

# CSC373 Summer '22

## Tutorial 5

July 7, 2022

### Q1 Standard Form

Consider the following linear program.

$$\begin{array}{ll}\min & 4x + 3y - 6z \\ \text{s.t.} & y - 3z \geq 2x + 2 \\ & 3x + 2y + 5z = 10 \\ & x, z \geq 0\end{array}$$

- (a) Convert this LP into the standard form.
- (b) Write the dual of the LP from Part (a).

### Q2 Simple Scheduling with Prerequisites (SSP)

You are given  $n$  jobs with a list of durations  $d_1, d_2, \dots, d_n$ . For every pair of jobs  $(i, j)$ , you are also given a boolean  $p_{i,j}$ : if this is true, then job  $i$  must finish before job  $j$  can begin (i.e. job  $i$  is a prerequisite for job  $j$ ).

Your goal is to find start times  $s_1, s_2, \dots, s_n$  for the jobs (no job can start earlier than time 0) such that the total time to complete all jobs is minimized while ensuring that the prerequisite constraints are met. Write a linear program to solve this problem.

### Q3 Integer Linear Programming

Suppose you are writing down a binary integer linear program (i.e., an optimization problem with a linear objective, linear constraints, and each variable taking a value in  $\{0, 1\}$ ). Three of the binary variables in your program are  $x$ ,  $y$ , and  $z$ ; you have already placed the constraint:  $x, y, z \in \{0, 1\}$ .

Now, you want to encode the following relationships between  $x$ ,  $y$ , and  $z$ . Show how to do so using linear constraints. Briefly justify your answers.

- (a) Logical AND,  $z = x \wedge y$ : You want  $z$  to be 1 whenever both  $x$  and  $y$  are 1, and 0 otherwise.
- (b) Logical OR,  $z = x \vee y$ : You want  $z$  to be 1 whenever at least one of  $x$  and  $y$  is 1, and 0 otherwise.
- (b) Logical NOT,  $z = \neg x$ : You want  $z$  to be 1 whenever  $x$  is 0, and 0 otherwise.