Midterm 2 grades have been released. These are the markers' feedback for the questions. Please read these carefully *before* submitting a remark request. Also, note that remark requests will be considered only for an incorrect *application* of the marking scheme.

1a. [10] The solution is to: Create a node for each injured person and a node for each medical facility; connect the source to every person node with an edge of capacity one; connect any person node to a facility node that is at most distance D away; connect every facility node to the sink with an edge with capacity equal to its vacancy; and finally find a max-flow in this network. Submissions that did this were awarded full (or close to full, depending on minor errors) marks. Most students got this part correct.

[2 marks] for correct vertices (source, sink, person, medical facility)

[3 marks] for correct connections (person -> medical only if distance less than D)

[3 marks] for correct capacities

[2 marks] for correctly stating the objective of finding max-flow

1b. [5] Note the two directions that were to be shown:

[1 mark] for stating valid flow <=> valid assignment means the assignment is optimal

[2 marks] for valid assignment => valid flow

[2 marks] for valid flow => valid assignment

Some students failed to argue why the max-flow on the network maximizes the number of treated patients. They showed that a flow leads to a valid assignment but not the other direction.

2. This one was the slightly more challenging question of the three.

2a. [10] The solution idea is to connect a source vertex s to all cities with capacity p_i (amount produced), connect all cities to a sink vertex t with capacity q_i (number of sales), and use the input graph with $c_{i,j}$ capacity (amount transferred). Then, run a maximum flow and interpret the flows on each edge suitably. Overall, 6 marks are for a valid description of the proposed network flow, 1 mark for stating running a max-flow algorithm, and then 3 marks for pointing out the solution to the original problem once a max-flow has been found.

Some students stopped at the point of describing their flow network. The question asked for an algorithm, and we need to run a maximum flow algorithm (-1 point if not mentioned) and return the production and sales of each city and the amount they transferred to each city (-1 for each missing).

Some students made an extra assumption that each produced item can only be transferred *once* to a neighbouring city, but this is not correct: it can be transferred from one city to the next multiple times. These solutions lost 1 point if the rest was in order.

2b. [10] The key idea for doubling capacities was to introduce $x_{i,j} \in \{0,1\}$ variables indicating if the edge (i, j) is doubled in capacity or not. Let $f_{i,j} \in \mathbb{Z}_{\geq 0}$ denote the number of products transferred along edge (i, j), then we can bound it by $f_{i,j} \leq \left(1 + x_{i,j}\right) \cdot c_{i,j}$ (worth 2 points). This way, if $x_{i,j} = 1$, the flow can go as high as $2 \cdot c_{i,j}$ or only up to $c_{i,j}$ if not. To limit the number of doubled edges we can have $\sum_{\{i,j\}} x_{i,j} \leq m$ (worth 2 points). The rest is simply writing the maximum flow as a linear program. Objective (worth 2 points): max$\sum i \, f_{s,i}$ (total flow from source s, each variable in this sum indicates the amount produced in city i).

Constraints: in-flow = out-flow condition (worth 2 points), i.e., for each city, the production amount + received items from other cities = the sale amount + transferred items to other cities (some students did not handle this correctly). The other 2 points for defining the variables and their valid range of values.

Some students did not introduce variables for flow amounts and tried solving the problem only using $x_{i,j}$'s and limit capacities $c_{i,j}$'s, which was significantly incorrect and lost at least 4 points.

Some treated the production and sale flow like other transfer edges. They had different limits, i.e., p_i and q_i and their capacity couldn't be doubled (lost 2 points).

Some did not have a linear program, e.g., multiplied two variables or used a maximum function (lost 2 points for this if the rest was correct).

3. Most students did well on this question.

3a. [10] The solution consists of a cost minimization objective and 4 kinds of constraints: (1) a demand lower-bound on s, (2) flow conservation over all vertices except s and t, (3) lower-bound on the flow of each edge, (4) capacity upper-bound for each edge. Each objective/constraint is worth 2 marks.

For the constraints, one common mistake was forgetting to add the scope (e.g., for all $e \in E$). 1 mark is deducted for each missing/incorrect scope description. Note that the flow conservation constraint should exclude s and t.

3b. [5] To argue that the solution is correct, there are three key points: (1) a valid flow corresponds to a feasible solution to the LP, (2) a feasible solution to the LP yields a valid flow, (3) the objective of the LP is aligned with the objective of the min-cost flow problem. The first two are worth 2 marks each and the last is worth 1. Simply stating these three points is enough to earn you full marks on this sub-question.

A somewhat common oversight by students was to simply restate what each constraint is in English (like Q2). This is not quite sufficient as a justification of correctness and 2 marks are deducted for this. Note that even though (1) and (2) are straightforward consequences of the constraints being correct, the overall correctness can directly be implied only after writing *both* statements (1) & (2) and as such, were needed to be stated *explicitly*.

Thanks,

Marking Team