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Recommendation system for climate informed urban design under model uncertainty

Ido Nevat[a,*], Lea A. Ruefenacht[b], Heiko Aydt[b]

[a] TUMCREATE, Singapore
[b] Singapore-ETH Centre (SEC), Singapore

ABSTRACT

We develop a novel *recommendation system* for optimal urban design from a socio-economic and Outdoor Thermal Comfort (OTC) perspectives. Currently, urban planners and designers do not have quantitative tools or methods at their disposal to incorporate various important effects into consideration in a systematic way. Our framework is based on *risk measures* which quantify and take into account various important design criteria such as OTC, spatial use in the form of *exposure maps* and the investment cost of the design. Our framework combines those criteria into a spatio-temporal risk measure to assess the performance of candidate urban design options. This is used as the basis for our optimal design problem formulation. We then formulate the problem as an optimisation problem which is easy to solve and has a clear interpretation. The objective of this paper is therefore twofold: on the one hand, we develop technical knowledge and methodologies to assist urban planners develop and prioritize competing urban design strategies. On the other hand, we demonstrate the importance of incorporating the uncertainty in climate models into the utility function when making policy decisions. To illustrate how our framework can be used in practice we present a real-world study, which is based on a set of urban design strategies that aim to improve the OTC of a specific site in Singapore. The ENVI-met micro-climate model has been used in order to calculate the spatio-temporal OTC process. We show how our framework can assist decision-makers make more informed and interpretable choices on how to select the optimal design option and where to allocate best their investment/resources.

1. Introduction

An important challenge faced by environmental policy makers and urban designers is to choose the optimal design strategy to improve the Outdoor Thermal Comfort (OTC) of an urban space. Urban design strategies or measures can have a significant impact on micro-climate and OTC, and should therefore be considered carefully. In particular, OTC awareness in urban spaces is gaining attention because it is directly related with the quality of life and livability of a city (Hartz, 2012; Lam et al., 2016). When designing an urban space, the designer has the difficult and complex task of balancing contradicting requirements. These include not only the aesthetic aspects of the design, but also its practicality, from social (i.e. attractiveness), environmental (i.e. thermal comfort) to monetary (i.e. investment costs) points of view. Due to the complex nature of the climate, it is difficult for a designer to consider of all the various variables which control the OTC and their complex interactions. As a result, in many cases the thermal comfort aspects are not the most important criteria and other design considerations take precedence. However, in cities with a tropical rainforest climate, such as the case of Singapore, the high and uniform temperatures and high humidity all year long have a great impact on people's thermal comfort and also people's wellbeing.

1.1. Literature review

While the theoretical framework of decision theory has been well understood and used in the last century (Raiffa, 1974; Berger,

* Corresponding author.
E-mail address: ido.nevat@tum-create.edu.sg (I. Nevat).

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2013), not many works which develop a climate-responsive urban design framework exist. An interesting attempt to semi-automate the urban design task was proposed in (Chirkin and Koenig, 2016). In this paper the authors developed interactive machine learning algorithm for urban design and the problem was formulated as a multiple-criteria optimization problem. They then developed a learning-based model where changes to designs are submitted by the user, and has many parallels with reinforcement learning models, with human rewards. In (Nouri, 2015) the authors suggest an approach to address the thermal comfort levels in public spaces. In (Chatzidimitriou and Yannas, 2016), the influence of urban morphology and urban design parameters affect on pedestrian thermal comfort in cities has been studied. In (Taleghani et al., 2016) the authors compared the impact of four heat mitigation strategies on micro-meteorology and the thermal comfort of pedestrians in a neighborhood in eastern Los Angeles County. They performed micro-meteorological simulations for an extreme heat day assuming widespread adoption of each mitigation strategy. In (Taleghani, 2017) the authors reviewed the impact of different heat mitigation strategies on the pedestrians thermal comfort in the context of urban and micro-climate. In (van Hooff et al., 2014) the authors performed a computational analysis of six different passive climate change adaptation measures at the building component scale using dynamic thermal simulations. The main aim of the study is to assess the performance of these measures to reduce the number of overheating hours in residential buildings. In (Algeciras et al., 2016) the authors analyse the contribution of street configuration towards the improvement of thermal comfort at pedestrian level, in the Old Town of Camaguey in Cuba. In (Sodoudi et al., 2018) the authors investigated the influence of spatial configurations of green areas on the micro-climate and thermal comfort. In (Tapias and Schmitt, 2014) the authors proposed an automated tool to explore design spaces of urban forms according to measurements and empirical findings on the relationship between the outdoor thermal comfort, the micro-climate conditions and the building geometries.

1.2. Sources of uncertainty in climate models

In many cases climate models are numerical approximations to fluid dynamical equations which are forced by parameterisations of physical processes. Climate models involve multiple sub-models and many parameters, which are never perfectly known and need to be estimated from data. Data (observations) are always distorted, missing and incomplete. While it is possible that each of the sub-models may provide reasonably accurate output, it does not guarantee that the combined modeling system will be accurate, as error propagation can degrade the overall performance. It is very possible that the output of the climate model has very significant errors, due to the cumulative uncertainty which propagates in the system. Uncertainty in climate models is often divided into three broad categories: initial conditions, boundary condition, and model uncertainty.

- Parametrisation: the most simple example of the parameters uncertainty is the Local Climate Zone (LCZ) classification (Stewart and Oke, 2012). The LCZ in itself is a simplification of reality and transforms the continuous parameters space into a quantised one (via the introduction of classes). Moreover, the classification itself is usually based on distorted images (eg. remote sensing) and the classification of the LCZ contains many errors, which can be quite significant (Xu et al., 2017).
- Boundary conditions: to set boundary conditions, observations from weather stations, satellites etc. need to be collected. These observations are not perfect and can be distorted due to many reasons. It is therefore a common approach to model these as a noisy version of reality.
- Climate model inaccuracy: as mentioned earlier, climate models are imperfect, and are prone to errors. In addition, solving the model equations usually involves some form of numerical approximation (Räisänen, 2007; Barron, 1995).

1.3. Limitations of current approaches

At the moment, there are no formal procedures that directly aid urban designers to incorporate thermal comfort considerations into their design in a holistic way (with the exception of the work in (Chirkin and Koenig, 2016)). While there have been many works which use simulation tools to asses the OTC under different urban designs (see (Chen and Ng, 2012) for a comprehensive review), there is a lack of a rigorous and coherent mathematical framework which covers important attributes of the design, as well as the thermal comfort and cost of implementation. In particular, we highlight the following aspects which are lacking:

1) There is no single metric which represents and summarises the quality of an urban design strategy as a function of the spatial-temporal OTC process.
2) There is no clear definition of spatial importance of the urban locations, from a social, economic or other points of view.
3) There is no decision theoretic framework which incorporates the environmental, social and economical aspects into a single coherent methodology.
4) Most importantly, all current approaches neglect the fact that all climate models are imperfect and introduce estimation errors.

Therefore, a decision theoretic framework is required as well as suitable for OTC-responsive urban design. The purpose of this paper is to address these limitations in a rigorous way. To this end we develop a novel unified framework, that is based on *decision theory*, to select the optimal design measure for a particular urban form, while taking into account specific climatic conditions, spatial use, as well as the cost of implementation. Each design measure receives a “score” which reflects its overall quality (in terms of risk), and the design with the highest quality (lowest risk) is chosen. To develop our framework we borrow ideas from well established fields of research, such as *decision theory* (Savage, 1972) and *risk theory* (McNeil et al., 2005). The basic approach we follow in this paper for the development of our decision mechanism is based on the work of (Koch, 2017; Koch, 2018; Ahmed, 2017), where various spatial risk measures and their theoretical properties have been analyzed. The choice of *risk measures* as our framework is motivated by the fact that these are well understood and widely used in economics and social science. To quantify and express our utility function, we use the notion of *spatial risk measures* which have been introduced in (Föllmer, 2014) and further explored in (Koch, 2017; Koch, 2018; Ahmed, 2017). We introduce the notion of cost process which is a combination of the underlying OTC process (e.g., Physiologically Equivalent Temperature (PET)), the *damage function* and *exposure map*. We then integrate the resulting process of both spatial and temporal domains to obtain scalar-valued random variable which reflects a notion of risk due to a particular design. We then formulate the optimisation problem as a linear combination of population summaries of the risk combined with the cost of implementation of the strategy.

1.4. Research questions

Urban design is a fundamental aspect of modern life which has and will have a significant effect on people's livability. Any such decision has economic, social and environmental impacts. It is therefore why there is a need to develop a holistic formulation of this problem and provide simple and tractable solutions which can be easily conveyed to decision makers which may not be expert in urban design, climatology and statistical methods.

Therefore, the research questions we address in this paper are: Given a set of candidate urban designs, how to:

1) mathematically formulate the urban design problem, which takes into account the uncertainty (eg. modeling errors) in the climate model?
2) score and rank each of the urban design candidates in a meaningful way via a utility function under climate model uncertainty?

1.5. Contributions

We make the following contributions:

C1. We formulate the urban design problem under decision theory and risk models. We borrow concepts and ideas from the well developed theories which are widely used in economics and operational research to re-formulate the optimal urban design problem.
C2. We introduce the notions of *damage function*, *exposure map* and *loss process* in order to quantify the *Normalized Temporarily - Spatially Aggregated Loss* which represents the quality of a specific urban design.
C3. We propose a new risk measure which summarises the spatio-temporal quality of a specific design from its thermal comfort perspective.
C4. Our framework integrates climate science, social and economic models for the application of optimal urban design in a firm theoretical footing and systematic way.
C5. We developed and implemented the *climate informed urban design decision tool* which can be used by urban planners and designers to evaluate the thermal quality of the different design options.

This paper is organized as follows: in Section II we present the fundamental definitions of the risk measure we utilise. In Section III·B we present the objective function for the optimal design and the algorithm to solve the problem. In Section V we present a detailed and practical example of our framework at a site in Singapore.

The following notation is used throughout, boldface upper case letters denote matrices, boldface lower case letters denote column vectors, and standard lower case letters denote scalars. By $x := y$ we mean that x is defined as y. The Probability Mass Function (PMF) of a discrete random variable $A \in \mathscr{A}$ is denoted by $Pr(A = a)$, $a \in \mathscr{A}$ and $\sum_{a \in \mathscr{A}} Pr(A = a) = 1$. By $\Pr(A = a|\lambda)$ we denote the conditional probability that the random variable A is equal to a, given the value of λ. The operator $\mathbb{E}[A] := \sum_{a \in \mathscr{A}} \Pr(A = a)a$ is the statistical mean of a discrete random variable A, and $\mathbb{E}_x[g(x)] := \int_{x \in \mathscr{X}} g(x)p(x)\mathrm{d}x$ is the statistical mean of a function of the continuous random variable x with pdf $p(x)$.. The symbols ∪ and ∩ denote the union and intersection of two sets, and ϕ denotes an empty set. By $y = \mathscr{D}(\alpha)$ we denote a functional, meaning that $\mathscr{D}$: $\mathscr{Y} \rightarrow \alpha$ is a mapping of a parameter α to an output value y.

The notation $\mathbf{1}(x)$ is the Iverson bracket, defined as:

$$\mathbf{1}(x > \lambda) = \begin{cases} 1, \text{ if} x > \lambda \\ 0, \text{ Otherwise.} \end{cases}$$

2. Thermal comfort based decision making for urban design

In this Section we present the theoretical Decision Making framework for our optimal urban design selection. We first provide useful definitions which will be the core of our decision making methodology. We then define the spatio-temporal loss process followed by a definition of the *Normalized Temporarily - Spatially Aggregated Loss* random variable. Finally, we provide our risk measures which are statistical summaries of the loss random variable.

2.1. Formal definition of decision making

In urban design, the set of possible designs is a variable which is directly under the control of the architect or urban designer and is strictly related to the thermal comfort. It is the specific choice of urban design strategy, that is, an action in the decision theory terminology, that represents a policy that the urban designer can perform. We assume the presence of a single agent (decision maker) which can make a single design choice (action). Next we define our notion of spatio-temporal risk measure, which makes explicit the contribution of the space-time in the risk measurement. Our definition is the same as in (Koch, 2017; Koch, 2018), where we also introduce the temporal domain.

Definition 1. (Spatio-temporal risk measure).

A spatial risk measure is a function R that associates to any spatio-temporal region $\mathscr{X} \times \mathscr{T}$ and to any distribution $P \in \mathscr{P}$ a real number:

$$R: \mathscr{X} \times \mathscr{T} \times \mathscr{P} \to \mathcal{R}. \tag{1}$$

See (Koch, 2017; Koch, 2018) for more details and the related axioms spatial risk measures. In the setting of our problem, the distribution space $\mathscr{P}$ contains all the uncertainty regarding the climate model.

2.2. Decision making under uncertainty

Some clarification regarding the sources of uncertainty are in order. In this paper we concentrate on a particular case of *scientific uncertainty*. In our model, the sources of uncertainty are manifested by two different sources:

1) The climate uncertainty which we model via *weather types* (see Definition 4).
2) The climate model uncertainty, that is, the imperfections in the estimation of the environmental parameters (see Definition (5)).

Our goal is then to find the urban design which performs “best on average”, that is, minimises the following objective function:

$$\widehat{A} = \arg\min_{A \in \mathscr{A}} (\pi \mathbb{E}[L(W; A, \mathscr{X}, \mathscr{T})] + (1 - \pi) B(A)), \tag{2}$$

where $L(W; A, \mathscr{X}, \mathscr{T})$ is the *aggregated Loss Variable* corresponding to the normalized total (spatial and temporal) loss (economic, social and environmental) over region $\mathscr{X}$ and time $\mathscr{T}$ due to a specific urban design scenario A (see Definition 9), $B(A)$ is the cost of implementing design A and $0 \leq \pi \leq 1$ is a weight which balances between the environmental and cost preferences.

In the rest of the paper we shall formulate this problem in accordance with our thermal comfort goals. To begin our formulation of the problem, we first present the building blocks of our spatio-temporal risk measure model:

1) A physical region of interest, denoted by $\mathscr{X} \subset \mathbb{R}^2$. Examples include a whole city or a residential neighborhood.
2) A time duration of interest, denoted by $\mathscr{T} \subset [0,\infty)$. Examples include a whole year or a single day.
3) A set $\mathscr{A} := [A_1,...,A_{|\mathscr{A}|}]$ of $|\mathscr{A}|$ actions which could be taken in the form of urban design scenarios, such as shaded plazas, building porosity or green facades (see Definition 2).
4) A set $\mathscr{W} = [W_1,..., W_{|\mathscr{W}|}]$ of $|\mathscr{W}|$ random weather types and their corresponding Probability Density Mass (PMF), denoted $\{Pr(W = w)\}_{w=1}^{|\mathscr{W}|}$, $\sum_{w=1}^{|\mathscr{W}|} Pr(W = w) = 1$.
5) Spatio-temporal Thermal Comfort Index (TCI) process under each urban design $Z(\mathbf{x}, t; A)$ (See Definition 3).
6) Damage function $\mathscr{D}(\alpha; \mathbf{x}, t)$ (See Definition 6).
7) Exposure process $E(\mathbf{x}, t)$ (See Definition 7).
8) Each action $A \in \mathscr{A}$ incurs a budgetary cost $B(A)$. Examples include implementation or maintenance costs.

The conceptual framework is presented in Fig. 1. Each of the $|\mathscr{A}|$ urban designs (“actions” in decision theory terminology) serves as an input to the climate model, together with the random weather types. Each of the urban designs is then given a risk measure score which is based on the damage function and exposure map. Finally, the optimisation engine sorts the urban designs according to their scores and chooses the one which has the minimal score.

We concentrate on the case where the choice of the optimal action to be taken is decided once and a single action is implemented. The more general case where a sequence of actions as well as the times of execution is to be chosen (which could be considered as policy making) is also very interesting and important, but is outside the scope of this paper. We now provide a set of definitions which will be the basis for the derivation of the loss process and the risk measures.

Definition 2. (Urban Design Scenarios).

Urban design scenarios is a finite set, denoted $\mathscr{A}$, of potential urban design solutions and strategies, based on certain assumptions chosen

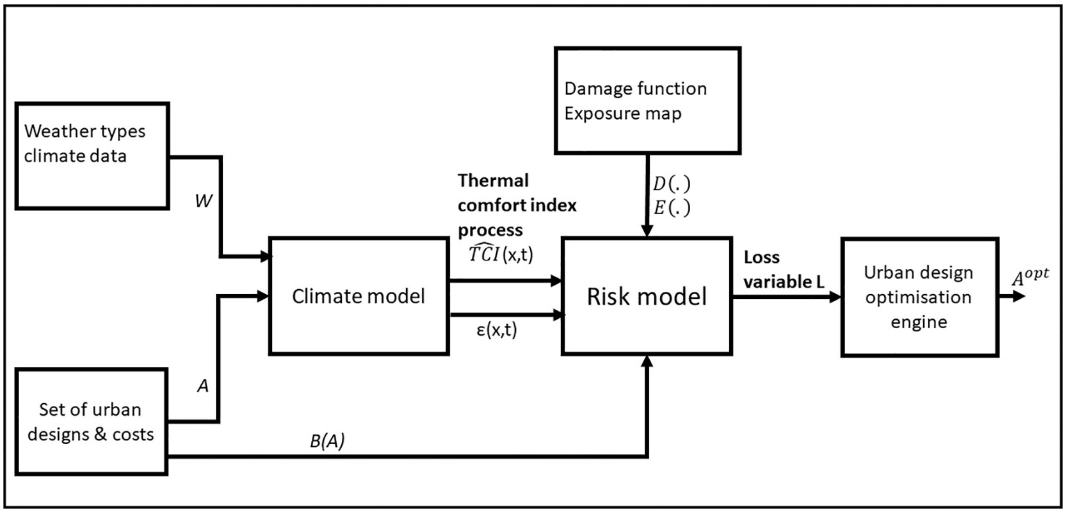

Fig. 1. System model of optimal urban design.

by the designer. Scenarios are used to evaluate the effects of one or more solutions in order to support short and long term planning.

Urban design is the process of designing and shaping the urban form, with the goal of making urban areas functional, attractive, and economic viable. In contrast to architecture, which focuses on the design of individual buildings, urban design deals with the larger scale of groups of buildings, streets and public spaces, whole neighbourhoods and districts, and entire cities.

2.3. Spatio-temporal thermal process

There is a multitude of thermal comfort related indices which have been proposed in the literature, see for example an overview in (de Freitas and Grigorieva, 2015). We do not wish to choose a particular index, but instead, we allow our framework to work with any thermal comfort index the user wishes to endorse. We therefore provide the following definition of thermal comfort index, which is general enough to represent all the widely used thermal comfort indices.

Definition 3. (Thermal Comfort Index (TCI) process).

Thermal Comfort Index (TCI) process is the spatio-temporal process which represents the thermal comfort at any location of interest $\mathbf{x} \in \mathscr{X}$ *and any time* $t \in \mathscr{T}$. *The TCI process is parametrised by an urban design scenario* $A \in \mathscr{A}$. *The TCI process is denoted by* $Z(\mathbf{x}, t; A)$: $\mathscr{X} \times \mathscr{T} \times \mathscr{A} \to \mathbb{R}$, *is a random process due to the implicit random weather type* $W \in \mathscr{W}$ *which affects the TCI process.*

We note here that it is not our intention to propose a specific TCI, but instead, we provide the user the freedom to choose whatever TCI process they would like to use. These include the well known thermal comfort indices, such as *Physiological Equivalent Temperature* (PET), *Universal Thermal Climate Index* (UTCI) and others, as well as heat stress indices, such as *Wet-Bulb Globe Temperature* (WBGT) and *Effective Temperature* (ET). The calculation of the TCI process can be done in various ways, but all require a climate model. Examples include, the use of physics based models such as ENVI-met (Bruse et al., 2009), OpenFOAM (Greenshields, 2015) or statistical models, such as spatial regression models (Nevat et al., 2013; Nevat et al., 2015). It is not the scope of this work to propose which model to use and we leave this to the discretion of the user. We note here that clearly although the scope of this work is optimal urban design through the lens of thermal comfort considerations, one could consider other processes in the same manner. For example, one could be interested in pollution levels of CO_2 or other particle concentrations. To make the relationship between the weather type W and the TCI explicit, we denote the TCI as $Z(\mathbf{x}, t|\, w; A)$. This notation should be interpreted as the TCI process values, conditional on a particular choice of weather type w and parameterised by a particular choice of urban design A.

2.4. Climate model and uncertainty quantification

Since we do not have a direct access to the TCI process values, it is common to use a simulation tool which estimates the TCI process These models cannot provide error-free estimates of the TCI, and therefore, one needs to account for the uncertainty in the TCI estimated values. We incorporate these uncertainties in two ways:

1) Weather types: we partition the climate into a finite set of *weather types*, which is a partition of the d-dimensional climatic conditions space, denoted $\mathscr{Y} \subset \mathbb{R}^d$, into a (finite) set, $|\mathscr{W}|$, of centroids (which we call "weather types") in $\mathscr{Y}$. The probability of the occurrence of each weather type is given by $\Pr(W = w)$ (See Definition 4).
2) Climate model estimation errors: we model the errors in the model via a simple *truth* + *error* model (See Definition. 5).

Definition 4. (Weather Types).

Weather Types is a finite set, denoted $\mathscr{W} = [W_1, \ldots, W_{|\mathscr{W}|}]$*, of size* $|\mathscr{W}|$*. Each element in* $\mathscr{W}$*,* W_i *corresponds to a subset of* $\mathscr{Y}$*,* K_i*, such that for the set* $\mathscr{K} = [K_1, \ldots, K_{|\mathscr{W}|}]$*, it follows that:* $\cup_{w=1}^{|\mathscr{W}|} K_w = \mathscr{S}$ *and* $K_{w'} \cap K_w = \phi, \forall w' \neq w$*. We then assume that the probability of each weather type is given by* $\Pr(W = w)$*.*

Definition 5. (Climate model estimation errors).

We model the climate model via *the following simple additive error model:*

$$Z(\mathbf{x}, t; A) = \widehat{Z}(\mathbf{x}, t; A) + \varepsilon(\mathbf{x}, t; w), \tag{3}$$

where $\widehat{Z}(\mathbf{x}, t; A)$ is the estimated TCI process (that is, the output of the weather simulation tool) and $\varepsilon(\mathbf{x}, t; w)$ is the spatio-temporal process which represents the estimation error of the TCI process. We note that we explicitly encode the parameters of the error process with the particular weather type, w. This means that different weather types result in different statistical properties of the error process. We further assume that the error is temporarily and spatially independent as is given by:

$$\varepsilon(\mathbf{x}, t; A) \overset{\text{i.i.d}}{\sim} \mathscr{N}(0, \sigma_w^2). \tag{4}$$

The value $\sigma_w{}^2$ *depends on the accuracy of the climate model and the weather type and can be easily calculated using multiple calibration procedures (*Taleghani et al., *2014). While here we explicitly assume that the error follows a Normal distribution, this can be trivially extended to any other parametric distribution. The statistical justification for the normality assumption stems from the fact that in many practical cases the output of the climate model is averaged over multiple days, allowing one to invoke the Central Limit Theorem (CLT).*

Next, we define the *damage function*, which is a point-wise transformation (possibly non-linear) of the TCI process. The *damage function* is designed to capture the implication of a specific TCI value on the subjects, in our case, the people.

2.5. Damage function

The *damage function* (sometimes referred to as *vulnerability function*) is a feature which represents the “damages” stemming from an environmental event. The term “damages” should be understood as a notion of a negative effect that the particular value of the TCI has on society (e.g. people, the environment). This damage could be related to limitation of performing outdoor activities, degradation of health, economic loss due to increased hospitalization of the population due to heat-stress and other. In general, the *damage function* reflects the welfare degradation effect to the public due to thermal conditions. The idea is to map of the environmental variable under consideration, in our case TCI process, to a loss (e.g. social or economic loss) via a damage function. The use of *damage function* in climate change economics is widely used, see for example (Berger et al., 2017; Pindyck, 2013).

Definition 6. (Damage function).

A damage function is a point-wise mapping $D: \mathbb{R} \times \mathscr{X} \times \mathscr{T} \to \Xi \subseteq \mathbb{R}$ *of the environmental process (ie. CTCI process) to a space* Ξ *(which can be continuous or discrete), and is denoted by* $\mathscr{D}(\alpha; \mathbf{x}, t)$*.*

We note here that the definition above is very general in the sense that the *damage function* can change as a function of space and time. In the following we shall restrict the *damage function*, without loss of generality, to a simpler of the form $\mathscr{D}(\alpha; \mathbf{x}, t) = \mathscr{D}(\alpha)$.

Typical examples of *damage function* include the following:

1) Indicator *damage function*: $\mathscr{D}(\alpha) = \mathbf{1}(\alpha \geq \lambda), \lambda \in \mathbb{R}$.
2) Parabolic *damage function*: $\mathscr{D}(\alpha) = a\alpha^2 + b\alpha + c, \{a, b, c\} \in \mathbb{R}$.
3) Staircase *damage function*: $\mathscr{D}(\alpha) = \sum_{i=0}^{n} a_i \mathbf{1}(\alpha \in \mathscr{A}_i)$, $n \geq 0$, $a_i \in \mathbb{R}$, and $\mathscr{A}_i$ are intervals with the properties $\mathscr{A}_i \cap \mathscr{A}_j = \phi, \forall i \neq j$, and $\cup_{i=0}^{n} \mathscr{A}_i = \mathbb{R}$.

One practical example of *damage function* which is widely used can be found in the quantification of heat-stress related morbidity and mortality rates, see for example (Paravantis et al., 2017). In this case, the input to the *damage function* would be the heat-stress index, and the output, the morbidity/mortality rates. This could be on a continuous scale, categorical (according to the severity) or a combination of both. We provide two illustrative examples of such *damage functions*, one continuous and one categorical. In Fig. 2 the left column depicts a continuous *damage function*. In this case the input parameter is mapped into an output which quantifies the number of mortalities. This could also be mapped into a dollar value when used by an insurance company trying to quantify its losses due to climate related claims. The other example, depicted in the right column, shows categorical outcomes in the form of *Heat stroke, Heat exhaustion, Heat syncope, Heat cramps* and *Heat fatigue*. These categories are widely used in the medical literature and may not cause deaths, but would lead to increase in the number of hospitalization rates, which just as before, can be quantified in dollar value.

We note here that it is not in the scope of this paper to endorse, recommend or justify a particular *damage function*. Instead we provide a generic framework to utilise any *damage function*, and leave it to the user to decide what the *damage function* should be.

2.6. Exposure process

The *exposure process* is a spatio-temporal process which represents the “importance” of each location in space over time. The term “importance” means how each location is valued (or used) by the user. For example, a parking lot may be considered as unimportant since people are not expected to spend time there. In contrast, a sport field or a playground could be considered by the user as having

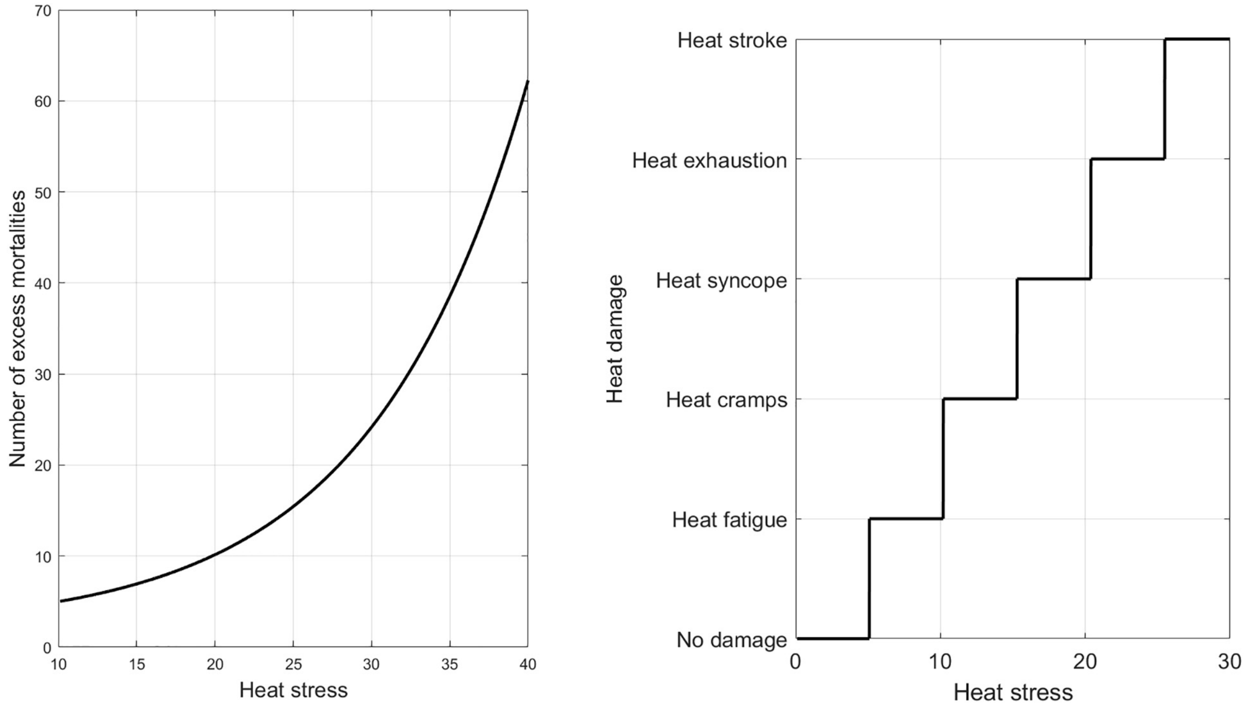

Fig. 2. Two examples of *damage functions* for heat stress related damages. The left column depicts a continuous *damage function* and the right column a discrete (categorical) *damage function*.

high importance. A walkway may be considered by the user as medium important. The *exposure process* encodes those aspects into the risk model.

Definition 7. (Exposure process).

The Exposure process, $E: \mathscr{X} \times \mathscr{T} \to \mathbb{R}$*, represents some notion of population density and involves demographic and economic conditions. The exposure map is denoted by* $E(\mathbf{x}, t)$*.*

In Fig. 3 we illustrate an example of an exposure map where the colors represent the spatial importance of each location.

We note here that both the *damage function* and the *exposure process* could be trivially generalised to vary with space and time, as well as by the action taken, without any conceptual modification. For the sake of ease of notation, we do not pursue this in this paper.

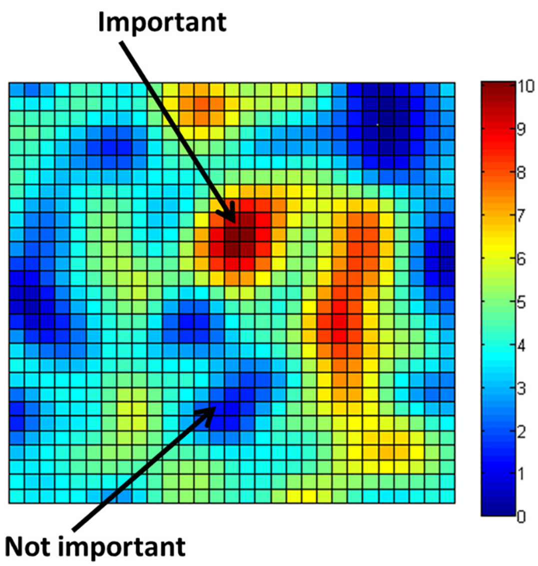

Fig. 3. An example of an exposure map: space is partitioned into small cells and each cell contain a value which represents its social importance.

2.7. Spatio-temporal Loss process

The *Loss process* process is a combination of the *damage function* applied to the TCI process, multiplied by the *exposure process*, resulting in a single spatio-temporal process.

Definition 8. (Loss Process (LP)).
The Loss Process is defined as a point-wise product of the damage function and exposure function, parameterised on a particular urban design scenario A:

$$\begin{aligned}\Omega(\mathbf{x},t;A) &:= \mathscr{D}(Z(\mathbf{x},t;A)) \times E(\mathbf{x},t)\\ &\overset{(3)}{=} \mathscr{D}(\widehat{Z}(\mathbf{x},t;A) + \varepsilon(\mathbf{x},t;A)) \times E(\mathbf{x},t)\end{aligned}$$

The *Loss Process* is a random process due to the fact that the weather types have a fixed set of probabilities, representing their frequency of appearance, and also, $\varepsilon(\mathbf{x},t;A)$ is a random process. This means that the *Loss Process* is a mixture of spatio-temporal Gaussian Processes.

Now that we have defined the *loss process*, we can ask the following question: how should one use the *loss process* to make an optimal decision? To answer this question, a decision rule needs to be defined. Before the decision rule can be defined, we first need to characterise the *loss process*. We do that in the next Section.

3. Loss variable characterisation & quantification

We want to assign a ranking in the form of a single number which summarises the “quality” of each design scenario. This can be understood as the consequence of taking action $a \in \mathscr{A}$ The general setting of this quality would be as follows:

$$L(W;A,\mathscr{X},\mathscr{T}) := \mathscr{S}(\Omega(\mathbf{x},t;A)),$$

where $\mathscr{S}(\Omega(\mathbf{x},t;A))$ is a summary of the spatio-temporal process $\Omega(\mathbf{x},t;A)$. Note that $L(W;A,\mathscr{X},\mathscr{T})$ is a random variable. Example of typical summaries include:

1) Maximum: $\mathscr{S}(\Omega(\mathbf{x},t;A);\mathscr{X},\mathscr{T}) := \max(\Omega(\mathbf{x},t;A))$, $\mathbf{x} \in \mathscr{X}$, $t \in \mathscr{T}$.
2) Median: $\mathscr{S}(\Omega(\mathbf{x},t;A);\mathscr{X},\mathscr{T}) := \text{med}(\Omega(\mathbf{x},t;A))$, $\mathbf{x} \in \mathscr{X}$, $t \in \mathscr{T}$, where $\text{med}([\alpha_1,\ldots,\alpha_K])$ is the Median of $\alpha_{1:K}$.
3) Max-Min: $\mathscr{S}(\Omega(\mathbf{x},t;A);\mathscr{X},\mathscr{T},\alpha) := \alpha\max(\Omega(\mathbf{x},t;A)) + (1-\alpha)\min(\Omega(\mathbf{x},t;A))$, $\mathbf{x} \in \mathscr{X}$, $t \in \mathscr{T}$, $\alpha \in (0,1]$.

In the following we present our choice for the summary of the *loss process*. The calculation of the summaries defined before can also be applied in a similar manner.

3.1. Normalized temporarily - spatially aggregated loss

A very intuitive such summary which we adopt in this paper is the average behavior of $\Omega(\mathbf{x},t;A)$. We denote this as the *aggregated Loss Variable* which corresponds to the normalized total (spatial and temporal) loss (economic, social and environmental) over region $\mathscr{X}$ and time $\mathscr{T}$ due to a specific urban design scenario A. It is of interest for making the optimal urban design choice (in some well defined sense).

Definition 9. (Normalized Temporarily - Spatially Aggregated Loss).
The Normalized Temporarily - Spatially Aggregated Loss over $\mathscr{X}$ and time duration $\mathscr{T}$ is defined as:

$$\begin{aligned}L(W,\Omega(\mathbf{x},t;A);A) &:= \frac{1}{|\mathscr{X}| \times |\mathscr{T}|}\int_{\mathscr{X}}\int_{\mathscr{T}} \Omega(\mathbf{x},t;A)\mathrm{d}\mathbf{x}\ \mathrm{d}t\\ &= \frac{1}{|\mathscr{X}| \times |\mathscr{T}|}\int_{\mathscr{X}}\int_{\mathscr{T}} (\mathscr{D}(\widehat{Z}(\mathbf{x},t;A) + \varepsilon(\mathbf{x},t;A)) \times E(\mathbf{x},t))\mathrm{d}\mathbf{x}\ \mathrm{d}t.\end{aligned}$$

The quantity $\int_{\mathscr{X}}\int_{\mathscr{T}} \Omega(\mathbf{x},t;A)\mathrm{d}\mathbf{x}\ \mathrm{d}t$ represents the aggregate loss over a region $\mathscr{X}$ and time period $\mathscr{T}$.

The most common way of quantifying the loss variable is via its first moment, that is, its statistical expectation, as defined in the following Lemma:

Lemma 1. *The Mean Risk is given by*:

$$\begin{aligned}
\mathcal{R}\ \ &(W,\Omega(\mathbf{x},t;A);A) := \mathbb{E}_{W,\varepsilon(\mathbf{x},t;A)}[L(W,\Omega(\mathbf{x},t;A);A)]\\
&= \sum_{w=1}^{|\mathscr{W}|}\int_{\varepsilon(\mathbf{x},t;A)} \Pr(W=w)\int_{\mathscr{X}}\int_{\mathscr{T}}\frac{1}{|\mathscr{X}|\times\mathscr{T}}p(\Omega(\mathbf{x},t;A))\Omega(\mathbf{x},t\mid W;A)\mathrm{d}\mathbf{x}\ \mathrm{d}t\ \ \mathrm{d}\Omega(\mathbf{x},t;A)\\
&= \frac{1}{|\mathscr{X}|\times\mathscr{T}}\sum_{w=1}^{|\mathscr{W}|}\left\{\Pr(W=w)\int_{\mathscr{X}}\int_{\mathscr{T}}\left(\int_{\varepsilon(\mathbf{x},t;A)}\mathscr{D}(Z(\mathbf{x},t;A))\mathrm{d}\varepsilon(\mathbf{x},t;A)\right)E(\mathbf{x},t)\mathrm{d}\mathbf{x}\ \mathrm{d}t\right\}\\
&= \frac{1}{|\mathscr{X}|\times\mathscr{T}}\sum_{w=1}^{|\mathscr{W}|}\left\{\Pr(W=w)\int_{\mathscr{X}}\int_{\mathscr{T}}\mathbb{E}_{\varepsilon(\mathbf{x},t;A)}[\mathscr{D}(\widehat{Z}(\mathbf{x},t\mid W=w;A)+\varepsilon(\mathbf{x},t;A))]E(\mathbf{x},t)\mathrm{d}\mathbf{x}\ \mathrm{d}t\right\},
\end{aligned} \tag{5}$$

where we used Fubini's theorem in order to change the order of the integrals.

Clearly, the exact calculation of the risk depends on the choice of the *damage function*, and is therefore expressed generically as $\mathbb{E}_{\varepsilon(\mathbf{x},t;A)}[\mathscr{D}(\widehat{Z}(\mathbf{x},t\mid W=w;A)+\varepsilon(\mathbf{x},t;A))]$. In Section IV we derive the exact expression of the risk for various general *damage functions*.

3.2. Optimal urban design via mean risk minimisation

Now that we have a well defined way of ranking designs we can formulate the optimal design selection problem, by finding the urban design which minimises the risk measure in (5), as well as the budgetary cost of implementation. This is given in (2) as follows:

$$\mathscr{U}(\mathscr{X},\mathscr{T},A,\pi) := \pi\mathcal{R}(W,\Omega(\mathbf{x},t;A);A) + (1-\pi)B(A) \tag{6}$$

where $0 \le \pi \le 1$, and $B(A)$ is the budgetary cost of implementing the A-th urban design. The optimal design (from a finite set of pre-defined set $\mathscr{A}$) is given by the solution to:

$$\widehat{A} = \arg\min_{A\in\mathscr{A}} \mathscr{U}(\mathscr{X},\mathscr{T},A,\pi,\lambda). \tag{7}$$

4. Risk derivation for specific damage functions

To complete our framework, we now show how the expectation in (5) can be calculated for a few examples of *damage functions*. Those include the *Acceptance set*, *Staircase criterion* and *Polynomial damage functions*. For *damage functions* which do not have a closed form expression of the expectation, we develop Taylor series expansion to approximate the damage function and calculate the expectation, as presented by *Generic functions*.

4.1. Acceptance set

The *Acceptance set damage function* is binary: 0 if the TCI process is within a predefined interval and 1 otherwise.

$$\mathscr{D}_1(\alpha;\lambda_1,\lambda_2) = \begin{cases}0, & \lambda_1 \le \alpha \le \lambda_2\\ 1, & \text{Otherwise}\end{cases} \tag{8}$$

The expectation in (5) is given by

$$\begin{aligned}
\mathbb{E}_{\varepsilon(\mathbf{x},t;A)}[\mathscr{D}_1(Z(\mathbf{x},t\mid W=w;A))] &= \mathbb{E}_{\varepsilon(\mathbf{x},t;A)}[\mathscr{D}_1(Z(\mathbf{x},t\mid W=w;A))]\\
&= \mathbb{E}_{\varepsilon(\mathbf{x},t;A)}[\mathscr{D}_1(\widehat{Z}(\mathbf{x},t\mid W=w;A)+\varepsilon(\mathbf{x},t;A))]\\
&= 1\times\Pr(\lambda_1 \ge Z(\mathbf{x},t\mid W=w;A) \cup Z(\mathbf{x},t\mid W=w;A) \ge \lambda_2)\\
&= 1-\Pr(\lambda_1 \le Z(\mathbf{x},t\mid W=w;A) \le \lambda_2)\\
&= 1-\Pr(\lambda_1 \le \widehat{Z}(\mathbf{x},t\mid W=w;A)+\varepsilon(\mathbf{x},t;A) \le \lambda_2)\\
&= 1-\Pr(\lambda_1-\widehat{Z}(\mathbf{x},t\mid W=w;A) \le \varepsilon(\mathbf{x},t;A) \le \lambda_2-\widehat{Z}(\mathbf{x},t\mid W=w;A))\\
&= 1-\Pr\left(\frac{\lambda_1-\widehat{Z}(\mathbf{x},t\mid W=w;A)}{\sigma_w} \le \nu \le \frac{\lambda_2-\widehat{Z}(\mathbf{x},t\mid W=w;A)}{\sigma_w}\right)\\
&= 1-\Phi\left(\frac{\lambda_2-\widehat{Z}(\mathbf{x},t\mid W=w;A)}{\sigma_w}\right)+\Phi\left(\frac{\lambda_1-\widehat{Z}(\mathbf{x},t\mid W=w;A)}{\sigma_w}\right),
\end{aligned} \tag{9}$$

where $\nu \sim \mathscr{N}(0,1)$. [width = 19 cm].

Proposition 1. *The Risk measure due to damage function $\mathscr{D}_1(\alpha;\lambda_1,\lambda_2)$ is given by*:

$$\begin{aligned}&\mathcal{R}(W,\Omega(\mathbf{x},t;A);A)\\&=\frac{1}{|\mathcal{X}|\times\mathcal{T}}\sum_{w=1}^{|\mathcal{W}|}\left\{\Pr(W=w)\int_{\mathcal{X}}\int_{\mathcal{T}}\left(1-\Phi\left(\frac{\lambda_2-\widehat{Z}(\mathbf{x},t\mid W=w;A)}{\sigma_w}\right)+\Phi\left(\frac{\lambda_1-\widehat{Z}(\mathbf{x},t\mid W=w;A)}{\sigma_w}\right)\right)E(\mathbf{x},t)\mathrm{d}\mathbf{x}\ \mathrm{d}t\right\}\\&=\frac{1}{|\mathcal{X}|\times\mathcal{T}}\sum_{w=1}^{|\mathcal{W}|}\left\{\Pr(W=w)\int_{\mathcal{X}}\int_{\mathcal{T}}\left(\Phi\left(\frac{\lambda_1-\widehat{Z}(\mathbf{x},t\mid W=w;A)}{\sigma_w}\right)-\Phi\left(\frac{\lambda_2-\widehat{Z}(\mathbf{x},t\mid W=w;A)}{\sigma_w}\right)\right)E(\mathbf{x},t)\mathrm{d}\mathbf{x}\ \mathrm{d}t\right\}\\&+\int_{\mathcal{X}}\int_{\mathcal{T}}E(\mathbf{x},t)\mathrm{d}\mathbf{x}\ \mathrm{d}t.\end{aligned}$$

4.2. Staircase set

The *Staircase set damage function* is a multilevel response for a set of thresholds $\{\lambda_k\}_{j=1}{}^{k}$, where $\lambda_j \geq \lambda_k, \forall j \geq k$, and the corresponding values $\{\gamma_k\}_{j=1}{}^{k}$.

$$\mathcal{D}_2(\alpha;\lambda_1,\lambda_2,\cdots,\lambda_k,\gamma_1,\gamma_2,\cdots,\gamma_k)=\begin{cases}\gamma_1, & \lambda_1\geq\alpha\\ \gamma_2, & \lambda_1\leq\alpha\leq\lambda_2\\ \vdots, & \vdots\\ \gamma_k, & \lambda_K\leq\alpha\leq\lambda_{K+1}\end{cases}\tag{10}$$

where $\lambda_0 = -\infty$, $\lambda_{K+1} = \infty$. The expectation in (5) is given by

$$\begin{aligned}\mathbb{E}_{\varepsilon(\mathbf{x},t;A)}[\mathcal{D}_2(Z(\mathbf{x},t\mid W=w;A))]&=\sum_{k=0}^{K}\Pr(\lambda_k\leq Z(\mathbf{x},t\mid W=w;A)\cap Z(\mathbf{x},t\mid W=w;A)\leq\lambda_{k+1})\times\gamma_k\\&=\sum_{k=0}^{K}\Pr(\lambda_k\leq\widehat{Z}(\mathbf{x},t\mid W=w;A)+\varepsilon(\mathbf{x},t;A)\cap\widehat{Z}(\mathbf{x},t\mid W=w;A)+\varepsilon(\mathbf{x},t;A)\leq\lambda_{k+1})\times\gamma_k\\&=\sum_{k=0}^{K}\left(\Phi\left(\frac{\lambda_{k+1}-\widehat{Z}(\mathbf{x},t\mid W=w;A)}{\sigma_w}\right)-\Phi\left(\frac{\lambda_k-\widehat{Z}(\mathbf{x},t\mid W=w;A)}{\sigma_w}\right)\right)\times\gamma_k.\end{aligned}\tag{11}$$

Proposition 2. *The Risk measure due the damage function* $\mathcal{D}_2(\alpha;\lambda_1,\lambda_2)$ *is given by*:

$$\begin{aligned}&\mathcal{R}(W,\Omega(\mathbf{x},t;A);A)=\frac{1}{|\mathcal{X}|\times\mathcal{T}}\\&\times\sum_{w=1}^{|\mathcal{W}|}\left\{\Pr(W=w)\int_{\mathcal{X}}\int_{\mathcal{T}}\left(\sum_{k=1}^{K}\left(\Phi\left(\frac{\lambda_{k+1}-\widehat{Z}(\mathbf{x},t\mid W=w;A)}{\sigma_w}\right)-\Phi\left(\frac{\lambda_k-\widehat{Z}(\mathbf{x},t\mid W=w;A)}{\sigma_w}\right)\right)\times\gamma_k\right)E(\mathbf{x},t)\mathrm{d}\mathbf{x}\ \mathrm{d}t\right\}.\end{aligned}$$

4.3. Polynomial function

The *Polynomial function damage function* is given by:

$$\mathcal{D}_3(\alpha;\lambda_1,\lambda_2,\cdots,\lambda_k)=\sum_{n=0}^{k-1}\lambda_n\alpha^n.\tag{12}$$

The expectation in (5) is given by

$$\begin{aligned}\mathbb{E}_{\varepsilon(\mathbf{x},t;A)}[\mathcal{D}_3(Z(\mathbf{x},t\mid W=w;A))]&=\mathbb{E}_{\varepsilon(\mathbf{x},t;A)}\left[\sum_{n=0}^{k-1}\lambda_n(Z(\mathbf{x},t\mid W=w;A))^n\right]\\&=\mathbb{E}_{\varepsilon(\mathbf{x},t;A)}\left[\sum_{n=0}^{k-1}\lambda_n(\widehat{Z}(\mathbf{x},t\mid W=w;A)+\varepsilon(\mathbf{x},t;A))^n\right]\\&=\sum_{n=0}^{k-1}\lambda_n\mathbb{E}_{\varepsilon(\mathbf{x},t;A)}[(\widehat{Z}(\mathbf{x},t\mid W=w;A)+\varepsilon(\mathbf{x},t;A))^n].\end{aligned}\tag{13}$$

To solve these expectations we require the moments of non-central Normal random variable, presented next:

Lemma 2. (Moments of Normal random variables (Papoulis and Pillai, 2002)).

The p-th moment of $X \sim N(\mu_x, \sigma_x{}^2)$ *is given by*

$$\mathbb{E}[x^p] = \sigma_x^p(-i\sqrt{2})^p U\left(-\frac{1}{2}p, \frac{1}{2}, -\frac{1}{2}\left(\frac{\mu_x}{\sigma_x}\right)^2\right),$$

where $U(a, b, z)$ is the Tricomi's confluent function (Tricomi, 1952).

Plugging this expression into (13) leads to the following result.

Proposition 3. *The Risk measure due to damage function $\mathscr{D}_3(\alpha; \lambda_1, \dots, \lambda_k)$ is given by:*

$$\mathcal{R}(W, \Omega(\mathbf{x}, t; A); A) = \frac{1}{|\mathscr{X}| \times \mathscr{T}} \times \sum_{w=1}^{|\mathscr{W}|} \left\{ \Pr(W = w) \int_{\mathscr{X}} \int_{\mathscr{T}} \sum_{n=0}^{k-1} \lambda_n \sigma_w^n (-i\sqrt{2})^n U\left(-\frac{1}{2}n, \frac{1}{2}, -\frac{1}{2}\left(\frac{\widehat{Z}(\mathbf{x}, t \mid W = w; A)}{\sigma_w}\right)^2\right) E(\mathbf{x}, t) \mathrm{d}\mathbf{x} \; \mathrm{d}t \right\}.$$

4.4. Generic functions (approximation):

In some cases, due to the complexity of the *damage function*, there will be no simple way to calculate the expectation in closed form. To overcome this we develop a series expansion approximation of the *damage function* which will allow to approximate the expectation. The Taylor series expansion of the function $D(\alpha)$ about a point $\alpha = \alpha_*$ is

$$\begin{aligned} D(\alpha) &= D(\alpha_*) + \frac{dD(\alpha_*)}{\alpha}(\alpha - \alpha_*) + \frac{1}{2}\frac{d^2D(\alpha_*)}{\alpha^2}(\alpha - \alpha_*)^2 + \frac{1}{3!}\frac{d^3D(\alpha_*)}{\alpha^2}(\alpha - \alpha_*)^3 + \cdots \\ &= \sum_{n=0}^{\infty} \frac{D^{(n)}(\alpha_*)}{n!}(\alpha - \alpha_*)^n, \end{aligned} \tag{14}$$

where $D^{(n)}(\alpha_*)$ is the n-th derivative of $D(\alpha)$ with respect to α at point α_*. We set $\alpha_* = \widehat{Z}(\mathbf{x}, t \mid W = w; A)$ and write down the first three terms of the series:

$$\begin{aligned} D(\alpha) \approx D(\widehat{Z}(\mathbf{x}, t \mid W = w; A)) &+ \frac{dD(\widehat{Z}(\mathbf{x}, t \mid W = w; A))}{\alpha}(\alpha - \widehat{Z}(\mathbf{x}, t \mid W = w; A)) \\ &+ \frac{1}{2!}\frac{d^2D(\widehat{Z}(\mathbf{x}, t \mid W = w; A))}{\alpha^2}(\alpha - \widehat{Z}(\mathbf{x}, t \mid W = w; A))^2. \end{aligned} \tag{15}$$

Then, the expectation in (5) is given by:

$$\begin{aligned} &\mathbb{E}_{\varepsilon(\mathbf{x},t;A)}[\mathscr{D}(\widehat{Z}(\mathbf{x}, t \mid W = w; A) + \varepsilon(\mathbf{x}, t; A))] \\ &\overset{(15)}{\approx} \mathbb{E}_{\varepsilon(\mathbf{x},t;A)}\left[D(\widehat{Z}(\mathbf{x}, t \mid W = w; A)) + \frac{dD(\widehat{Z}(\mathbf{x}, t \mid W = w; A))}{\alpha}\varepsilon(\mathbf{x}, t; A) + \frac{1}{2!}\frac{d^2D(\widehat{Z}(\mathbf{x}, t \mid W = w; A))}{\alpha^2}\varepsilon(\mathbf{x}, t; A)^2\right] \\ &= D(\widehat{Z}(\mathbf{x}, t \mid W = w; A)) + \frac{dD(\widehat{Z}(\mathbf{x}, t \mid W = w; A))}{\alpha}\underbrace{\mathbb{E}_{\varepsilon(\mathbf{x},t;A)}[\varepsilon(\mathbf{x}, t; A)]}_{0} \\ &+ \frac{1}{2}\frac{d^2D(\widehat{Z}(\mathbf{x}, t \mid W = w; A))}{\alpha^2}\mathbb{E}_{\varepsilon(\mathbf{x},t;A)}[\varepsilon(\mathbf{x}, t; A)^2] \\ &= D(\widehat{Z}(\mathbf{x}, t \mid W = w; A)) + \frac{1}{2}\frac{d^2D(\widehat{Z}(\mathbf{x}, t \mid W = w; A))}{\alpha^2}\sigma_w^2. \end{aligned} \tag{16}$$

Proposition 4. *The Risk measure due to the damage function $\mathscr{D}_3(\alpha; \lambda_1, \lambda_2)$ is given by:*

$$\mathcal{R}(W, \Omega(\mathbf{x}, t; A); A) \approx \frac{1}{|\mathscr{X}| \times \mathscr{T}} \times \sum_{w=1}^{|\mathscr{W}|} \left\{ \Pr(W = w) \int_{\mathscr{X}} \int_{\mathscr{T}} \left(D(\widehat{Z}(\mathbf{x}, t \mid W = w; A)) + \frac{1}{2}\frac{d^2D(\widehat{Z}(\mathbf{x}, t \mid W = w; A))}{\alpha^2}\sigma_w^2 \right) E(\mathbf{x}, t) \mathrm{d}\mathbf{x} \; \mathrm{d}t \right\}.$$

5. Simulations

In this Section we illustrate how our recommendation system can be used in practice using a detailed numerical analysis.

The purpose of this section is to illustrate how our framework could be used in practice. It is not intended to make explicit design recommendations for this specific study area, but rather, to complement the methodology developed in the previous sections and illustrate how all the aspects of the framework should be considered. It is important to understand that since our framework is quite general, there are many specific choices that the user needs to make and tailor to the particular requirements of the system. It is also important to note that our framework does not:

1) select which urban designs should be considered.
2) decide which weather types to choose.
3) select a climate model.
4) set the cost of implementation of each urban design.
5) decide what the *exposure map* should be.
6) which *damage function to use*.
7) how to set the value of the weight π.

These aspects are all outside the scope of this paper and should be chosen judicially by the user. However, in order for us to present our framework we make specific choices which help illustrate how the framework can work in practice.

In addition, the following comments should be carefully considered:

1) We do not specify which TCI to use, but instead, we provide the user the freedom to choose whatever TCI process they would like to use (see Definition 3 and the discussion that follows).
2) It is not the scope of this work to propose which climate model to use and we leave this to the discretion of the user (see Definition 3 and the discussion that follows)).
3) The scope of this paper is optimal urban design through the lens of thermal comfort considerations. However, one could consider other processes in the same manner. For example, one could be interested in the level of pollution by investigating the concentration of CO_2 or other polluting particles.
4) The choice of an appropriate *damage function* is paramount. We provided a generic framework to utilise any *damage function*, and leave it to the user to decide what the *damage function* should be. More information about how to design *damage functions* can be found here (Prahl et al., 2015; Prahl et al., 2016).
5) The choice of the weight can and will affect the final decision, as depicted in Fig. 9. The choice of the weight shoul'd be chosen to reflect the preferences of the urban designer which needs to analyse the problem in a holistic way.
6) The importance of choosing the appropriate *damage function* for the particular problem at hand is crucial. Our results in Figs. 9–10 show how the use of different *damage functions* leads to a different choices of optimal design.

These aspects are all outside the scope of this paper and should be chosen judicially by the user. However, in order for us to illustrate our framework we provide specific choices.

We begin by providing the background of the study area, then explain the Envi-met simulation set-up and the estimation procedure of the weather types, followed by the urban design scenarios. Finally we demonstrate the optimal urban design for a few choices of *damage functions.* We illustrate how our risk measure framework operates by providing some specific examples and their interpretations. We note that a presentation of the implementation of the tool can be found here https://www.youtube.com/watch?.

5.1. Study area

Singapore is an island city-state located around a degree north of the Equator in Southeast Asia, with an area of 721.5 km^2 and a population of 5.61 million inhabitants. Due to the island's climate, classified as a tropical rainforest climate, it is characterized by uniform temperature and pressure, high humidity and abundant rainfall all year long. The two monsoon seasons are characterized by heavy rainfalls and higher wind speeds - Northeast monsoon from December to March and Southwest monsoon from June to September. The hottest period of the year is from May to June (mean daily temperature of 26.0C), the coolest from December to January (mean daily temperature of 27.8C). The study area is a relative young public housing estate built in 2004/2005 located Northeast of the island, in the planning area “Punggol” and subzone “Punggol Central”. The selected residential estate is composed by a central courtyard surrounded by six similar residential building towers and two similar car parks. The towers have approximately 20 floors and the car parks six floors. The courtyard offers a variety of uses such as a badminton field, two playgrounds, an urban gym, two shaded pavilions, walkways, green areas (with trees, bushes and grass), and parking lots, see Fig. 4. These spatial uses have been categorized by levels of importance (5-scale), see Table 1 and translated into an exposure map, see Fig. 5.

5.2. Simulation set-up

ENVI-met is a micro-climate model commonly used to assess the OTC of the urban environment. It simulates surface-plant-atmosphere interactions with a spatial resolution between 0.5 and 10 m. Airflow is solved with the standard $k - \varepsilon$ turbulence closure model for the Reynolds Averaged Navier-Stokes (RANS) equations. The model, with certain approximation, considers a complete radiation budget (i.e. direct, reflected and diffused solar radiation and longwave radiation). Although the region of interest is focused on a courtyard (as described previously), a bigger domain has been modelled and simulated in ENVI-met V4.3 to consider the

Fig. 4. The study area of Punggol is a residential estate, composed of a central courtyard surrounded by six similar residential building towers and two similar car parks.

Table 1
Exposure map - spatial uses and their associated scores.

Scale	Description	Numerical value
Very important	Playground, urban gym and badminton field	5
Important	Shaded pavilion	4
Neutral	Walkways and sitting area	3
Unimportant	Green area and transition space	2
Very unimportant	Parking lot and road	1

Fig. 5. Exposure map of the Punggol estate representing the spatial use of the area via 5-scale categorical values, see Table 1.

influence of the surrounding urban elements on the micro-climate of the courtyard. The horizontal spatial resolution of the domain was 3 × 3 m. The vertical grid size was 2 m in the first five lowest levels (i.e. up to 10 m. a.g.l.). From this height upwards the vertical grid size increased with a telescoping factor of 18%. Despite the surface of the courtyard is close to 8,500 m^2 (close to be a rectangle of ≈170 × 50 m), the whole model domain covered a horizontal area of 399 × 414 m and extended in the vertical to 122m.

In the current simulations all buildings had the same material (i.e. concrete slab) and only grass was included as vegetative element in the domain. The non-vegetated ground was covered with red brick road, concrete pavement and traditional black asphalt road.

Meteorological conditions were extracted from weather stations located at Seletar and Changi (¡10 km distant from the case

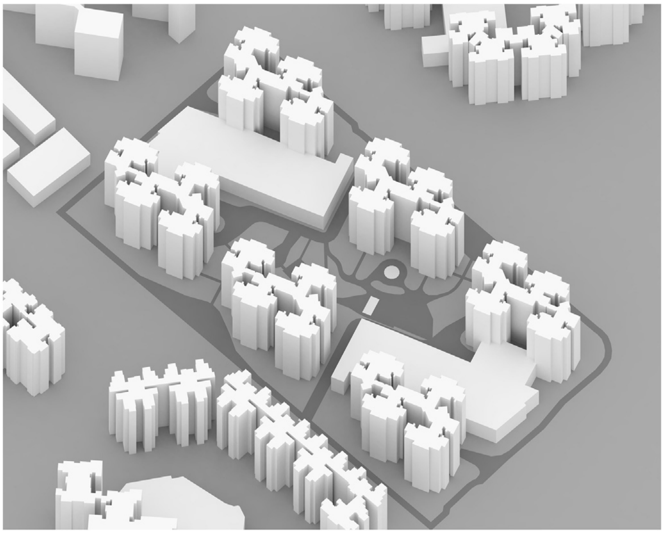

Fig. 6. The baselinecase study model, before the implementation of any action.

study). These were clustered into ten representative weather types. In the simulations, the variables air temperature (Ta) and relative humidity (RH) were considered with their hourly profiles while the rest of the meteorological parameters were considered as constants (i.e. wind speed (WS), wind direction (WD), and Cloud Cover). Table 3 summarises the meteorological parameters for ENVI-met as calculated from the cluster analysis of the weather types (WTs). The simulations were launched at 4 : 00am LAT (Local Apparent Time, GMT + 7), approximately 2 hours before sunrise, and run for 40 hours. Outputs of the model were analyzed for the last 24 hours of simulation (i.e. a complete diurnal cycle) so as to guarantee a suitable spin-up of the model. A validation of the model was performed by comparing simulation results with measured data deriving from a dedicated in-field monitoring campaign. The case study model in his current condition, before the implementation of any action aimed to improve the thermal comfort, is presented in Fig. 6. The weather types classification is based on a clustering procedure described in Acero et al. (Acero et al., 2018). These are representative climatic conditions that affect the area of study. By using weather types we reduce significantly the time to estimate the long-term impact on OTC.

5.3. Urban design scenarios

The urban designs we consider are presented in Table 2. This set of 15 elements constitutes our possible set of actions $\mathscr{A}$ and the corresponding set of budgetary costs $B(A)$. The current condition of the existing residential estate is defined as the *baseline*. The different design strategies are each defined as *scenario*. These scenarios aim to improve the OTC of the residents within the courtyard, through the use of passive cooling solutions. Examples include the increase of urban ventilation (i.e. building porosity), reduction of incoming solar radiation (i.e. shading) and lowering the surface and ambient temperature (i.e. cool materials and vegetation). The (hypothetical) cost of implementing each of the urban designs is defined in Table 2. These figures are used merely for the purpose of presentation and do not represent the “true” costs which need to be carefully calculated, and are outside the scope of this paper.

Table 2
List of urban scenarios and their associated hypothetical costs.

Scenario	Description	Hypothetical implementation cost [normalized units of effort]
Baseline	Current condition	1
S_1	Removing car parks	9
S_2	Partially removing car parks	8
S_3	Green roofs on car parks	2
S_4	Green facades on car parks	2
S_5	Green facades on towers at 4 m height	3
S_6	Green facades on towers at 6 m height	3
S_7	Green facades on towers at 10 m height	5
S_8	33% tree coverage in courtyard	4
S_9	33% tree coverage in courtyard without car parks	6
S_{10}	66% tree coverage in courtyard	5
S_{11}	66% tree coverage in courtyard without car parks	7
S_{12}	Void decks on ground floor of towers	9
S_{13}	Urban canopy over courtyard	10
S_{14}	Urban canopy over courtyard without car parks	11

Table 3
Meteorological parameters associated to each WTs.

	Weather types									
	1	2	3	4	5	6	7	8	9	10
Simulation reference date	15th Dec	15th Feb	15th Apr	15th Mar	15th Oct	15th Jan	15th Sep	15th Jul	15th Jun	15th Mar
Wind Speed (m/s) (10 m a.g.l.)	0.8	3.7	1.5	0.7	0.4	2.1	1.6	2.4	1.7	4.8
Wind direction (∘)	350.4	34.3	88.8	353.8	172.8	1.1	228.5	177.6	236.3	22.3
Minimum air temperature (∘C)	25.9 (6:00 LAT)	26.7 (6:00 LAT)	27.3 (6:00 LAT)	25.9 (6:00 LAT)	26.9 (6:00 LAT)	25.6 (6:00 LAT)	26.7 (6:00 LAT)	27.5 (6:00 LAT)	26.1 (6:00 LAT)	26.4 (6:00 LAT)
Maximum air temperature(∘C)	29.2 (12:00 LAT)	31.9 (12:00 LAT)	33.7 (13:00 LAT)	31.6 (13:00 LAT)	31.4 (13:00 LAT)	27 (15:00 LAT)	31.8 (14:00 LAT)	33.6 (13:00 LAT)	33.5 (14:00 LAT)	32.4 (13:00 LAT)
Minimum relative humidity(%)	71.6 (12:00 LAT)	59.9 (13:00 LAT)	54.0 (13:00 LAT)	60 (13:00 LAT)	65.8 (12:00 LAT)	81.3 (15:00 LAT)	60.2 (14:00 LAT)	53 (13:00 LAT)	50.3 (14:00 LAT)	52.2 (14:00 LAT)
Maximum relative humidity (%)	87.9 (6:00 LAT)	89.6 (6:00 LAT)	87.8 (5:00 LAT)	88.1 (6:00 LAT)	89.2 (6:00 LAT)	87.2 (6:00 LAT)	85 (6:00 LAT)	84.5 (6:00 LAT)	83.5 (4:00 LAT)	83.9 (6:00 LAT)
Cloud cover (oktas)	7.2	6.9	7	7.1	7	7.6	7.1	6.9	7	6.8
Specific humidity (2500 m)	9.8 g/kg (corresponding to year 2016 mean value)									
$Pr(W = w)$	0.1042	0.0774	0.0744	0.1161	0.1042	0.0476	0.1220	0.0833	0.1429	0.1280

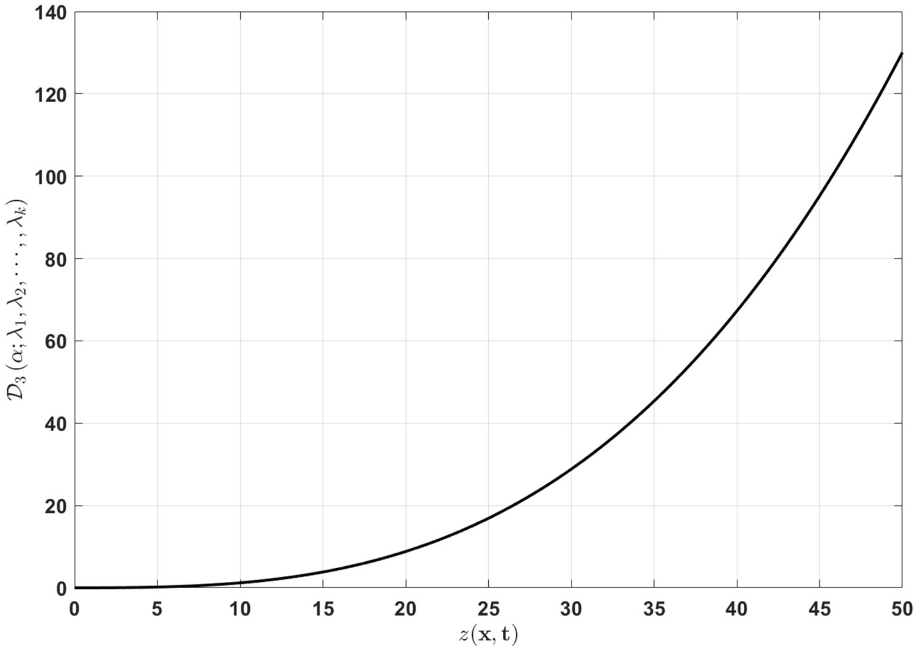

Fig. 7. Example of $\mathscr{D}_3(\alpha; \lambda_1,..., \lambda_k)$.

5.4. Climate sensitive urban design decision tool simulation results

We now illustrate our *climate sensitive urban design decision tool* via a few examples. To this end we still need to specify the *damage function* $\mathscr{D}(\alpha)$, and the weight π in (6). We illustrate our results based on three types of *damage functions* $\mathscr{D}_1(\alpha; \lambda_1, \lambda_2)$, $\mathscr{D}_2(\alpha; \lambda_1, \lambda_2, \cdots,, \lambda_k)$ and $\mathscr{D}_3(\alpha; \lambda_1, \lambda_2, \cdots, \lambda_k)$ as defined in the precious Section. The parameters are set as follows:

1) *Acceptability criterion* $\mathscr{D}_1(\alpha; \lambda_1, \lambda_2)$: $\lambda_1 = 25$, $\lambda_2 = 27$.
2) *Staircase criterion* $\mathscr{D}_2(\alpha; \lambda_1, \lambda_2, \cdots,, \lambda_k)$: $\lambda_1 = 21$, $\lambda_2 = 22$, $\lambda_3 = 23$, $\lambda_4 = 24$, $\lambda_5 = 25$, $\lambda_6 = 27$, $\lambda_7 = 27$, $\lambda_8 = 28$ and $\gamma_1 = 0$, $\gamma_2 = 1$, $\gamma_3 = 2$, $\gamma_4 = 3$, $\gamma_5 = 4$, $\gamma_6 = 5$, $\gamma_7 = 6$, $\gamma_8 = 7$.
3) *Polynomial function* $\mathscr{D}_3(\alpha; \lambda_1, \lambda_2, \cdots, \lambda_k)$: $\lambda_1 = 1 \times 10^{-3}$, $\lambda_2 = 3 \times 10^{-3}$, $\lambda_3 = 1 \times 10^{-3}$, $\lambda_4 = 1 \times 10^{-3}$, and the damage function is presented in Fig. 7. For this case we have that the first four moments are given by:

$$n = 0: \mathbb{E}_{\varepsilon(\mathbf{x},t;A)}[(\widehat{Z}(\mathbf{x}, t \mid W = w; A) + \varepsilon(\mathbf{x}, t; A))^0] = 1,$$
$$n = 1: \mathbb{E}_{\varepsilon(\mathbf{x},t;A)}[(\widehat{Z}(\mathbf{x}, t \mid W = w; A) + \varepsilon(\mathbf{x}, t; A))^1] = \widehat{Z}(\mathbf{x}, t \mid W = w; A),$$
$$n = 2: \mathbb{E}_{\varepsilon(\mathbf{x},t;A)}[(\widehat{Z}(\mathbf{x}, t \mid W = w; A) + \varepsilon(\mathbf{x}, t; A))^2] = \widehat{Z}(\mathbf{x}, t \mid W = w; A)^2 + \sigma_w^2,$$
$$n = 3: \mathbb{E}_{\varepsilon(\mathbf{x},t;A)}[(\widehat{Z}(\mathbf{x}, t \mid W = w; A) + \varepsilon(\mathbf{x}, t; A))^3] = \widehat{Z}(\mathbf{x}, t \mid W = w; A)^3 + 3\widehat{Z}(\mathbf{x}, t \mid W = w; A)\sigma_w^2.$$

Remark 1. We note that the selection of the damage functions and the weights is for demonstration purposes and are not to be understood as "optimal" in any sense. The selection of these parameters should be tailored to the particular utility function the decision maker is interested in. More information about how to design damage functions can be found here (Prahl et al., 2015; Prahl et al., 2016).

We also note that for ease of presentation all the scores were re-noramalised to lie within the range of [0,100].

1) *Impact of the weight* π *on the utility function (6):* The choice of the weight π controls and balances between the environmental-social aspects of the design which are captured by $\mathscr{R}(W, \Omega(\mathbf{x}, t; A); A)$, and the cost of implementation $B(A)$. As π approaches 1, the environmental-social aspects are more heavily weighed compared with the cost of implementation. To illustrate this we present a study where we utilsed the *damage function* $\mathscr{D}_2(\alpha; \lambda_1, \lambda_2, \cdots,, \lambda_k)$, for various values of π. The results are presented in Fig. 8, where the symbol × shows which urban scenario has the best score. As we observe, for different values of π, different urban designs are chosen. This result illustrates the importance of choosing the value of π judicially with domain experts in order to reflect the balance between the environmental-social and cost of implementation aspects.

2) *Impact of the climate model uncertainty* σ *on the utility function (6):* We now study the importance of incorporating the climate model uncertainty σ into the utility function. To this end we compare two cases:

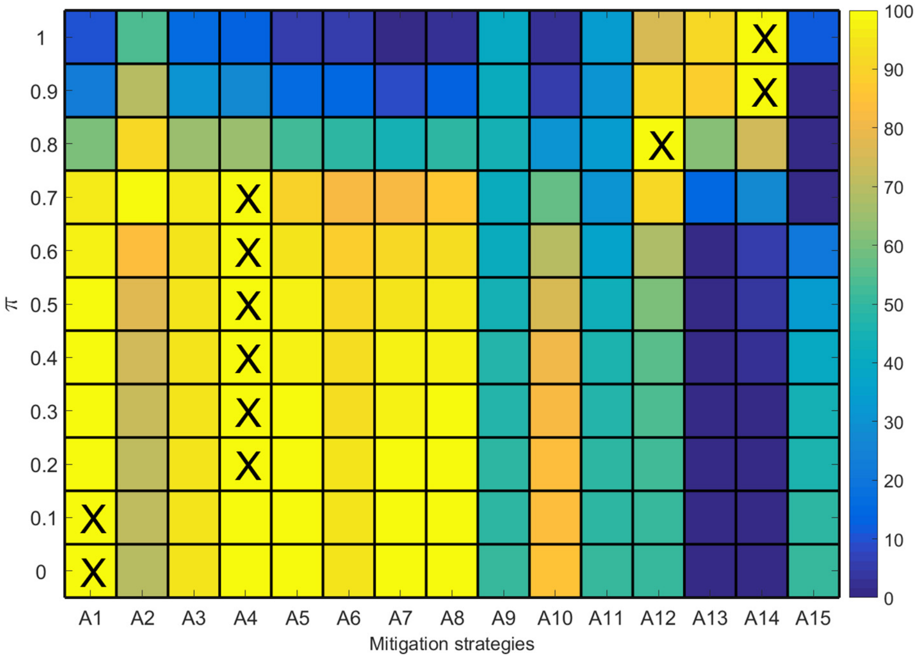

Fig. 8. Risk matrix as a function of $\pi = \{0,0.1,\ldots,1\}$ for $\mathscr{D}_2(\alpha;\lambda_1,\lambda_2)$. The symbol × shows which urban scenario has the best score for each value of π.

1) Case 1: we neglect the climate model uncertainty by setting $\sigma = 0$.
2) Case 2: we incorporate the climate model uncertainty by setting $\sigma = 1$.

We study this effect for two *damage functions* $\mathscr{D}_1(\alpha;\lambda_1,\lambda_2)$ and $\mathscr{D}_2(\alpha;\lambda_1,\lambda_2,\cdots,\lambda_k)$, with the same parameters as before. In order to isolate the impact of the implementation costs, we set $\pi = 1$ (no cost consideration). The results are presented in Figs. 9–10, respectively. The results show the important result that incorporating climate model uncertainty can lead to different solution then when neglecting its effect. In the first case, when not considering the uncertainty, the optimal choice is $A10$, while when considering the uncertainty, the optimal choice is $A12$. In the second case, when not considering the uncertainty, the optimal choice is $A14$, while when considering the uncertainty, the optimal choice is $A12$.

These examples illustrate the importance of the choice of the weights and how they can affect the final decision. The choice of the

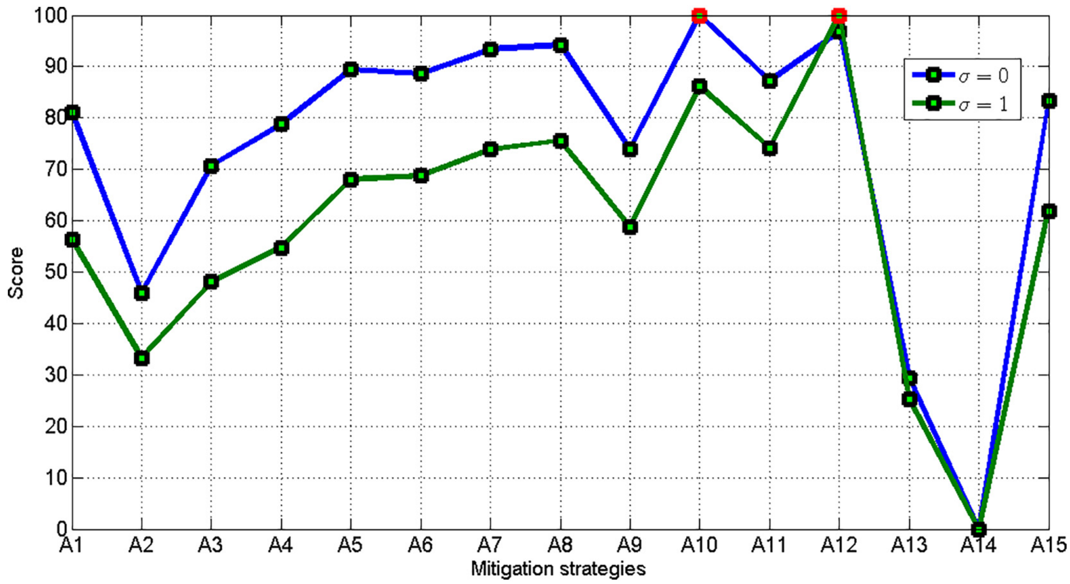

Fig. 9. Optimal urban design under $\mathscr{D}_1(\alpha;\lambda_1,\lambda_2)$ for $\sigma = \{0,1\}$.

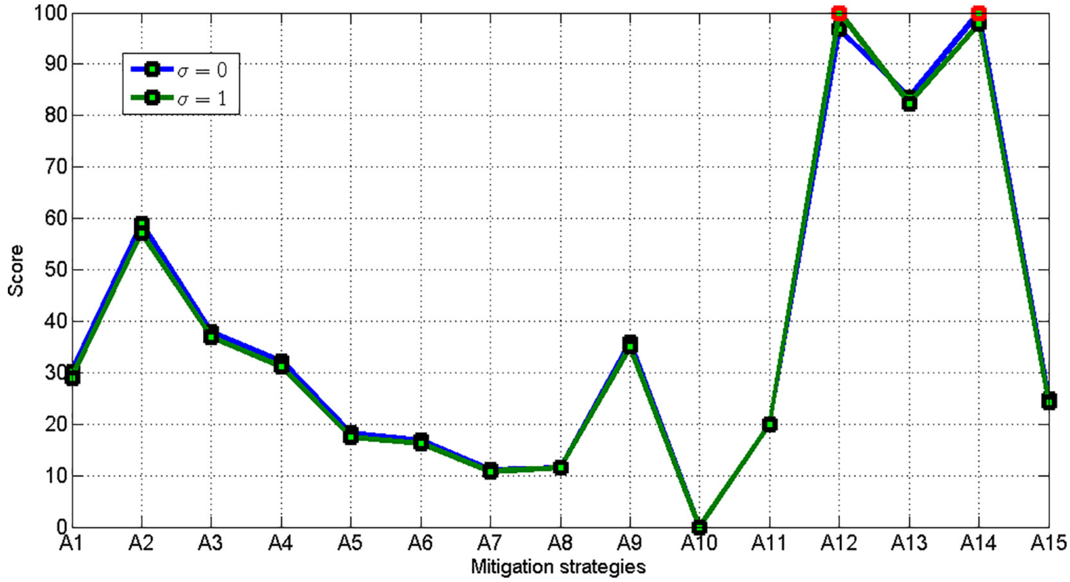

Fig. 10. Optimal urban design under $\mathscr{D}_2(\alpha; \lambda_1, \lambda_2)$ for $\sigma = \{0, 1\}$.

weights should be chosen to reflect the preferences of the urban designer which needs to analyse the problem in a holistic way.

5.5. Summary

The purpose of this section was to illustrate how the recommendation system can be used in practice. It is not intended to provide a specific conclusion as to which of the scenarios should be chosen, but instead, provides a holistic description of the system. The tool which we developed provides the decision maker much flexibility and various choices and those should be considered carefully by the decision maker.

6. Conclusions

We developed a rigorous, principled framework for choosing the optimal urban design which incorporates various criteria such as Outdoor Thermal Comfort (OTC), spatial use in form of exposure maps and monetary costs of the design. We have derived the spatial risk measures under different classes of *damage functions* and demonstrated the impact that the climate model uncertainty can have on the choice of the optimal urban design.

Declaration of Competing Interest

None.

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