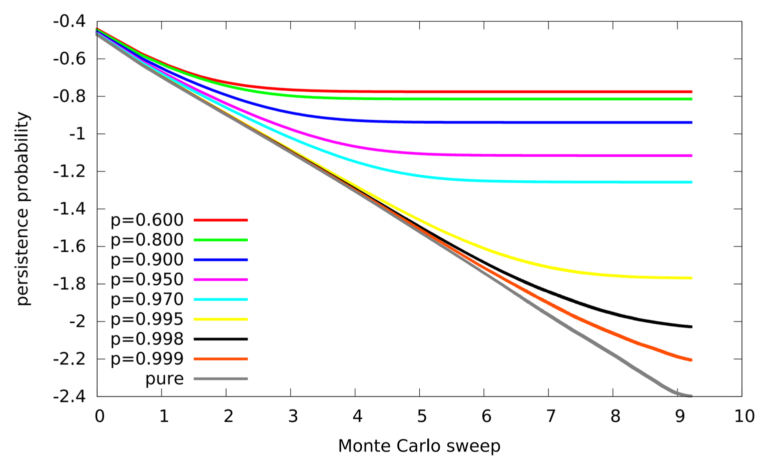


Figure 3: *Log-log plots, P(t) vs t, weak dilution.*

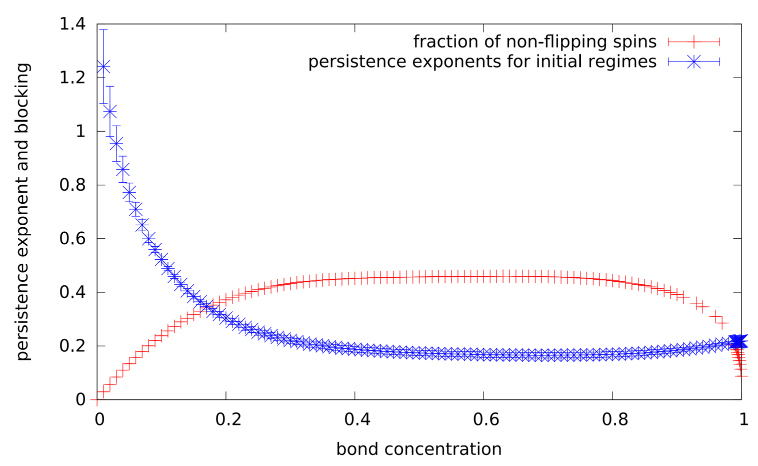


Figure 4: *Blocking and persistence exponents in initial regimes. Blocking peaks ~ 0.46 at ~ 0.63. θ is minimum 0.16(1) for $p \sim 0.7$.*

Blocking vs Coarsening

The frozen persistence in pure model is resulted from domain growth, completely different from blocking in disordered model. Blocking involves meta-stable states having groups of spins are separated from the dynamics of the system.

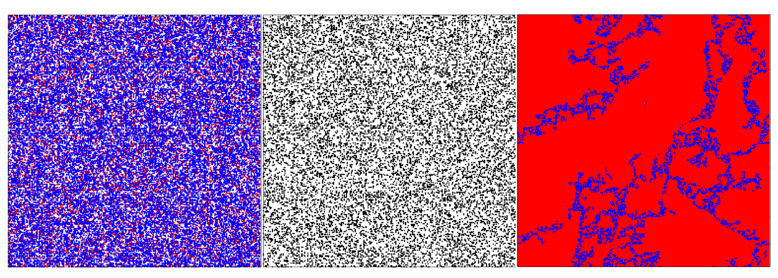


Figure 5: *Bond-dilute, left, spins -1, after 3000 MCS red, and after 5000 MCS blue. middle, blocked spins. Right most: the dynamics in pure Ising has stopped but still non-flipping spins.*

Random-bond Ising

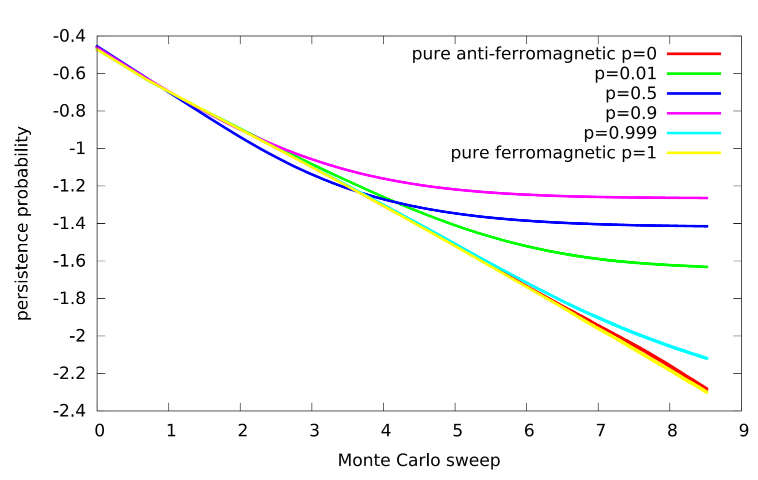


Figure 6: *Log-log plot of persistence probability for some bond concentrations.*

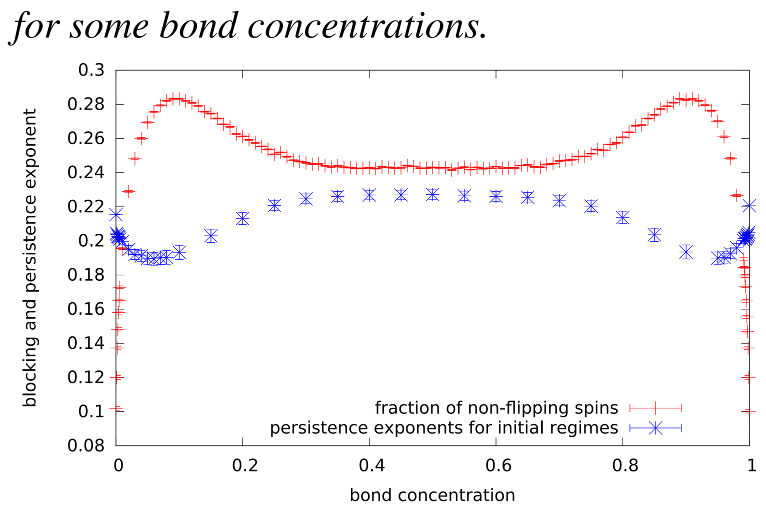


Figure 7: *Blocking and persistence exponents for the initial regime. The symmetry about $p = 0.5$ the spin glass model.*

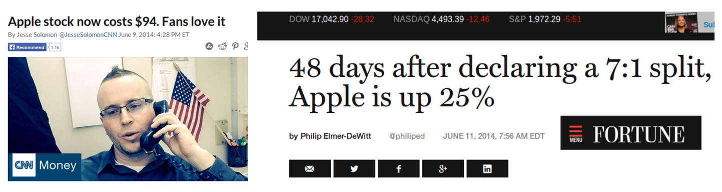
Financial Markets

Financial markets can be treated as a physical system with constituents are traders interact with each other through their investing strategies and there are some market forces like corporate actions, political news, etc. acting on every trader's investment.

Persistence in financial markets is the portion of shares (tickers) haven't changed the trend in raising or lowering price.

Figure 8: *Hanoi stock exchange.*

Figure 9: *Ho Chi Minh stock exchange.*



Corporate Actions

Stock split, e.g. 7 for 1, means, every shareholder earns extra 7 shares for each share they have, each share's price then reduced 7 times, the net amount of the shareholder's capital doesn't change. Share price at \$93.70 cheaper than that at \$645.57 should be more popular for people to buy. The close price drop sharply, \$645.57 to \$93.70, but the adjusted one increases, \$91.77 to \$93.24.

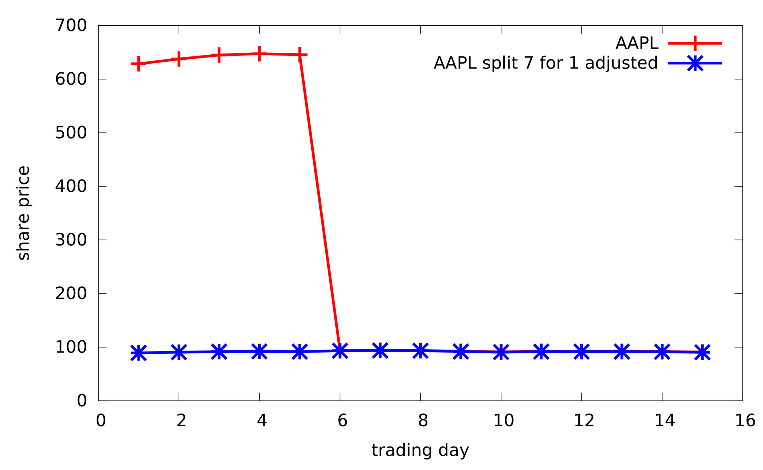


Figure 10: *Apple's stock split 7:1, June 9th.*

Dividend is usually paid in cash to the company's shareholders. Aberdeen paid \$6.75 to its shareholders, for the close price's behaviour, it appears to be a drop from \$435.50 to \$429.60, however, the adjusted close price increases from \$428.75 to \$429.60.

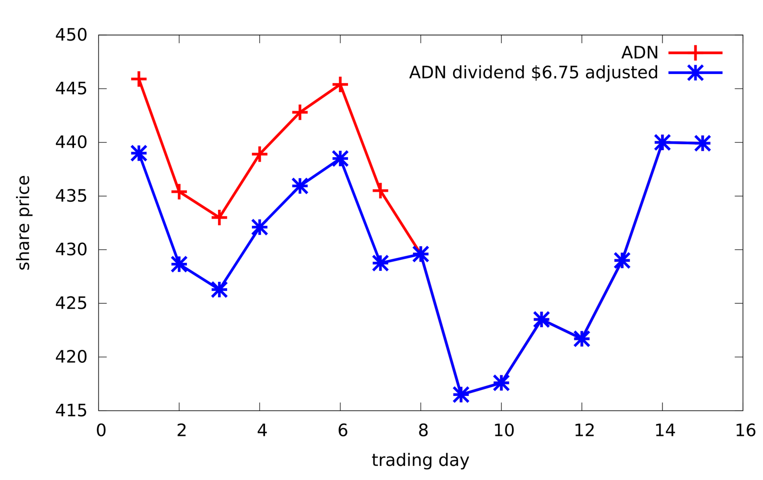


Figure 11: *Aberdeen's dividend \$6.75, May 14th.*

NASDAQ 100

Double power law decay found in NASDAQ 100 index with an initial exponent is $\theta \sim 0.39$ and a subsequent exponent of $\theta \sim 0.49$ consistent with the results found in FTSE 100 index. This would suggest universality of double power law decay in financial markets.

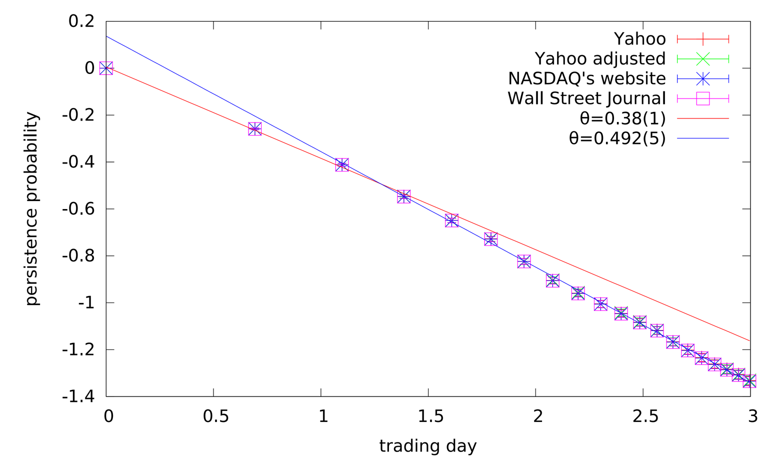


Figure 12: *Log-log plot, 69 tickers, December 20th 2010.*

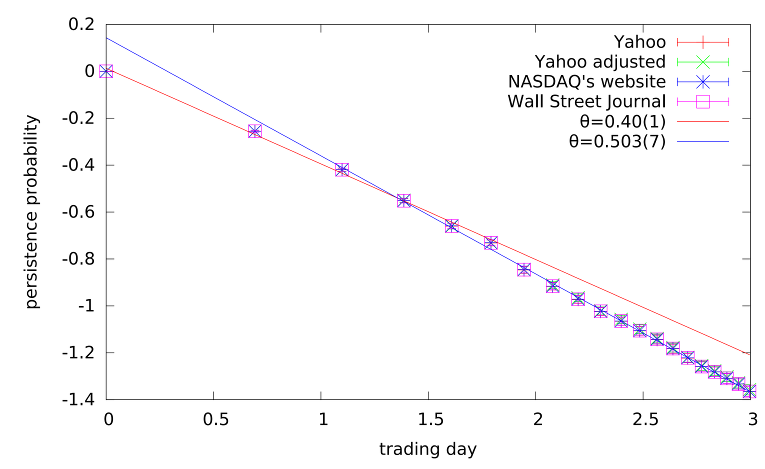


Figure 13: *Log-log plot, 73 tickers, December 19th 2011.*

Conclusion

1. A spin model, each site locates a trader, buying for spin +1, selling for -1, for long time, the present of blocking is not suitable for the trader would have infinite amount of capital.
2. Universal double power law decay in financial markets observed with an initial exponent is $\theta \sim 0.39$ and a subsequent exponent of $\theta \sim 0.49$.
3. The transition between two exponents suggests a fraction of traders change their investing strategies after about 5 days corresponding to a trading week.
4. Two of several corporate actions, stock split and dividend don't have any notable effect on persistence's behaviour. Action of delisting companies from the index also don't affect persistence. These may support the nature of a market that purely reflects demand and supply.

Future Work

1. Study global persistence probability in Ising models, and return in the world's major indices.
2. Examine some Vietnamese indices like VN30, VN100, etc.
3. Minute to minute data in NASDAQ 100 should give a huge amount of data to check the end of a trading day's effect.
4. Glauber dynamics at non-zero temperatures.
5. Other various dynamics, e.g. conserved Kawasaki dynamics, modelling a square lattice of spins with conserved Kawasaki in which every spin site represents a trader and these traders with their strategies exchange shares characterized by temperature, bond concentration and external field, should be appropriate for a market purely reflects demand and supply.