236EX5

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1 Proof of Correctness

Define P(n):Let A be a list of integers, let p, q be integers, let n = p - q, $0 \leq p < q \leq len(A)$, IndexMin(A, p, q) terminates and returns the index of a minimum element in A[p: q].

Base case: Let n = 1. So q - p = 1, that means A[p: q] contains only one element, that is the element with index p. So the smallest element in A[p: q] is the only element with index p. Hence P(1) holds.

Inductive Step: Let n ¿ 1.so q - p ¿ 1, it means that A[p: q] contains more than one elements. Let j $\in N, 1 \leq j < n$, P(j) holds. WTP: for all n $\in N, n \geq 1$, P(n) holds.

Since n ¿ 1, so q - p ¿ 1, the if statement on Line1 is false.Goes to Line 4 else statement part. It runs Line 4-10.
On line 4, $m = \lfloor \frac{p+q}{2} \rfloor \Rightarrow p < m < q \ (by\ hint)$
So $0 \leq p < m \leq len(A)$.
Also, since m > p, so $1 \leq m - p < n$.
By line 5 and inductive hypothesis, j is the index of the smallest element in A[p: m]
Since m < q, so $0 \leq m < q \leq len(A)$. Also, $1 \leq q - m < n$
By line 6 and inductive hypothesis, k is the index of the smallest element in A[m: q]
So there are two cases.
Case 1: if A[j] $\leq$ A[k],so j is the smallest element in A[p: q], then by line 8, j is returned.
Case 2: if A[j] $\geq$ A[k],so k is the smallest element in A[p: q], then by line 10, k is returned.

2 Proof

2.1 a

loop invariant LI(k):
i)The elements of A[0: k] are rearranged in sorted(nondecreasing) order
ii) $0 \leq k \leq len(A) - 1$
iii)All the elements of A[0: k] is less than or equal to all the elements of A[k: len(A) - 1]

2.2 b

m = len(A)-1-k

2.3 c

Suppose the loop terminates. By the second condition of loop invariant, $0 \leq k \leq len(A) - 1$
when the while loop ends, k $\geq len(A) - 1$
$\Rightarrow$ k=len(A)-1
By the first condition of loop invariant, all the elements of A[0: k] are rearranged so they are sorted(nondecreasing) order.
Since k = len(A)-1, so k is the last index of A$\Rightarrow$ all the elements of A are rearranged in sorted(nondecreasing)order.