Dept. of Computer Science
University of Toronto

Term Test # 2
(Duration: 50 minutes)

CSC 373 H1
Summer 2021

Student Number: |_|_|_|_|_|_|_|_|_|_|_|_|_|

Last (Family) Name(s): _____

First (Given) Name(s): _____

Write the following statement on the first page of the exam:

"In submitting this exam, I confirm that my conduct during this exam adheres to the Code of Behaviour on Academic Matters. I confirm that I did NOT act in such a way that would constitute cheating, misrepresentation, or unfairness, including but not limited to, using unauthorized aids and assistance, personating another person, and committing plagiarism."

On the first page of your answer sheet, please write down your full name and student number. Also write down the above Academic Integrity Statement.

This test consists of 2 questions.

In your answers, you may use without proof any result or theorem covered in lectures, tutorials, homework, tests, or the textbook, as long as you give a clear statement of the result(s)/theorem(s) you are using. You must justify all other facts required for your solutions.

Write up your solutions carefully! In particular, use notation and terminology correctly and explain what you are trying to do — part marks *will* be given for showing that you know the general structure of an answer, even if your solution is incomplete.

If you are unable to answer a question (or part), you will get 10% of the marks for that question (or part) if you leave it blank, and 20% of the marks if you write "I don't know" and nothing else — you will **not** get those marks if your answer contains contradictory statements (such as "I don't know" followed or preceded by parts of a solution that have not been crossed off).

MARKING GUIDE

1: _____/20

2: _____/20

TOTAL: _____/40

Good Luck!

OVER...

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Question 1. [20 MARKS]

You have a business with several offices, and you want to lease phone lines to connect them up with each other. The phone company charges different amounts of money to connect different pairs of offices. You want a set of lines that connects all your offices and has the minimum total cost.

Part (a) [8 MARKS]

Write a polynomial-time algorithm to solve the above problem. Justify the correctness of your algorithm and compute its runtime.

Part (b) [12 MARKS]

The phone company informed you that one of the phone lines that you are using (from your solution in part (a)) has some technical problem and they won't be able to service that line. Write an algorithm to modify the existing solution to come up with a new solution, i.e., a new set of lines that connects all your offices and has the minimum total cost given the restriction. Justify the correctness of your algorithm and compute its runtime.

Your algorithm for part (b) must modify the existing solution, and not rerun the algorithm from part (a). For full marks, your algorithm for part (b) must be more efficient than your algorithm from part (a).

Question 2. [20 MARKS]

The University of Toronto assigns Chief Presiding Officers (CPOs) to invigilate final exams. The office in charge of organizing final exams has the following information about exams and CPOs:

- Each exam has a date and a time, where time is one of "morning", "afternoon" or "evening". Each combination of dates and times (eg. "the evening of July 22") is called an *examination period*.
- Each CPO has a list of examination periods when the CPO is available. CPOs cannot be assigned to slots when they are not available.
- Each CPO has a maximum number of exam periods that the CPO can invigilate, which is usually (but not necessarily) less than or equal to the number of examination periods when the CPO is available. This number can be different for different CPOs.
- Each CPO can invigilate at most two exams on a single day.
- If an examination period has ℓ exams, there should be $\lceil 1.1\ell \rceil$ CPOs available for that period (10% extra), in case some CPOs cannot make their shifts due to last minute emergencies.

Part (a) [10 MARKS]

You are provided with a list of the CPOs along with their availabilities and work preferences, the examination days, the examination periods, and the number of exams in each slot. Design a polynomial-time algorithm using network flow that either assigns each CPO to examination slots satisfying the constraints listed above, or reports that an assignment is not possible.

Part (b) [10 MARKS]

Briefly justify the correctness and runtime of the algorithm designed in part (a) using properties of network flow.