**Student Number:** 1 0 0 4 0 2 4 5 4 8

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**Write the following statement on the first page of the exam:**

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This test consists of 2 questions.

In your answers, you may use without proof any result or theorem covered in lectures, tutorials, homework, tests, or the textbook, as long as you give a clear statement of the result(s)/theorem(s) you are using. You must justify all other facts required for your solutions.

Write up your solutions carefully! In particular, use notation and terminology correctly and explain what you are trying to do — part marks *will* be given for showing that you know the general structure of an answer, even if your solution is incomplete.

If you are unable to answer a question (or part), you will get 10% of the marks for that question (or part) if you leave it blank, and 20% of the marks if you write "I don't know" and nothing else — you will **not** get those marks if your answer contains contradictory statements (such as "I don't know" followed or preceded by parts of a solution that have not been crossed off).

Marking Guide

# 1: ______/20

# 2: ______/20

TOTAL: ______/40

 *Good Luck!* 

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1.

a) procedure Maximize_value(B, G, P):

① sort all games $g_i \in G$ by relative prices $g_1, g_2, \ldots, g_n$ $(P_1 \geq P_2 \geq \cdots \geq P_n)$

② Res = { }

③ m = B

④ for i = 1 to n do

⑤     if m > 0 then

⑥         Res = Res ∪ $\{g_i\}$

⑦         m = m − 10

⑧ return Res

first sort the games in prices, add to result set with the game that has largest price, which means the game has largest profit. then do loop until there is no more budget money.

b) complexity: $\Theta(n\lg n)$

The first line sort takes $\Theta(n\lg n)$.

the for loop from line 4 takes $\Theta(n)$.

so $\Theta(n\lg n) + \Theta(n) + \Theta(1)$ (return)

$\Rightarrow \Theta(n\lg n)$

c) Let $i$ denotes number of games checked from $G$ into result set.

Want to show: part a) algorithm is optimal

Base case:

$k = 0$, trivially true, all the optimal solution holds

$k=1$, since by our part a), we choose game with highest price $P_1$, profit $= P_1 - 10$ which is maximum, so solution is optimal.

Induction Hypothesis, Let $0 \leq i \leq n$, $OPT_i$ is the optimal solution.

Induction step: consider the (n+1)th game.
WTP: $OPT_{n+1}$ is optimal

case 1: the rest budget $B' = B - 10n$
if $B' < P_{n+1}$, budget is not enough.
In such a case, $OPT_{n+1} = OPT_n$ since game $g_{n+1}$ would not be in the solution set. So it is still optimal

case 2: $B' = B - 10n > P_{n+1}$, the budget is enough to purchase $g_{n+1}$. Since for the rest games $\{g_{n+1}, \ldots\}$ by our algorithm, $P_{n+1}$ is the maximum price among the rest games, $\Rightarrow P_{n+1} - 10$ is maximum profit among rest games

$\Rightarrow OPT_{n+1} = OPT_n \cup \{g_n\}$ is optimal since $OPT_n$ and $\{g_n\}$ is optimal.

d) No, it does not work.

Eg: $B = 10$

$g = \{g_1, g_2, g_3\}$ $P = \{24, 16, 16\}$

by part a) it would choose $g_1$,

the $24 - 24/6 = 18$

but the maximum way is to choose

$g_2, g_3$, $16/4 + 16/4 = 8$

profit: $16 + 16 - 8 = 24$

2.

a) $O(i,j,b)$ be optimal solution for ith category with b budget, purchase j games

$0 \le b \le B, \quad 0 \le j \le n, \quad 0 \le i \le m.$

$$O(i,j,b) = \begin{cases} \max\{O(i,j-1,b), O(i-1,j-1,b-\xi_{ij}) + (P_{g_j} - \xi_{g_j})\} & \xi_{g_j} < b \\ O(i,j-1,b) & \xi_{g_j} > b \\ 0 & b = 0 \end{cases}$$

b) Let M be a 3D array where

M[i,j,b] indicates the maximum value that purchases j games of i categories with b budget.

$0 \le i \le m$
$0 \le j \le n \quad 0 < b \le B$
$j \le i$

c)
$$M[i,j,b] = \begin{cases} \max\{M[i,j-1,b], M[i-1,j-1,b-\xi_{ij}] + (P_{g_j} - \xi_{g_j})\} & \xi_{g_j} \le b \\ M[i,j-1,b] & \xi_{g_j} > b \\ 0 & b = 0 \end{cases}$$

d) procedure iterative_games(G, P, C, Q, B)

① Define M

```
for b = 0, 1, ..., B:
    M[i, j, b] = 0
    for i = m, m-1, ..., 1:
        for j = 1, 2, ..., n:
            if b < q_{g_j}:
                M[i, j, b] = M[i, j-1, b]
            else:
                M[i, j, b] = max{ M[i, j-1, b],
                  M[i-1, j-1, b - q_{g_j}] + c(g_j - q_{g_j}) }
```

Complexity: $O(Bmn)$