A Compositional Approach to Generative Modeling of Network Paths

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***Abstract*—Path generation on network is a crucial research field across various domains, including traffic planning and management, routing optimization in communication networks, and relationship analysis in social networks. Although many studies have used deep learning or generative AI methods to model paths and have achieved promising results, the robustness and interpretability of such models are largely unexplored. This limits the application of path generation algorithms on noisy real-world data and their trustworthiness in downstream tasks. To address this issue, we leverage the insight that it's easy to learn a complex path by decomposing it into representative subpaths, each carrying local routing information. Building on this idea, we propose PathletVAE, a compositional generative model that assembles atomic subpaths from a jointly optimized dictionary. The model, based on a variational auto-encoder framework with a linear decoder, is easy to implement in different scenarios. Moreover, the incorporation of sparse dictionary representation enhances the model's robustness under noise and offers explainability through the statistics and semantics of dictionary subpaths. Our method has demonstrated its effectiveness in multiple scenarios. For urban taxi trajectory generation, it beats strong baselines by 35.4% and 26.3% in terms of the alignment between generated data and the original data on two real-world path datasets. Moreover, we show its novel application scenario in Mixture-of-Experts network pruning, with results that not only significantly outperform baseline method but also demonstrate flexibility in manipulating capabilities across different domains.**

***Index Terms*—compositional learning, pathlet, paths on network, variational autoencoder, model compression**

I. Introduction

Paths on networks are a fundamental data type pervasive across physical and digital domains. They appear in vehicle movements on road networks [28], data packet routing in communication systems [23], and in specific computational contexts such as expert activation paths in Mixture-of-Experts network. Recently, generative models have become a mainstream and advantageous method for the effective modeling of path data [30]. These models work by learning the patterns and distributions inherent in the data, enabling them to perform tasks such as data augmentation and manipulation. For instance, in privacy-preserving path modeling on urban road network, path generation models play a significant role in this context as they are capable of generating synthetic paths that mimic the patterns of real-world datasets [21]. Controlled path generation is also useful for downstream tasks such as routing optimization in communication network [23, 2] and travel route planning [26].

Various methods have been proposed for generating paths on a network. In these approaches, paths are either generated recursively using sequence models such as LSTM [33] or Transformer [29], or as whole entities through generative models, such as diffusion on graph neural networks [10] and masked autoencoders [38]. However, these methods primarily focus on how well the generated paths match the distribution of the training data and their performance in downstream tasks. The generation process, however, lacks interpretability due to the non-linearity of deep networks. It is also challenging to explain the generated paths in terms of their semantic meaning, as the learned representations are dense and not easily interpretable. Few path generation studies address the challenges of noise and missing data in real-world datasets. For instance, traffic path data may suffer from issues like hardware instabilities, environmental interference, signal obstruction, low-frequency sampling, and privacy-related data omissions. These problems can introduce sampling bias, hindering model generalization.

To address the aforementioned challenges, we introduce the concept of compositional learning to the generative modeling of paths, whose basic idea is illustrated in Figure 1. This idea is inspired by a common observation in path datasets: there are numerous shared subpaths that act as fundamental components of paths, akin to how words are the building blocks of a corpus. The original path data can be decomposed into basic units to form a dictionary, from which new paths can be generated by selecting and combining elements. An ensuing question is: what is the optimal decomposition of the path into atomic parts? This question was first studied in trajectory compression. Analogous to the principle of Minimum Description Length (MDL), [7] proposed to reconstruct the dataset using the smallest dictionary and the fewest parts for each path, and refer to the dictionary elements as "pathlets". Building on this idea, we propose a deep generative model that models complex paths via the sparse representation of pathlets. This sparse representation not only enhances the robustness of the model [34, 32], but also makes the path generation process explicit, allowing paths to be explained by the semantics of their containing pathlets.

Our generative framework, named PathletVAE, models the path generation process using a Variational Autoencoder (VAE) component and a linear decoder component. The VAE models the distribution of binary representation vectors, indicating whether a pathlet is used. While the linear decoder con-

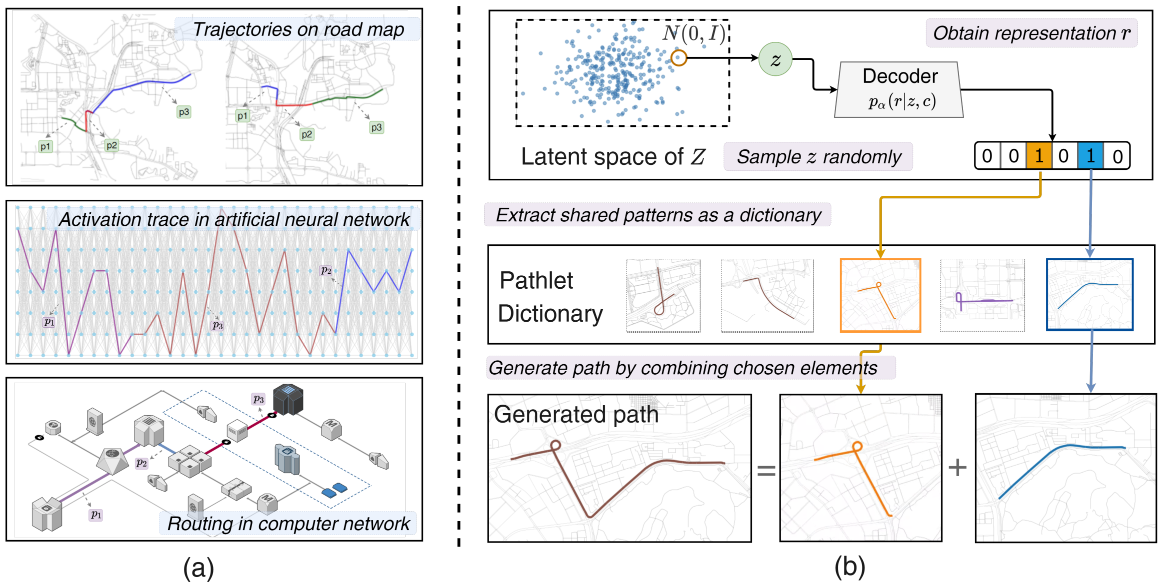

Fig. 1. The left side shows that in different application scenarios, paths on networks can often be decomposed into combinations of multiple basic units. The right side takes urban road network scenario as an example to present the fundamental idea of our method: by learning shared patterns in the data to obtain a dictionary and then generating by recombining elements from the dictionary, the inherent sparsity makes the algorithm more robust and understandable.

verts these vectors into full paths. During the training process, the pathlet dictionary and the VAE model for representation vectors are jointly learned. Several regularization constraints are proposed to ensure the optimality of the learned dictionary.

We applied the proposed method to two specific scenarios. The first scenario involves paths in city road networks. We evaluated our method on two real-world datasets and simulated scenarios under varying noise levels. The experimental results show that the data generated by our method exhibits significantly higher similarity to real data compared to other methods. Furthermore, we validated the effectiveness of our method in downstream tasks such as conditional generation. The second scenario focuses on token routing in Mixture of Experts (MoE) networks. By applying our path generative model, we discovered inherent patterns in token routing related to the tokens and expert weights. Learning these patterns enhanced the interpretability of MoE models, enabled more effective model compression, and allowed us to adjust the model's capabilities in specific domains.

Overall, our main contributions can be summarized as follows:

- We propose a path generation framework that combines the strong distribution fitting capabilities of deep learning models with the robustness of sparse combinatorial learning, leveraging the advantages of both approaches. Additionally, we comprehensively evaluate the performance of the method on real-world datasets.
- Experimental results in the urban road network scenarios indicate that our method achieves better Jensen-Shannon Divergence (JSD) scores than previous approaches. Moreover, we examined the algorithm's utility in downstream tasks, visualized important pathlets, and discussed the interpretability of the method.
- Using our path generative model, we uncover fundamental patterns in token routing that are influenced by both the tokens and the weights assigned to experts. Understanding these patterns not only enhances the interpretability of MoE models but also facilitates effective model compression and enables the customization of the model's capabilities for specific domains.

II. Related work

A. Path generative model on networks.

Path generation on networks is a fundamental problem in many domains, where the goal is to generate one or more paths on a given network. The generated paths should often meet objectives such as minimizing the distance of distribution between true data and generated data [30], while adhering to constraints like feasibility (e.g., connectivity), and diversity (e.g., generating multiple distinct paths). For example, in traffic simulation, it is used to create path datasets that replicate real-world traffic patterns, enabling urban mobility analysis and transportation optimization [8, 20]. In path prediction [19], it involves forecasting future paths based on initial conditions, which is critical for autonomous driving and pedestrian safety.

Path generation techniques can be grouped into two broad categories: traditional methods and data-driven methods. Classical methods for path generation on networks are primarily based on search and optimization techniques. Search-based methods include well-known algorithms such as Dijkstra's algorithm [11] for single-source shortest paths, A-star [16] for heuristic-based pathfinding, and Bellman-Ford [5] for handling networks with negative edge weights. These methods are robust and theoretically grounded, making them suitable for