Estimation en traitement d'antenne

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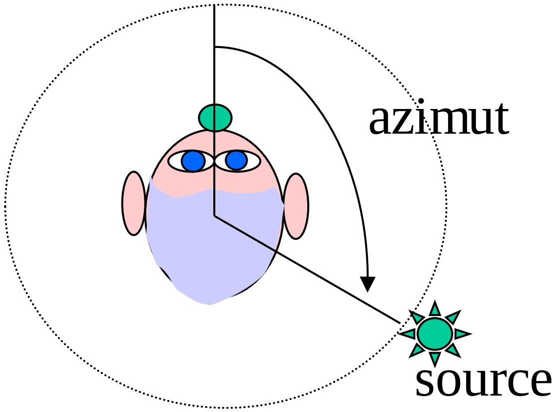
- Méthodes de sous espace (potentiel et faiblesse)
- Maximum de vraisemblance (complexité)
- Sélection de l'ordre
- Erreur de modèle et calibration

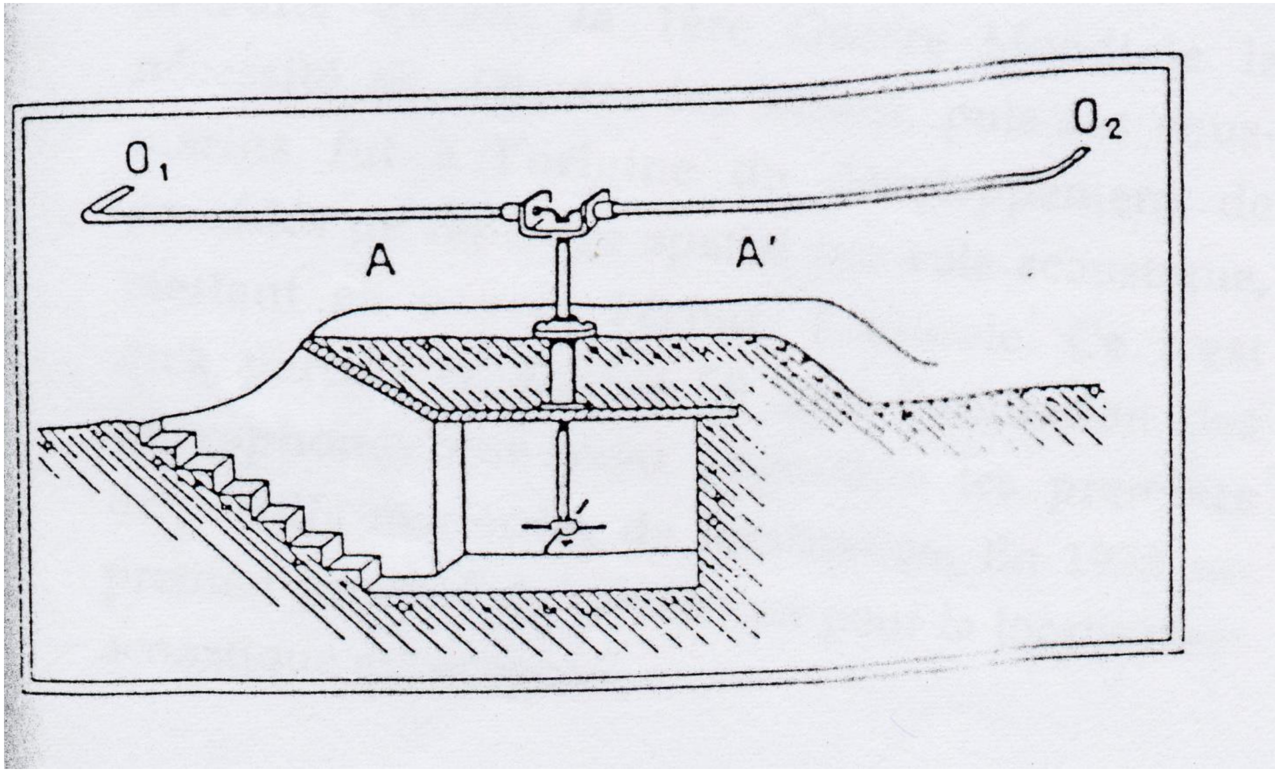
Radioastronomie
Géolocalisation
Localisation avec algorithmes parcimonieux
Application au cyclone Georges (complexité, robustesse, performances)

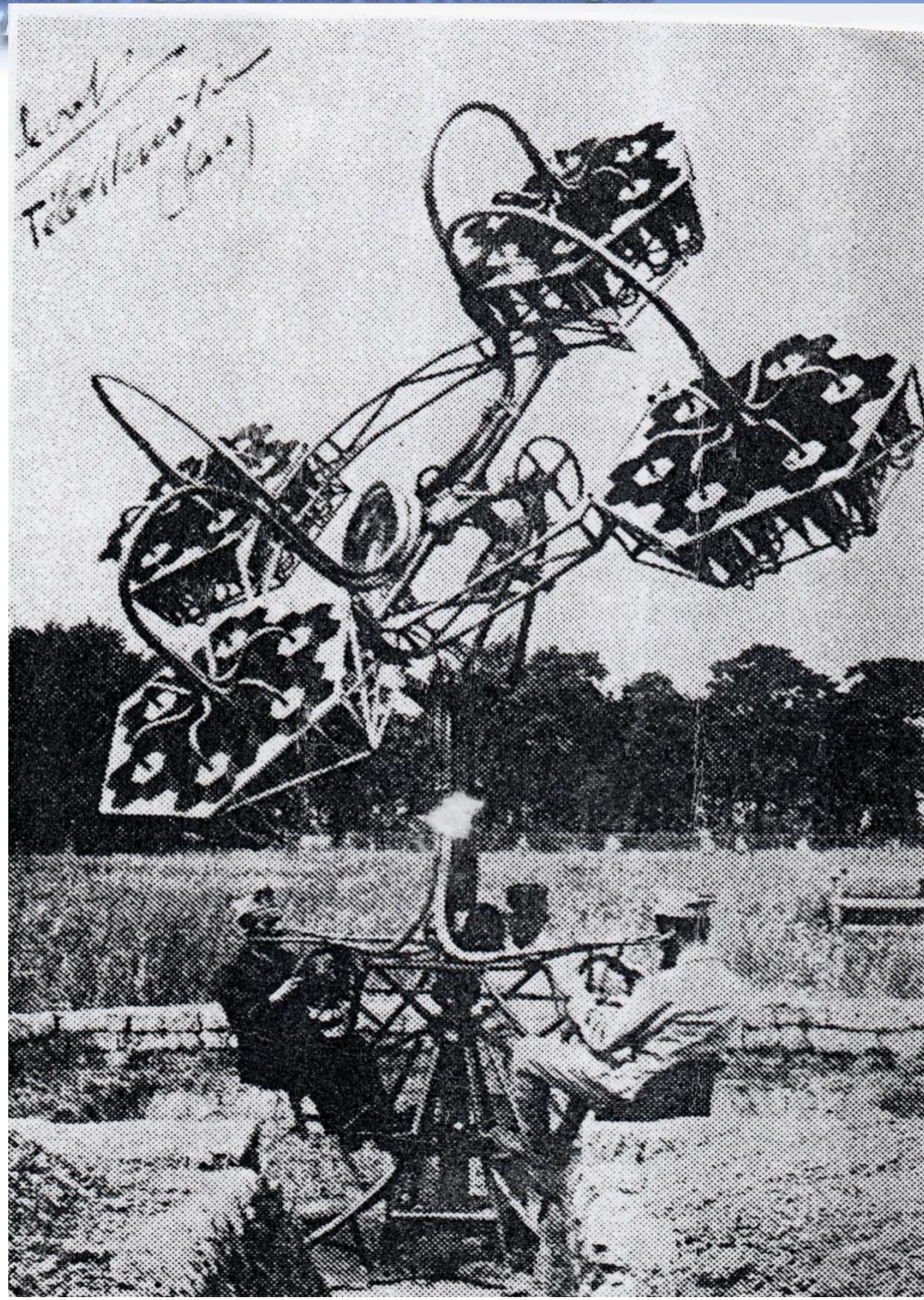
- Array signal processing



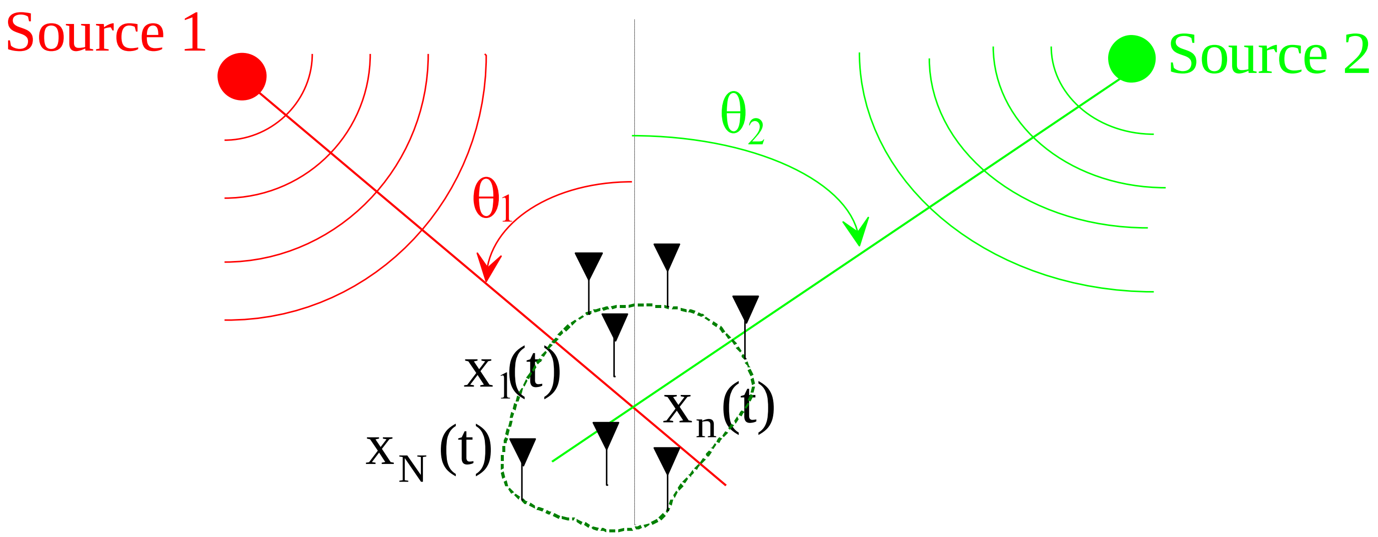
<u>Sensibility in Azimut</u> :

• phase and intensity


O_1
O_2
A
A'



LE TRAITEMENT D'ANTENNE



Estimation :

- **Localisation**
- **Détection**
- **Séparation**
- **Identification**

$$\mathbf{x}(t) = \sum_{m=1}^{M} \mathbf{a}(\theta_m) \cdot s_m(t) + \mathbf{b}(t)$$

$$\mathbf{x}(t) = A(\theta) \cdot \mathbf{s}(t) + \mathbf{b}(t) = \mathbf{y}(t) + \mathbf{b}(t)$$

Two Types of Beamformers

- Method 1: Single sensor with directional response due to reflector, aperture size, baffles, pipes, etc.

Green Bank Telescope, National Radio Astronomy Observatory, West Virginia.

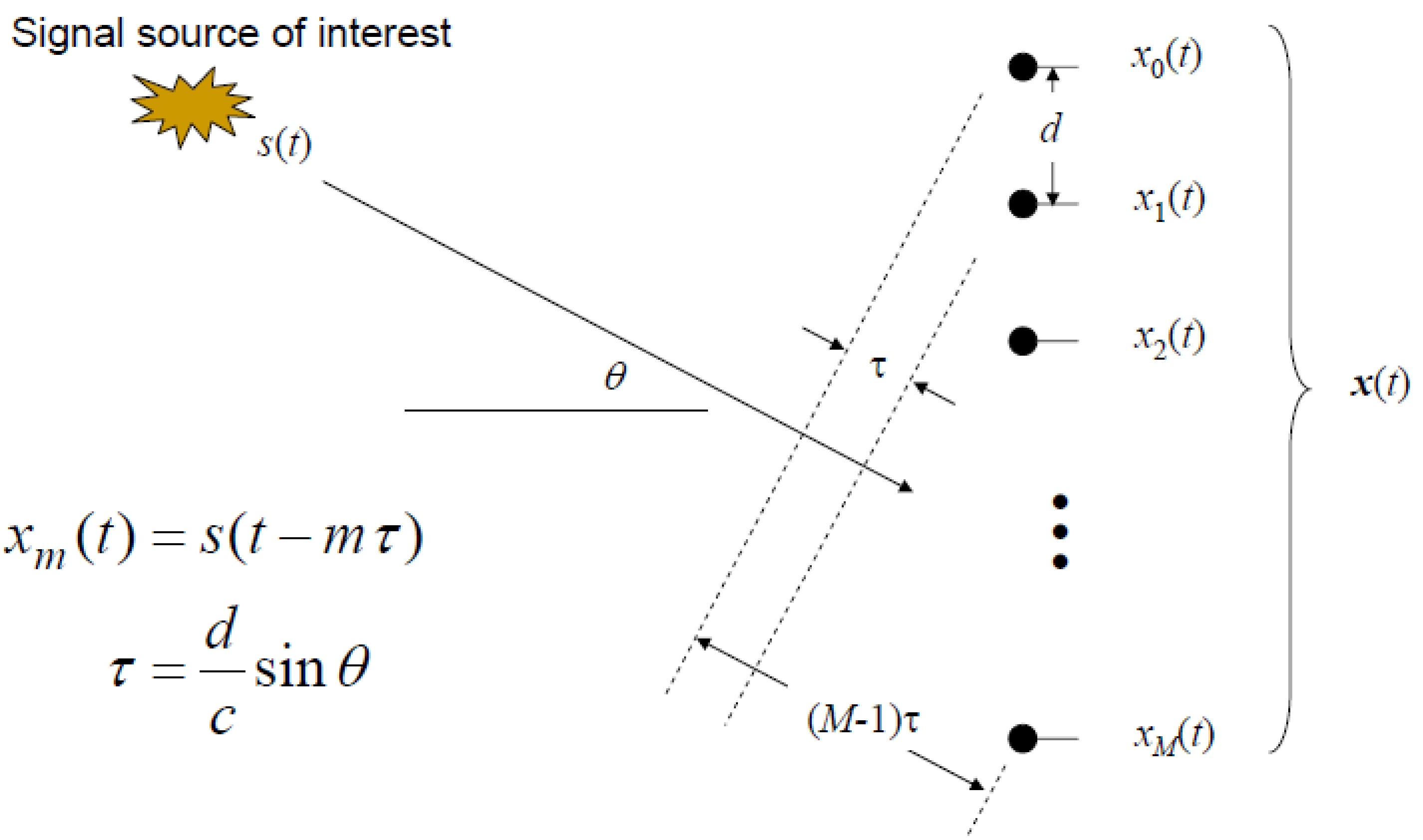
100 m clear aperture. Largest fully steerable antenna in the world.

Two Types of Beamformers (cont.)

- Method 2: Sensor arrays. Used in SONAR, RADAR, communications, medical imaging, radio astronomy, etc.

Line array of directional sensors
Westerbork Synthesis Array Radio Telescope, (WSRT) the Neterlands.

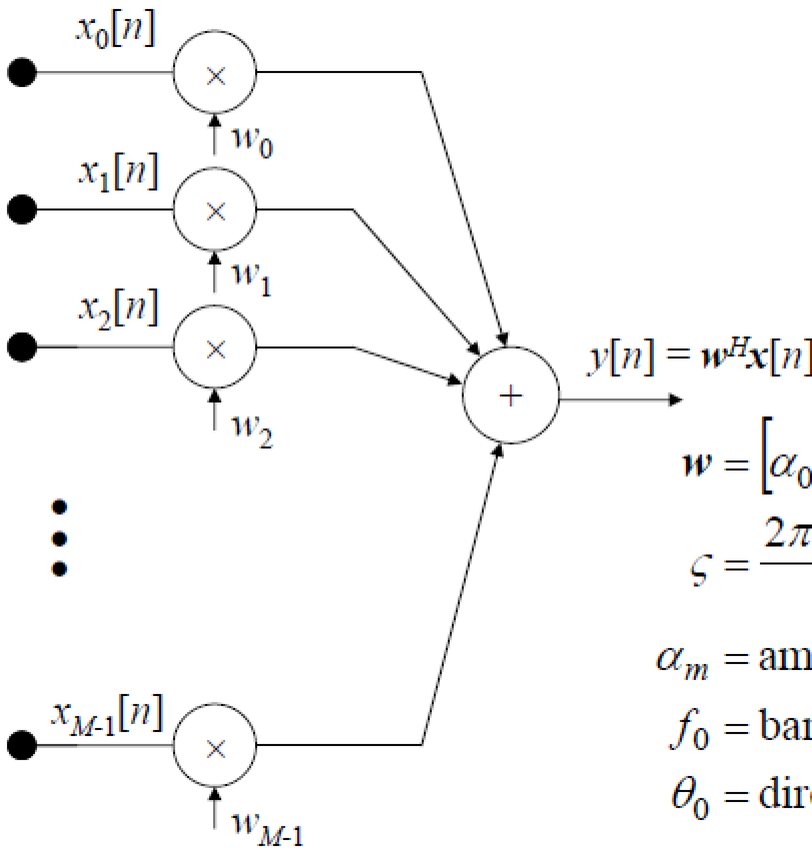
The Uniform Line Array



$$x_m(t) = s(t - m\tau)$$

$$\tau = \frac{d}{c}\sin\theta$$

Narrowband Phased Array



$$\mathbf{w} = \left[\alpha_0, \alpha_1 e^{-j\varsigma}, \cdots, \alpha_{M-1} e^{-j(M-1)\varsigma}\right]^T,$$

$$\varsigma = \frac{2\pi f_0 d}{c}\sin\theta_0,$$

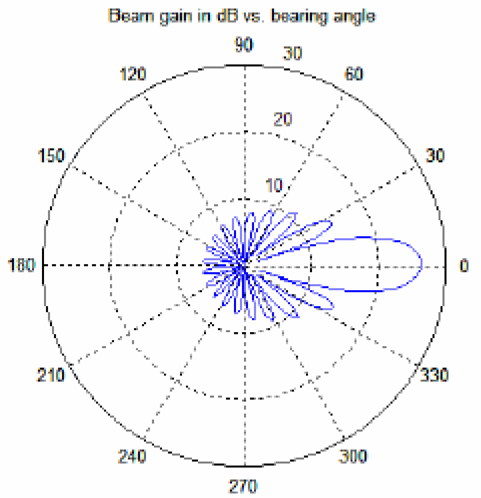
α_m = amplitude weight for sensor m,

f_0 = bandpass center frequency, Hz,

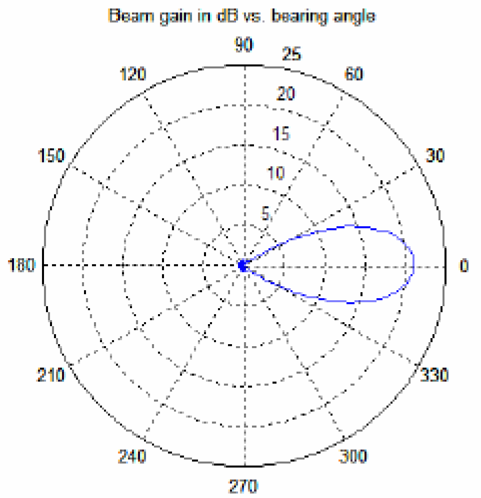
θ_0 = direction of max response.

Beamforming is Spatial Filtering

- Sensors in any wave propagation medium (acoustic, electromagnetic) can form a response pattern with higher sensitivity in desired directions.

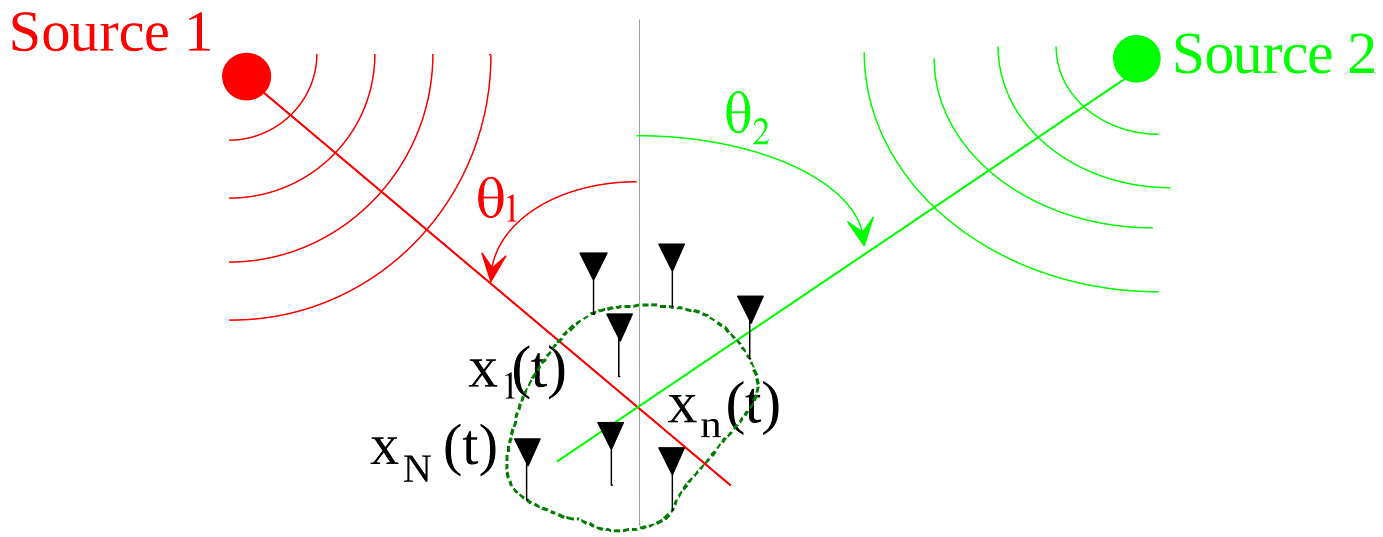


Pencil beam response, no windowing



Pencil beam response, Hamming window

LE TRAITEMENT D'ANTENNE



Estimation :

- **Localisation**
- **Détection**
- **Séparation**
- **Identification**

$$\begin{aligned}\mathbf{x}(t) &= \sum_{m=1}^{M} \mathbf{a}(\theta_m) s_m(t) + \mathbf{b}(t) \\ &= A(\boldsymbol{\theta})\mathbf{s}(t) + \mathbf{b}(t)\end{aligned}$$

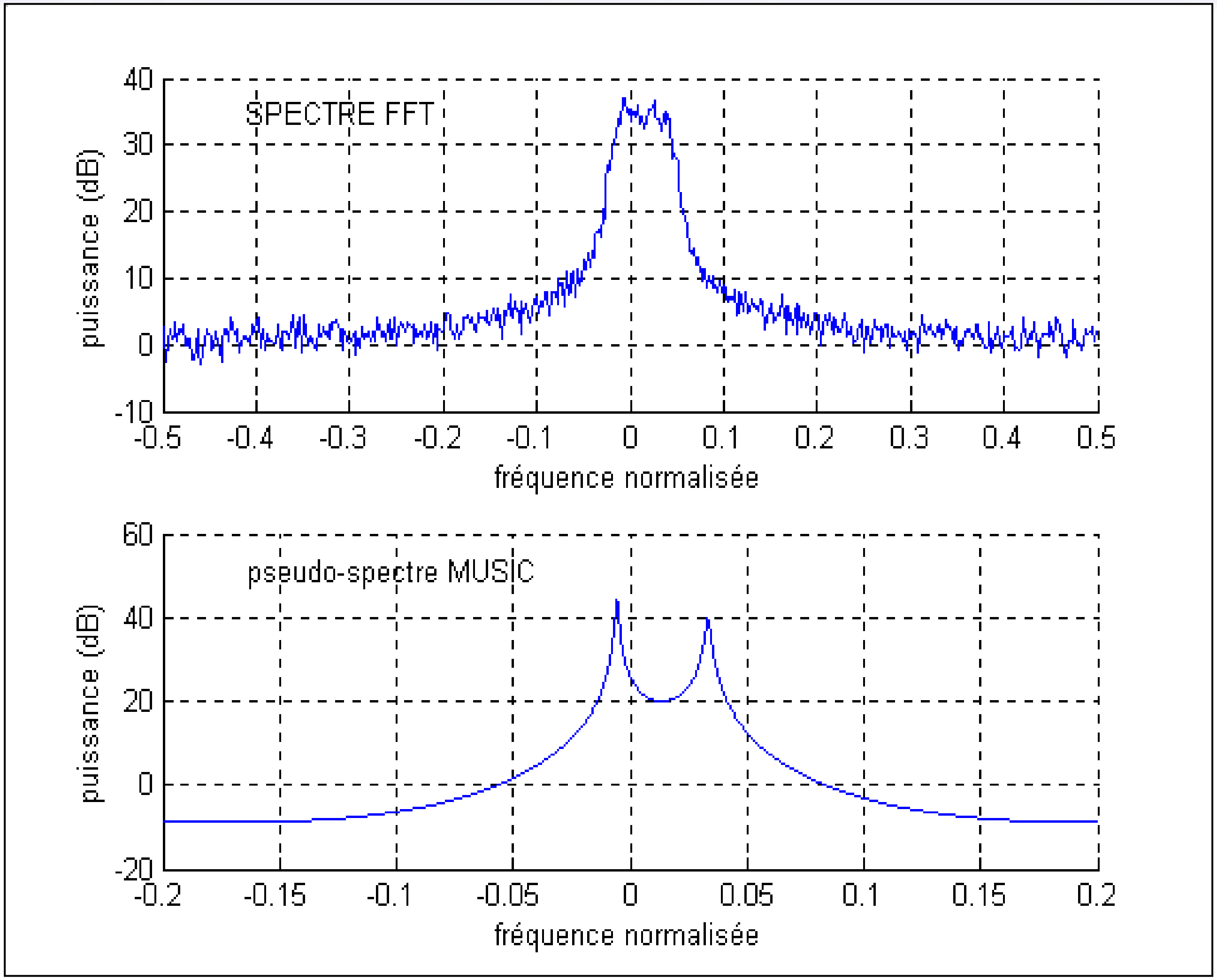
Les méthodes de sous espace : la méthode MUSIC

$$\mathbf{x}(t) = \mathbf{y}(t) + \mathbf{b}(t) = \sum_{m=1}^{M} \mathbf{a}(\theta_m)\, s_m(t) + \mathbf{b}(t), \quad \mathbf{x}(t) \in \mathbb{C}^N \quad \text{et} \quad \mathbf{y}(t) \in \mathbb{C}^M \qquad N > M$$

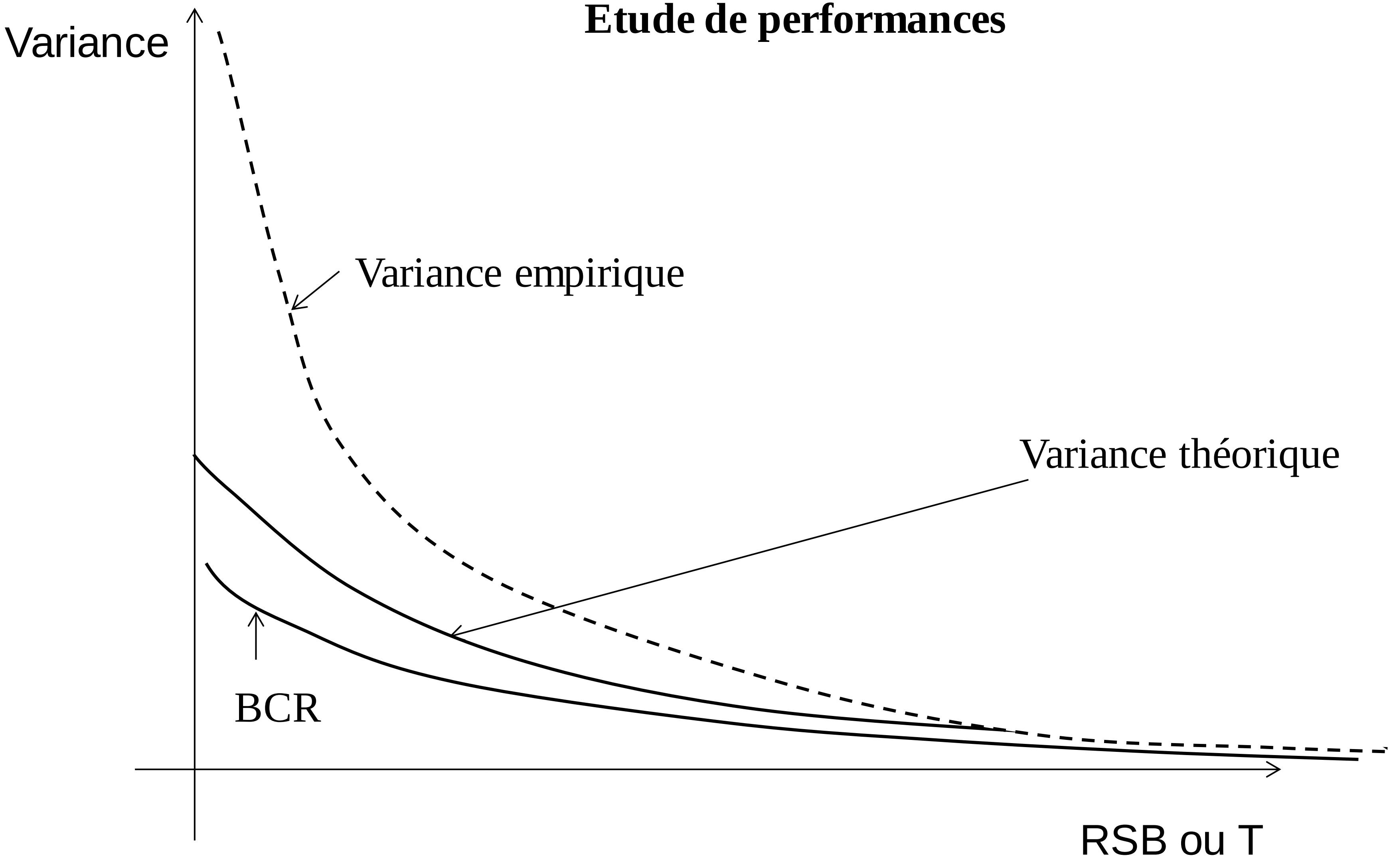
$$\hat{R}_{\mathbf{x}} = \sum_{t=1}^{T} \mathbf{x}(t)\, \mathbf{x}(t)^H = \hat{\mathbf{E}}_S \hat{\mathbf{\Lambda}}_S \mathbf{E}_S^H + \hat{\mathbf{E}}_B \hat{\mathbf{\Lambda}}_B \hat{\mathbf{E}}_B^H$$

$$R_{\mathbf{x}} = E\left[\mathbf{x}(t)\, \mathbf{x}(t)^H\right] = E\left[\hat{R}_{\mathbf{x}}\right] = \mathbf{E}_S \mathbf{\Lambda}_S \mathbf{E}_S^H + \mathbf{E}_B \mathbf{\Lambda}_B \mathbf{E}_B^H$$

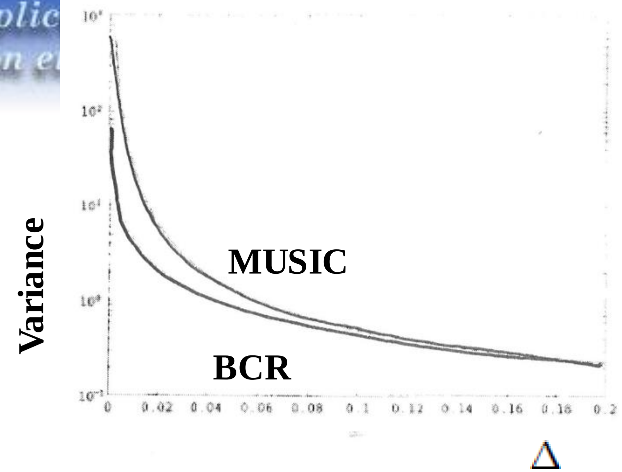
$$\begin{array}{rcl} Vect(\mathbf{E}_S) & = & Vect(\mathbf{A}) \\ Vect(\mathbf{E}_B) & \perp & Vect(\mathbf{A}) \end{array} \Longrightarrow \hat{\theta} = \arg\min_{\theta} \frac{1}{\mathbf{a}(\theta)^H\, \hat{\mathbf{E}}_B \hat{\mathbf{E}}_B^H \mathbf{a}(\theta)} = \arg\min_{\theta} \frac{1}{\mathbf{a}(\theta)^H\ \Pi_B\ \mathbf{a}(\theta)}$$


SPECTRE FFT
puissance (dB)
40
30
20
10
0
-10
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
fréquence normalisée
pseudo-spectre MUSIC
puissance (dB)
60
40
20
0
-20
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
fréquence normalisée

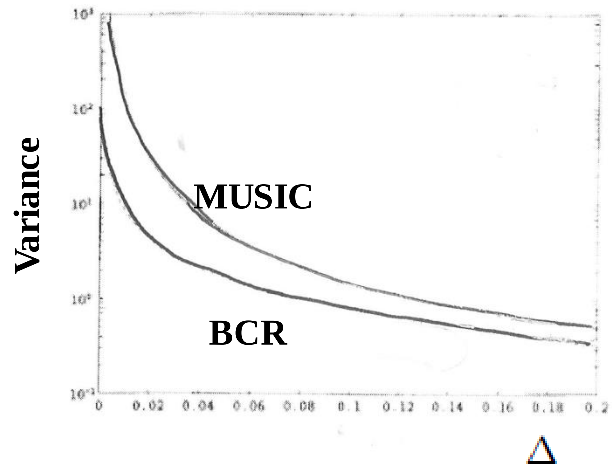
Etude de performances



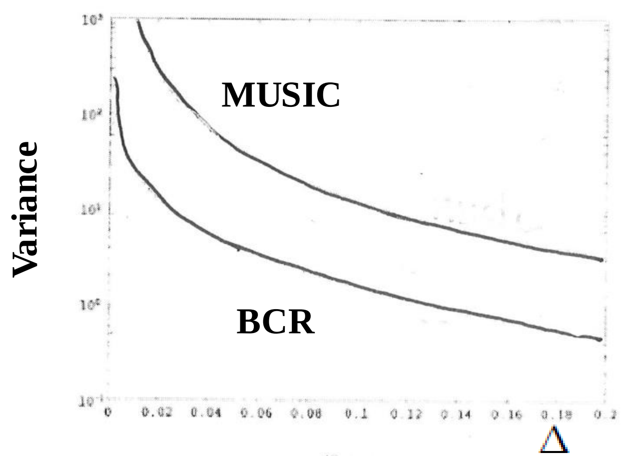
BCR : Borne Cramer Rao



$\rho = 0.5$

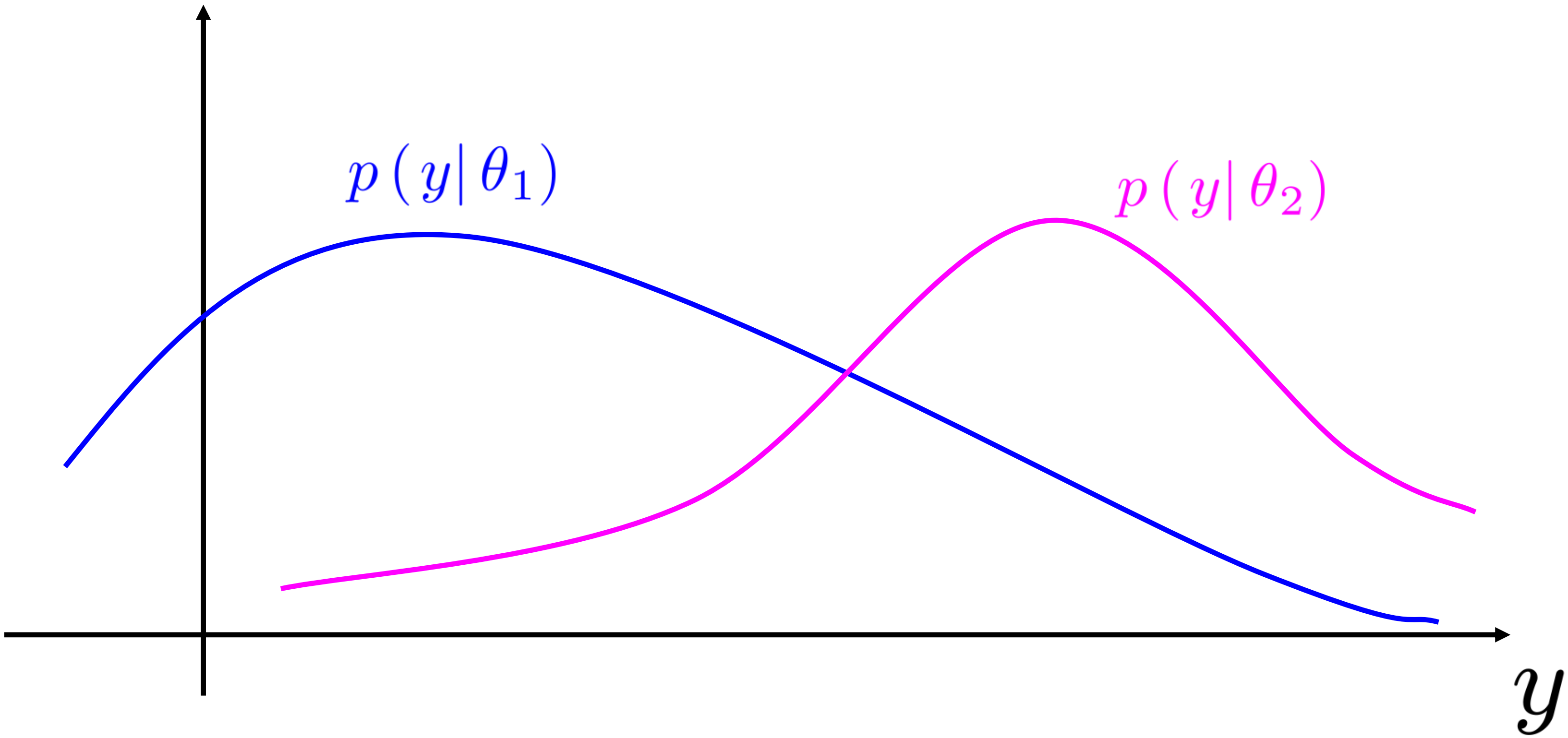


$\rho = 0.9$

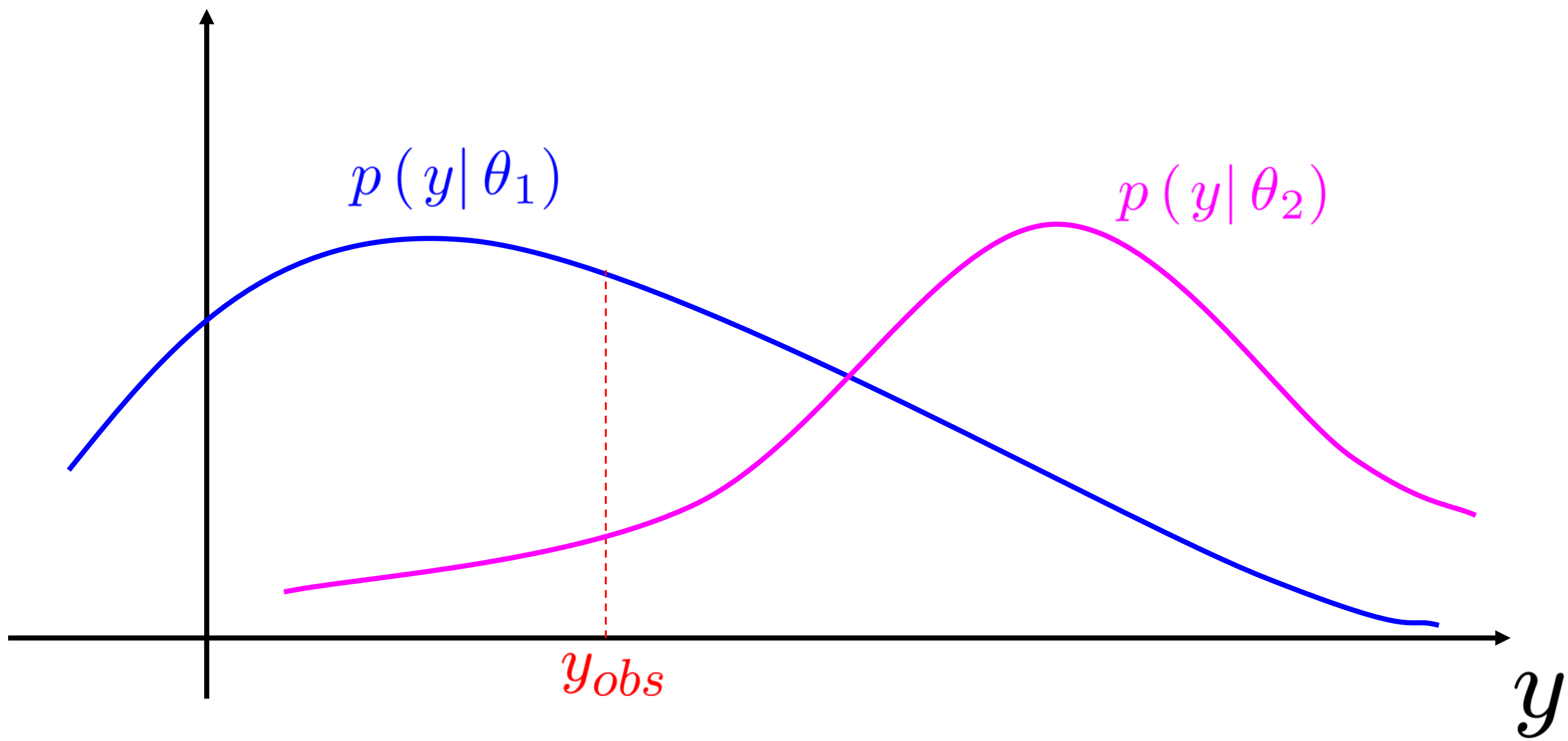


$\rho = 0.99$

Maximum de vraisemblance : principe

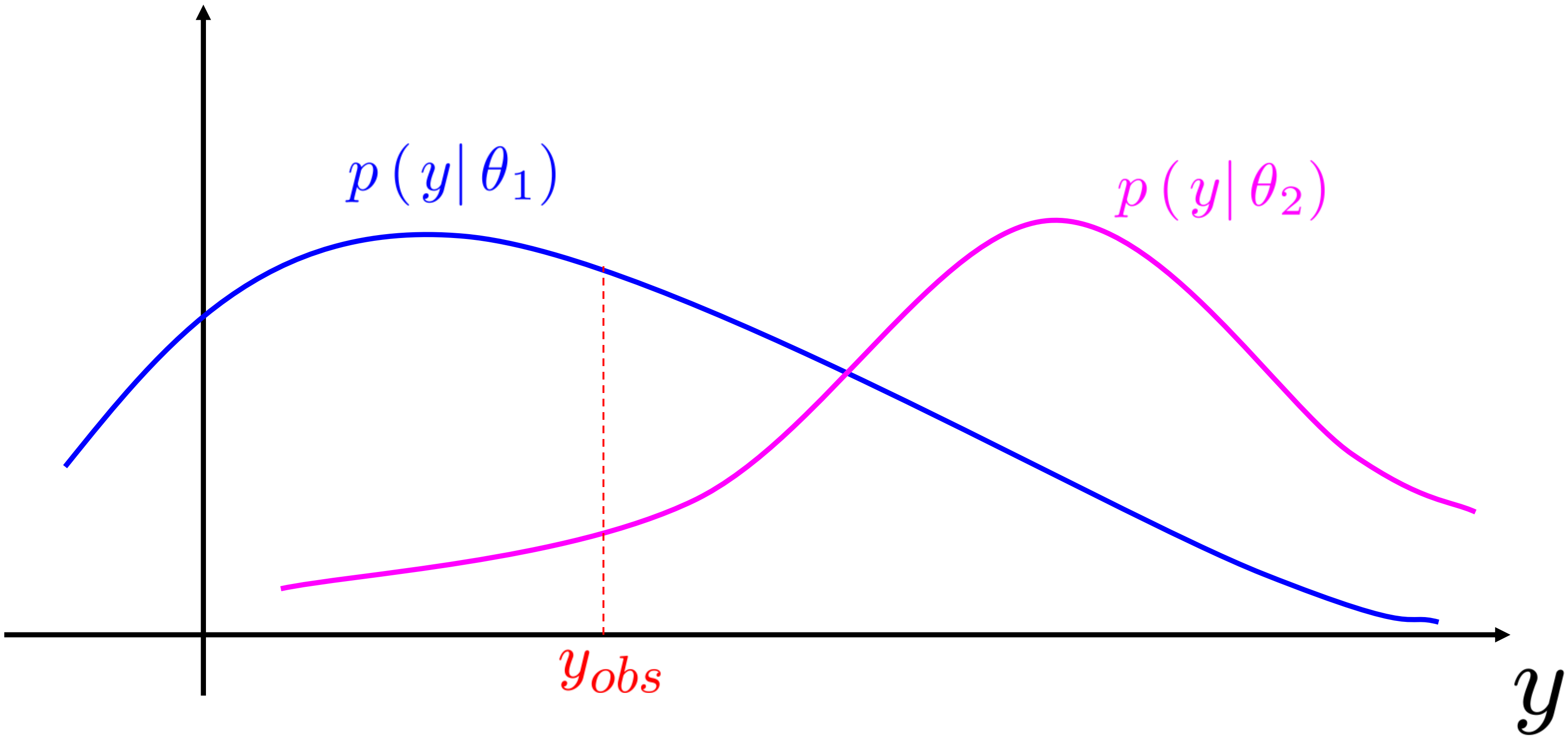


Maximum de vraisemblance : principe



$$\hat{\theta}_{ML}\left(y_{obs}\right) = \theta_1$$

Maximum de vraisemblance : principe



Généralisation $$\hat{\boldsymbol{\theta}}_{ML}(\mathbf{y}_{obs}) = \arg\max_{\boldsymbol{\theta}} p(\mathbf{y}_{obs}|\boldsymbol{\theta})$$

Maximum de vraisemblance en traitement d'antenne

$$\mathbf{y}(t) = \mathbf{A}(\boldsymbol{\theta})\,\mathbf{s}(t) + \mathbf{b}(t) \qquad t = 1 \ldots N$$

Hypothèses

$\mathbf{s}(t) \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \boldsymbol{\Sigma}_s)$	$\mathbf{s}(t) \qquad t = 1 \ldots N$
aléatoires	déterministes
Paramètres à estimer	Paramètres à estimer
$\boldsymbol{\theta}, \sigma^2, \boldsymbol{\Sigma}_s$	$\boldsymbol{\theta}, \sigma^2, \mathbf{s}_1, \ldots, \mathbf{s}_N$
Maximum de Vraisemblance stochastique	Maximum de Vraisemblance déterministe

Maximum de Vraisemblance stochastique

$$\mathbf{y}(t) = \mathbf{A}(\boldsymbol{\theta})\,\mathbf{s}(t) + \mathbf{b}(t) \quad \mathbf{s}(t) \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \boldsymbol{\Sigma}_s)$$

$$\mathbf{y}(t) \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \boldsymbol{\Sigma}_y)$$

$$\boldsymbol{\Sigma}_y = \mathbf{A}(\boldsymbol{\theta})\,\boldsymbol{\Sigma}_s\mathbf{A}^H(\boldsymbol{\theta}) + \sigma^2\mathbf{I}$$

$$\begin{cases} \hat{\boldsymbol{\theta}}_{MVS} = \arg\min\limits_{\boldsymbol{\theta}} \left|\mathbf{A}(\boldsymbol{\theta})\,\hat{\mathbf{R}}_s\mathbf{A}^H(\boldsymbol{\theta}) + \hat{\sigma}^2\mathbf{I}\right| \\ \hat{\sigma}^2 = \dfrac{1}{M-K} Trace\left\{\boldsymbol{\Pi}_{\mathbf{A}}^{\perp}(\boldsymbol{\theta})\,\hat{\boldsymbol{\Sigma}}_y\right\} \\ \hat{\mathbf{R}}_s = \mathbf{A}^{\dagger}\left(\hat{\boldsymbol{\Sigma}}_y - \hat{\sigma}^2\mathbf{I}\right)\mathbf{A}^{\dagger H} \end{cases}$$

Borne de Cramér-Rao Stochastique

$$\mathbf{BCRS}(\boldsymbol{\theta}_0) = \frac{\sigma^2}{2N}\left[\mathrm{Re}\left\{\mathbf{H}(\boldsymbol{\theta}_0) \odot \boldsymbol{\Sigma}_s\mathbf{A}^H(\boldsymbol{\theta}_0)\,\boldsymbol{\Sigma}_y^{-1}\mathbf{A}(\boldsymbol{\theta}_0)\,\boldsymbol{\Sigma}_s\right\}\right]^{-1}$$

Maximum de Vraisemblance déterministe

$$\mathbf{y}(t) = \mathbf{A}(\boldsymbol{\theta})\,\mathbf{s}(t) + \mathbf{b}(t) \qquad \mathbf{s}(t) \qquad t = 1 \ldots N$$

déterministes

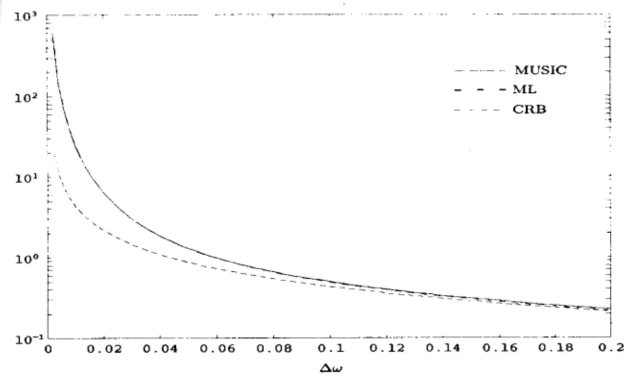
$$\mathbf{y}(t) \sim \mathcal{N}_{\mathbb{C}}\left(\mathbf{A}(\boldsymbol{\theta})\,\mathbf{s}(t),\, \boldsymbol{\Sigma}_y\right)$$

$$\boldsymbol{\Sigma}_y = \sigma^2 \mathbf{I}$$

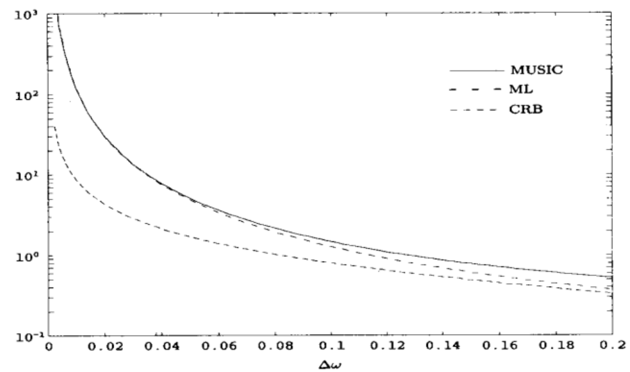
$$\begin{cases} \hat{\boldsymbol{\theta}}_{MVD} = \arg\min_{\boldsymbol{\theta}} Trace\left\{\boldsymbol{\Pi}_{\mathbf{A}}^{\perp}(\boldsymbol{\theta})\,\hat{\boldsymbol{\Sigma}}_y\right\} \\ \hat{\boldsymbol{\Sigma}}_y = \dfrac{1}{N}\displaystyle\sum_{t=1}^{N} \mathbf{y}(t)\,\mathbf{y}^H(t) \end{cases}$$

Borne de Cramér-Rao Déterministe

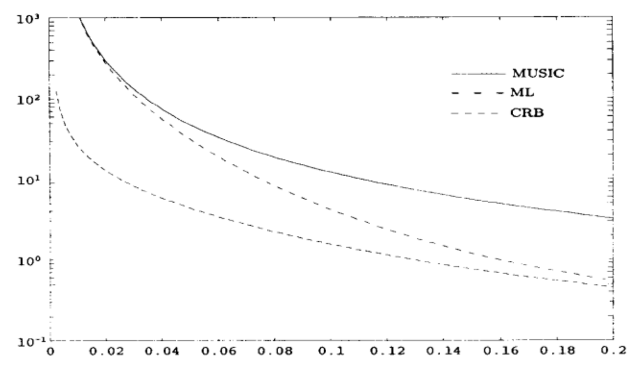
$$\mathbf{BCRD}(\boldsymbol{\theta}_0) = \frac{\sigma^2}{2N}\left[\mathrm{Re}\left\{\mathbf{H}(\boldsymbol{\theta}_0) \odot \hat{\boldsymbol{\Sigma}}_s\right\}\right]^{-1}$$



$\rho = 0.5$



$\rho = 0.9$

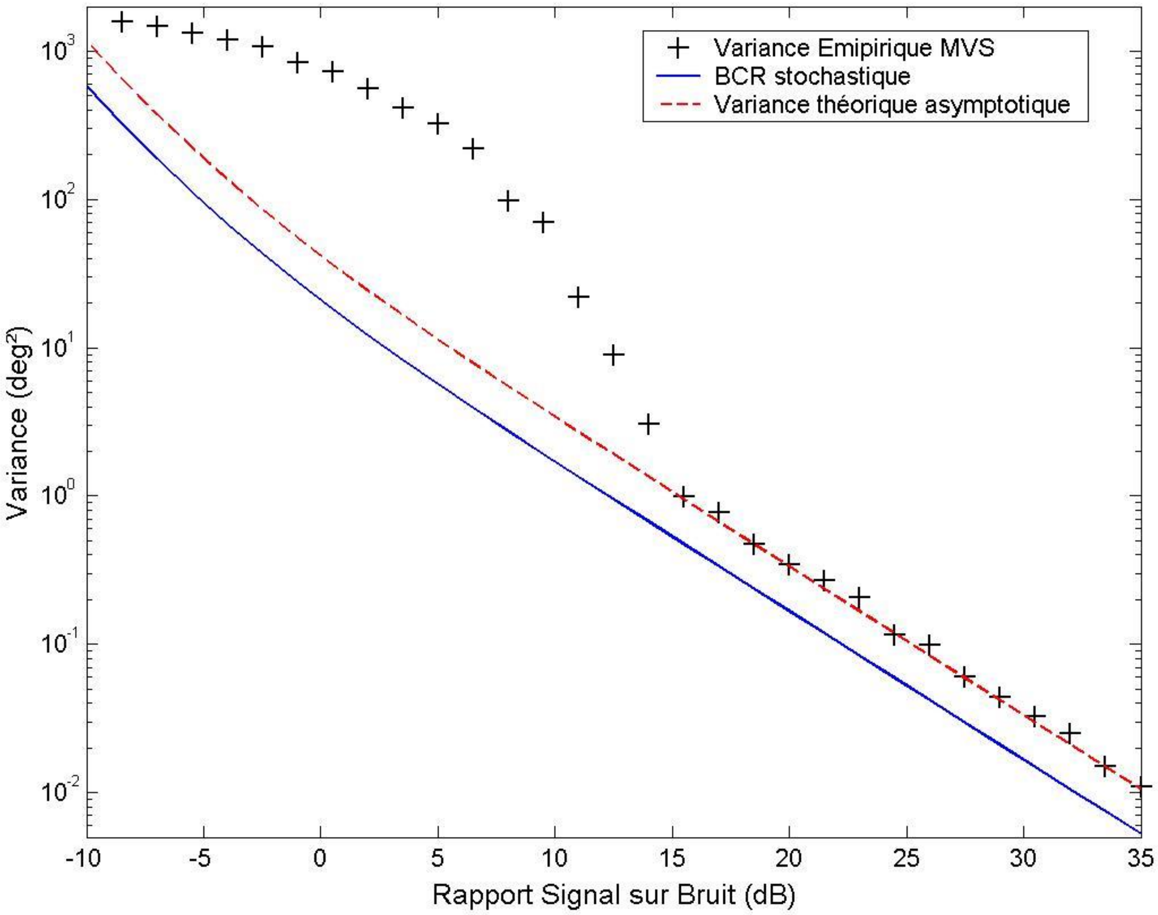


$\rho = 0.99$

$m = 5$

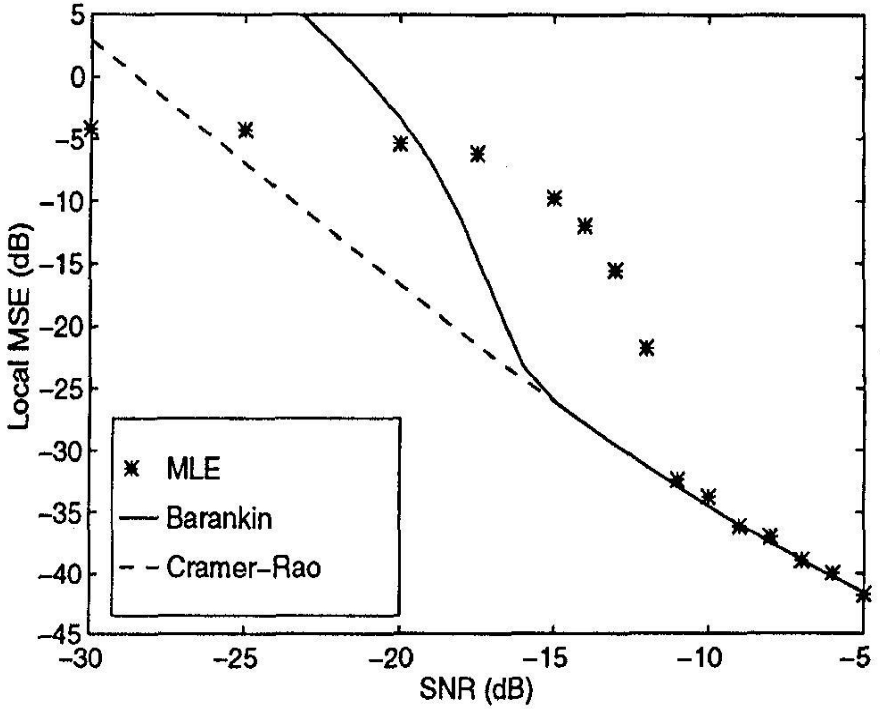
Performances asymptotiques

Cas particulier : scénario une source

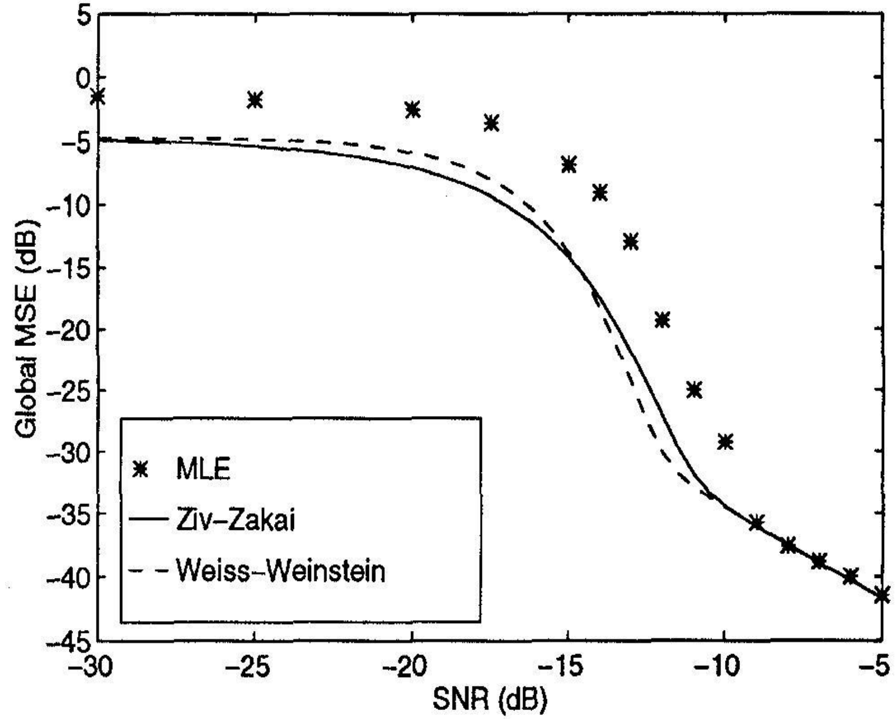


- ALU ($\lambda/2$) : M = 4 capteurs
- N = 2 observations
- 1 source 0°
- 1000 Monte Carlos

BORNES MINIMALES



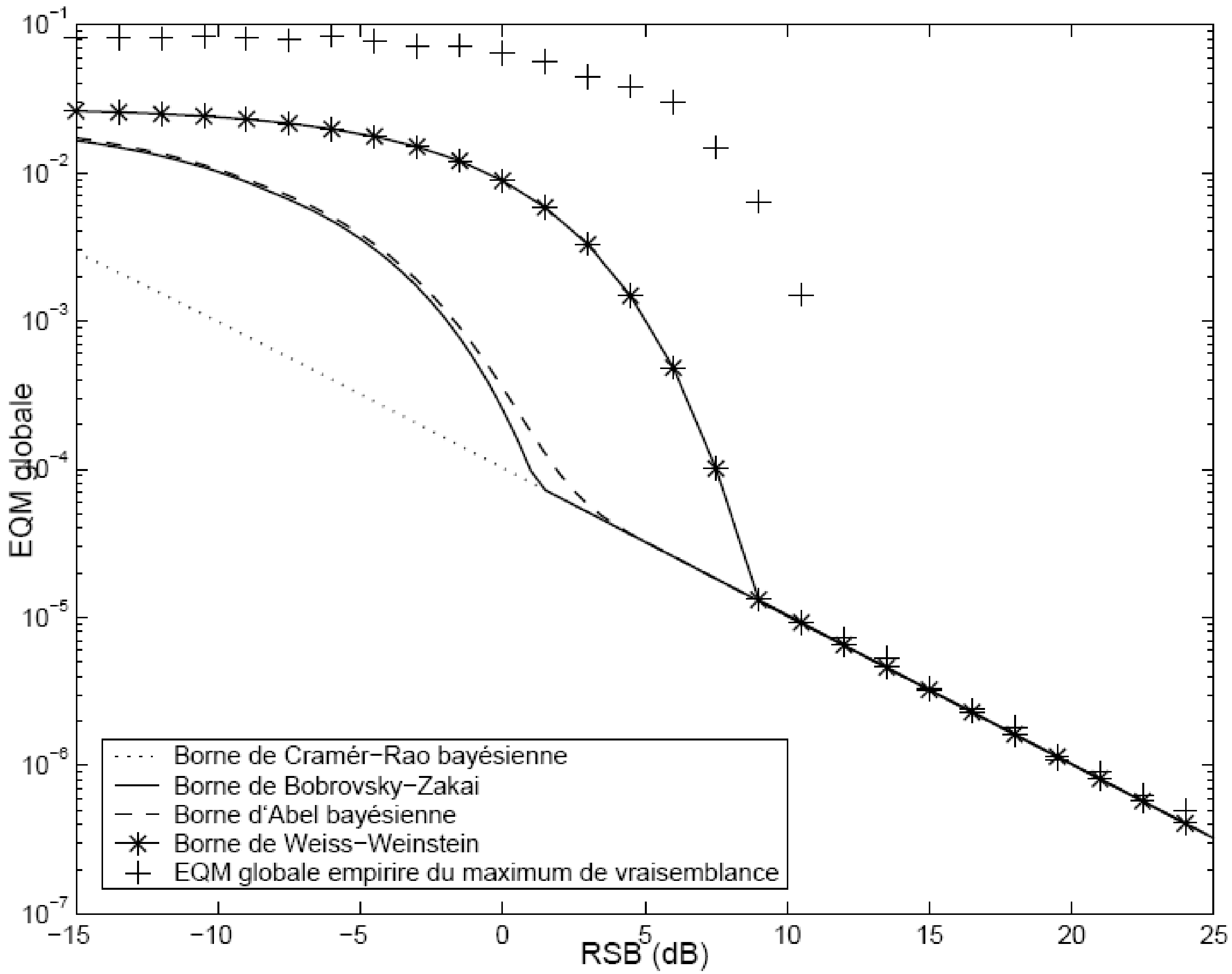
Paramètres Déterministes



Paramètres aléatoires

Bornes minimales en traitement du signal

Application à la synchronisation (analyse spectrale)

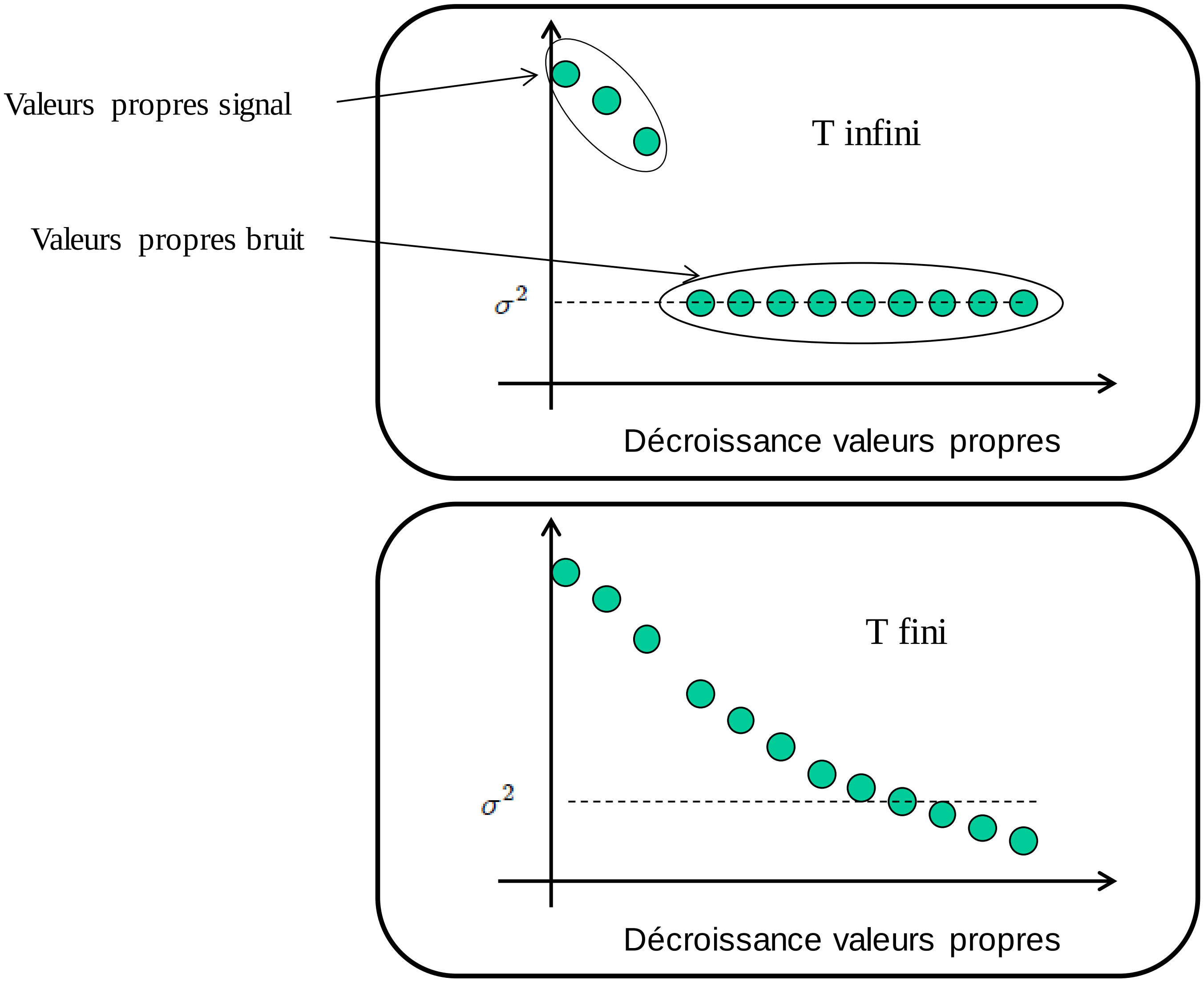


20 observations

Maximum de Vraisemblance

EQM Globale

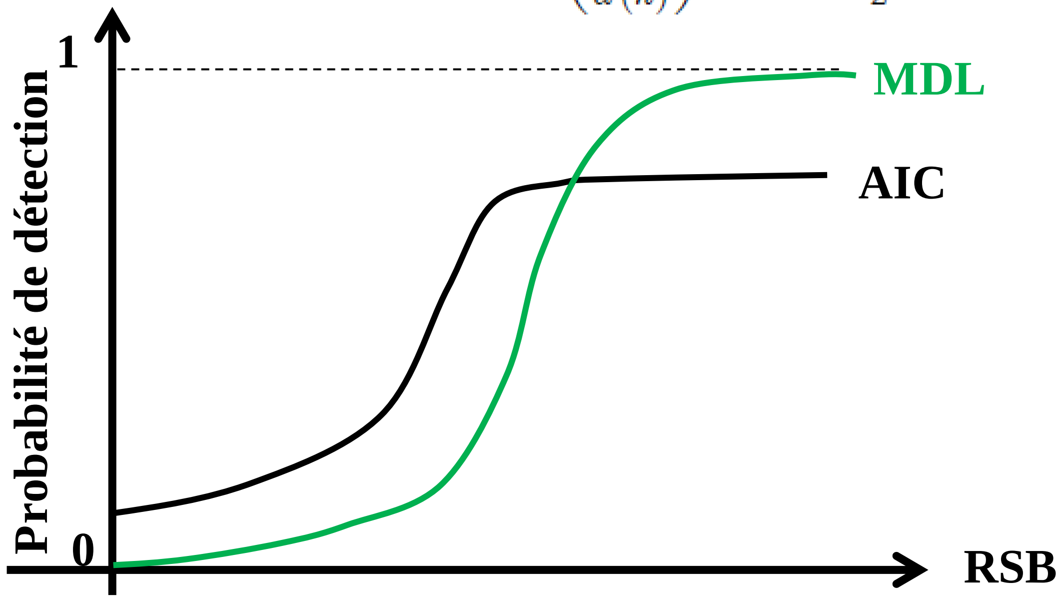
Sélection de l'ordre



Sélection fondée sur la théorie de l'information

Akaike(74) $AIC(k) = -LV(k) + NPL = -\log\left(\frac{g(k)}{a(k)}\right)^{T(N-k)} + k(2N-k)$

Rissanen et Schwartz(78) $MDL(k) = -\log\left(\frac{g(k)}{a(k)}\right)^{T(N-k)} + \frac{1}{2}k(2N-k)\log(T)$



Réglage de la probabilité de fausse alarme ⇒ **Théorie de la décision**

Rapport de vraisemblance $RV(k) = \left(\frac{a(k)}{g(k)}\right)^{T(N-k)} \lessgtr s(k)$

Radar
Communications
Radiogoniométrie
Tomographie
Imagerie
Identification

Déterministe
Aléatoire

Déterministe
Aléatoire
Paramétré

Gaussien
Impulsionnel
Hiérarchique

$$\mathbf{x}(t) = A(\boldsymbol{\theta})\mathbf{s}(t) + \mathbf{b}(t)$$

$\{\boldsymbol{\theta}\}$

$\{\mathbf{x}\}$

Modèle statistique

$\boldsymbol{\theta}_0$

$\mathbf{x}_{obs}$

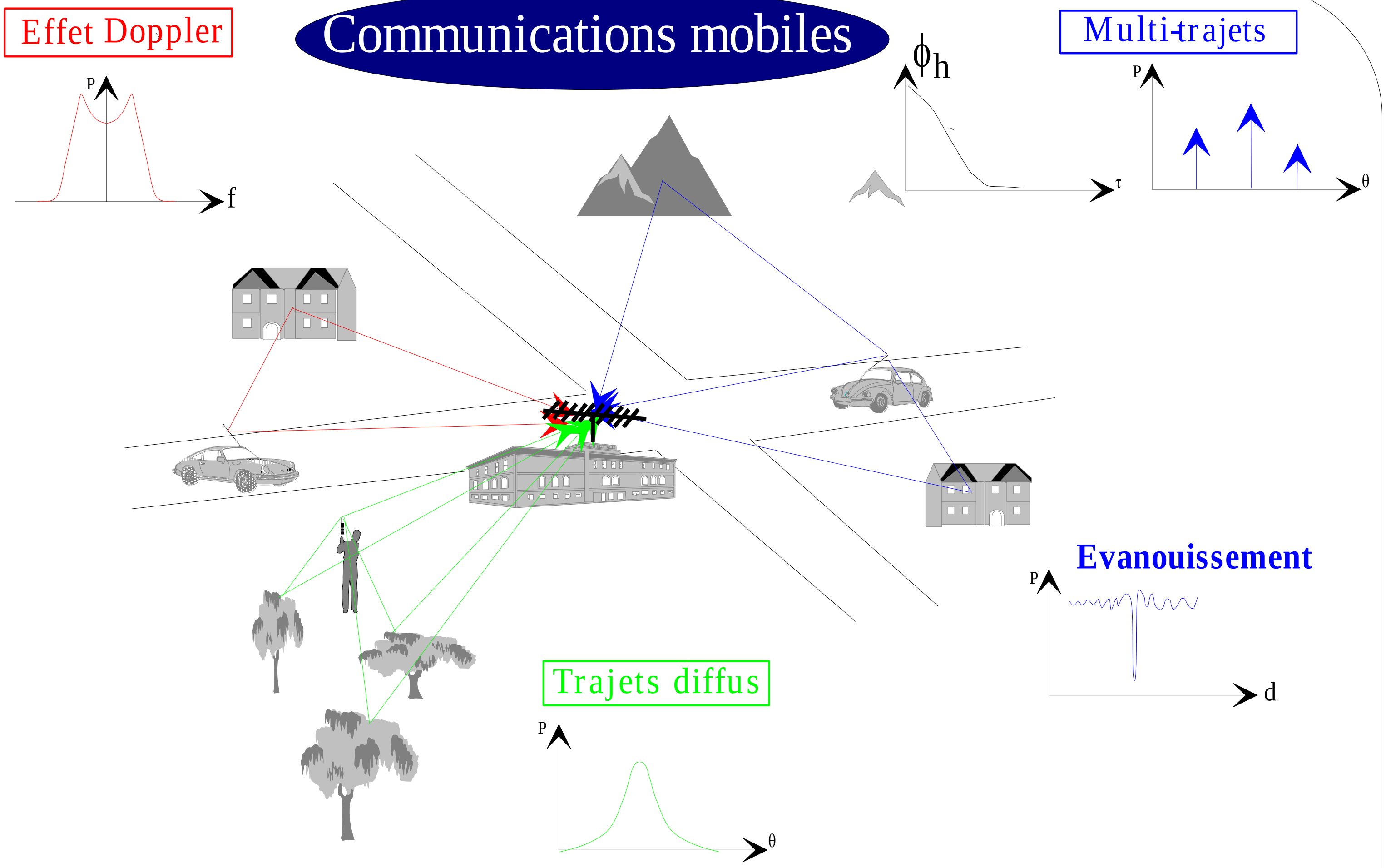
$\widehat{\boldsymbol{\theta}}$

$$\widehat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \{f(\boldsymbol{\theta})\}$$

Règle d'estimation

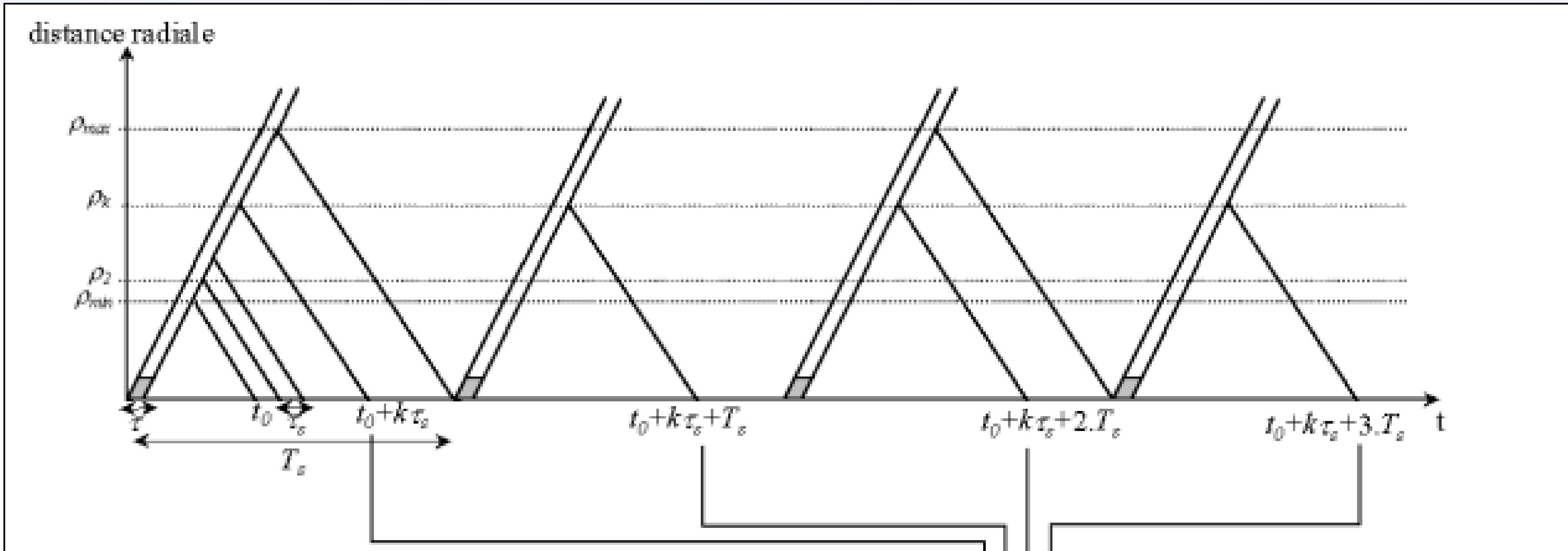
- **Mise en œuvre** (=> Maximum de Vraisemblance)
- **Performances** (=> Bornes minimales)
- **Robustesse** (=> Erreurs de modèles)

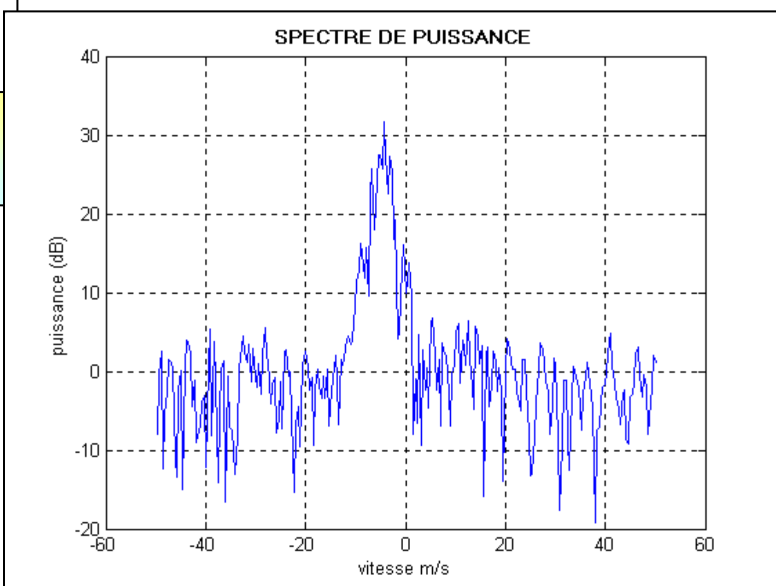
Communications mobiles

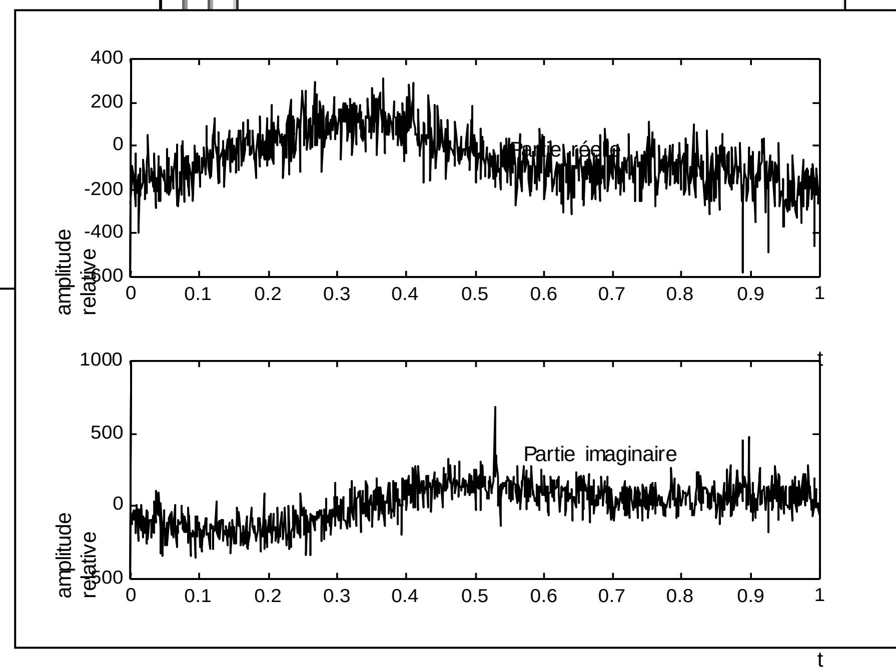


$$h(\tau, t_0) = \sum_{m=1}^{M} \alpha_m(t_0)\, \delta(\tau - \tau_m(t_0))$$

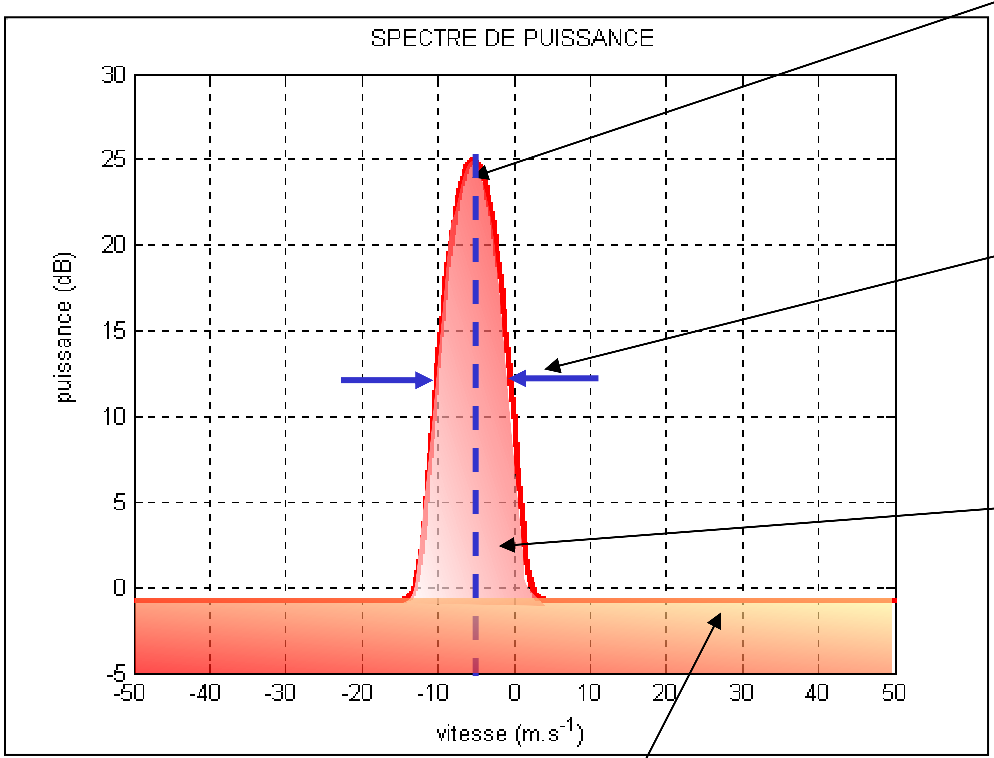
Estimation d'échos Doppler par radars strato-troposphériques
Le cyclone Georges


distance radiale
ρ_{max}
ρ_k
ρ_1
ρ_{min}
τ
t_0
τ_s
T_s
$t_0+k\tau_s$
$t_0+k\tau_s+T_s$
$t_0+k\tau_s+2.T_s$
$t_0+k\tau_s+3.T_s$
t


SPECTRE DE PUISSANCE
puissance (dB)
vitesse m/s
40
30
20
10
0
-10
-20
-60
-40
-20
0
20
40
60


Partie réelle
amplitude relative
400
200
0
-200
-400
-600
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
t
Partie imaginaire
amplitude relative
1000
500
0
-500
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
t

Paramètres d'intérêt



Vent moyen (Doppler)

Renseigne sur la valeur et le sens du vent radial

Ecart-type de l'écho

Mesure la dispersion des vitesses par rapport à la vitesse moyenne

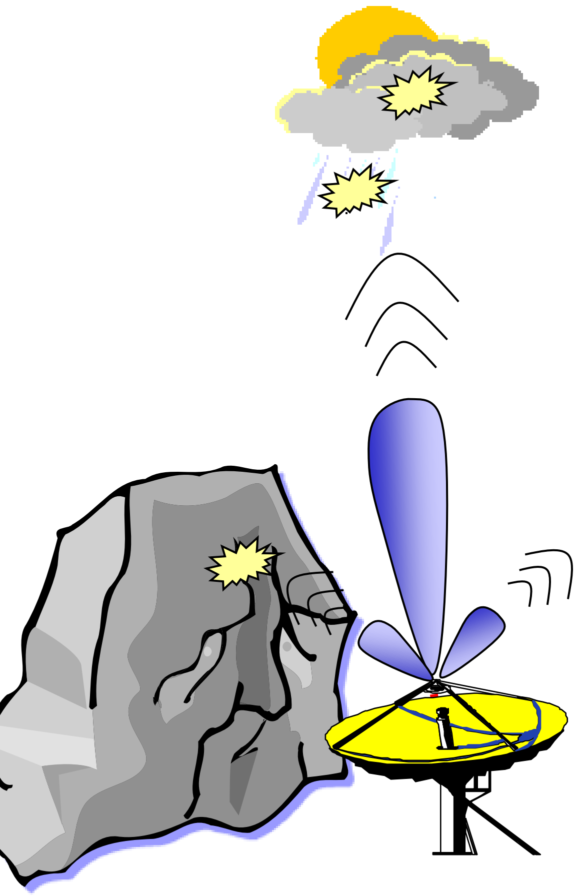
Puissance de l'écho

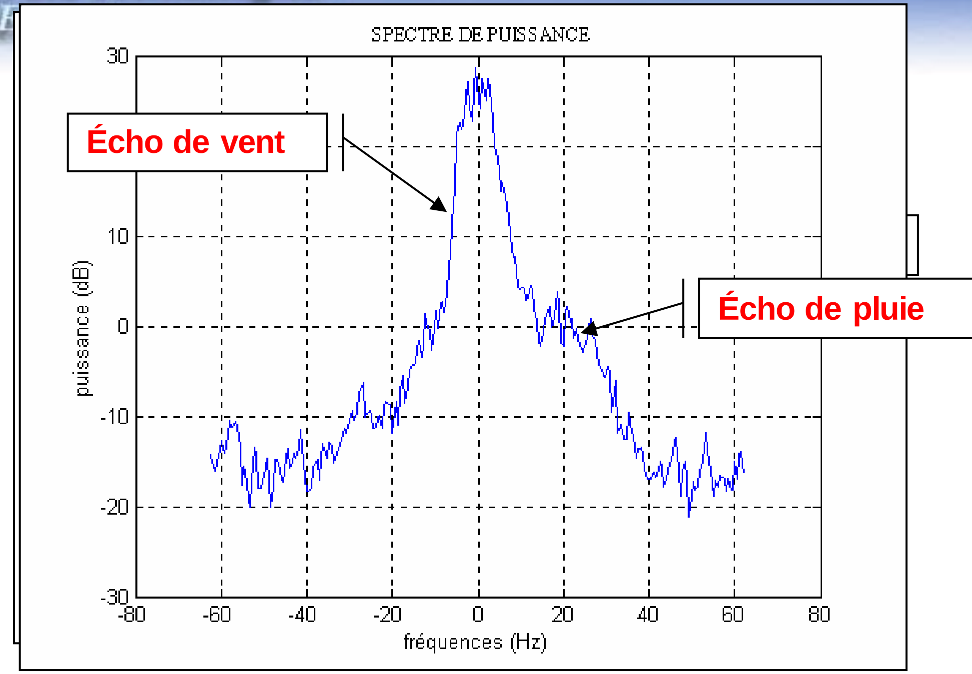
Donne une estimation de la réflectivité qui est reliée à l'intensité de la turbulence

Puissance de bruit

Paramètre de nuisance

$$\underbrace{P_0 \;\; ; \;\; v_0 = -\frac{\lambda}{2} f_0 \;\; ; \;\; \sigma_0}_{\text{3 paramètres d'intérêt à estimer}}$$





En cas d'échos multiples fortement superposés, peut-on estimer correctement les paramètres d'intérêt ?

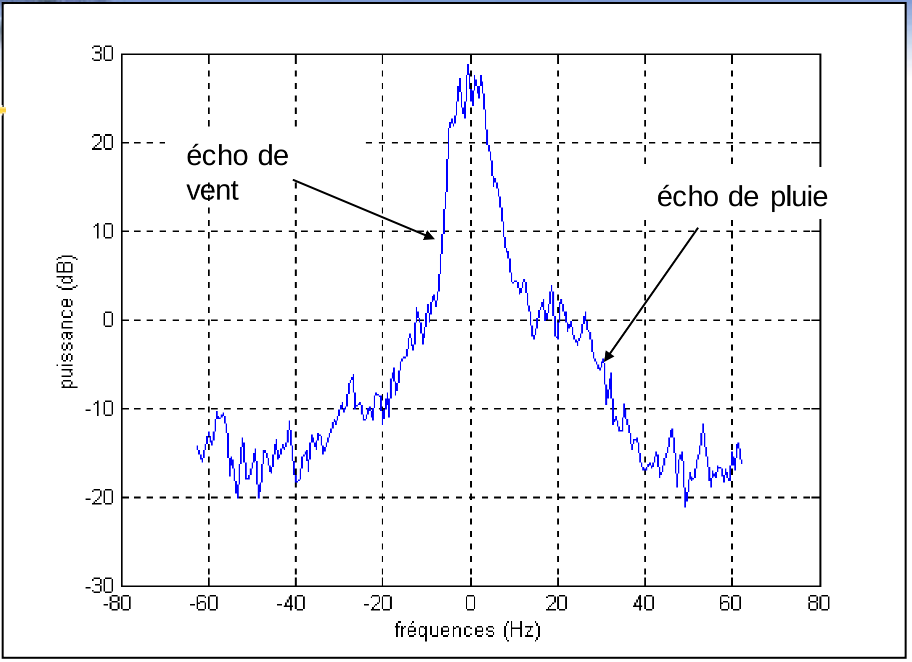
Modèle paramétrique des échos radar

HYP :LES ECHOS SONT GAUSSIENS

Spectre de puissance

$$P_s(f) = \sum_{i=1}^{N} S_i(f) + \sigma_b^2,$$

$$S_i(f) = \frac{P_i}{\sqrt{2\pi}.\sigma_i} e^{-\frac{1}{2}\left(\frac{f-f_i}{\sigma_i}\right)^2}$$



$$\mathbf{x} = \mathbf{y} + \mathbf{b} \quad \begin{cases} \mathbf{y} \text{ vecteur aléatoire gaussien centré} \\ \mathbf{b} \text{ bruit blanc complexe gaussien indépendant de } \mathbf{y}, \text{de variance } \sigma_b^2 \end{cases}$$

$$\mathbf{x} \in \mathcal{N}(0, \mathbf{R}_x)$$

$$\mathbf{R}_x(\boldsymbol{\mu}) = E\left(\mathbf{x}\mathbf{x}^H\right) = \sum_{i=1}^{N} \mathbf{R}_{y_i}\left(P_i, \omega_i, \sigma_i^2\right) + \sigma_b^2 \mathbf{I}$$

$$\mathbf{R}_{\mathbf{y}_i}\left(P_i, \omega_i, \sigma_i^2\right) = P_i.\mathbf{A}(\omega_i)\mathbf{B}\left(\sigma_i^2\right)\mathbf{A}^H(\omega_i)$$

$$\mathbf{A}(\omega_i) = diag\left(1, e^{j\omega_i T_s}, \ldots, e^{j(m-1)\omega_i T_s}\right)$$

$$\mathbf{B}_{k,l}\left(\sigma_i^2\right) = e^{-2\pi^2 \sigma_i^2 (k-l)^2 T_s^2}$$

Vecteur paramètre

$$\boldsymbol{\mu} = \left[P_1\ \omega_1\ \sigma_1^2 \ldots P_N\ \omega_N\ \sigma_N^2\ \sigma_b^2\right]^T$$

Estimation conjointe des moments spectraux

EXPLOITER LA STRUCTURE DE $\mathbf{R}_x$

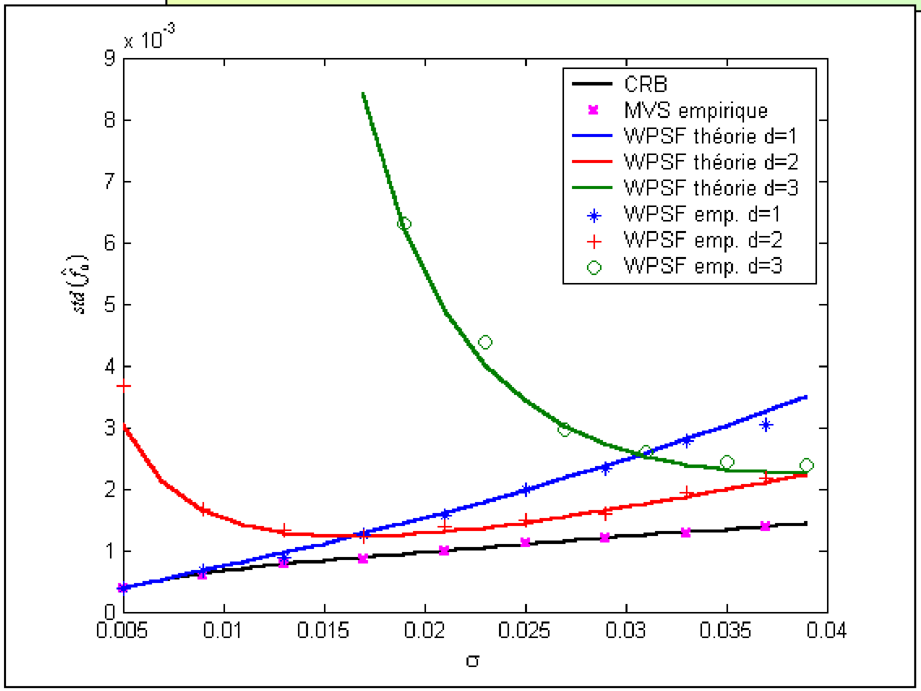
- Facile à calculer
- Porte toute l'information statistique
- Elle dépend des paramètres d'intérêt

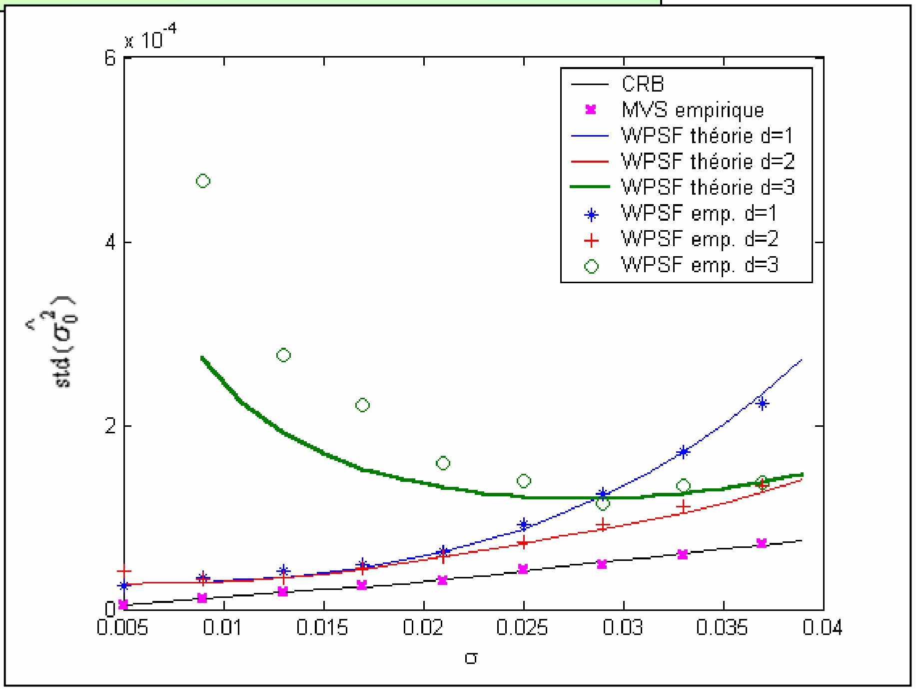
DEUX APPROCHES RETENUES

- **ESTIMATEUR DU MAXIMUM DE VRAISEMBLANCE (MV)**
- **GENERALISATION MULTI-DIMENSIONNELLE DES METHODES DE SOUS-ESPACE**

Simulations – cas de réalisations indépendantes

Performances empiriques et théoriques des estimateurs WPSF et MVS dans le cas d'un écho gaussien

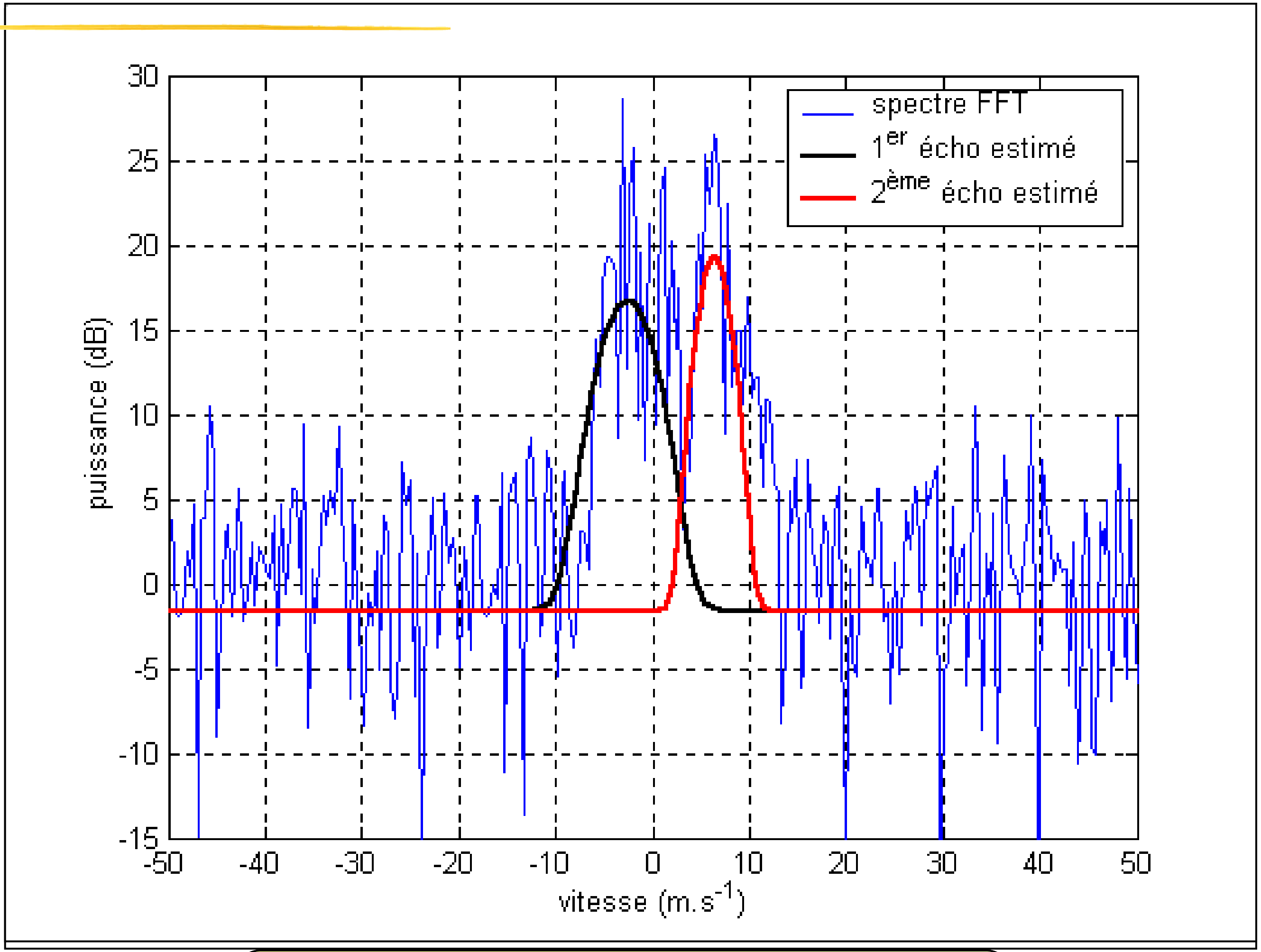




Pour WPSF

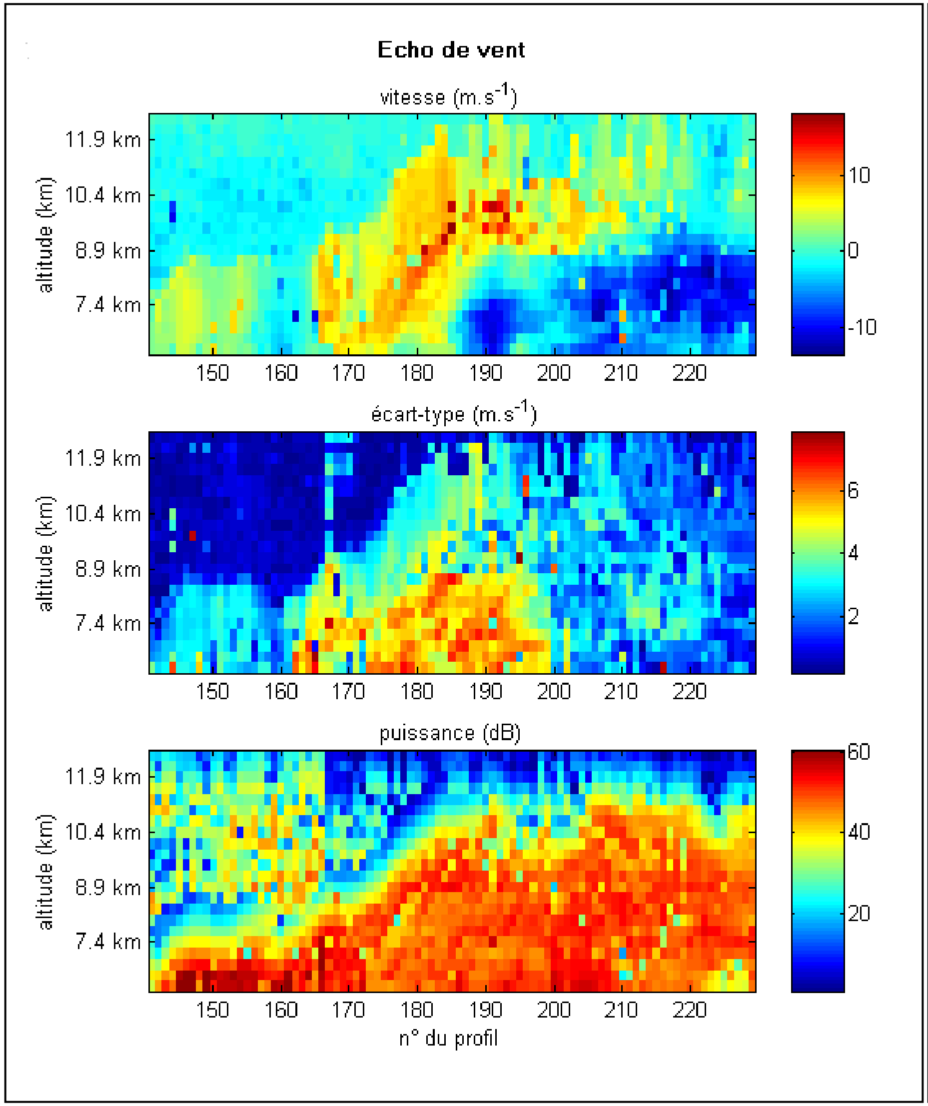
- Choix de *d* crucial
- Non optimal

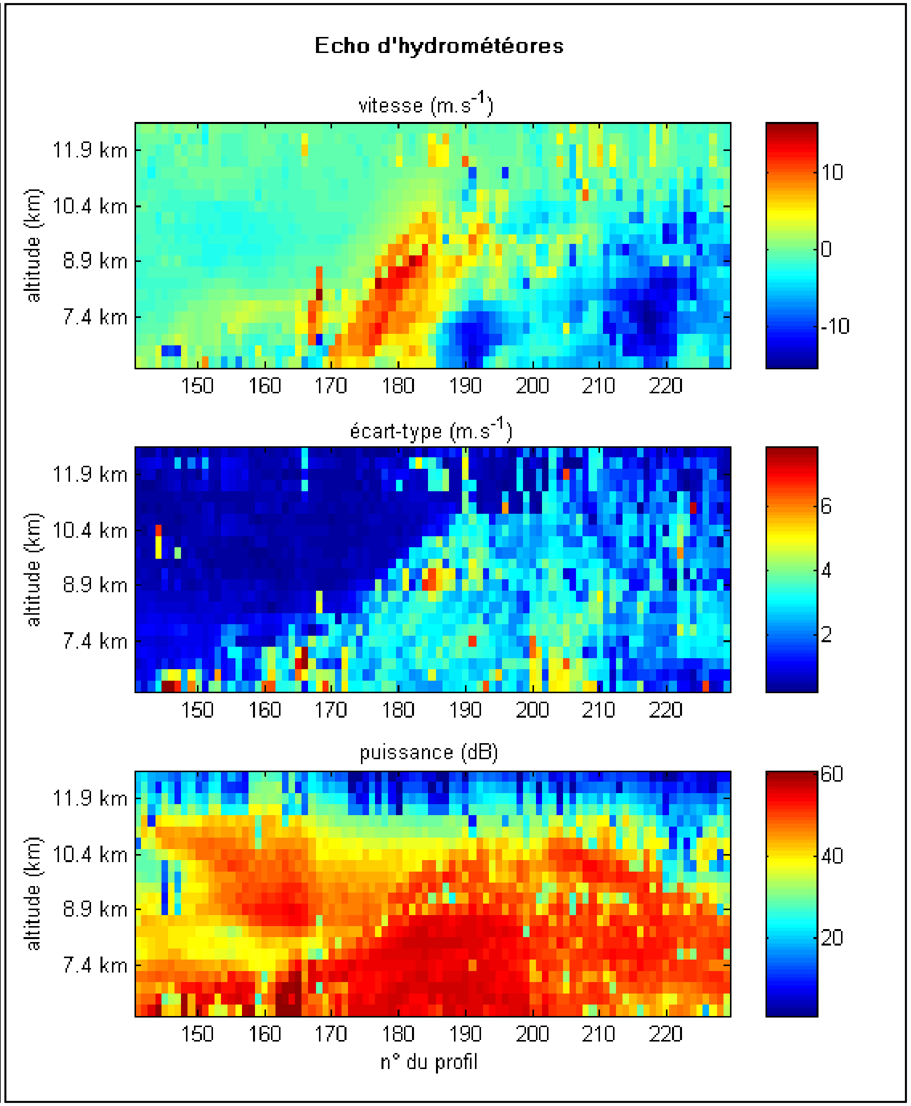
Validation sur données réelles



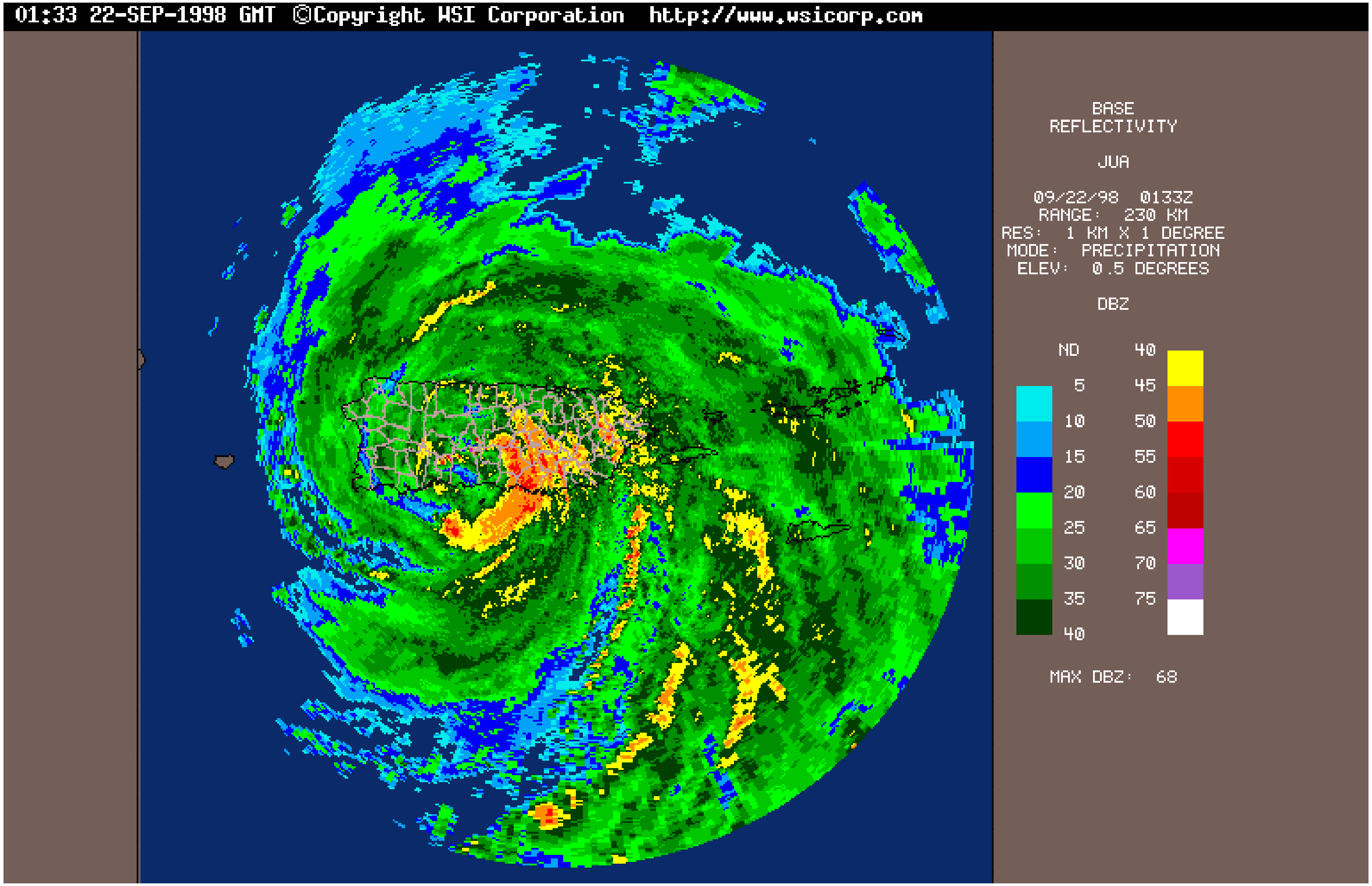
Estimation MVS d'un signal UHF avec *M*=16384, N_{coh}=8 et N_{inc}=8

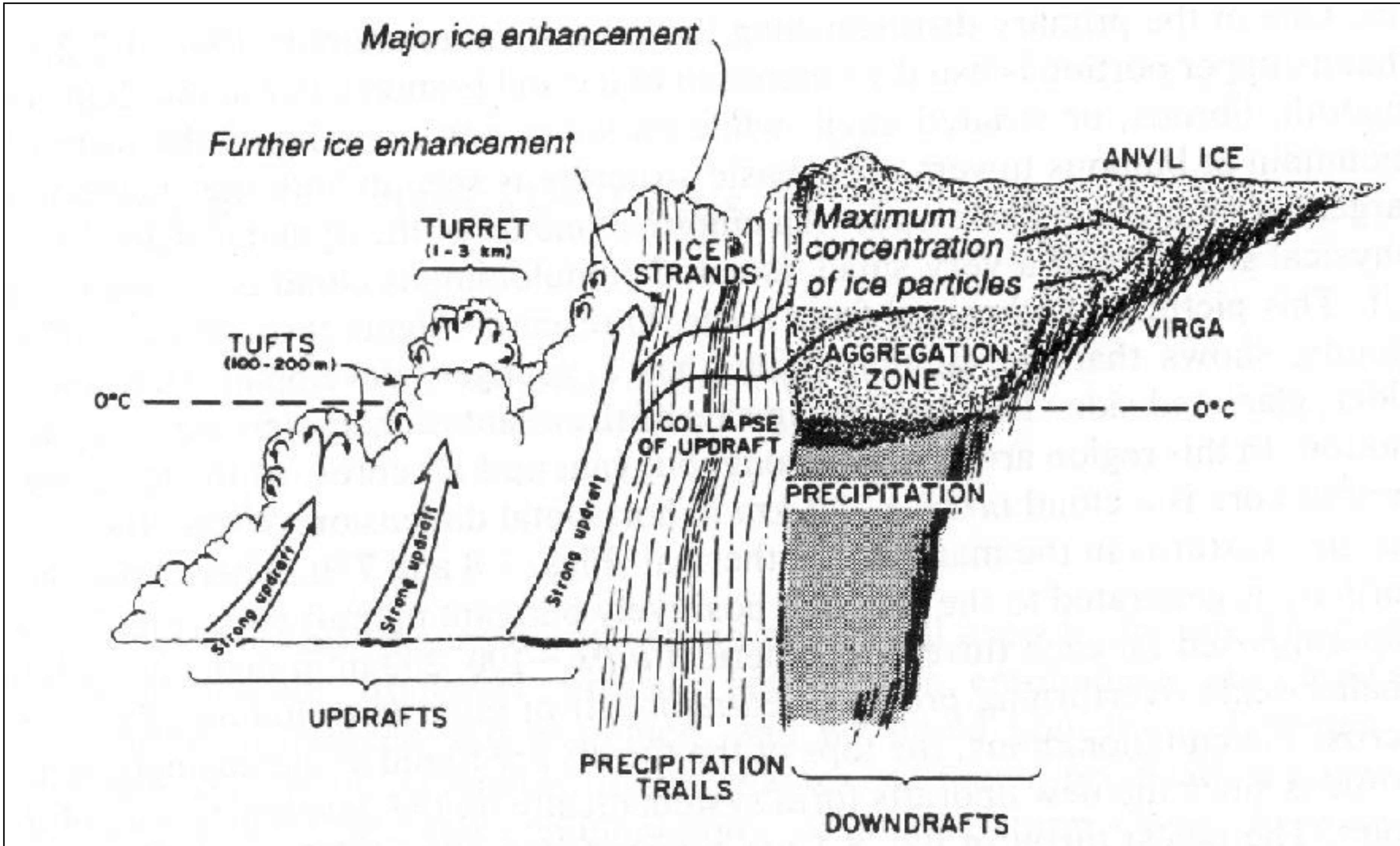
Restitution MVS des échos UHF de vent et de pluie





Validation sur les données réelles du cyclone Georges




Major ice enhancement
Further ice enhancement
TURRET
(1 - 3 km)
TUFTS
(100 - 200 m)
0°C
ICE STRANDS
Maximum concentration of ice particles
ANVIL ICE
VIRGA
AGGREGATION ZONE
0°C
COLLAPSE OF UPDRAFT
PRECIPITATION
Strong updraft
Strong updraft
Strong updraft
UPDRAFTS
PRECIPITATION TRAILS
DOWNDRAFTS

Issues

- Performance
- Computational complexity
- Robustness

Radar: Space Surveillance

- 5184 transmit elements
- 4660 receive elements

The extended invariance principle (EXIP)

Optimiser $L(\mathbf{x}, f(\boldsymbol{\eta}))$

1 $\hat{\boldsymbol{\eta}} = \min_{\boldsymbol{\eta}} L(\mathbf{x}, f(\boldsymbol{\eta}))$

2 $\boldsymbol{\xi} = f(\boldsymbol{\eta})$

$\hat{\boldsymbol{\xi}} = \min_{\boldsymbol{\xi}} L(\mathbf{x}, \boldsymbol{\xi})$

?

$$\hat{\boldsymbol{\eta}}' = \min_{\boldsymbol{\eta}} ||\hat{\boldsymbol{\xi}} - f(\boldsymbol{\eta})||^2_{\mathbf{W}}$$

- f, fonction injective ($\dim(\boldsymbol{\xi}) \geq \dim(\boldsymbol{\eta})$)
- Choix naturel : MCP.
- Choix du poids $\mathbf{W}$?
- Comportement de $\hat{\eta}'$?

EXIP [Stoica et al., 1989]

Si $f(\hat{\eta}) \to \hat{\xi}$, alors $\hat{\eta}'$ converge en probabilité vers $\hat{\eta}$ avec $\mathbf{W} = E\left[\frac{\partial^2 L(\mathbf{x}, \boldsymbol{\xi})}{\partial \boldsymbol{\xi} \partial \boldsymbol{\xi}^T}\right]$.

Estimation avec ajustement d'une matrice de covariance structurée

Modèle

$$\mathbf{x}(t) = A(\boldsymbol{\theta})\,\mathbf{s}(t) + \mathbf{b}(t) \qquad t = 1, ..., T$$

Première étape

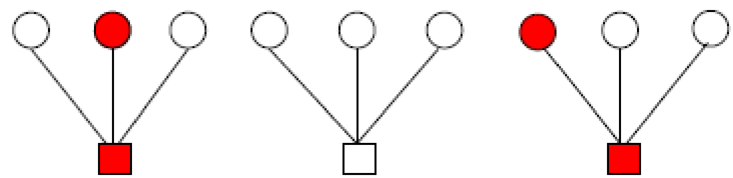
$$\hat{R} = \sum_{t=1}^{T} \mathbf{x}(t)\,\mathbf{x}(t)^{H}$$

Seconde étape

$$\left\{ \begin{array}{c} \hat{r} = vec(\hat{R}) \Rightarrow \hat{\gamma} = J\hat{r} \\ \gamma(\theta) = Jr(\theta) \end{array} \right\} \Longrightarrow \hat{\theta} = \arg\min_{\theta} \left(\hat{\gamma} - \gamma(\theta)\right)^{H} C^{-1} \left(\hat{\gamma} - \gamma(\theta)\right)$$

Blood tests (Dorfman 1943)

N individuals. Probability of infection p<<1. Pool the tests:



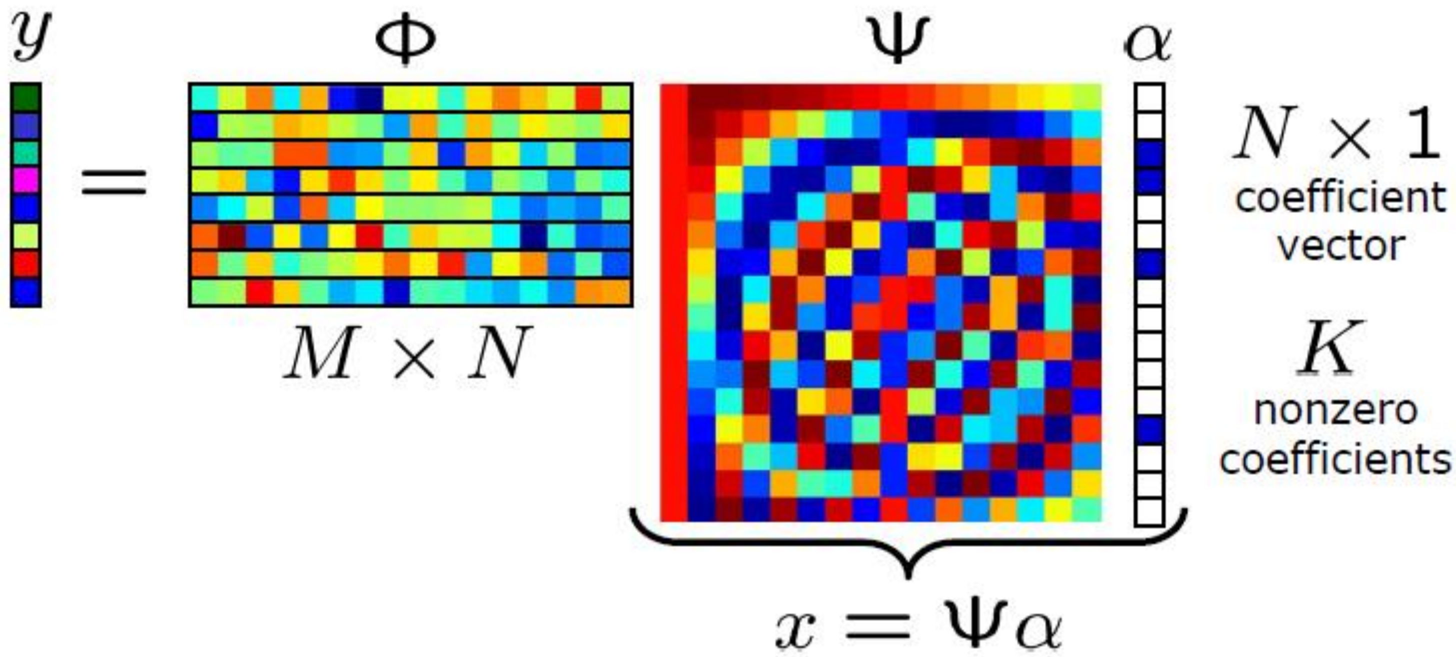
N/k groups of k persons, two-stage testing

$$\overline{T} = \frac{N}{k} + \left[1 - (1-p)^k\right] \frac{N}{k}\, k$$

Minimal for $k \sim \frac{1}{\sqrt{p}}$, gives $\overline{T} \sim 2\sqrt{p}N$

CS Paradigm

- K-term quality from just $K \log(N)$ measurements



- Robust and widely applicable