Study of persistence behavior in Ising models and the similarity in financial markets

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Abstract

Persistence with exponent $\theta \sim 0.19(3)$ in simple Ising models were numerically reviewed, coarsening and blocking phenomena were demonstrated. Studying shares of companies regularly listed in NASDAQ 100 shown power law decay in persistence probability that is similar in Ising models with two exponents, the initial decay with $\theta \sim 0.38(1)$ and the second with $\theta \sim 0.503(7)$. Both close price and adjusted close price were studied to examine some corporate actions and found no notable long-term effect.

Keywords: persistence, financial market, Ising, share, disorder, NASDAQ

Introduction

Persistence probability decays in power law as $P(t) \sim t^{-\theta}$ in pure ferromagnetic 2d Ising model was first numerically estimated to be $\theta \sim 0.22(3)$ in Ref. [1], the first experimental measurement for $\theta \sim 0.19(3)$ in planar Ising model was made in Ref. [2] with $\theta \sim 0.19(3)$, theoretical efforts in approximating $\theta \sim 0.19$ reported in Ref. [3]. Coarsening dynamics in Ref. [4, 5]. Blocking was studied in [6, 7, 8]. Persistence in financial markets by considering constituents of an index first introduced in [9].

The Hamiltonian of an Ising system

$$H = -\sum_{\langle ij \rangle} J_{ij} s_i s_j,$$

$< ij >$ demonstrating that the summation takes over the nearest neighbours of i only, $i = \overline{1, N}$. The pure ferromagnetic Ising model has $J_{ij} = J = 1$. For a disordered Ising model, J_{ij} is assumed to be independent identically distributed random variables following the distribution $P\left(J_{ij}\right) = (1-p)\,\delta\left(J_{ij} + \lambda J\right) + p\delta\left(J_{ij} - J\right), p \in [0,1]$ [7] p, the concentration of a ferromagnetic bond. $\lambda = 0$ for the bond diluted model and $\lambda = 1$ gives the random bond Ising $+/-J$ model.

Glauber dynamics at zero temperature was used to flip spins,

$$P\left(s_i \rightarrow s_i'\right) = e^{-\Delta H/T} / \left(1 + e^{-\Delta H/T}\right) = \begin{cases} 0 & \Delta H > 0 \\ 0.5 & \Delta H = 0 \\ 1 & \Delta H < 0 \end{cases} \quad [7]$$

For an Ising system, persistence probability P(t) is defined for the fraction of spin sites haven't changed sign upto some time, and time is measured in Monte Carlo update sweeps (MCS).

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Simulation parameters $L = 256, d = 2, J = 1, T = 0$ together with periodic boundary condition.

Pure Ising

After suffering a deep quench from a random initial configuration corresponding to high temperature at which the net magnetization is zero, the system tries to order locally. At zero temperature, the ferromagnetic Ising system has two possible ordered states, either all spins are up or they are all down. However, the two states have equal probability, they compete with each other, then the domain of both phases form, grow with time, and the system exhibit coarsening dynamics[2]. The length $L(t)$, the typical linear dimension of a growing domain is time dependent, $L(t) \sim t^{1/2}$ for the case of non-conserved order parameters[4], here the net magnetization with non-conserved Glauber dynamics. The patterns look statistically similar in these figures if the systems are rescaled by $L(t)$. For finite-size systems, a consequence of domain growth is that the dynamics will slow down and eventually stop leading to some spins that won't flip [Fig. 1], this will be discussed to be completely different from blocking phenomenon in disordered systems [Fig. 2]. In literature, the persistence probability in pure Ising model decays algebraically with an exponent θ as $P(t) \sim t^{-\theta}$. Therefore, $P(t) \sim L(t)^{-2\theta}$, for infinite systems, as L(t) tends to infinity, P(t) will eventually decay to zero. No blocking in pure system is consistent with the suggestion in Ref. [6] that in pure ferromagnetic system, every spin flips infinitely many times.

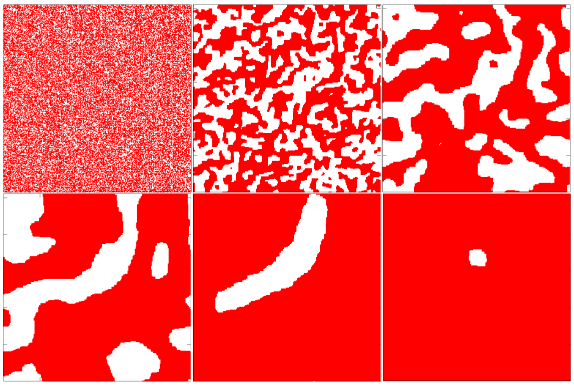

Figure 1: *Spin systems at 0, 10, 100 on the top and 400, 2000, 3900 MCS at the bottom, respectively.*

In this paper, the value of P(t) was averaged over 100 initial configurations, its behaviour is clearly an asymptotic power law decay and the value of persistence exponent was estimated to be $\theta \sim 0.2110(1)$ [Fig. 3]. Although the lattice is finite $L = 256$, the exponent is in good agreement with literature.

Bond-diluted Ising

In the disordered models, every measurement was averaged over 100 samples which include 4 disorders (i.e. bonding) and thermally averaged over 25 samples (i.e. initial configurations) for each disorder. There are three distinct regimes in persistence probability, initial behaviour is pure-like, i.e. decaying in power law, an intermediate regime in which the decay is non-algebraic and the final regime is the appearance of blocking [Figs. 4,5]. Persistence exponents for the initial regime are shown in [Fig. 6], it appears that the persistence exponents depend crucially on bond dilution and decrease smoothly from $\theta \sim 1.2(1)$ for $p = 0.01$ to $\theta \sim 0.16(1)$ for $p = 0.7$ and then increase slowly to $\theta \sim 0.2110(1)$ for the pure case. When p=0, P(t) decays very quickly to zero since spin sites have no connection, energy change is always zero. Persistence probability freezes after some amount of Monte Carlo sweeps, this is the so-called blocking phenomenon.

Blocking $p(\infty)$, the fraction of non-flipping spins increases from $p(\infty) = 0$ for p=0 to $p(\infty) \sim 0.460$ at $p \sim 0.63$ for a peak, and then reduces to the pure case with no blocking. Blocking is due to the fact that for disordered systems, spin sites flip only finitely many times leading to the formation of stable groups of spin, e.g. plaquettes[6] (small plaques), which are separated from the dynamics of the system, a proof for this fact was given in Ref. [6]. Physically, at zero temperature, energy-raising spin flip is forbidden and the system falls into a local minumum state which consists of stable groups of spins, and the system can not escape such metastable states to reach the ground state[5]. In the [Fig. 2], for bond-dilute (and also random-bond) model, the left shows that the dynamics keeps to continue while the persistence freezes, i.e. the configuration at 5000 MCS is different from that at 3000 MCS while the P(t) has frozen before the time 3000 MCS, and the right-most shows a number of spins that haven't flipped yet in the pure model with the stopped Glauber dynamics. When ferromagnetic bond concentration is close enough to the pure case, it is very sensitive to measure blocking because of growing domains due to the raise of ferromagnetism, however, the deflection from the pure case is clear even for $p = 0.999$ supporting the presence of blocking.

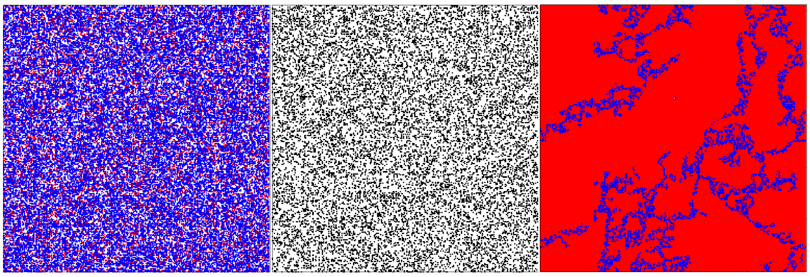

Figure 2: *Bond-dilute model, left, spins -1 after 3000 MCS are in red, and those after 5000 MCS are in blue; middle, blocked spins. Right: the dynamics in pure Ising has stopped but there're still non-flipping spins.*

Random-bond Ising

The random-bond Ising model is a composition of ferromagnetic and anti-ferromagnetic spins, the first tend to be aligned with their neighbours, and the latter tend to be anti-aligned with their neighbours. As expected, there are also three regimes in the persistence's behaviour similar to the bond-diluted case, in the [Fig. 7], persistence behaves similarly for the ferromagnetic and anti-ferromagnetic homogeneities with exponent $\theta \sim 0.21$, and from [Fig. 8], the symmetry about the bond concentration $p = 0.5$ which represents the spin glass model is clear. Consider the graph of blocking, two peaks that are symmetric, one is 0.2831(3) at $p \sim 0.09$ and the other is 0.2832(3) at $p \sim 0.91$, these are not inconsistent with results in Ref. [8]. Since initially the persistence decays in power law like, the exponents were extracted as in the [Fig. 8], it's the symmetry about $p = 0.5$ where the maximum exponent is 0.227(2), two symmetrically local minima, one is 0.189(3) at $p \sim 0.6$ and the other is 0.189(3) at $p \sim 0.95$.

Financial Market

Financial markets can be treated as a physical system with constituents are traders interact with each other through their investing's strategies and there are some market forces like corporate actions, political news, etc. acting on every trader's investment. Persistence in financial markets is considered here to be the portion of shares haven't change the trend in raising or lowering price. In Ref. [9], the authors extracted from the FTSE 100's historical data a power-law decay $P(t) \sim t^{-\theta}$ in persistence.

In this paper, historical close share prices from NASDAQ 100 index with regularly listing companies up to September 11^{th}2014 were collected to study, NASDAQ 100's constituent historical changes were directly extracted from Ref. [10]. For each period of active days considered, the close share prices of companies are mapped into a spin lattice. The close share prices on the first day in the period are selected as the base prices. The subsequent prices will be referenced to the base prices, if the subsequent price of a share i (i runs from 1 to N the number of the companies regularly listed in the index) is lower than the base price, the spin associated with the share i will have the value of -1 and vice versa. According to this rule, considering the price of shares on the second day gives the initial configuration for the spin lattice. Average over samples of period was implemented like in Ref.[9]. In addition to this average, we also did thermal average over initial configurations for each sample of period, i.e. the whole period in which the historical data being collected was divided into small periods of 21 days that correspond to one month of continuous trading, the above average was taken over such number of periods, then in each such period, initial price can be taken to be any day in the period. This 21-day-period is to have sufficient number of samples to take average.

To assure the collected data are reliable, several sources of data were used, i.e. data were collected directly from NASDAQ's website[10], from Yahoo Finance and from Wall Street Journal's website. Further step was due to the consistency among these sources, the adjusted close share price provided by Yahoo Finance was studied, the price was adjusted for stock split and dividend, the two of several corporate actions which are any event approved by a company's board of directors bringing material change to that company and its shareholders[11]. For example, if a stock split 7:1 is declared, the number of current shares of a shareholder will increase 7 times, the price for each share will reduce 7 times too, this means the net money doesn't change, and the total shares of the company that declares the stock split will increase 7 times and since the price is lower, more people are applicable for buying the shares. Practically, it appears to be a sharp drop from \$645.57 on June 6^{th} 2014 to \$93.70 June 9^{th} 2014 for AAPL's close price, however, on the latter day the stock split 7 for 1 was payable, so look at adjusted close share price, it's a raise of \$1.47 from \$91.77 to \$93.24. Some citations for the effect of stock split 7 for 1 announced on April 23^{rd} by Apple are in the articles *'Apple stock now costs \$94. Fan love it'[12]*, and *'48 days after declaring a 7:1 split, Apple (AAPL) is up 25%[13]'*. Dividend is usually paid in an amount of cash that a company pays for its shareholders, for example, on the 14^{th} of May 2014, Aberdeen Asset Management PLC (ADN)'s dividend of \$6.75 applied, the close price on the 13^{rd} of \$435.50 was down to \$429.60 on the next day, however, for the adjusted close price, a different behaviour is a \$0.85 increase from \$428.75 (which is \$6.75 subtracted from \$435.50) to \$429.60.

[Figs. 9,10] show double power law decays, initially with exponent $\theta \sim 0.39$ and subsequently $\theta \sim 0.49$ were observed. The reason for double power law may be due to the present of two kinds of traders, short-termed and long-termed as given in Ref. [14]. Although average over initial prices for various days was taken, the initial exponent seemed to last for $\sim$ 5 days corresponds to a trading week after which a fration of traders change their investment. Long time exponent $\theta \sim 0.5$ is in agreement with global persistence modeled at T_c in Ref. [15] and with results for FTSE 100 in Ref. [14].

Two mentioned corporate actions don't have any notable effect on the trend may support the nature of market that reflects the demand and supply, for example, when a stock split or a dividend is declared, a number of shareholders may wish to sell a portion of shares to expect some returns while other may like to buy some more or some other who may join the group of shareholders who possess the shares. We also observed no effect coming from the actions of delisting tickers that failed to satisfy some weighting criteria to be listed in the index.

Conclusion

Double power law decay in NASDAQ 100 observed with an initial exponent is $\theta \sim 0.39$ and following by an exponent of $\theta \sim 0.49$. A fraction of traders change their investing strategies after about 5 days corresponding to a trading week. Stock split, dividend and delisting don't contribute any notable effect on persistence's behaviour supports the nature of demand and supply in NASDAQ 100 index.

Acknowledgement

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Appendices

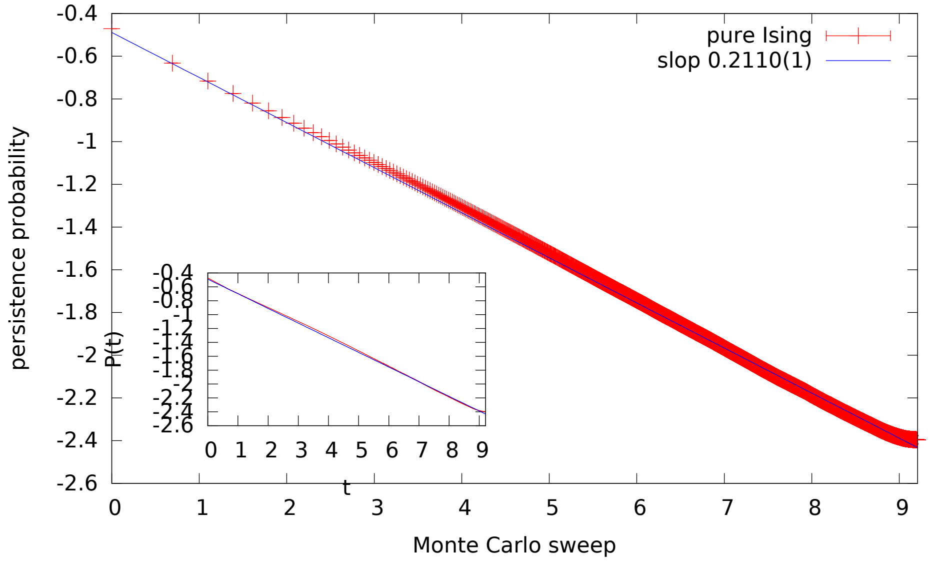

Figure 3: *Log-log plots of P(t) vs t for pure Ising model.*

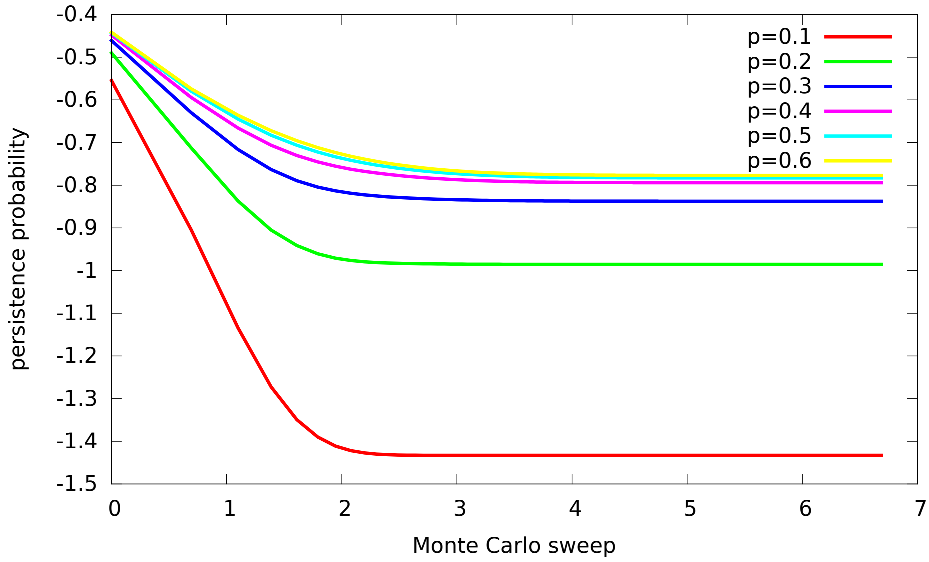

Figure 4: *Log-log plots of persistence probability for strong dilution.*

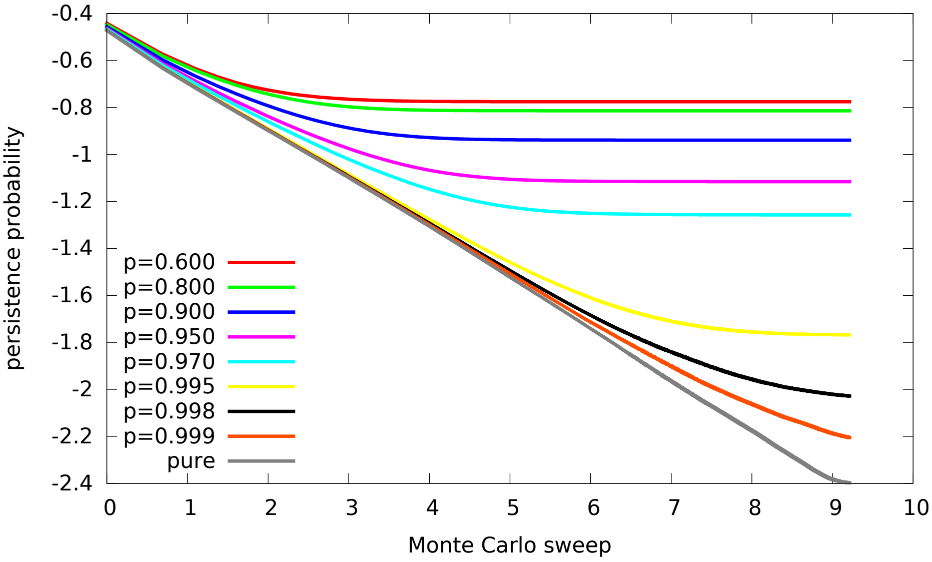

Figure 5: *Log-log plots of persistence probability for weak dilution.*

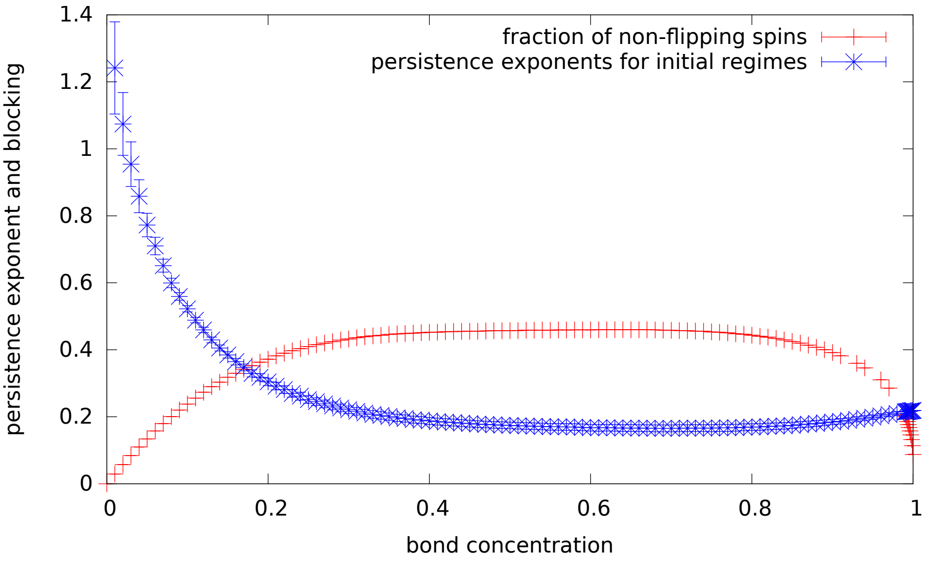

Figure 6: *Blocking in bond-diluted system and persistence exponents in initial regime.*

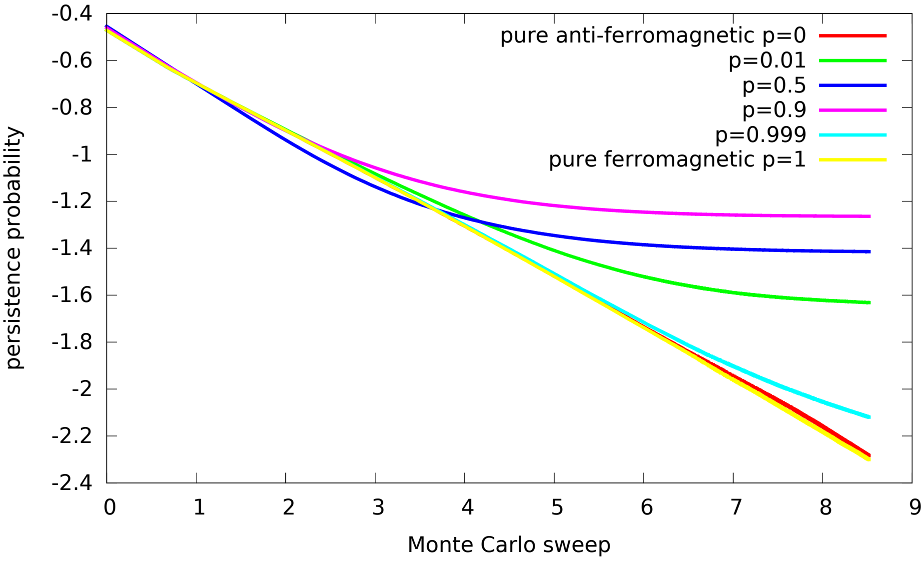

Figure 7: *Log-log plot of persistence probability for some bond concentrations, random-bond model.*

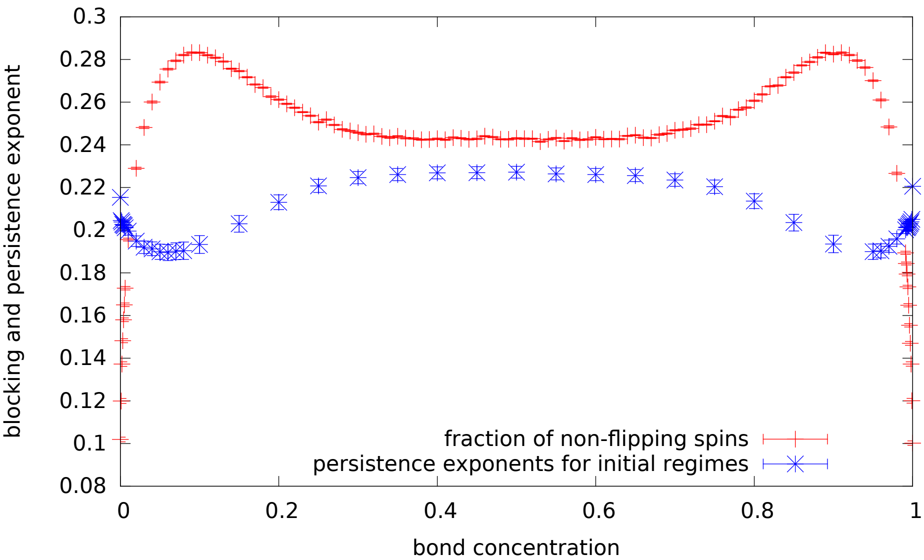

Figure 8: *blocking and persistence exponents for the initial regime in the random-bond model.*

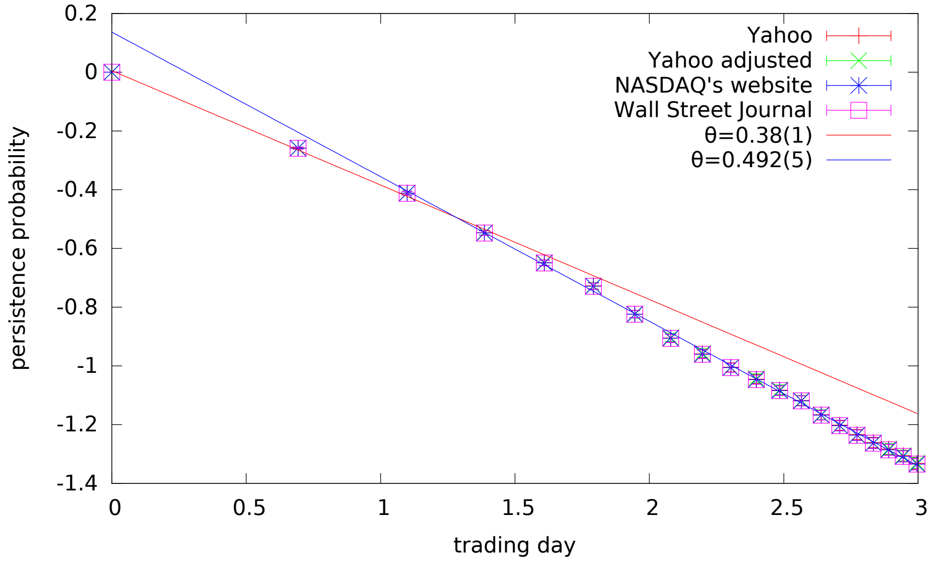

Figure 9: *Log-log plots, 69 tickers, from December 20th 2010 to September 11th 2014, periods of 21 days*

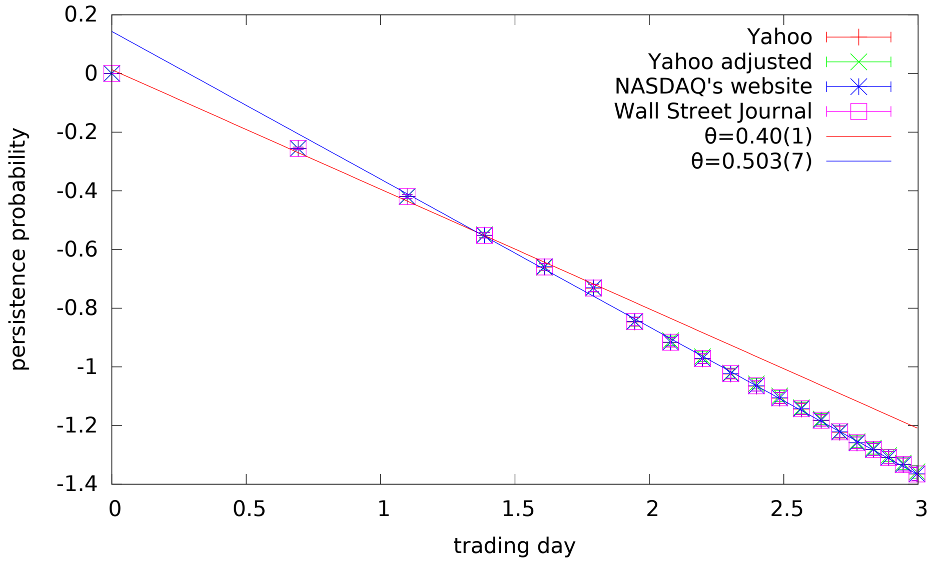

Figure 10: *Log-log plots, 73 tickers, from December 19th 2011 to September 11th 2014, periods of 21 days*