Ex2

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1. P(f): there exists f' $\in$ G such that f' and f are logically equivalent, and f' does not contain the $\wedge$ symbol.
   We want to prove that for all f $\in$ G, P(f) holds.

   Base Case: Let f = x, where x is a propositional variable that does not contain the $\wedge$ symbol
   Let f' = x. Then f' $\in$ G, f' does not contain the $\wedge$ symbol, and f' and f are logically equivalent.
   Therefore P(f) holds.

   Inductive step: Assume $f_1, f_2 \in$ G. By definition, $\neg f_1 \in G, (f_1 \vee f_2) \in G, and\ (f_1 \wedge f_2) \in$ G.
   Suppose P(f_1) and P(f_2), i.e., there exist $f_1', f_2' \in G$ such that f_1' is logically equivalent tof_1
   and f_2' is logically equivalent tof_2.
   $f_1' and\ f_2'$ does not contain $\wedge$ symbol **by IH.**
   WTP: (1) P($\neg f_1$), (2) P(($f_1 \vee f_2$)), (3) P(($f_1 \wedge f_2$)).

   Case 1: For f = $\neg f_1$, let $f' = \neg f_1'$.
   By IH, f_1' $\in$ G, and therefore f' $\in$ G
   By IH, f_1' does not contain the $\wedge$ symbol and therefore f' does not contain $\wedge$ symbol.
   By IH, f_1 is logically equivalent to f_1', and therefore f is logically equivalent to f'.
   Thus, P(f) = P($\neg f_1$) holds.

   Case 2: For f = ($f_1 \vee f_2$), let $f' = (f_1' \vee f_2')$.
   By IH, f_1', f_2' $\in$ G, and therefore f' $\in$ G
   By IH, f_1 and f_2' does not contain the $\wedge$ symbol and therefore f' does not contain $\wedge$ symbol.
   By IH, f_1 and f_2 are logically equivalent to f_1' and f_2' respectively, and therefore f is logically equivalent to f'.
   Thus, P(f) = P(($f_1 \vee f_2$)) holds.

   Case 3: For f = ($f_1 \wedge f_2$), let $f' = \neg(\neg f_1' \vee \neg f_2')$.
   By IH, since f_1', f_2' $\in$ G, then so are $\neg f_1'$ and $\neg f_2'$, as well as $(\neg f_1' \vee \neg f_2')$, and $\neg(\neg f_1' \vee \neg f_2')$. Thus f' $\in$ G
   Also, f' and f are logically equivalent. By IH, f_1' and f_2' do not contain the $\wedge$ symbol.
   Then f' does not contain the $\wedge$ symbol
   Thus, P(f) = P(($f_1 \wedge f_2$)) holds.