236A2

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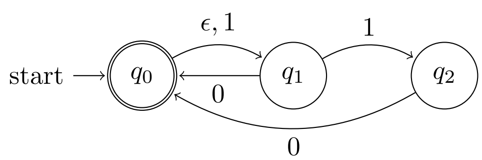
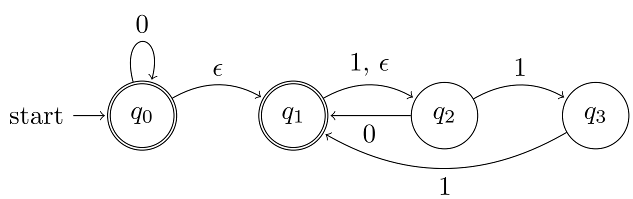
1 NFA

1.1 a

1.1.1 i



1.1.2 ii



1.2 b

1.2.1 i

(ab)*+(ba)*+(ab)(ab)*a+(ba)(ba)*b
Explanations: split the language into four parts, the first part is the infinite loop of ab, for example, ϵ, ab, abab,...
The second part is the infinite loop of ba, ϵ, ba, baba,...
The third part is the infinite loop of ab and ends with a, for example aba, ababa, abababa,...
The forth part is the infinite loop of ba ends with b, for example bab,babab,bababab,...

1.2.2 ii

a*ba*ba*+a*ba*ba*ba*
Explanations: split the language into two parts, the first part is the regular expression for string with exactly two b's and the second part is for regular expression with exactly three b's.

2 Proof

We need to prove in two directions, the first is to prove that (R*+S*)*$\subseteq$ (R+S)*
the second is (R+S)* $\subseteq$ (R*+S*)*

Part 1:
(1)R$\subseteq R + S \Rightarrow$ R* $\subseteq$ (R+S)*(By Fact 2)
(2)S$\subseteq R + S \Rightarrow$S* $\subseteq$ (R+S)*(By Fact 2)
According to (1) and (2), R*+S* $\subseteq$ (R+S)*
$\Rightarrow$(R*+S*)*$\subseteq$((R+S)*)*=(R+S)*

Part 2:
R$\subseteq$ R* and S$\subseteq$ S* $\Rightarrow (R + S)\subseteq$ R*+S*$\Rightarrow$ (R+S)* $\subseteq$(R*+S*)*

In conclusion, since (R*+S*)*$\subseteq$(R+S)* and (R+S)* $\subseteq$(R*+S*)*
so (R*+S*)* $\equiv$ (R+S)*

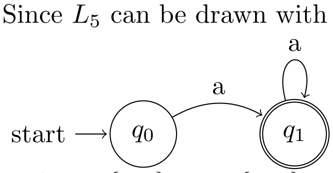
3 homomorphism

3.1 a

By question, we know that if L is a regular language over an alphabet $\sum$ and h is a homomorphism on $\sum$, then h(L) is also regular.
By contrapositive, we know that if h(L) is not regular language, then L is also not regular language.
Define homomorphism h: {a,b,c}* $\longrightarrow$ {0,1}* such that h(a)= ϵ, h(b) = 0,h(c) = 1 and $L_2 = \{a^i b^n c^n \ | i \geq 1, n \geq 0\}$
Then we have $h(L_2) = \{0^n 1^n | i \geq 1, n \geq 0\} = L_1$
By question, we know that $h(L_2) = L_1$ is not regular language, by the contrapositive statement, L_2 is not regular.

3.2 b

First, assume that L_3 is regular, then $\overline{L_3}$ is regular.
$L_4 = \overline{L_3} \cap L(a^*b^*c^*) = \{a^i b^j c^k | i \geq 1 \wedge j \neq k\}$
Since $\overline{L_3}$ and $L(a^*b^*c^*)$ is regular, so L_4 is regular by closure property
Define $L_5 = \{a^i | i \geq 1\}, L_6 = \{b^j c^k | j \neq k\}$
So $\overline{L_3} = L_5 \circ L_6$
Since L_5 can be drawn with DFA in finite states, so L_5 is regular language.



Define h:$\{b, c\}^*$ $\longrightarrow$ $\{0, 1\}^*$ such that h(b) = 0, h(c) = 1
since regular languages are closed under homomorphism, then h(L_6) = $\{0^j 1^k | j \neq k\}$ is regular
Let $L_7 = \overline{h(L_6)} \wedge L(0^*1^*) = \{0^n 1^n | n \geq 0\}$
By closure property, $h(L_6)$ is regular $\Rightarrow$ $\overline{h(L_6)}$ is also regular
Also, since L(0*1*) is regular, so by closure property, L_7 is regular.
However, by part a, $L_1 = L_7 whichisnotregular$, it is contradiction.
$\Rightarrow$ L_3 is not regular.

4 Pumping lemma

Suppose L is a regular language. Consider string w = $b^p a^{p+1}$, so w $\in L$ and $|w|$= 2p+1 ¿ p, so Pumping lemma will hold.
Thus, we can split string w into 3 parts w = xyz satisfying the conditions
i)$\forall i \geq 0, xy^i z \in L$
ii) $|y| > 0$
iii)$|xy| \leq p$
since w has p symbols of b. SO if w satisfies the last condition, it implies that x and y are all strings consists only b's.So z would be the rest of b's and a^{p+1}.
The second condition states that $|y| > 0$, implies that y has at least one b.
So x = b^r where $r \geq 0$
y = b^s where $s \geq 1$
z = $b^t a^{p+1}$ where $t \geq 0$
So p = r+s+t. The first condition implies that $xy^2 z \in L$
However, $xy^2 z = b^r b^s b^s b^t a^{p+1} = b^{r+2s+t} a^{p+1} = b^{p+s} a^{p+1}$
SO amount of b is greater than or equal to amount of a in $xy^2 z$, so $xy^2 z \notin L$ because $s \geq 1$. We got a contradiction. There fore, L is not regular language.