# CSC236 Exercise 4

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## 1 Question 1

Assume the contradiction: $L_2$ is regular. Then $\overline{L_2}$ is also regular by closure principle under complement. Also, since $1, 0$ are regular, $\mathcal{L}(0^*1^*)$ is also regular by closure principle under Kleene star and concatenation.
Let $L' = \overline{L_2} \wedge \mathcal{L}(0^*1^*)$. Note that $L'$ is also regular by closure principle under intersection. Therefore, we know that:

$$\begin{aligned} w \in L' &\Leftrightarrow w \in \overline{L_2} \wedge w \in \mathcal{L}(0^*1^*) \\ &\Leftrightarrow w = \{0^i 1^i \mid i \in \mathbb{N}\} \\ &\Leftrightarrow w \in L_1 \end{aligned}$$

That is, $L' = L_1$. But $L_1$ is not regular, and this contradiction indicates our assumption is false. As such, $L_2$ is non-regular.

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## 2 Question 2

Assume the contradiction: $L$ is regular. Then $L$ must satisfy conditions in Pumping Lemma, i.e. $\exists p \in \mathbb{N}^+, \forall w \in L, |w| \geq p$, then condition (1)-(4) holds.
Let $w = 1^{p^2}$. Note that $p^2$ is a perfect square and $|w| \geq p$. Assume $w$ satisfy condition (1)-(3), i.e. $\exists x, y, z, w = xyz, |xy| \leq p, |y| \geq 1$.
Since $w$ contains only 1's, we know that $w = xyz = 1^r 1^s 1^t$, where $r + s + t = p^2, r + s \leq p$, and $s \geq 1$. If we pumped $y$ twice, called it $\bar{w}$ we have:

$$\begin{aligned} |\bar{w}| &= r + 2s + t = p^2 + s \\ p^2 \leq p^2 + s &\leq p^2 + p && (|y| \leq |xy| \leq p) \\ &\leq (p+1)^2 \end{aligned}$$

Since $p^2 + s$ is in between $p^2$ and $(p+1)^2$, it cannot be a perfect square. So $\bar{w} \notin L$, which fails the Pumping Lemma. This contradiction indicates that our assumption is false.As such, $L$ is non-regular.

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