Hindawi Publishing Corporation
International Journal of Distributed Sensor Networks
Volume 2015, Article ID 120812, 10 pages
http://dx.doi.org/10.1155/2015/120812

Research Article

Approximation Algorithms for Maximum Link Scheduling under SINR-Based Interference Model

Zi-Ao Zhou and Chang-Geng Li

School of Physics and Electronics, Central South University, Changsha, Hunan 410083, China

Correspondence should be addressed to Chang-Geng Li; lcgeng@mail.csu.edu.cn

Received 7 February 2015; Revised 7 June 2015; Accepted 1 July 2015

Academic Editor: Stefano Savazzi

A fundamental problem in wireless networks is the maximum link scheduling (MLS) problem. In this problem, interference is a key issue and past researchers have shown that determining reception using Signal-to-Interference plus Noise Ratio (SINR) is more realistic than graph-based interference models. Unfortunately, the MLS problem has been proven to be NP-hard for SINR interference models. To date, several approximation algorithms have been proposed to solve MLS under the SINR-based interference model. However, most of these works do not have either an approximation bound or a distributed version. To this end, we present a novel scheduling method with a constant approximation ratio which is much simpler and only 1/28 of it in past research. The improvement of constant ϕ also offers a better MLS set. In addition, based on our centralized method, we present a polynomial time, randomized, distributed algorithm, which only requires estimates of the number of links, and maximum and minimum link lengths. We prove its correctness and show that it can compute a MLS with time complexity of $O(\log^2 n)$, where n is an estimate of the number of links.

1. Introduction

The capacity of a wireless network is directly proportional to the number of transmitting links in a certain time. This problem, called maximum link scheduling (MLS), can be succinctly described as follows: given a set of communication links L, derive a set S with maximum cardinality, where $S \subseteq L$. The key constraint is that links in S can be activated simultaneously only if the respective receiver of each transmission is able to receive correctly or with high probability; MLS is also referred to as one-slot scheduling [1] or maximum independent link set problem [2]. Unfortunately, this problem is made difficult by interference, an inherent characteristic of wireless transmissions. Unlike wired networks, signal interference bounds the maximum number of concurrent transmissions. Like other well-known scheduling problems (e.g., maximum throughput scheduling), MLS becomes NP-hard to solve when considering wireless interference, while similar problems in wired networks are solvable in polynomial time.

Past research on MLS has used different interference models, most of which are graph-based, for example, protocol interference and RTS/CTS models [3]. In a graph-based model, interference-free communications can be achieved by applying a coloring method on a conflict graph [4]. This model has produced a number of interesting results; see [4–8], but they are limited due to the overly idealistic assumption. To this end, in recent years, a more realistic interference model based on Signal-to-Interference plus Noise Ratio (SINR) has gained a lot of interest. Here, a signal is received successfully if the SINR at a receiver is above a threshold, which is set according to hardware and coding method. For example, in IEEE 802.11b, the minimum SINR corresponding to 11 and 1 Mbps are 10 and 4 dB, respectively [9].

Recently, many models and protocols have been proposed to solve the link scheduling problem in wireless networks. They can be grouped into two types: scheduling the largest set of noninterfering links from a given set (maximum link scheduling) and scheduling a given set of links using the smallest time length (minimum length scheduling). These problems are well understood when a wireless network is represented as a graph, where, given an interference range for each node, concurrent transmitting links are required to be outside. Graph-based scheduling algorithms usually employ coloring on the resulting conflict graph [4]. However, their performance is not ideal when nodes' reception is governed by a SINR model. This is well documented and has been shown theoretically as well as experimentally; see [10, 11].

The MLS problem over the SINR-based interference model is more challenging and has received a lot of interest recently. Goussevskaia et al. [12] proved that the MLS problem over the uniform power assignment is NP-hard and developed a $g(L)$-approximation algorithm, where $g(L)$ is the link diversity; see Section 2.2. The first constant approximation algorithm for MLS problem is proposed in [1]. However, as observed by Xu and Tang [13], its approximation bound and corresponding proof (Lemma 4.5 in [1]) are only correct in the absence of background noise. This motivated them to propose a new algorithm using the method in [1] to select independent links from a subset of short links, as per Euclidean distance. Yet, as shown in [2], their algorithm is also inaccurate, as the selected links may not be independent. The approximation bound 272 was calculated based on the set ϕ to 0.5 subjectively without any derivation. The comparison between the approximation bound they got and the one in [1, 13] is not rigorous. To the best of our knowledge, Halldórsson and Wattenhofer [14] proposed the first correct constant approximation algorithm for the MLS problem. In their algorithm, senders added into MLS are separated by at least a predefined interference range. The simple constant approximation algorithm proposed for the MLS problem in [14] assumes uniform power assignment, which only requires the interference received by a newly selected link to be lower-bounded by a constant. In [15], Halldórsson and Mitra extend [14] to the length-monotone, sublinear power assignments. Kesselheim [16] presents the first constant-approximate Maximal Independent Set (MIS) algorithm with power control. The best previous results depend on further network parameters such as the ratio of the maximum and the minimum distance between a sender and its receiver.

All the works reviewed thus far are centralized, and the studies do not clearly show how to develop a distributed version. The MIS algorithm in [17] is the first randomized and distributed MIS algorithm for the SINR-based interference model. But the time complexity of $O(n^2\log^3 n)$ is unacceptable. Ásgeirsson and Mitra attempt to solve MLS under SINR model in a distributed setting in [18] after [17]. This algorithm computes the MIS in time $O(\log\Delta)$ where Δ is the ratio between the largest and the smallest link in the network. Though their $O(\log\Delta)$ is an exponential improvement of the approximation factor, their model needs additional capabilities or assumptions. That means if another algorithm uses constant size messages without any additional capabilities or assumptions would be better. Recently, Pei and Anil Kumar [19] propose a fully distributed algorithm based on the centralized method in [2] with a time complexity of $O(\log^3 n)$. More recently, Pei and Vullikanti [20] develop the algorithm to the time complexity of $O(\log^2 n)$. However, we still could not verify the correctness of their algorithm because of gaps in Lemma IV.3; that is, in [20], they do not give reasons why the number of active nodes in the vicinity of any active node decreases by half after each phase with a high probability.

The most closely related work to ours is the method in [14]. However, in [14], Halldórsson and Wattenhofer apply the approach used in [1] to calculate the relative interference constant ϕ, their constant is much smaller than ours. For comparison, set α to 4 and β to 16; their constant ϕ is $1/10952$ which is only about $1/24$ of ours. That means that the size of their MLS set is much smaller than ours, both in terms of average and worst case. A bigger ϕ will permit more links to be added into the MLS set. This is one of the real crux of the MLS problems. Furthermore, the paper [14] does not give an approximation ratio and distributed implementation.

Henceforth, we design a centralized MLS schedule with a constant approximation ratio of $5\lceil(\phi^{-1/\alpha} + 2\beta^{-1/\alpha})^{\alpha}\rceil$, where ϕ is a constant, α is the path-loss exponent, and β is the minimum SINR threshold required for a message to be decoded successfully. Both the calculation method of phi and approximation ratio are improved. As the assumption before, the approximation ratio obtained in [1] (in the absence of noise) and in [13] (restricted to short links) is at least 138135 and 137890, respectively, with the same α and β, both of which are more than 28 times the approximation ratio 4882 of our algorithm.

A randomized and distributed algorithm with polynomial execution time has also been designed. Our distributed algorithm only requires an estimate of the number of links and the maximum and minimum link lengths. We then prove its correctness and show it can compute a MLS with time $O(\log^2 n)$, where n is an estimate of the number of links. To the best of our knowledge, it is one of the algorithms which have the best time complexity.

The remainder of this paper is organized as follows. In Section 2, we introduce the network model, some definitions and theories, and the problem formulation. In Section 3, we introduce our approximation algorithm and its distributed implementation. Lastly, Section 4 concludes the paper and presents further works.

2. Preliminaries

2.1. Network Model. Define a set of links as $L = \{l_1, \dots, l_n\}$, where each link l_v represents a communication channel from a sender s_v to a receiver r_v. Let $X(L)$ and $Y(L)$ be the set of senders and receivers of L, respectively; that is, $s_v \in X(L)$ and $r_v \in Y(L)$. We assume senders and receivers are distributed in the Euclidean plane. Let $d(u, v)$ denote the Euclidean distance between node u and v. For link l_v, let $d(l_v) = d(s_v, r_v)$ be its length. For link l_v and l_u, let $d(l_v, l_u) = d(s_v, r_u)$ denote the asymmetric length from link l_v to l_u. We will use $d_{\min}$ and $d_{\max}$ to denote the link with the smallest and the largest lengths, respectively. We assume the power level assignment is uniform whereby all senders transmit with power level P.

We adopt the SINR-based interference model, where a receiver r_v successfully receives a message from a sender s_v if and only if the following condition holds:

$$\frac{Pd(l_v)^{-\alpha}}{\sum_{l_u \in S \setminus \{l_v\}} Pd(l_u, l_v)^{-\alpha} + N} = \frac{1}{\sum_{l_u \in S \setminus \{l_v\}} d(l_u, l_v)^{-\alpha} / d(l_v)^{-\alpha} + N/Pd(l_v)^{-\alpha}} \geq \beta, \tag{1}$$

where α is the path-loss exponent, which defines how the signal fades away from its source. Considering realistic wireless propagation environments such as the line-of-sight propagation environment in the building, urban cellular wireless environment with shadow, and the environment with obstacle in the closed space, our approximation results use the assumption that $1.6 < \alpha \leq 6$. $\beta > 1$ denotes the minimum SINR required for a message to be received successfully, N is the ambient noise, and $S \subseteq L$ is the set of links that are active at the same time as link l_v.

We assume all communications are carried out in synchronized fixed length, time slots. In each time slot, a node can either listen or transmit. We assume nodes are able to measure the total received power from other nodes. Here, we define the received power $\mathrm{Th}(v)$ as the signal power that node v receives from other nodes, excluding ambient noise; that is, given a node set U, the received power of node v is calculated as $\mathrm{Th}(v) = \sum_{u \in U} (P/d(u,v)^{\alpha})$.

2.2. Definitions and Theories. We define *link diversity* $g(L)$ as $\lceil \log_{1+\epsilon}(d_{\max}/d_{\min}) \rceil$, where ϵ is a positive constant that is usually set to 1. We can partition set L into nonoverlapping link subclasses L_i, where $1 \leq i \leq g(L)$. Each subset $L_i = \{l_v \mid (1+\epsilon)^{i-1} d_{\min} < d(l_v) \leq (1+\epsilon)^i d_{\min}\}$ denotes the set of links with similar length. Let d_i denote the upper bound of link lengths in L_i; hence, $d_i = (1+\epsilon)^i d_{\min}$. For example, assume $d_{\max} = 16$ and $d_{\min} = 2$, which yield $g(L) = 3$ when $\epsilon = 1$. As a result, we will have three sets of links, L_1 to L_3, with length in the range $(2, 4]$, $(4, 8]$, and $(8, 16]$ respectively.

In a distributed environment, nodes use their shared estimates of minimum and maximum possible link length to replace $d_{\min}$ and $d_{\max}$. As shown in [19], the minimum link length is constrained by a device's dimension, empirically at least 0.1 meter ($d_{\min}$); the maximum link length depends on the network type and is usually bounded by 105 meters ($d_{\max}$). It implies that, in most cases, when $\epsilon = 1$, we have $g(L) \leq 10$. Furthermore, as discussed earlier, each link can compute which link subclass it belongs to using the estimates of minimum and maximum link length. The $g(L)$ is used for partitioning the links in L into disjoint link length classes. In practice, the exact number of links in a network may be unknown but can be approximated in advance. Here, we assume, in a distributed implementation, the approximate total number of links, $\hat{n}$, is an upper bound of a real number; that is, $\hat{n} = n^c$ for some constant $c \geq 1$.

A set I of nodes is said to be d-independent if the mutual distance of the nodes in I is greater than d [21]. A Maximal Independent Set (MIS) U is a d-independent set which is not a subset of any other d-independent sets.

Let R denote $(P/N\beta)^{1/\alpha}$. In the absence of interference, a link can communicate successfully if and only if $d(l_v) \leq R$. We thus refer to R as the *maximum transmission radius*.

Consider a link l_v and a set S of concurrently scheduled links l_v. By algebraic manipulation, inequality (1) holds if and only if

$$\frac{\sum_{l_u \in S \setminus \{l_v\}} d(l_u, l_v)^{-\alpha} / d(l_v)^{-\alpha}}{1/\beta - N/Pd(l_v)^{-\alpha}} \leq 1. \tag{2}$$

Motivated by inequality (2), we define relative interference $\mathrm{RI}(l_u, l_v)$ as follows:

$$\mathrm{RI}(l_u, l_v) = c_v \frac{d(l_u, l_v)^{-\alpha}}{d(l_v)^{-\alpha}}, \tag{3}$$

where $c_v = 1/(1/\beta - N/Pd(l_v)^{-\alpha}) = \beta/(1 - (d(l_v)/R)^{\alpha})$, since $R = (P/N\beta)^{1/\alpha}$. Note that we define $\mathrm{RI}(l_v, l_v) = 0$.

Then, with a slight abuse of notation, we define the relative interference of set S to link l_v as the sum of the relative interferences of the links in S on l_v. So, $\mathrm{RI}(S, l_v)$ is

$$\mathrm{RI}(S, l_v) = \sum_{l_u \in S \setminus \{l_v\}} c_v \frac{d(l_u, l_v)^{-\alpha}}{d(l_v)^{-\alpha}} = \sum_{l_u \in S \setminus \{l_v\}} \mathrm{RI}(l_u, l_v). \tag{4}$$

According to the definitions of relative interference, inequality (2) can be transformed as

$$\frac{\sum_{l_u \in S \setminus \{l_v\}} d(l_u, l_v)^{-\alpha} / d(l_v)^{-\alpha}}{1/\beta - N/Pd(l_v)^{-\alpha}} = \sum_{l_u \in S \setminus \{l_v\}} \mathrm{RI}(l_u, l_v) = \mathrm{RI}(S, l_v) \leq 1. \tag{5}$$

As shown in inequality (5), the link l_v succeeds if and only if the relative interference of S to the link l_v is at most one; that is, $\mathrm{RI}(S, l_v) \leq 1$.

A summary of notations used in this paper can be found in Notations section.

2.3. Problem Formulation. The MLS problem is a maximization problem whereby, given an input set of communication links L, we seek a subset of links $S \subseteq L$ whose cardinality is the largest and links can be scheduled simultaneously under the SINR-based interference model. As mentioned earlier, the MLS problem is NP-hard, and thus we focus on an approximation algorithm. We say an algorithm gives a C-approximation factor if it constructs a MLS set $S \subseteq L$ with $|S| \leq |\mathrm{OPT}(L)|/C$, where $\mathrm{OPT}(L)$ denotes an optimum solution for MLS.

3. Proposed Algorithm

We now present our constant approximation algorithm for MLS under the SINR-based interference model. We first introduce the centralized version and prove its correctness and approximation factor. After that, we present its distributed version.

(1) **input**: Set of links L
(2) **output**: Set S
(3) $L' \leftarrow$ links in L in the non-decreasing order of length
(4) $S \leftarrow \emptyset$
(5) **while** $L' \neq \emptyset$ **do**
(6) $\quad l_v \leftarrow$ first link in set L'
(7) $\quad$ **if** $\mathrm{RI}(S, l_v) \leq \phi$ **then**
(8) $\qquad S \leftarrow S \cup \{l_v\}$ and $L' \leftarrow L' \setminus \{l_v\}$
(9) $\quad$ **else**
(10) $\qquad L' \leftarrow L' \setminus \{l_v\}$
(11) **return** S

ALGORITHM 1: Centralized algorithm for MLS.

3.1. The Centralized Algorithm. Our centralized algorithm for MLS is outlined in Algorithm 1. The algorithm is associated with a constant ϕ, whose value will be determined later on. Algorithm 1 greedily schedules links in nonincreasing order of link length; that is, shortest link is scheduled first. A link in L is selected into set S if and only if its relative interference from S is no larger than ϕ (line (8) of Algorithm 1). On the contrary, a link is discarded if its relative interference from set S is larger than ϕ (line (10) of Algorithm 1). This process repeats until all links in L are considered. Next, we prove the correctness of Algorithm 1 and derive its approximation factor.

3.1.1. Analysis. We begin our analysis by firstly presenting one key property for a pair of links; this property is captured by Lemmas 1 and 2. In particular, the property states that for a pair of links, for example, l_v and l_u, the distance between their respective sender is lower-bounded by a constant if their relative interference is upper-bounded by a constant ϕ. Then, following directly from Lemmas 1 and 2, we use Theorem 3 to prove our centralized algorithm is valid when ϕ is set to a proper value. Together with Lemma 4, Theorem 5 proves our centralized algorithm has a constant approximation factor.

Lemma 1. *Given a pair of links l_v and l_u, if the relative interference $RI(l_u, l_v)$ is upper-bounded by a constant $0 < \phi < 1$, that is, $RI(l_u, l_v) \leq \phi$, the distance between two senders $d(s_v, s_u)$ is lower-bounded by $(1/\phi^{1/\alpha} - 1)c_v^{1/\alpha} d(l_v)$.*

Proof. Since $\mathrm{RI}(l_u, l_v) = c_v(d(l_u, l_v)/d(l_v))^{-\alpha} \leq \phi$, it implies that $d(l_u, l_v) \geq (1/\phi^{1/\alpha})c_v^{1/\alpha} d(l_v)$. Recall that $c_v = \beta/(1 - (d(l_v)/R)^{\alpha}) > 1$. Then, using the triangular inequality, we have the required lemma:

$$\begin{aligned} d(s_u, s_v) &> d(l_u, l_v) - d(l_v) > \left(\frac{1}{\phi^{1/\alpha}} c_v^{1/\alpha} - 1\right) d(l_v) \\ &> \left(\frac{1}{\phi^{1/\alpha}} - 1\right) c_v^{1/\alpha} d(l_v). \end{aligned} \tag{6}$$

□

Lemma 2. *Given a pair of links l_v and l_u where $d(l_v) \leq d(l_u)$, if the relative interference $RI(l_v, l_u)$ is upper-bounded by a constant ϕ, the distance between two senders, namely, $d(s_v, s_u)$, is lower-bounded by $(1/\phi^{1/\alpha} - 1)c_v^{1/\alpha} d(l_v)$.*

Proof. Since $\mathrm{RI}(l_v, l_u) \leq \phi$, as per Lemma 1, we have $d(s_u, s_v) > (1/\phi^{1/\alpha} - 1)c_u^{1/\alpha} d(l_u)$. Note that $d(s_u, s_v) = d(s_v, s_u)$ and $c_v \leq c_u$ whenever $d(l_v) \leq d(l_u)$. Thus, we have

$$\begin{aligned} d(s_v, s_u) &> \left(\frac{1}{\phi^{1/\alpha}} - 1\right) c_u^{1/\alpha} d(l_u) \\ &\geq \left(\frac{1}{\phi^{1/\alpha}} - 1\right) c_v^{1/\alpha} d(l_v). \end{aligned} \tag{7}$$

□

Theorem 3. *Algorithm 1 provides a valid solution when $\phi \leq 1/(2 + (\rho + 1)^{1/\alpha})^{\alpha}$, where $\rho = 8(2/(\alpha - 2) + 1/(\alpha - 1) + 3)$.*

Proof. Let S_v^- denote the set of links shorter than link l_v, that is, those added to the MLS set S before l_v, and let S_v^+ be the set of links longer than l_v, that is, those added to S after l_v. When link l_v is added to S, the relative interference $\mathrm{RI}(S_v^-, l_v)$ is no larger than ϕ (line (8) of Algorithm 1). Therefore, to make sure S is a valid set satisfying inequality (1), we need to show $\mathrm{RI}(S_v^+, l_v)$ is less than $1 - \phi$. In the following, we partition the nodes into different rings, then compute the relative interference from each ring, and sum them up to show $\mathrm{RI}(S_v^+, l_v)$ is upper-bounded by $1 - \phi$.

Using Lemmas 1 and 2 and the fact that the length of links in S_v^+ is larger than $d(l_v)$ and c_v is an increasing function of $d(l_v)$, the distance of any two senders of links in set $S_v^+ \cup \{l_v\}$ is larger than $(1/\phi^{1/\alpha} - 1)c_v^{1/\alpha} d(l_v)$. We partition the senders in S_v^+ into concentric rings Ring^k with width $(1/\phi^{1/\alpha} - 1)c_v^{1/\alpha} d(l_v)$ around sender s_v. Rings Ring^k contain all senders s_u of links in S_v^+, for which $k(1/\phi^{1/\alpha} - 1)c_v^{1/\alpha} d(l_v) \leq d(s_u, s_v) < (k + 1)(1/\phi^{1/\alpha} - 1)c_v^{1/\alpha} d(l_v)$. The first ring Ring^0 does not contain any senders of links in S_v^+. We now consider all senders $s_u \in \mathrm{Ring}^k$ for some integer $k > 0$.

First, we consider the distance between any senders s_u in Ring^k and s_v. As per the construction of rings, we have $d(s_u, s_v) \geq k(1/\phi^{1/\alpha} - 1)c_v^{1/\alpha} d(l_v)$ for ring Ring^k. Note that

$c_v > 1$. Using the triangular inequality, we can lower bound $d(l_u, l_v)$ for Ring^k as follows:

$$\begin{aligned} d(l_u, l_v) &> d(s_u, s_v) - d(l_v) \\ &> \left(\frac{1}{\phi^{1/\alpha}} - 1\right) c_v^{1/\alpha} d(l_v) - d(l_v) \\ &> \left(\frac{1}{\phi^{1/\alpha}} - 2\right) c_v^{1/\alpha} d(l_v). \end{aligned} \tag{8}$$

Next, observing that, for any senders s_u in Ring^k, the disk centered at s_u with a radius of $(1/2)(1/\phi^{1/\alpha} - 1)c_v^{1/\alpha} d(l_v)$ is nonoverlapping with other senders in Ring^k, and such a disk is fully contained in an extended ring of Ring^k, with an extra width of $(1/2)(1/\phi^{1/\alpha} - 1)c_v^{1/\alpha} d(l_v)$ at each side of Ring^k. Then, by referring to the ratio between the area of this extended ring and the disk, the number of senders contained in Ring^k is upper-bounded by $8(2k+1)$.

The total relative interference coming from Ring^k is then bounded by

$$\begin{aligned} \text{RI}\left(\text{Ring}^k, l_v\right) &\le \sum_{s_u \in \text{Ring}^k} \text{RI}(l_u, l_v) \\ &< 8(2k+1) c_v \left(k\left(\frac{1}{\phi^{1/\alpha}} - 2\right) c_v^{1/\alpha} \frac{d(l_v)}{d(l_v)}\right)^{-\alpha} \\ &= 8(2k+1) k^{-\alpha} \left(\frac{1}{\phi^{1/\alpha}} - 2\right)^{-\alpha}. \end{aligned} \tag{9}$$

Summing up the relative interferences over all rings yields

$$\begin{aligned} \text{RI}(S_v^+, l_v) &\le \sum_{k=1}^{\infty} \text{RI}\left(\text{Ring}^k, l_v\right) \\ &\le \left(\frac{1}{\phi^{1/\alpha}} - 2\right)^{-\alpha} \sum_{k=1}^{\infty} 8(2k+1) k^{-\alpha} \\ &< \left(\frac{1}{\phi^{1/\alpha}} - 2\right)^{-\alpha} 8\left(\frac{2}{\alpha-2} + \frac{1}{\alpha-1} + 3\right). \end{aligned} \tag{10}$$

Using $\text{RI}(S_v^-, l_v) \le \phi$ and inequality (10), if the following inequality holds, Algorithm 1 provides a valid solution:

$$\text{RI}(S_v^-, l_v) + \text{RI}(S_v^+, l_v) \le (\tau - 2)^{-\alpha} \rho + \tau^{-\alpha} \le 1, \tag{11}$$

where $\phi = \tau^{-\alpha}$ and $\rho = 8(2/(\alpha-2) + 1/(\alpha-1) + 3)$. Since $(\tau-2)^{-\alpha}\rho + \tau^{-\alpha} < (\tau-2)^{-\alpha}\rho + (\tau-2)^{-\alpha}$, if the following inequality holds, inequality (11) must also hold:

$$(\tau-2)^{-\alpha} \rho + (\tau-2)^{-\alpha} \le 1. \tag{12}$$

Therefore, by inequality (12), we have the following bound:

$$\tau \ge 2 + (\rho+1)^{1/\alpha}. \tag{13}$$

According to inequality (13), when $\phi \le 1/(2+(\rho+1)^{1/\alpha})^{\alpha}$, inequality (11) holds; that is, Algorithm 1 provides a correct solution. □

Lemma 4. *Given links l_v and l_u and a set of links W whose senders lie in the disk centered at sender s_v of l_v with the radius of $d(s_v, s_u)$, if $RI(W, l_v) \le 1$ and $|W| \ge \lceil(\phi^{-1/\alpha} + 2\beta^{-1/\alpha})^{\alpha}\rceil$, one has $RI(l_u, l_v) \le \phi RI(W, l_v)$.*

Proof. Considering the fact that for any sender s_w of link l_w in W, $d(s_w, s_v) \le d(s_v, s_u)$, and $\text{RI}(W, l_v) \le 1$, we have $|W|\text{RI}(l_u, l_v) \le 1$. Using the triangular inequality, we get

$$\begin{aligned} |W| c_v \left(\frac{d(s_u, s_v) + d(l_v)}{d(l_v)}\right)^{-\alpha} &\le |W| \text{RI}(l_u, l_v) \\ = |W| c_v \left(\frac{d(l_u, l_v)}{d(l_v)}\right)^{-\alpha} &\le 1. \end{aligned} \tag{14}$$

Using inequality (14) and the fact that $c_v \ge \beta$, we have

$$\begin{aligned} \left(1 + \frac{d(s_u, s_v)}{d(l_v)}\right)^{\alpha} &\ge \beta |W| \ge \beta \left\lceil \left(\phi^{-1/\alpha} + 2\beta^{-1/\alpha}\right)^{\alpha} \right\rceil \\ &\ge \left(2 + \left(\frac{\beta}{\phi}\right)^{1/\alpha}\right)^{\alpha}. \end{aligned} \tag{15}$$

Hence, we get $d(s_u, s_v)/d(l_v) \ge 1 + (\beta/\phi)^{1/\alpha}$. Let q denote $1 + (\beta/\phi)^{1/\alpha}$. Next, using the triangular inequality, it follows that, for any $l_w \in W$,

$$\begin{aligned} \frac{d(l_u, l_v)}{d(l_w, l_v)} &\ge \frac{d(s_v, s_u) - d(l_v)}{d(s_v, s_w) + d(l_v)} \ge \frac{d(s_v, s_u) - d(l_v)}{d(s_v, s_u) + d(l_v)} \\ &= 1 - \frac{2}{d(s_v, s_u)/d(l_v) + 1} \ge \frac{q-1}{q+1}. \end{aligned} \tag{16}$$

From inequality (16), we have

$$\begin{aligned} \text{RI}(W, l_v) &= c_v \sum_{l_w \in W} \left(\frac{d(l_w, l_v)}{d(l_v)}\right)^{-\alpha} \\ &\ge c_v \sum_{l_w \in W} \left(\frac{q+1}{q-1} \frac{d(l_u, l_v)}{d(l_v)}\right)^{-\alpha} \\ &= |W| \left(\frac{q-1}{q+1}\right)^{\alpha} \text{RI}(l_u, l_v). \end{aligned} \tag{17}$$

Then using the fact that $q = 1 + (\beta/\phi)^{1/\alpha}$ and $|W| \ge \lceil(\phi^{-1/\alpha} + 2\beta^{-1/\alpha})^{\alpha}\rceil$, we have

$$\text{RI}(W, l_v) \ge \frac{1}{\phi} \text{RI}(l_u, l_v). \tag{18}$$

□

Theorem 5. *Algorithm 1 is a 5μ-approximation algorithm, where μ is $\lceil(\phi^{-1/\alpha} + 2\beta^{-1/\alpha})^{\alpha}\rceil$.*

Proof. Before outlining our proof, we first introduce a method used by [2] to construct μk disjoint subsets of links in the optimal MLS set $\text{OPT}(L)$, where k is the cardinality of the MLS set S outputted by Algorithm 1. Let W denote the set of links in $\mathrm{OPT}(L)$. For each $1 \le i \le k$ and $1 \le j \le \mu$, initialize W' to W and each W_{ij} to be empty set. Then, repeat the following iterations for each i and j. For sender s_i of link $l_i \in S$, include into subset W_{ij} a link $l_{w_1} \in W'$ with the closest sender s_{w_1} to s_i, and draw six closed 60°-sectors originating at s_i such that one of six boundary rays goes through node s_{w_1}, as shown in Figure 1. For each of the left four sectors not containing s_{w_1}, include into W_{ij} a link in W' whose sender is closest to s_i in this sector. After that, remove W_{ij} from W' and repeat the above iterations. By construction, each W_{ij} contains at most five links; that is, $|W_{ij}| \le 5$. To sum up all links in W_{ij}, we get $\sum_{i=1}^{k}\sum_{j=1}^{\mu}|W_{ij}| \le 5\mu k$. Hence, to prove this theorem, we can prove $W = \bigcup_{i=1}^{k}\bigcup_{j=1}^{\mu} W_{ij}$.

By contradiction, assume $W \setminus \bigcup_{i=1}^{k}\bigcup_{j=1}^{\mu} W_{ij}$ is nonempty, and an arbitrary link l_v is in this set. Hence, for any node $s_i \in X(S)$, after μ iterations, node l_v will not be selected into any W_{ij} by assumption. That is, for each $1 \le i \le k$ and $1 \le j \le \mu$, there must be a node $s_w \in X(W_{ij})$ such that $d(s_i, s_w) \le d(s_i, s_v)$, and the angle $\angle s_w s_i s_v \le 60^\circ$. Let U_i denote the set of links whose sender lies in intersection of the two disks centered at s_i and s_v with radius $d(s_i, s_v)$. Then U_i contains at least μ links in W; else link l_v must be chosen into some W_{ij}.

According to Lemma 4, for any $1 \le i \le k$, $\mathrm{RI}(l_i, l_v) \le \phi\mathrm{RI}(U_i, l_v)$. Recall that $\bigcup_{i=1}^{k} U_i \subseteq W$ and $\mathrm{RI}(W, l_v) \le 1$ by assumption, and thus we have

$$\mathrm{RI}(S, l_v) = \sum_{i=1}^{k}\mathrm{RI}(l_i, l_v) \le \sum_{i=1}^{k}\phi\mathrm{RI}(U_i, l_v) \le \phi\mathrm{RI}(W, l_v) \le \phi. \tag{19}$$

Therefore, the relative interference of S to link l_v is at most ϕ. This means that link l_v should not have been removed which is a contradiction. So $W = \bigcup_{i=1}^{k}\bigcup_{j=1}^{\mu} W_{ij}$ and $|W| \le 5\mu k$. □

3.2. The Distributed Algorithm. In this section, we present a randomized, distributed implementation for Algorithm 1. We then prove its correctness and calculate its time complexity.

Our distributed algorithm is illustrated in Algorithm 2, in which we set $d_i = (1+\epsilon)^i d_{\min}$ and $c_i = \beta/(1-(d_i/R)^\alpha)$. Let S_i be the set of links that are selected into the MLS set S in L_i, and let S_i^- (resp., S_i^+) be the set of links that are selected into the MLS set S before (resp., after) links in S_i. Initially, we have $S = \emptyset$, and $\sigma(v) = 0$ for any $v \in X(L) \cup Y(L)$, where $\sigma(v)$ is used as an indicator to decide whether a link needs to be selected into S. Specifically, a link l_v is added into S if and only if $\sigma(s_v) = \sigma(r_v) = 1$.

The algorithm then sweeps through the link classes in $g(L)$ rounds. The ith round, where $i \in [1, g(L)]$, consists of three steps: (1) it checks the relative interference constraint for receivers r_v in $Y(L_i)$ (line (9) to (12) in Algorithm 2); (2) it selects a set of $(\phi^{-1/\alpha}-1)c_i^{1/\alpha}d_i$-independent senders s_v from $X(L_i)$ (line (14) to (16) in Algorithm 2); and (3) it selects into

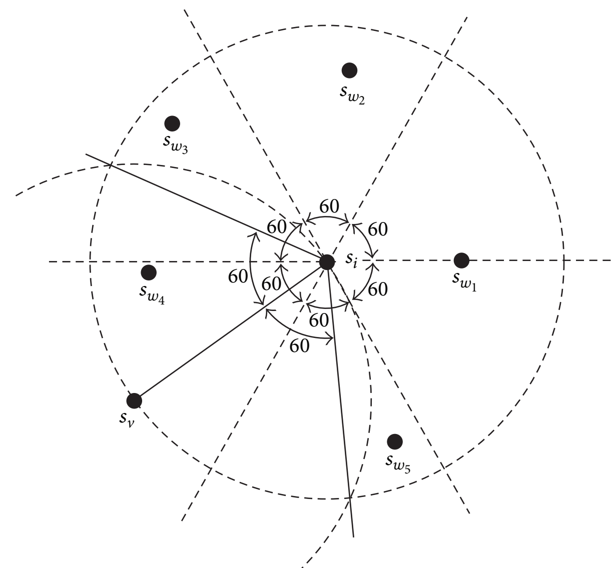

Figure 1: An illustration of Theorem 5.

S link l_v satisfying $\sigma(s_v) = \sigma(r_v) = 1$ (line (18) to (25) in Algorithm 2).

In the first step, only links l_v in L_i satisfying the relative interference constraint, that is, $\mathrm{RI}(S_i^-, l_v) \le \phi$, can be selected into S; that is, according to (3), the interference received by receiver r_v must be no larger than $\phi P/c_v d(l_v)^\alpha$. Considering the fact that $d(l_v) \le d_i$ and $c_v \le c_i$ for set class L_i, we can replace this interference bound with $\phi P/c_i d_i^\alpha$. All senders of links in S will send a message with power P in the first step and any receiver r_v of links in L_i senses the channel. Note that the value of $\mathrm{Th}(r_v)$ is equal to the interference r_v received. Hence, if $\mathrm{Th}(r_v) \le \phi P/c_i d_i^\alpha$, we set $\sigma(r_v)$ to "1."

The second step is used to select a MIS set from $X(L_i)$ with a mutual distance larger than $(\phi^{-1/\alpha}-1)c_i^{1/\alpha}d_i$. The intuition behind this step is to ensure that the relative interference received by $l_v \in S_i$ from other links in $S_i \cup S_i^+$ does not exceed $1-\phi$; that is, $\mathrm{RI}(S_i \cup S_i^+, l_v) \le 1-\phi$. In this step, all senders of links in L_i need to compete to get into the MIS set using the MIS algorithm in [22]. The MIS algorithm in [22] randomized and distributed MIS algorithm for the SINR-based interference model. As shown in [22], it only requires an estimate of the number of nodes. In our case, the number of nodes in $X(L_i)$ is bounded by n. Observing that the MIS algorithm in [22] requires nodes to have adjustable power levels, we assume that, in Algorithm 2, the second step will use variable power levels, for example, $(\phi^{-1/\alpha}-1)^\alpha c_i d_i^\alpha N\beta$, but the other steps use uniform power assignment P. Briefly, at any time during the execution, a node can be in one of four states. At first, a node joins the waiting state $\mathscr{W}$ in which it only listens for messages. If a node does not become covered by a MIS node, it will join the active state $\mathscr{A}$. In state $\mathscr{A}$, a node joins state $\mathscr{B}$ by sending a message with an increasing probability. If a node transmits this message successfully, it will join state $\mathscr{B}$, whereas its neighbors in state $\mathscr{A}$ that receive the said message will restart the algorithm, returning to

(1) **input:** Set of links L
(2) **output:** MLS set S
(3) // *initialize* (*one-slot*) //
(4) $S \leftarrow \emptyset$
(5) $\sigma(v) \leftarrow 0, \forall v \in X(L) \cup Y(L)$
(6) **for** $i \leftarrow 1$ to $g(L)$ **do**
(7) // *1st step*: *RI Constraints* (*one-slot*) //
(8) Senders of links in S transmit in this slot
(9) **if** $v \in Y(L_i)$ **then**
(10) v senses the channel
(11) **if** $Th(v) \leq \phi P / c_i d_i^{\alpha}$ **then**
(12) $\sigma(v) \leftarrow 1$
(13) // *2nd step*: *Spatial Constraints* ($O(\log^2 n)$-*slots*) //
(14) Perform the MIS algorithm of [22] with mutual distances larger than $(\phi^{-1/\alpha} - 1)c_i^{1/\alpha} d_i, \forall v \in X(L_i)$
(15) **if** $v \in X(L_i) \cap \mathrm{MIS}$ **then**
(16) $\sigma(v) \leftarrow 1$
(17) // *3rd step*: *Decision* (*two-slots*) //
(18) **if** $v \in X(L_i)$ and $\sigma(v) = 1$ **then**
(19) v sends a REQUEST containing its ID and receiver's ID
(20) **else**
(21) v listens to the channel in this slot
(22) **if** $v \in Y(L_i), \sigma(v) = 1$ and receives REQUEST from its sender **then**
(23) v transmits an ACK
(24) if $v \in X(L_i), \sigma(v) = 1$ and receives ACK from its sender **then**
(25) Add its link to S
(26) **return** S

Algorithm 2: Distributed algorithm for MLS.

the initial waiting state $\mathscr{W}$. After joining state $\mathscr{B}$, nodes will compete to join state $\mathscr{M}$ to become a MIS member.

After finding the MIS set for nodes in $X(L_i)$, all nodes v in the MIS will set $\sigma(v)$ to "1" (lines (15) to (16) in Algorithm 2).

For the third step, only links $l_v \in L_i$ with $\sigma(r_v) = \sigma(s_v) = 1$ are added into S. This decision step is conducted by a two-slot transmitting/receiving procedure. In the first slot, each sender s_v in $X(L_i)$ with $\sigma(s_v) = 1$ transmits a REQUEST message containing its ID and its receiver's ID, and each receiver $r_v \in Y(L_i)$ listens to the channel (lines (18) to (21) in Algorithm 2). In the next slot, only the receivers $r_v \in Y(L_i)$ with $\sigma(r_v) = 1$ receiving the REQUEST message from the senders s_v transmit an ACK message, and the senders add themselves into S after receiving the ACK (lines (22) to (25) in Algorithm 2). We will prove that all communications in this step are interference-free later on.

3.2.1. Analysis. Theorem 8 follows directly from Lemmas 6 and 7. Lemma 6 shows that the relative interference experienced by selected links in L_i and those links selected after them is lower-bounded by $1 - \phi$. Lemma 7 proves that the communications in the third step of our distributed algorithm are interference-free. Together with Lemmas 6 and 7, we use Theorem 8 to prove that our distributed algorithm is correct. Additionally, Theorem 10 is used to determine the time complexity.

Lemma 6. *Given a link $l_v \in L_i$, if l_v is selected into S of Algorithm 2, the relative interference $RI(S_i \cup S_i^+, l_v)$ does not exceed $1 - \phi$.*

Proof. According to the first step of Algorithm 2, for any links l_v in S_i, their relative interference $\mathrm{RI}(S_i^-, l_v)$ does not exceed ϕ. Assume that $l_u \in S_j$ and $j > i$. Considering Lemma 1 and $l_v \in S_j^-$, the distance between s_u and s_v must be larger than $(1/\phi^{1/\alpha} - 1)c_u^{1/\alpha} d(l_u)$. Note that $d(l_u) > d_i$ and c_u is an increasing function of $d(l_u)$. Hence, we have that for any pair of links belonging to two different subclasses S_i and S_j, where $j > i$, the mutual distance between their sender is larger than $(1/\phi^{1/\alpha} - 1)c_i^{1/\alpha} d_i$.

Considering the fact that for any pair of links in the same subclass S_j, the mutual distance between their sender is larger than $(1/\phi^{1/\alpha} - 1)c_j^{1/\alpha} d_j$ by the second step of Algorithm 2. Since c_j and d_j are increasing function of j, we have that for any pair of links in the same subclass S_j, where $j \geq i$, the mutual distance between their sender is no less than $(1/\phi^{1/\alpha} - 1)c_i^{1/\alpha} d_i$.

To summarize, we have that, for any pair of links l_u and l_v in $S_i \cup S_i^+$, the mutual distance of their sender s_u and s_v is

$$d(s_u, s_v) \geq \left(\frac{1}{\phi^{1/\alpha}} - 1\right) c_i^{1/\alpha} d_i. \quad (20)$$

Using the same method in Theorem 3, we can get the following upper bound for $\mathrm{RI}(S_i \cup S_i^+, l_v)$ based on inequalities (8), (9), and (10):

$$\mathrm{RI}\left(S_i \cup S_i^+, l_v\right) < \left(\frac{1}{\phi^{1/\alpha}} - 2\right)^{-\alpha} 8\left(\frac{2}{\alpha-2} + \frac{1}{\alpha-1} + 3\right). \tag{21}$$

Recall that $\phi \leq 1/(2+(\rho+1)^{1/\alpha})^{\alpha}$, where $\rho = 8(2/(\alpha-2) + 1/(\alpha-1)+3)$, and thus $\mathrm{RI}(S_i \cup S_i^+, l_v) < 1-\phi$. □

Lemma 7. *The two-slot transmitting/receiving mechanism in the third step of Algorithm 2 is correct.*

Proof. To prove the correctness of this lemma, we only need to show that all communications can be executed successfully in two slots, that is, interference-free. We prove this lemma by considering two cases: communications in the first slot and communications in the second slot.

In the first case, only senders $s_v \in X(L_i)$ with $\sigma(s_v) = 1$ are allowed to send a REQUEST to their corresponding receivers r_v. Recall that if $\sigma(s_v) = 1$, sender s_v must belong to the d-independent set by the second step of Algorithm 2, and the mutual distance between any pair of senders in the d-independent set is no less than d, where $d = (1/\phi^{1/\alpha} - 1)c_i^{1/\alpha} d_i$. Denote by D_i the set of links in L_i whose senders belong to the d-independent set. Using inequalities (9) and (10) and triangular inequality, we have the relative interference received by l_v from other links in D_i:

$$\begin{aligned}
\mathrm{RI}\left(D_i, l_v\right) &= c_v \sum_{l_u \in D_i \setminus \{l_v\}} \left(\frac{d\left(l_u, l_v\right)}{d\left(l_v\right)}\right)^{-\alpha} \\
&\leq c_v \sum_{l_u \in D_i \setminus \{l_v\}} \left(\frac{d\left(s_u, s_v\right) - d\left(l_v\right)}{d\left(l_v\right)}\right)^{-\alpha} \\
&\leq c_i \sum_{l_u \in D_i \setminus \{l_v\}} \left(\frac{d\left(s_u, s_v\right) - d_i}{d_i}\right)^{-\alpha} \\
&< \left(\frac{1}{\phi^{1/\alpha}} - 2\right)^{-\alpha} 8\left(\frac{2}{\alpha-2} + \frac{1}{\alpha-1} + 3\right) \\
&< 1-\phi < 1.
\end{aligned} \tag{22}$$

That is, the transmissions in the first slot are interference-free.

For the second case, only receivers $r_v \in Y(L_i)$ with $\sigma(r_v) = 1$ that have received the REQUEST message from their corresponding senders s_v are allowed to send the ACK to s_v. As mentioned above, sender s_v belongs to the d-independent set. That is, only receivers of links l_v in D_i are allowed to transmit in the second slot. Let $\overline{l_v}$ denote the link l_v with transmission direction inverted, that is, from r_v to s_v. Likewise, let $\overline{D_i}$ denote the set of links in D_i with transmission direction inverted. Note that, if $l_v \in D_i$, we have $\overline{l_v} \in \overline{D_i}$, $d(l_v) = d(\overline{l_v})$, and $c_v = \beta/(1-(d(l_v)/R)^{\alpha}) = \beta/(1-(d(\overline{l_v})/R)^{\alpha})$. Then, using inequality (22) and triangular inequality, we get

$$\begin{aligned}
\mathrm{RI}\left(\overline{D_i}, \overline{l_v}\right) &= c_v \sum_{\overline{l_u} \in \overline{D_i} \setminus \{\overline{l_v}\}} \left(\frac{d\left(\overline{l_u}, \overline{l_v}\right)}{d\left(\overline{l_v}\right)}\right)^{-\alpha} \\
&\leq c_v \sum_{\overline{l_u} \in \overline{D_i} \setminus \{\overline{l_v}\}} \left(\frac{d\left(s_u, s_v\right) - d\left(\overline{l_u}\right)}{d\left(\overline{l_v}\right)}\right)^{-\alpha} \\
&= c_v \sum_{l_u \in D_i \setminus \{l_v\}} \left(\frac{d\left(s_u, s_v\right) - d\left(l_u\right)}{d\left(l_v\right)}\right)^{-\alpha} \\
&\leq c_i \sum_{l_u \in D_i \setminus \{l_v\}} \left(\frac{d\left(s_u, s_v\right) - d_i}{d_i}\right)^{-\alpha} < 1.
\end{aligned} \tag{23}$$

The transmissions in the second slot are also interference-free. Hence, the lemma is true. □

Theorem 8. *Algorithm 2 provides a valid solution.*

Proof. For any links l_v in S_i, where $1 \leq i \leq g(L)$, $\mathrm{RI}(S_i^-, l_v) \leq \phi$ by the first step of Algorithm 2. According to Lemma 6, we have $\mathrm{RI}(S_i \cup S_i^+, l_v) < 1-\phi$. Therefore, $\mathrm{RI}(S, l_v) = \mathrm{RI}(S_i^-, l_v) + \mathrm{RI}(S_i \cup S_i^+, l_v) < 1$. □

Lemma 9. *The total time to compute a MIS at each stage is $O(\log^2 n)$.*

Proof. Please see [22]. □

Theorem 10. *The time complexity of Algorithm 2 is $O(\log^2 n)$.*

Proof. As shown in Algorithm 2, the initializing step only takes one slot for all links. For links in L_i, lines (8) to (24) take $O(\log^2 n) + 3$ slots to select valid links into S, and, in total, there are $g(L)$ subclasses. To sum up, Algorithm 2 takes $1 + (O(\log^2 n) + 3)g(L)$ time slots. □

3.3. Remarks on Distributed MIS Algorithm. The second step of Algorithm 2 is to compute a MIS set which has been extensively studied and many distributed algorithms have been proposed [23–25]. However, most of these methods compute the MIS set by modeling it as a graph, for example, the unit disk graph (UDG). A SINR-based interference model is fundamentally different because the cumulative interference may result in a node v failing to receive a message even when only one neighbor of v transmits. Due to this difference, it is impractical to adopt a graph-based, distributed MIS algorithm for our problem, which assumes the SINR-based interference model.

To the best of our knowledge, the first MIS algorithm designed for the SINR-based interference model is proposed by Yu et al. [22]. It is a randomized distributed method that can compute the MIS set in time $O(\log^2 n)$. Another possible distributed MIS algorithm that we can use in the second step of Algorithm 2 is presented in [26]. Yu et al. [26] adapt the deterministic MIS algorithm of [24] by carefully choosing power levels. As proven in [26], their methods can compute the MIS set in time $O(\log n)$, which will improve the time complexity of Algorithm 2 to $O(\log n)$.

4. Conclusion

In this paper, we have studied the maximum link scheduling problem under the SINR interference model. The goal is to maximize the set of concurrent links. To address this problem, we design a constant approximation algorithm under the assumption of uniform transmission power. With the same assumption, the constant approximation ratio of our algorithm is $5\lceil(\phi^{-1/\alpha} + 2\beta^{-1/\alpha})^{\alpha}\rceil$. Both the constant ϕ and ratio have improved by 24 times at least when comparing with excellent related algorithms. We also present its distributed implementation that is dependent only on the maximum and minimum link length.

There are still a number of open problems, such as power control, multihop traffic scheduling and routing, analog network coding, and models beyond SINR such as log-normal shadowing. As a future work, we are currently looking into a distributed algorithm with lower time complexity. The use of our solution in arbitrary power assignment and 3D space is also possible future works.

Notations

L:	Set of links
$g(L)$:	Link diversity
$X(L)$:	Senders of links in L
n:	Number of links
$Y(L)$:	Receivers of links in L
R:	Maximum transmission radius
$d(u,v)$:	Distance of node u and v
c_v:	$\beta/(1-(d(l_v)/R)^{\alpha})$
$d(l_v)$:	Length of l_v
RI:	Relative interference
$d(l_v,l_u)$:	$d(s_v,r_u)$
ϕ:	Constant defined in Theorem 3
α:	Path-loss exponent
d_i:	$(1+\epsilon)^i d_{\min}$
β:	SINR threshold
c_i:	$\beta/(1-(d_i/R)^{\alpha})$
N:	Ambient noise
P:	Transmission power.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] O. Goussevskaia, R. Wattenhofer, M. M. Halldórsson, and E. Welzl, "Capacity of arbitrary wireless networks," in *Proceedings of the 28th Conference on Computer Communications (IEEE INFOCOM '09)*, pp. 1872–1880, Rio de Janeiro, Brazil, April 2009.

[2] P. J. Wan, X. Jia, and F. Yao, "Maximum independent set of links under physical interference model," in *Wireless Algorithms, Systems, and Applications*, Lecture Notes in Computer Science, pp. 169–178, Springer, Berlin, Germany, August 2009.

[3] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388–404, 2000.

[4] K. Jain, J. Padhye, V. N. Padmanabhan, and L. Qiu, "Impact of interference on multi-hop wireless network performance," in *Proceedings of the 9th Annual International Conference on Mobile Computing and Networking (MobiCom '03)*, pp. 66–80, ACM, San Diego, Calif, USA, September 2003.

[5] M. Alicherry, R. Bhatia, and L. E. Li, "Joint channel assignment and routing for throughput optimization in multiradio wireless mesh networks," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 11, pp. 1960–1971, 2006.

[6] M. Kodialam and T. Nandagopal, "Characterizing the capacity region in multi-radio multi-channel wireless mesh networks," in *Proceedings of the 11th Annual International Conference on Mobile Computing and Networking (MobiCom '05)*, pp. 73–87, ACM, Cologne, Germany, September 2005.

[7] S. Sujay Sanghavi, L. Bui, and R. Srikant, "Distributed link scheduling with constant overhead," *ACM SIGMETRICS*, vol. 35, no. 1, pp. 313–324, 2007.

[8] P. J. Wan, "Multiflows in multihop wireless networks," in *Proceedings of the 10th ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc '09)*, pp. 85–94, New Orleans, La, USA, May 2009.

[9] Official homepage of the IEEE 802.11 working group, http://grouper.ieee.org/groups/802/11/.

[10] R. Maheshwari, S. Jain, and S. R. Das, "A measurement study of interference modeling and scheduling in low-power wireless networks," in *Proceedings of the 6th ACM Conference on Embedded Networked Sensor Systems (SenSys '08)*, pp. 141–154, Raleigh, NC, USA, November 2008.

[11] T. Moscibroda, R. Wattenhofer, and Y. Weber, "Protocol design beyond graph-based models," in *Proceedings of the Workshop on Hot Topics in Networks (HotNets '06)*, Irvine, Calif, USA, November 2006.

[12] O. Goussevskaia, Y. A. Oswald, and R. Wattenhofer, "Complexity in geometric SINR," in *Proceedings of the 8th ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc '07)*, pp. 100–109, ACM, Montreal, Canada, September 2007.

[13] X. H. Xu and S. J. Tang, "A constant approximation algorithm for link scheduling in arbitrary networks under physical interference model," in *Proceedings of the 2nd ACM International Workshop on Foundations of Wireless Ad Hoc and Sensor Networking and Computing (FOWANC '09)*, New Orleans, La, USA, May 2009.

[14] M. M. Halldórsson and R. Wattenhofer, "Wireless communication is in apx," in *Proceedings of the 36th International Colloquium on Automata, Languages and Programming (ICALP '09)*, pp. 525–536, Rhodes, Greece, July 2009.

[15] M. M. Halldórsson and P. Mitra, "Wireless capacity with oblivious power in general metrics," in *Proceedings of the 22nd Annual ACM-SIAM Symposium on Discrete Algorithms (SODA '11)*, pp. 1538–1548, San Francisco, Calif, USA, January 2011.

[16] T. Kesselheim, "A constant-factor approximation for wireless capacity maximization with power control in the SINR mode," in *Proceedings of the 7th International Symposium on Algorithms for Sensor Systems*, San Francisco, Calif, USA, January 2011.

[17] M. Dinitz, "Distributed algorithms for approximating wireless network capacity," in *Proceedings of the IEEE INFOCOM*, pp. 1–9, IEEE, San Diego, Calif, USA, March 2010.

[18] E. I. Ásgeirsson and P. Mitra, "On a game theoretic approach to capacity maximization in wireless networks," in *Proceedings of the IEEE INFOCOM*, pp. 3029–3037, Shanghai, China, April 2011.

[19] G. Pei and V. S. Anil Kumar, "Efficient algorithms for maximum link scheduling in distributed computing models with sinr constraints," in *Proceedings of the 31st Annual IEEE International Conference on Computer Communications (IEEE INFOCOM '12)*, Orlando, Fla, USA, March 2012.

[20] G. Pei and A. K. S. Vullikanti, "Distributed approximation algorithms for maximum link scheduling and local broadcasting in the physical interference model," in *Proceedings of the 32nd IEEE Conference on Computer Communications (IEEE INFOCOM '13)*, pp. 1339–1347, Turin, Italy, April 2013.

[21] P. J. Wan, L. Wang, and O. Frieder, "Fast group communications in multihop wireless networks subject to physical interference," in *Proceedings of the 6th IEEE International Conference on Mobile Adhoc and Sensor Systems (MASS '09)*, pp. 526–533, IEEE, Macau, China, October 2009.

[22] D. Yu, Y. Wang, Q.-S. Hua, and F. C. M. Lau, "Distributed $(\Delta+1)$-coloring in the physical model," in *Algorithms for Sensor Systems*, vol. 7111 of *Lecture Notes in Computer Science*, pp. 145–160, 2012, http://i.cs.hku.hk/~qshua/algosensorsfullversion.pdf.

[23] T. Moscibroda and R. Wattenhofer, "Maximal independent sets in radio networks," in *Proceedings of the 24th Annual ACM Symposium on Principles of Distributed Computing (PODC '05)*, pp. 148–157, ACM, Las Vegas, Nev, USA, July 2005.

[24] J. Schneider and R. Wattenhofer, "What is the use of collision detection (in wireless networks)?" in *Distributed Computing*, vol. 6343 of *Lecture Notes in Computer Science*, pp. 133–147, Springer, Berlin, Germany, 2010.

[25] J. Schneider and R. Wattenhofer, "A log-star distributed maximal independent set algorithm for growth-bounded graphs," in *Proceedings of the 27th ACM Symposium on Principles of Distributed Computing (PODC '08)*, Toronto, Canada, August 2008.

[26] D. Yu, Y. Wang, Q. S. Hua, and F. C. M. Lau, "Distributed local broadcasting algorithms in the physical interference model," in *Proceedings of the 7th IEEE International Conference on Distributed Computing in Sensor Systems (DCOSS '11)*, Barcelona, Spain, June 2011.