Problem Set 1

Siwei Tang, Yuanyue Qiao, Zheqiao Zhang

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1 Translating statement

(a)

$\forall$ t $\in$ T, Canadian(t) $\Rightarrow$ ($\neg$Stanley(t))

(b)

$\exists$ d $\in$ D, $\forall$ t $\in$ T, BelongsTo(t,d) $\Rightarrow$ ($\neg$Canadian(t))

(c)

$\exists$ d $\in$ D, $\forall$ t $\in$ T, Stanley(t) $\Rightarrow$ BelongsTo(t,d)

(d)

$\forall$ t $\in$ T, $\exists$ d_1 $\in$ D, (BelongsTo(t,d_1) $\wedge$ ($\forall \mathrm{d}_2 \in D, (BelongsTo(t,\mathrm{d}_2) \Rightarrow \mathrm{d}_1 = d_2$))

(e)

$\forall$ d $\in$ D, $\exists$ t_1 $\in$ T,$\exists$ t_2 $\in$ T, (BelongsTo(t_1,d) $\wedge BelongsTo(\mathrm{t}_2$,d) $\wedge \mathrm{t}_1 \neq \mathrm{t}_2 \wedge (\forall \mathrm{t}_3 \in T(BelongsTo(\mathrm{t}_3$,d)$\Rightarrow$ (t_3 = $t_1 \vee \mathrm{t}_3 = t_2$))))

2 Even and odd function

(a)

Even(f): $\forall$ x $\in \mathbb{R}, f(x) = f(-x), where f : \mathbb{R} \rightarrow \mathbb{R}$
Odd(f): $\forall$ x $\in \mathbb{R}, f(x) = -f(-x), where f : \mathbb{R} \rightarrow \mathbb{R}$

(b)

$\forall$ f: $\mathbb{R} \rightarrow \mathbb{R}$, $\forall$g: $\mathbb{R} \rightarrow \mathbb{R}$, (Odd(f) $\wedge$ Odd(g)) $\Rightarrow$ ($\forall$ f $\times$ g: $\mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \times \mathbb{R}$, Even(f $\times$ g))

c)

Consider f(x) = 0, $\forall$ x $\in \mathbb{R}$
since f is both even and odd function $\Rightarrow$ f(x) = f(-x) = -f(-x)
$\Rightarrow$ f(-x) = 0
if f(x) = -f(-x) = 0, $\forall$ x $\in \mathbb{R}$
$\Rightarrow$ f(-x) = 0, $\forall$ x $\in \mathbb{R}$
$\Rightarrow$ f is both even and odd

d)

$\forall$ f: $\mathbb{R} \to \mathbb{R}$, $\exists$ f_1: $\mathbb{R} \to \mathbb{R}$,$\exists$ f_2: $\mathbb{R} \to \mathbb{R}$, f = f_1 + f_2 $\wedge(Even(\mathrm{f}_1) \wedge \mathrm{Odd}(f_2))$

e)

$$
\begin{aligned}
f_1 &= \sum_{i=0}^{n} (a_{2i}x^{2i}) \\
f_2 &= \sum_{k=1}^{n} (a_{2k-1}x^{2k-1}) \\
f(x) &= a_0 + a_1x + a_2x^2 + ... \\
&= (a_0 + a_2x^2 + ...) + (a_1x + a_3x^3 + ...) \\
&= \sum_{i=0}^{n} (a_{2i}x^{2i}) + \sum_{k=1}^{n} (a_{2k-1}x^{2k-1}) \\
&= f_1 + f_2 \\
f_1(-x) &= \sum_{i=0}^{n} (a_{2i}x^{2i}) = f_1(x) \\
&\Rightarrow f_1 \text{ is even function} \\
f_2(-x) &= \sum_{k=1}^{n} (a_{2k-1}(-x)^{2k-1}) = \sum_{k=1}^{n} (-a_{2k-1}x^{2k-1}) = -f_2(x) \\
&\Rightarrow f_2 \text{ is odd function}
\end{aligned}
$$

f)

$$
\begin{aligned}
g_1 &= \frac{1}{2}(g(x)+g(-x)) = \frac{1}{2}(2^{-x}+2^{x}) \\
g_2 &= \frac{1}{2}(g(x)-g(x)) = \frac{1}{2}(2^{x}-2^{-x}) \\
g(x) &= 2^{x} \\
&= \frac{1}{2}*2^{x}+\frac{1}{2}*2^{x}+\frac{1}{2}*2^{-x}-\frac{1}{2}*2^{-x} \\
&= \frac{1}{2}(2^{-x}+2^{x})+\frac{1}{2}(2^{x}-2^{-x}) \\
&= g_1+g_2 \\
g_1(-x) &= \frac{1}{2}(2^{-x}+2^{x}) = g_1(x) \\
&\Rightarrow g_1 \text{ is even function} \\
g_2(-x) &= \frac{1}{2}(2^{-x}-2^{x}) = -g_2(x) \\
&\Rightarrow g_2 \text{ is odd function}
\end{aligned}
$$

3 Choosing a universe and predicates

a)

Let S = $\mathbb{N} \times \mathbb{N}$ be the domain of P
Define P(x,y) as x $\leq$ y.
For statement 1, P(x, 165) $\Rightarrow$ P(x, 1) is False, since for x = 3, we have x = 3 $\leq$ 165,$\Rightarrow$ P(x, 165) is True, but x = 3 $\geq$ 1 $\Rightarrow$ P(x, 1) is False.Hence $\forall$ x $\in$ $\mathbb{N}$, P(x,165) $\Rightarrow$ P(x,1) is False.
For statement 2,($\forall$ x $\in$ $\mathbb{N}$, P(x,165)) is false, since there exists x = 166 > 165. Hence statement2 is always true.

b)

Let S = T = $\mathbb{N}$
Define P(x,y) as x $\geq$ y on S $\times$ T which is $\mathbb{N} \times \mathbb{N}$.
Define Q(x) as x > 1 on S which is $\mathbb{N}$.
For statement 1, it is equivalent to $\forall$ x $\in$ S, $\exists$ y $\in$ T, $\neg P(x,y) \vee Q(x)$
$\neg P(x,y) is\ x<y, let\ y=x+1, \neg P(x,y)\ is\ True,\ so\ statement1\ is\ True.$
$For\ statement2,\ it\ is\ equivalent\ to$$\forall$ x $\in$ S, ($\forall$ y $\in$ T, $\neg P(x,y)$) $\vee\ Q(x)$
$if\ x=0,\ 0<1,\ so\ Q(x)\ is\ False,\ if\ y=0,\ \neg P(x,y)\ is\ False\ since\ 0\ cannot\ smaller\ than\ 0,\ so\ statement2\ is\ False.$

4 Working with infinity

a)

$\exists\ n_0 \in \mathbb{N}, \forall n \in \mathrm{N}, (Prime(n)) \vee (n \leq \mathrm{n}_0)$

b)

$\forall \mathrm{a}_0 \in \mathrm{N}\ , \exists \mathrm{a} \in \mathrm{N}, (a > \mathrm{a}_0) \wedge (\exists p \in \mathrm{N}, Prime(p) \wedge Prime(p+a) \wedge (\forall n \in \mathrm{N}, (p < n < p+a) \Rightarrow \neg Prime(n)))$

c)

$\exists \mathrm{a}_0 \in \mathrm{N}, \forall a \in \mathrm{N}, a > \mathrm{a}_0 \Rightarrow (\forall p \in \mathrm{N}, \neg prime(p) \vee \neg prime(p+a) \vee (\exists n \in \mathrm{N}, (p < n < p+a) \wedge prime(n)))$