Assignment 1

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April 15, 2021

Reference

Not applicable.

Question 4

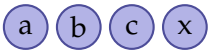
Tree implementation of the disjoint set ADT.

- $T1$ use union-by-weight without path compression
- $T2$ use union-by-weight with path compression
- $T3$ use union-by-rank with path compression

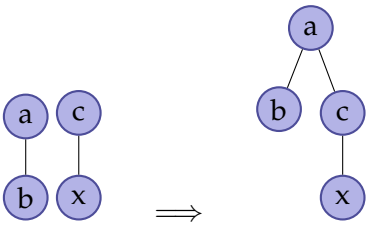
a) n = 8

Since both $T1$ and $T2$ are of the same union approach – union-by-weight, and only difference is with or without path compression, we shall make difference on both trees when it comes to a FIND-SET operation.

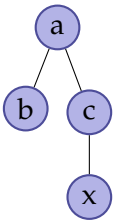
$Step1$: call MAKE-SET 4 times



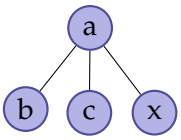
$Step2$:call UNION 3 times
UNION(a, b)
UNION(c, x)
(by now, two sets formed and each weight are of 2.)
UNION(a, c)
(since both $T1$ and $T2$ are of the same union approach – union-by-weight, there is no difference of change before any FIND-SET operation takes.)



$Step3$: call FIND-SET 1 time
T1: since no path compression taken placed, FIND-SET would not change the tree representation.



T2: since path compression is needed, FIND-SET would change the tree representation when FIND-SET(x).



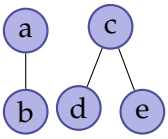
b) n = 9

Since $T2$ and $T3$ have different UNION approaches while both use path compression, all we need to make sure is to use UNION operation to make difference on $T2$ and $T3$, and no FIND-SET operation is needed.

$Step1$: call MAKE-SET 5 times

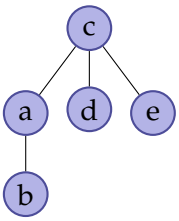
$Step2$: call UNION 3 times

UNION(a, b)
UNION(c, d)
UNION(c, e)
(so far, both $T2$ and $T3$ are of the same shape.)



$Step3$: call UNION once again
UNION(a, c)

T2: Since set a has lighter weight than the set c, c becomes the new representative, and it results into a tree as followed:



T3: Since set a has the same rank as the set c, while both rank are of 1. However, a is alphabetically ranked ahead than c, a becomes the new representative, and it results into a tree as followed: