Journal of Network and Computer Applications 77 (2017) 64–72

Contents lists available at ScienceDirect

Journal of Network and Computer Applications

journal homepage: www.elsevier.com/locate/jnca

SINR based shortest link scheduling with oblivious power control in wireless networks

Baogui Huang[a,b], Jiguo Yu[a,b,*], Xiuzhen Cheng[c], Honglong Chen[d], Hang Liu[e]

[a] School of Information Science and Engineering, Qufu Normal University, Rizhao, Shandong 276826, China
[b] Institute of Network Computing, Qufu Normal University, Rizhao, Shandong 276826, China
[c] Department of Computer Science, The George Washington University, Washington DC 20052, USA
[d] College of Information and Control Engineering, China University of Petroleum, Qingdao, Shandong 266580, China
[e] Department of Electrical Engineering & Computer Science, The Catholic University of America, Washington DC 20052, USA

ARTICLE INFO

Keywords:
Wireless network
Shortest link scheduling
SINR
Oblivious power assignment

ABSTRACT

In this paper, we consider shortest link scheduling (SLS), a fundamental problem in wireless networks to improve the network performance, under the signal-to-interference-plus-noise-ratio (SINR) constraints. It is challenging to design efficient SLS algorithms due to the intrinsic non-locality of SINR. However, if two transmission links are far away from each other, the interference of one link on the other should be small under the SINR model. This motivates us to consider the interference only in a local area, which decreases the difficulty of designing link scheduling under SINR, by partitioning the links into disjoint local link sets with a certain distance away from each other, such that independent scheduling inside each local link set is possible. Based on this idea, we propose a novel approximation algorithm PPSLS (Plane Partition based Shortest Link Scheduling) for SLS with oblivious power control. Theoretical analysis and simulations demonstrate the correctness and effectiveness of the proposed algorithm.

1. Introduction

As a fundamental problem in wireless networks, link scheduling is crucial for improving the network performance through maximizing throughput and fairness. Wireless networks have been employed in a variety of applications and have become more and more important nowadays. However, wireless communication resources such as spectrum is deficient. Therefore, many communication links have to share a common channel, resulting in a significant interference among concurrent transmissions. One effective technique to reduce interference and enhance network performance is to allocate different time slots for concurrent transmission links.

Link scheduling plays an essential role especially when the network has stringent quality of service restrictions. Generally speaking, link scheduling mainly includes three sub-problems: *maximum link scheduling* (*MLS*) (e.g. Goussevskaia et al., 2007, 2009; Halldórsson and Mitra, 2014; Huang et al., 2014; Deng et al., 2015; Zhou and Li, 2015), *maximum weighted link scheduling* (*MWLS*) (e.g. Goussevskaia et al., 2007; Xu et al., 2010; Joo et al., 2013; Wan et al., 2014), and *shortest link scheduling* (*SLS*) (e.g. Wan et al., 2010; Goussevskaia et al., 2014; Wang et al., 2015; Yu et al., 2016). Given a set of communication link requests $L = \{l_1, l_2, \ldots, l_n\}$, with l_i denoting the ith link request, MLS seeks to compute the largest feasible subset $S \subseteq L$ of links that can be scheduled simultaneously without interference. If each link is assigned a weight, MWLS computes a feasible subset whose weighted sum is the maximum. SLS is represented by $S = (S_1, S_2, \ldots, S_T)$, where S_t denotes a subset of links of L, designated to time slot t, with T being referred to as the *length* or *latency* of the schedule. In other words, SLS intends to compute a link schedule of the shortest length for L. Note that when the weight of each link is equal to one, MWLS is equivalent to MLS.

Interference models and power assignment are two important parameters that should be considered in link scheduling algorithm design and analysis. In this paper, we adopt the physical interference model, in which a signal is received successfully if and only if the Signal to Interference plus Noise Ratio (SINR) at the receiver is above a threshold depending on hardware and physical layer technologies. This model is claimed to be more practical, and has been used to study scheduling problems since the year 2006 (Moscibroda et al., 2006; Moscibroda and Wattenhofer, 2006; Halldórsson, 2012; Goussevskaia et al., 2007, 2009; Wan et al., 2010, 2014; Xu et al., 2010; Joo et al., 2013; Zhou et al., 2014). However, due to the non-locality of SINR, it is difficult to design a link scheduling algorithm with the SINR constraint, and the link scheduling problems under SINR are NP-hard (Goussevskaia et al., 2009; Gupta and Kumar, 2000). In fact, one must

* Corresponding author at: School of Information Science and Engineering, Qufu Normal University, Rizhao, Shandong 276826, China.
E-mail addresses: hjbaogui@126.com (B. Huang), jiguoyu@sina.com (J. Yu), cheng@gwu.edu (X. Cheng), chenhl@upc.edu.cn (H. Chen), liuh@cua.edu (H. Liu).

http://dx.doi.org/10.1016/j.jnca.2016.10.012
Received 13 April 2016; Received in revised form 9 October 2016; Accepted 18 October 2016
Available online 24 October 2016
1084-8045/

take energy efficiency into account when designing algorithms for wireless networks, such as ad hoc wireless network (Li et al., 2007) and wireless sensor network (Xie and Wang, 2014), since the batteries are low power and not recharged or replaced during a mission. For power assignment, we adopt the *oblivious* power control, in which the transmission power of a link depends only on the length of the link. The two most frequently used power assignment strategies, namely the *uniform* (*or fixed*) *assignment* and the *linear assignment* (which ensures that the signals received at the intended receivers are identical), are special cases of oblivious power assignment.

In this paper, we tackle the challenges of SLS in wireless networks under the SINR interference model. We mainly consider the interference in a local area and propose a localized link scheduling algorithm with oblivious power assignment. Our algorithm design is motivated by the following two observations. (1) If the distance between two links is sufficiently large, their mutual interference is small, and thus, the two links might be able to transmit simultaneously; (2) A short link can tolerate a large interference but a long link can only tolerate a small interference.

We first classify the links into groups according to their lengths, and then partition the links of each group into disjoint local link sets, whose distances are large enough, such that independent scheduling inside each local link set is possible. By this way we successfully decouple the global interference constraints for the implementation of localized scheduling, which lowers the difficulty to design link scheduling algorithms under the SINR model.

We also consider oblivious power assignment to links, which can conserve energy to the limit while improving the network throughput. Extensive theoretical analysis is conducted to justify our algorithm design and investigate its performance, and *the results state that the proposed algorithm has a constant performance ratio if the link length diversity is a constant*. To the best of our knowledge, this is the first localized solution on the SLS problem under the SINR interference model.

The rest of the paper is organized as follows. We discuss the related work in Section 2. Section 3 presents the model and definitions. In Section 4, we propose an algorithm for the shortest link scheduling problem under the SINR model and demonstrate its correctness and effectiveness by theoretical analysis. In Section 5 we show the efficiency of our algorithm by simulations. Finally, we summarize the paper in Section 6.

2. Related work

There have been a few results on link scheduling under the SINR constraint (e.g. Wan et al., 2012; Halldórsson, 2012; Goussevskaia et al., 2007, 2009; Wan et al., 2010, 2014; Xu et al., 2010; Joo et al., 2013; Zhou et al., 2014; Blough et al., 2010). Although we focus on the SLS problem, we give a brief review on the MLS problem because it is closely related to our work.

Goussevskaia et al. (2007) presented the first NP-hard proof of link scheduling under the SINR model and proposed an $O(g(L))$ factor approximation algorithm for both MWLS and SLS with a uniform power assignment, where $g(L) = \log(l_{\max}/l_{\min})$ is called the *link length class diversity*, and $l_{\max}$ and $l_{\min}$ denote the length of the longest and the shortest link, respectively. Later, Goussevskaia et al. (2009) developed a constant approximation ratio algorithm for the MLS problem and derived an $O(\log n)$ factor SLS algorithm by directly applying the MLS algorithm *OneSlotSchedule*, where n is the total number of links. The interference model used in Goussevskaia et al. (2007, 2009) is an approximation of the SINR model, in which the effect of noise is neglected. When ignoring the ambient noise, SINR is simplified to SIR, in which the transmission range of a link is infinite; thus the possible number of link length classes is infinite as well. Blough et al. (2010) proposed the first SLS algorithm under the exact SINR model. They defined a class of links called "black-gray" links, whose lengths are equal or near to the maximum transmission range of the sender. The approximation bound of the proposed algorithm is heavily affected by the "black-gray"links. If few or no "black-gray" links are present, the approximation bound is a constant. However, if relatively more "black-gray" links appear in the wireless network, the approximation bound becomes looser. In the extreme case, in which all the links to be scheduled are "black-gray", the approximation bound is $O(n)$, with n being the number of links. The problem of MWLS under the physical interference model with oblivious power assignment in wireless networks was studied in Xu et al. (2010), which presented an algorithm to find multiple sets of well-separated links and then select the one with the largest weight by using partition and shifting strategies. Wang et al. (2015) improved the results of Blough et al. (2010) by introducing hypergraph model and selecting more than one links from a square to schedule simultaneously. The upper bound of the scheduling length is $O(\Delta_{\max})$, where $O(\Delta_{\max})$ is the maximal number of links in a square, which is $O(n)$ in the extreme case of all the n links in one square. Furthermore, the side length of a square is related to $\Delta_{\max}$ while $\Delta_{\max}$ depends on the size of square. The intersecting relationship is paradoxical.

Note that the algorithms mentioned above are all centralized. Due to the global characteristic of the SINR model, designing a distributed link scheduling algorithm is a challenging problem.

Distributed link scheduling has been studied in Le et al. (2010) and Pei and Vullikanti (2012). Le et al. (2010) proposed a distributed greedy maximal link scheduling algorithm under interference localization. A link l only performs scheduling coordination inside a circle area named interference neighborhood of the link. However, the trivial procedure for determining the interference neighborhood is centralized. Moreover, link l and other links need to calculate their cumulative interference in an iterated procedure, which is impractical in a large scale network. Pei and Vullikanti (2012) and Gupta and Kumar (2000) proposed a local distributed scheduling and power control algorithm under the SINR model, achieving an $O(g(L))$ approximation factor in the throughput region. Note that the uniform power assignment for all links was adopted, and thus the links in the same link class were assigned the same power. Similar to Le et al. (2010), a local interference region was defined in Pei and Vullikanti (2012), in which the SINR feasible set was computed.

By combining the partition with shifting strategies into a pick-and-compare scheme, Zhou et al. (2014) presented a class of localized scheduling algorithms with provable throughput guarantee subject to the physical interference constraints. The algorithm under the linear power setting was the first localized algorithm that achieves at least a constant fraction of the optimal capacity region subject to the physical interference constraints. The algorithm under the uniform power setting was the first localized algorithm with a logarithmic approximation ratio to the optimal solution. The basic idea of the algorithm is to create a set of disjoint local link sets in which the scheduling can be done independently without violating the global interference constraints. The distance of two cells is determined by the longest link, which leads to a looser approximation factor. Moreover, the links of the feasible set S are picked from sub-squares, and some links that are in a super-subsquare but out of the sub-squares that may transmit concurrently with S are not picked, which also contributes to the loose approximation factor.

The main differences of our work from the most related ones are summarized as follows.

- In Goussevskaia et al. (2007), Blough et al. (2010), and Le et al. (2010), the algorithms are under the uniform power assignment rather than the general oblivious power assignment. Uniform power assignment is simple but has low efficiency. To ensure that the longest link transmits successfully, all links are assigned a large power, which is not necessary especially to the short links. Moreover, large transmit power results in large interference. In

Pei and Vullikanti (2012), the link set is partitioned into subsets, and links in the same subset are assigned a uniform power. In this paper, we adopt the oblivious power assignment, which assigns powers to the links depending on their lengths. Since oblivious power assignment includes uniform and mean power assignments as special cases, our algorithm is also suitable for uniform power assignment.
- In Xu et al. (2010) and Zhou et al. (2014), the plane is partitioned into small cells and the width of each cell is fixed for all links. However, links with different lengths can tolerate different interference, and short links can tolerate strong interference. Therefore the distance between two shorter links in the same SINR-feasible set may be smaller. Based on this fact, the widths of the cells should vary with the link lengths. In this paper, we partition the links into different classes according to their lengths. The network region is partitioned into cells and the width of a small cell is related to the length class. It is easy to compute a SINR feasible set since the interference is localized in a small cell.
- In Goussevskaia et al. (2007), Xu et al. (2010), and Blough et al. (2010), only one link is picked from each small cell, which obviously is not an effective method. In fact, when the small cells are far away from each other, we can select more than one link from each small cell. In Wang et al. (2015), more than one link selected from a small cell. However, the side length of the cell is uncertain, since it depends on the number of receivers in a cell and the number of receivers is uncertain. In this paper, we partition the network region into small cells and the side length of the cell is determined. *we* propose a *OneSlotScheduling* algorithm that can greedily pick one or more possible links within a small cell to form a SINR-feasible set for each slot, which significantly enhance the scheduling performance.

3. Model and definition

We consider a wireless network consisting of a number of nodes that communicate with each other over a single wireless channel. Assume that all the nodes are arbitrarily deployed in a 2-dimensional Euclidean plane, and that each node knows its position. We further assume that time is slotted, with the length of each slot being able to support the transmission of one unit traffic. Given a set of link requests $L = \{l_1, l_2, \ldots, l_n\}$, where each link request $l = (s, r)$ represents a transmission from a sender s to a receiver r, we intend to minimize the total number of time slots to transmit all requests in L. The length of link $l = (s, r)$, denoted by $d(l) = d(s, r)$, is the Euclidean distance between s and r. When no ambiguity arises, we simply use l to denote $d(l)$. The asymmetric distance from link $l' = (s', r')$ to $l = (s, r)$ is the distance from s' to r, denoted by $d(l'l) = d(s', r)$. Furthermore, we use $l_{\max}$ and $l_{\min}$ to denote the lengths of the longest and the shortest links in L, respectively. Assume that each link has a unit-traffic demand, and model the case of non-unit traffic demand by replicating the links.

SINR model: The signal power received at the receiver r of link $l = (s, r)$ is P_l/l^α, where P_l denotes the transmission power of l and $\alpha > 2$ denotes the path loss exponent. Such a transmission is successful if the strength of the desired signal at r is at least a factor $\beta \geq 1$ higher than that of simultaneous transmissions plus ambient noise N at r. This is the Signal-to-Interference-plus-Noise-Ratio (SINR) model, which can be formally defined as follows: a transmission over a link $l = (s, r)$ succeeds if and only if the following condition holds:

$$\frac{P_l/l^\alpha}{N + \sum_{l' \in S \setminus \{l\}} P_{l'}/d^\alpha(l'l)} \geq \beta, \tag{1}$$

where S is the set of concurrently scheduled links at the same time slot. If each link in S satisfies (1), then S is called a *SINR-feasible* link set.

An important factor that should be taken into account is the transmission power of a link. A signal can be transmitted to a distant receiver with a large power. However, a large power implies large interference and large energy consumption, which leads to the decrease of the life time and capacity of the wireless network. If the transmission power is small, the transmission distance of the signal is small as well. Then the signal may need multiple hops from its sender to receiver, which results in a large transmission delay. In this paper, we adopt the *oblivious power assignment policy*, where the power depends only on the length of the link. In other words, the transmission power of link l, denoted by P_l, equals $c \cdot l^{k\alpha}$, where $c > 0$ and k $(0 \leq k \leq 1)$ are constants. In fact, if k=0, each link uses the same power, which is called the *uniform power assignment* or *fixed power assignment*. When $k = 1/2$ and k=1, the oblivious assignments are called *mean power assignment* and *linear power assignment*, respectively. The oblivious power assignment ensures that no multi-hop transmissions is needed by L.

Interference: Let $l = (s, r)$ and $l' = (s', r')$ be two links that can transmit concurrently. Then the *interference of* l' *on* l is defined as $I_{l'}(l) = P_{l'}/d^\alpha(l'l)$. Similarly, the *accumulative interference* of the feasible set S on link $l \in S$ is defined as $I_S(l) = \sum_{l' \in S \setminus \{l\}} P_{l'}/d^\alpha(l'l)$. Based on these definitions, the SINR model can be denoted as

$$\frac{P_l/l^\alpha}{N + I_S(l)} \geq \beta, \tag{2}$$

where N is the ambient noise.

We use $I_{\max}(l)$ to denote the maximum interference link l can tolerate, i.e., if link l transmits successfully, the interference of the feasible set S on link l is at most $I_{\max}(l)$. Obviously, $I_{\max}(l) = \frac{P_l}{\beta \cdot l^\alpha} - N$. In this paper, we seek a partition of the link set L into a minimum number of SINR-feasible subsets, so that each subset of links can be concurrently scheduled in one time slot.

Relative interference (Halldórsson and Wattenhofer, 2009): The *relative interference* (*RI*) of link $l' = (s', r')$ on link $l = (s, r)$ is the increase in the inverse of the SINR at l caused by l', namely $RI_{l'}(l) = \frac{P_{l'}/d^\alpha(l'l)}{P_l/l^\alpha}$.

Affectance (Halldórsson and Wattenhofer, 2009): Let $c(l) = \frac{\beta}{1 - (\beta N l^\alpha)/P_l}$ indicate the extent to which the ambient noise approaches the required signal at receiver r. The *affectance* of link l, caused by a set S of links, is the sum of the relative interferences of the links in S on l, scaled by $c(l)$, i.e.,

$$a_S(l) = c(l) \sum_{l' \in S \setminus \{l\}} RI_{l'}(l). \tag{3}$$

Note that the set S is a SINR-feasible set if and only if for $\forall\ l \in S$, we have $a_S(l) \leq 1$.

p-Signal set (Halldórsson and Wattenhofer, 2009): A *p-signal set* or schedule is a link set where the affectance of any link is at most $1/p$.

Obviously, a SINR-feasible set S is a 1-signal set.

4. Shortest link scheduling with oblivious power control

In this section, we focus on the design of a shortest link scheduling algorithm with oblivious power control. We first outline the basic ideas of our algorithm design and present the corresponding pseudo-codes. Then we prove the correctness and the effectiveness of our algorithm by theoretical analysis and simulation study.

4.1. Plane partition based shortest link scheduling (PPSLS)

First, we partition the set of links $L = \{l_1, l_2, \ldots, l_n\}$ into disjoint link length classes according to the lengths of the links. Formally, let $L = L_0 \cup L_1 \cup \cdots \cup L_{g(L)-1}$, where $g(L) = \lceil \log(l_{\max}/l_{\min}) \rceil$ denotes the length diversity. For each link length class, $L_i = \{l | 2^i l_{\min} \leq l < 2^{i+1} l_{\min}\}$, $i = 0, 1, \ldots, g(L) - 1$. In practical scenarios, the diversity $g(L)$ is usually a small constant.

Next, we perform the following 5 steps for each link length class L_i:

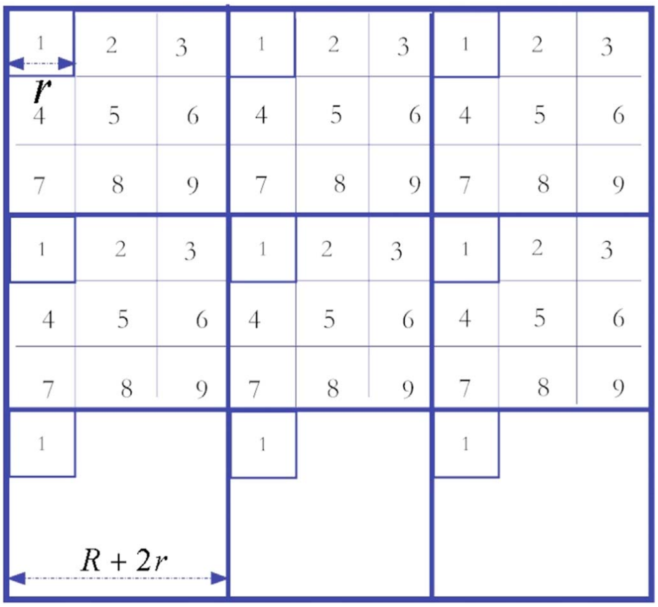

Fig. 1. Steps 1–3: Partition the plane into large squares and partition each large square into small cells.

1. Partition the plane covering all links of L_i into large squares with a width $R + 2r$, where $r = 2^{i+1}l_{\min}$, as shown in Fig. 1. The value of R is determined later.
2. Partition each large square into small cells with a width r. The number of small cells in all large squares is identical.
3. All the small cells in a large square are numbered regularly from 1 to $\lceil R/r + 2\rceil^2$, as shown in Fig. 1. The distance of two cells with the same serial number is at least $R + r$. Here we point out that, R, r, and the number of small cells in a large square, are different for different length classes.
4. Determine the small cell at which a link belongs to. There are four spatial relations between a link l and a small cell A_{cell}, as shown in Fig. 2.
   (i) If l resides in A_{cell} entirely, l belongs to A_{cell} (e.g. l_1);
   (ii) If l is completely out of A_{cell}, l dose not belong to A_{cell} (e.g. l_5);
   (iii) If l does not completely reside in A_{cell} and the segment of l residing in A_{cell} is at least one half of the length of l, l belongs to A_{cell} (e.g. l_2); and

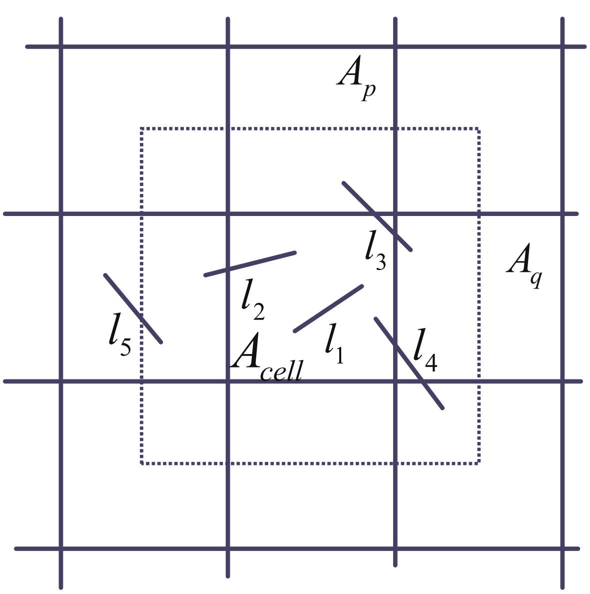

Fig. 2. Step 4: Determining the cells to which each link belongs.

   (iv) If l stretches across the borders of three cells A_p, A_{cell}, and A_q, with its source in A_p and destination in A_q, and the segments of l in A_p and A_q are both shorter than one half of its length, l participates in the scheduling of A_{cell} (e.g. l_3). This step is summarized by the pseudo code given in Algorithm *LinkHoming*.
5. Links that belong to the cells with the same serial number compute a local schedule by performing the *OneSlotScheduling* algorithm (Algorithm 3) for each time slot t and derive a SINR feasible set S_t. This process repeats until all links in the link length class L_i are scheduled. Let $A_m^{\bar{k}}$ denote the $\bar{k}$ th small cell in the mth large square. Then all links in $A_m^{\bar{k}}$ compute a SINR-feasible set $S_t^{A_m^{\bar{k}}}$ at time slot t by calling Algorithm *OneSlotScheduling*. The links in $\bigcup_{m=1}^{\tau} S_t^{A_m^{\bar{k}}}$ can transmit simultaneously, with τ being the total number of large squares.

Algorithm 1. LinkHoming.

Input : Links in L_i and the small cells
Output: The small-cell to which each link $l \in L_i$ belongs
for $\forall\, l = (s, r) \in L_i$ **do**
  if s and r are in the same small cell A_{cell} **then**
    l belongs to A_{cell};
  else
   if s (or r) is in A_{cell} and the segment in A_{cell} is at least one half of the length of l **then**
     l belongs to A_{cell};
    else
     if s and r are not in A_{cell} but l intersects with A_{cell} **then**
       if both segments out of A_{cell} are shorter than one half of the length of l **then**
         l belongs to A_{cell};
       end if
     end if
    end if
  end if
end for
Return

These five steps are summarized by our algorithm termed *Plane Partition based Shortest Link Scheduling* (*PPSLS*), whose pseudo-code is presented in algorithm *PPSLS* (Algorithm 2).

Algorithm 2. PPSLS.

Input: A set L of links arbitrarily deployed in the Euclidean plane
Output : A link schedule $S_1, S_2, \ldots, S_T$, such that all links in a feasible set S_t can transmit simultaneously
Let
$L = L_0 \cup L_1 \cup \cdots \cup L_{g(L)-1},\ \ L_i = \{l | 2^i l_{\min} \le l < 2^{i+1}l_{\min}\},\ i = 0,\ ;$
$1, \ldots, g(L) - 1$
t=0;
for each $L_i \ne \emptyset$ **do**
  Partition the plane into large squares with width $R + 2r$;
  Partition each large square into small cells with width $r = 2^{i+1}l_{\min}$;
  Label all small cells in each large square with numbers $1, 2, 3, \ldots, \lceil R/r + 2\rceil^2$;
  Call *LinkHoming* to identify the small cell to which each link $l \in L_i$ belongs;
  repeat
   for $\bar{k} = 1$ to $\lceil R/r + 2\rceil^2$ **do**
     $t = t + 1$;
     $S_t = \emptyset$;
     for m=1 to τ **do**

```
            S_t^{A_m^k} = OneSlotScheduling(A_m^k);
            S_t = S_t ∪ S_t^{A_m^k};
            L_i = L_i \ S_t^{A_m^k};
        end for
      end for
    until all links in L_i are scheduled;
end for
Return S_1, S_2, …, S_t;
```

4.2. Theoretical performance analysis

Next, we prove that the algorithm PPSLS is correct and has an $O(g(L))$ approximation ratio to the optimal solution.

We consider a SINR-feasible set S_t of time slot t. Without loss of generality, assuming that $S_t \subseteq L_i$, i.e., the SINR-feasible set S_t is composed of the links that belong to the link length class L_i. For any link $l \in S_t$ that belongs to $A_m^{\bar{k}}$, l is affected not only by the interference of the simultaneous transmission links inside of $A_m^{\bar{k}}$, which is denoted by $I_{in}(l)$, but also by the interference of the simultaneous transmission links in $S_t \setminus S_t^{A_m^{\bar{k}}}$, which is denoted by $I_{out}(l)$. If $I_{in}(l) + I_{out}(l) \le I_{\max}(l)$, then l can transmit successfully. Let $I_{in}(l) = (1-\varepsilon)I_{\max}(l)$, where $0 < \varepsilon < 1$, which can be guaranteed by algorithm *OneSlotScheduling* (see Theorem 5). If $I_{out}(l) \le \varepsilon I_{\max}(l)$, then the PPSLS algorithm is correct. For this purpose, we first prove the following lemmas.

Lemma 1. *The number of simultaneous transmission links in a small cell is bounded. Let OPT_{ub} be an optimal SINR - feasible set of a small cell, then* $|OPT_{ub}| \le \frac{(2\sqrt{2}+2)^\alpha}{(1-\varepsilon)\cdot\beta} + 1$.

Proof. Without loss of generality, we consider $OPT_{ub} \subseteq L_i$, where $L_i = \{l | 2^i l_{\min} \le l < 2^{i+1} l_{\min}\}$. Let $l' = (s', r') \in OPT_{ub}$ be the shortest link in OPT_{ub} and $l = (s, r) \in OPT_{ub}$ be any other arbitrary link in OPT_{ub}, as shown in Fig. 3. One can easily derive that $d(ll') = d(s, r') \le (\sqrt{2}+1)\cdot r$.

The cumulative interference of OPT_{ub} on l' is denoted by $I_{OPT_{ub}}(l')$. Then

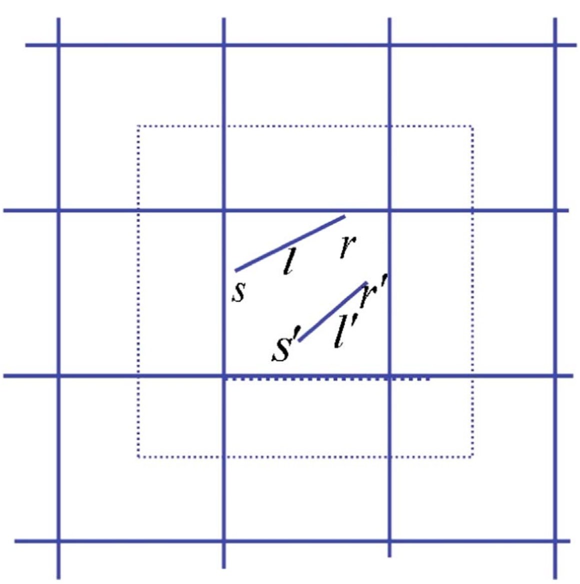

Fig. 3. l and l' can transmit simultaneously, where $l \ge l'$.

$$I_{OPT_{ub}}(l') = \sum_{l \in OPT_{ub} \setminus \{l'\}} I_l(l') = \sum_{l \in OPT_{ub} \setminus \{l'\}} \frac{P_l}{d^\alpha(ll')}$$
$$\ge \sum_{l \in OPT_{ub} \setminus \{l'\}} \frac{P_l}{((\sqrt{2}+1)\cdot r)^\alpha} \ge (|OPT_{ub}|-1)\cdot\frac{P_{L_i}^{\min}}{((\sqrt{2}+1)\cdot r)^\alpha}.$$

where $P_{L_i}^{\min}$ denotes the minimum power assigned to the link length class L_i.

On the other hand, since

$$\frac{P_{l'}/l'^\alpha}{N + (1-\varepsilon)I_{\max}(l')} \ge \beta,$$

we have

$$(1-\varepsilon)I_{\max}(l') \le \frac{P_{l'}}{\beta\cdot l'^\alpha} - N \le \frac{P_{l'}}{\beta\cdot l'^\alpha}.$$

Also note that

$$I_{OPT_{ub}}(l') \le I_{\max}(l') \le \frac{P_{l'}}{(1-\varepsilon)\cdot\beta\cdot l'^\alpha},$$

we have

$$(|OPT_{ub}|-1)\cdot\frac{P_{L_i}^{\min}}{((\sqrt{2}+1)\cdot r)^\alpha} \le \frac{P_{l'}}{(1-\varepsilon)\cdot\beta\cdot l'^\alpha}.$$

Therefore, we have

$$(|OPT_{ub}|-1) \le \frac{P_{l'}}{P_{L_i}^{\min}}\frac{((\sqrt{2}+1)\cdot r)^\alpha}{(1-\varepsilon)\cdot\beta\cdot l'^\alpha}.$$

Under oblivious power assignment, i.e., $P = c\cdot l^{k\alpha}$, $0 \le k \le 1$, we have $P_{l'} \ge P_{L_i}^{\min}$ since $l' \in L_i$. Consequently,

$$\frac{P_{l'}}{P_{L_i}^{\min}}\frac{((\sqrt{2}+1)\cdot r)^\alpha}{(1-\varepsilon)\cdot\beta\cdot l'^\alpha} \le \frac{c\cdot(l')^{k\alpha}}{c\cdot(2^i l_{\min})^{k\alpha}}\frac{((\sqrt{2}+1)\cdot 2^{i+1} l_{\min})^\alpha}{(1-\varepsilon)\cdot\beta\cdot l'^\alpha}$$
$$= \frac{l'^{k\alpha}\cdot(2\sqrt{2}+2)^\alpha\cdot(2^i l_{\min})^\alpha}{l'^\alpha\cdot(1-\varepsilon)\cdot\beta\cdot(2^i l_{\min})^{k\alpha}} = \frac{(2\sqrt{2}+2)^\alpha}{(1-\varepsilon)\cdot\beta}\cdot$$
$$\frac{(2^i l_{\min})^{(1-k)\alpha}}{(l')^{(1-k)\alpha}} \le \frac{(2\sqrt{2}+2)^\alpha}{(1-\varepsilon)\cdot\beta}.$$

The last inequality holds since $l' \ge 2^i l_{\min}$.

This proves that $|OPT_{ub}| \le \frac{(2\sqrt{2}+2)^\alpha}{(1-\varepsilon)\cdot\beta} + 1$.□

Next, we prove that $I_{out}(l')$ is bounded.

Lemma 2. *Let $l' \in S_t^{A_m^{\bar{k}}}$, where $S_t^{A_m^{\bar{k}}}$ denotes the feasible set of links picked from the small cell $A_m^{\bar{k}}$ at t. Let $I_{out}(l')$ denote the interference of $S_t \setminus S_t^{A_m^{\bar{k}}}$ on l', $I_{\max}^{L_i}$ denote the maximum interference the longest link of L_i can tolerate, and $P_{L_i}^{\max}$ denote the maximum power assigned to L_i:*
If $R \ge \left(\frac{8\cdot|OPT_{ub}|\cdot P_{L_i}^{\max}}{\varepsilon I_{\max}^{L_i}}\cdot\left(1 + \frac{\alpha-1}{\alpha-2} + \frac{\alpha}{\alpha-1}\right)\right)^{1/\alpha}$, *then* $I_{out}(l') \le \varepsilon I_{\max}^{L_i}$.

Proof. As shown in Fig. 4, we consider link l' in the small cell $A_m{}^1$ and figure out the bound of the interference placed on l' by all simultaneous transmission links in the small cells with serial number 1 besides $A_m{}^1$.

There are 8 closest small cells with serial number 1 around $A_m{}^1$, and the distance of a link in the 8 small cells and link l' is at least R. Thus, the sum of their interference on l' is at most $8\cdot|OPT_{ub}|\cdot\frac{P_{L_i}^{\max}}{R^\alpha}$. Then we consider the (at most) 16 small cells around the 8 closest small cells, and the distance of a link in these 16 small cells and link l' is at least $2R + 2r$. They contribute a total interference of at most $16\cdot|OPT_{ub}|\cdot\frac{P_{L_i}^{\max}}{(2R+2r)^\alpha}$ on link l'. Similarly, we can analyze the small cells with the same serial number but even farther away from $A_m{}^1$. Therefore, the total interference on a scheduled link l' can be upper-bounded by

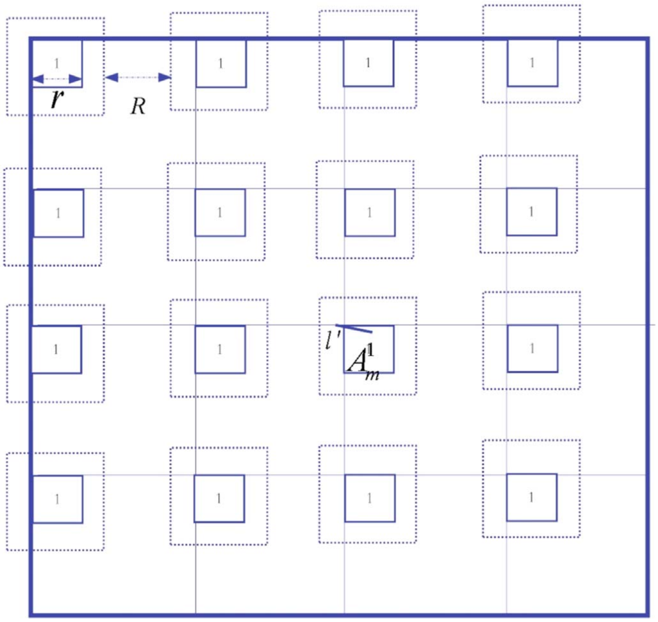

Fig. 4. The interference $I_{out}(l')$ is bounded.

$$I_{out}(l') = \sum_{l\in\bigcup_{n\neq m} S_t^{A_n^1}} I_l(l') = \sum_{l\in\bigcup_{n\neq m} S_t^{A_n^1}} \frac{P_l}{d^{\alpha}(ll')}$$

$$\leq \sum_{h=1}^{\infty}\left(8\,h\cdot \sum_{l\in\bigcup_{n\neq m} S_t^{A_n^1}} \frac{P_l}{(hR+(h-1)2r)^{\alpha}}\right)$$

$$= \sum_{h=1}^{\infty}\left(8\,h\cdot \sum_{l\in\bigcup_{n\neq m} S_t^{A_n^1}} \frac{P_l}{(h(R+2r)-2r)^{\alpha}}\right) \leq 8\cdot|OPT_{ub}$$

$$|\cdot\frac{P_{L_i}^{\max}}{R^{\alpha}} + \sum_{h=2}^{\infty}\left(8\,h\cdot \sum_{l\in\bigcup_{n\neq m} S_t^{A_n^1}} \frac{P_l}{((h-1)(R+2r))^{\alpha}}\right) \leq 8\cdot|OPT_{ub}$$

$$|\cdot\frac{P_{L_i}^{\max}}{R^{\alpha}} + \frac{8\cdot|OPT_{ub}|\cdot P_{L_i}^{\max}}{(R+2r)^{\alpha}}\cdot\sum_{h=2}^{\infty}\frac{h}{(h-1)^{\alpha}} \leq 8\cdot|OPT_{ub}$$

$$|\cdot\frac{P_{L_i}^{\max}}{R^{\alpha}} + \frac{8\cdot|OPT_{ub}|\cdot P_{L_i}^{\max}}{(R+2r)^{\alpha}}\cdot\left(\frac{\alpha-1}{\alpha-2}+\frac{\alpha}{\alpha-1}\right) \leq 8\cdot|OPT_{ub}$$

$$|\cdot P_{L_i}^{\max}\left(1+\frac{\alpha-1}{\alpha-2}+\frac{\alpha}{\alpha-1}\right)\cdot\frac{1}{R^{\alpha}}.$$

To ensure that $I_{out}(l') \leq \varepsilon I_{\max}^{L_i}$, we need

$$R \geq \left(\frac{8\cdot|OPT_{ub}|\cdot P_{L_i}^{\max}}{\varepsilon I_{\max}^{L_i}}\cdot\left(1+\frac{\alpha-1}{\alpha-2}+\frac{\alpha}{\alpha-1}\right)\right)^{1/\alpha}.$$

That is, if $R \geq \left(\frac{8\cdot OPT_{ub}|\cdot P_{L_i}^{\max}}{\varepsilon I_{\max}^{L_i}}\cdot(1+\frac{\alpha-1}{\alpha-2}+\frac{\alpha}{\alpha-1})\right)^{1/\alpha}$, we have $I_{out}(l') \leq \varepsilon I_{\max}^{L_i}$.□

Lemma 2 derives R, which is used in the partition to get the large squares. The following theorem proves that our algorithm PPSLS yields a SINR-feasible schedule of the given set of links L.

Theorem 3. *Let T be the total number of slots used by PPSLS. Then for any S_t, $t = 1, 2, \ldots, T$, obtained by PPSLS, is a SINR − feasible set.*

Proof. Without loss of generality, let $S_t \subseteq L_i$. According to Lemma 1 and 2, for $\forall\ l \in S_t$ we have

$$I_{in}(l) + I_{out}(l) = (1-\varepsilon)I_{\max}(l) + I_{out}(l) \leq (1-\varepsilon)I_{\max}(l) + \varepsilon I_{\max}^{L_i}$$
$$\leq (1-\varepsilon)I_{\max}(l) + \varepsilon I_{\max}(l) = I_{\max}(l)$$

Therefore, link l can transmit successfully. Hence, the theorem holds.□

As mentioned earlier, $I_{in}(l) = (1-\varepsilon)I_{\max}$ can be guaranteed by *OneSlotScheduling*. In the following we prove this claim and present the algorithm *OneSlotScheduling*. If there are only a few links in $A_m^{\bar{k}}$, the SINR feasible set $S_t^{A_m^{\bar{k}}}$ can be derived by brute-force enumeration. However, in general, enumeration has an exponential time complexity. Thus $S_t^{A_m^{\bar{k}}}$ must be computed by an approximation algorithm when the number of links in $A_m^{\bar{k}}$ is large. Let *Enum* denote the upper bound of the number of links in a small cell that can tolerate the time complexity of enumeration.

Lemma 4. *Assume that l and l' belong to the same small cell and can transmit concurrently, then $d(l, l') > (\beta/(1-\varepsilon))^{1/\alpha} l^k l'^{1-k}$, and $d(l', l) > (\beta/(1-\varepsilon))^{1/\alpha} l'^k l^{1-k}$.*

Proof. The interference of l on l' is $I_l(l') = P_l/d^{\alpha}(l, l')$ and $I_{max}(l') = P_{l'}/(\beta d^{\alpha}(l')) - N$. Therefore,

$$P_l/d^{\alpha}(l, l') \leq (1-\varepsilon)(P_{l'}/(\beta d^{\alpha}(l')) - N) < (1-\varepsilon)(P_{l'}/(\beta d^{\alpha}(l'))).$$

Then we have,

$$d^{\alpha}(l, l') > \frac{\beta}{1-\varepsilon}\frac{P_l}{P_{l'}}d^{\alpha}(l') = \frac{\beta}{1-\varepsilon}\frac{cl^{k\alpha}}{cl'k\alpha}l'^{\alpha} = \frac{\beta}{1-\varepsilon}l^{k\alpha}l'^{(1-k)\alpha}.$$

That is,

$$d(l, l') > (\beta/(1-\varepsilon))^{1/\alpha} l^k l'^{1-k}.$$

Similarly, it can be proved that $d(l', l) > (\beta/(1-\varepsilon))^{1/\alpha} l'^k l^{1-k}$ holds.□

Lemma 4 indicates that when l' joins S, link l whose distance to l' is not larger than $(\beta/(1-\varepsilon))^{1/\alpha} l^k l'^{1-k}$ can never join S. That is, one of the necessary conditions of concurrent transmitting of l and l' is that the distance of l(or l') to l'(or l) is larger than $(\beta/(1-\varepsilon))^{1/\alpha} l^k l'^{1-k}$(or $(\beta/(1-\varepsilon))^{1/\alpha} l'^k l^{1-k}$).

Our *OneSlotScheduling* algorithm is presented in Algorithm 3, in which we adopt a greedy strategy to select a SINR feasible set. At each iteration, we select one unscheduled link randomly, say l, and judge whether or not l can join the current SINR feasible set. Assume that S is the SINR feasible set computed from the last iteration. If S does not impact the transmission of link l and the join of l does not impact every link in S, then l joins S (Lines 10 and 11). Once l joins S, link l' whose distance to l is not larger than $\lambda(l)(\beta/(1-\varepsilon))^{1/\alpha} l'^k l^{1-k}$ should be removed (Line 9). Where, $\lambda(l) = \left(\frac{3P_l}{P_l - \beta N l^{\alpha}}\right)^{1/\alpha}$ is called "*safety factor*". The function of "*safety factor*" is to reserver sufficient "space" to other links. On the other hand, the links affected greatly by S should be removed too to accelerate the implementation of *OneSlotScheduling* (Line 13). The pseudo-code of *OneSlotScheduling* is given in Algorithm 3.

Algorithm 3. OneSlotScheduling.

Input: Links in a small-cell A, denoted as L_A
Output: A SINR-feasible set in A
1 : n is the number of links in L_A
2:$S = \varnothing$;
3 : **if** $n \leq Enum$ **then**
4 : Compute S using brute-force enumeration;
5 : **else**
6 : **while** $L_A \neq \varnothing$ **do**
7 : Select one unscheduled link $l = (s, r) \in L_A$;
8 : $S = S \cup \{l\}$;
9: Remove all $l' = (s', r')$ from L_A with $d(l', l) = d(s', r) \leq \lambda(l)(\beta/(1-\varepsilon))^{1/\alpha} l'^k l^{1-k}$;
10 Remove all $l'' = (s'', r'')$ from L_A with $a_S(l'') > 2(1-\varepsilon)/3$;
11: Remove l from L_A;
12 : **end while**
13 : **end if**
14 : Return S;

In the following we prove that *OneSlotScheduling* yields a $1/(1-\varepsilon)$-signal set.

Theorem 5. *Algorithm OneSlotScheduling produces a $1/(1-\varepsilon)$–signal set.*

Proof. Let $\bar{S}$ be the intermediate $1/(1-\varepsilon)$-*signal* set after a iteration step. A link l joining $\bar{S}$ should satisfy two constraints.

(1) $\bar{S}$ does not impact on the transmission of l.

(2) l can not impact on the transmission of any link in $\bar{S}$.

Next, we examine whether or not a link l should join $\bar{S}$. Obviously, (1) holds since l is not deleted by line 10. Then, we prove that l can join $\bar{S}$ without impacting on the links in $\bar{S}$.

Note that, for $\forall\ l' \in \bar{S}, d(l, l') > \lambda(l')(\beta/(1-\varepsilon))^{1/\alpha} l^k l'^{1-k}$ holds, since l is not deleted by line 9 in a previous iteration. Therefore,

$$a_l(l') = c(l')RI_l(l') = c(l')\frac{P_l/d^\alpha(l, l')}{P_{l'}/l'^\alpha} = c(l')\frac{P_l}{P_{l'}}\frac{l'^\alpha}{d^\alpha(l, l')} < \frac{1-\varepsilon}{\beta\lambda^\alpha(l')}c(l') = (1-\varepsilon)/3.$$

Thus,

$$a_{\bar{S}\setminus\{l'\}\cup\{l\}}(l') = a_{\bar{S}\setminus\{l'\}}(l') + a_l(l') < 2(1-\varepsilon)/3 + (1-\varepsilon)/3 = 1-\varepsilon.$$

That is, l joins $\bar{S}$ without impacting on the links in $\bar{S}$. To sum up, l satisfies the two conditions and l can join $\bar{S}$.□

Next, we examine the approximation factor of PPSLS.

Theorem 6. *The number of time slots produced by PPSLS is an $O(g(L))$ approximation to the optimal solution.*

Proof. Let T_{OPT} denote the total number of time slots that an optimal algorithm needs, and T_{PPSLS} denote the total number of time slots PPSLS needs. Consider one small cell and a link class, say L_i. Denote by $|OPT^*|$ the maximum number of links that an optimal shortest link scheduling algorithm schedules in a small cell. Lemma 1 has shown that an optimal maximum link scheduling algorithm can schedule at most $|OPT_{ub}|$ links. Then, each shortest link scheduling algorithm schedules no more than $|OPT_{ub}|$ links at one time slot. That is, $|OPT^*| \le |OPT_{ub}|$. Assume that there are at most x links in a small cell. Then scheduling all the x links needs at least $x/|OPT^*|$ time slots. That is, $T_{OPT} \ge x/|OPT^*|$. On the other hand, *OneSlotScheduling* can schedule at least one link at each time slot. Then it needs at most x time slots to schedule all the x links.

Since the link set is partitioned into $g(L)$ classes, and for each link length class L_i, there are μ^2 different small cells, where $\mu = R/r + 2$, the total number of needed slots to schedule all links is at most $g(L)\mu^2 x$. That is, $T_{PPSLS} \le g(L)\mu^2 x$. Consequently, we have

$$\frac{T_{PPSLS}}{T_{OPT}} \le g(L)\mu^2|OPT^*| \le g(L)\mu^2|OPT_{ub}|.$$

Note that, μ and $|OPT_{ub}|$ are constants. Therefore, the approximation factor of PPSLS is $O(g(L))$. Furthermore, for a given wireless network, the length diversity $g(L)$ is a small constant.□

For shortest link scheduling problem, Blough et al. (2010) claimed a constant approximation factor, which is denoted as *BLOUappr*, $BLOUappr = |C_0| + 4\bar{k}\Delta_{\max}$, where $|C_0|$ is the number of links in class C_0, in which the links are scheduled sequentially, $\bar{k} = \lfloor \log_{1+\epsilon}(P/\beta N) \rfloor$ is the *SNR diversity* of links, and $\Delta_{\max}$ is the maximal number of links in a square. P is the transmitting power of a link. Since P is uniform, all links must be assigned the largest power, i.e., $P = \beta N l_{\max}^\alpha$ to ensure the correct packet reception. Therefore, $\bar{k} = \alpha \log_{1+\epsilon} l_{\max}$. Furthermore, we conclude that $BLOUappr = |C_0| + \frac{4\alpha((1+\epsilon)^{1/\alpha}+\sqrt{2}\mu)^\alpha \log_{1+\epsilon} l_{\max}}{\beta}$ by theorem 6 of Blough et al. (2010), where $\mu = 2\left(63\beta(1+\epsilon)\frac{\alpha-1}{\alpha-2}\right)^{1/\alpha}$, $\epsilon \ge \frac{1}{7}$. $|C_0|$ is concerned with the instance of input, and it is uncertain. Therefore, we ignore $|C_0|$. Here, let $\epsilon = 1$ and $l_{\min} = 1$. We compare our result, denoted as *PPSLSappr*, with *BLOUappr*. We use $\frac{BLOUappr}{PPSLSappr}$ as the approximation ratio of *PPSLSappr* to *BLOUappr*, Table 1 lists $\frac{BLOUappr}{PPSLSappr}$ with different parameters of α and β, and we conclude that PPSLS has tighter approximation factor than that of Blough et al. (2010).

Table 1
The ratio of *PPSLSappr* to *BLOUappr*.

Ratio					
α	β				
	5	10	20	50	100
2.5	5.4	10	16.8	28.5	34.6
3	7.8	**14.8**	27.1	55.4	84.1
4	11.9	22.7	43.1	100	182.7
6	15.2	28.2	52.7	121.3	229.5

5. Simulations

First, we report the simulation settings. Assume that nodes are arbitrarily deployed in a square area of 500 m*500 m or 1000 m*1000 m. There are 1000 links and the lengths of the links are arbitrarily distributed in the range from 1 m to 100 m. The threshold of SINR is 1 and the ambient noise is −70 dB. Due to the randomness of the node locations and link lengths, the results of the simulations are different. Thus we use an average of 50 runs to improve confidence.

We compare PPSLS with Goussevskaia et al. (2007) (GOU) and Blough et al. (2010) (BLOU). As shown in Fig. 5 and 6, PPSLS performs well when scheduling links in small- to medium-sized wireless networks and high-density wireless networks. Since the width of the small cells in Goussevskaia et al. (2007) and Blough et al. (2010) is very large, the number of cells in Goussevskaia et al. (2007) and Blough et al. (2010) is very small. Therefore, the number of links in one slot SINR-feasible schedule is small, which results in a bad scheduling result. Here we point out that the results are directly proportional to the number of small cells in Goussevskaia et al. (2007) and Blough et al. (2010), and are inversely proportional to the number of small cells in PPSLS. With the increase of α, the number of small cells increases in Goussevskaia et al. (2007) and Blough et al. (2010) and decreases in PPSLS. That is, the performance of all the three algorithms becomes better when α increases (see Fig. 7).

We notice that the results of Goussevskaia et al. (2007) and Blough et al. (2010) are sensitive to the ratio of the longest link length and the shortest link length, while those of PPSLS are independent of it (Fig. 8). Note that ε also influences the results (see Fig. 9). Links in a small cell compute a $1/(1-\varepsilon)$-signal scheduling. The number of links transmitting simultaneously in a small cell increases with the decrease of ε.

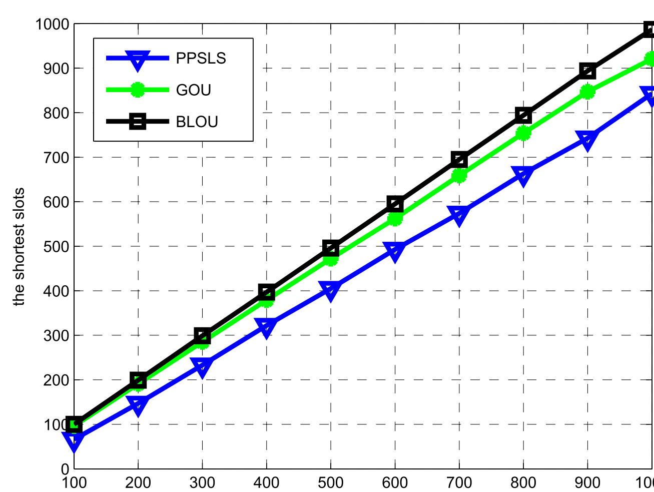

Fig. 5. Network area is 500*500, $\alpha = 3$.

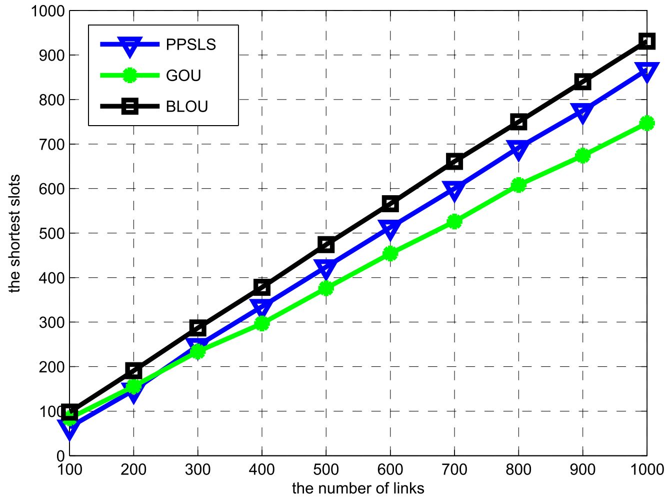

Fig. 6. Network area is 1000*1000, $\alpha = 3$.

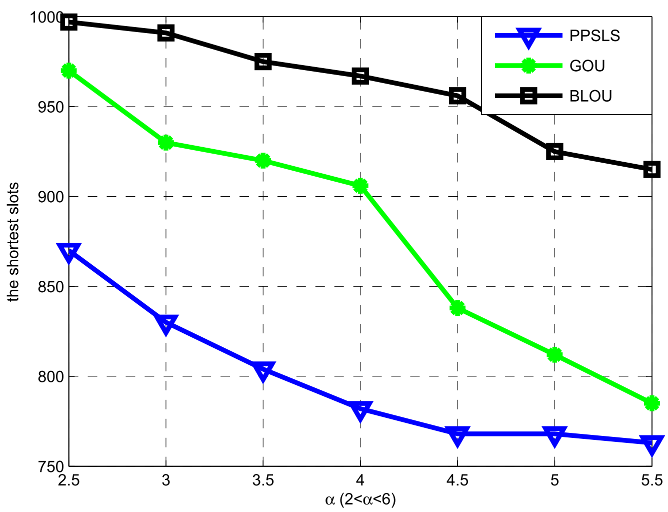

Fig. 7. Network area is 500*500, the number of links is 1000.

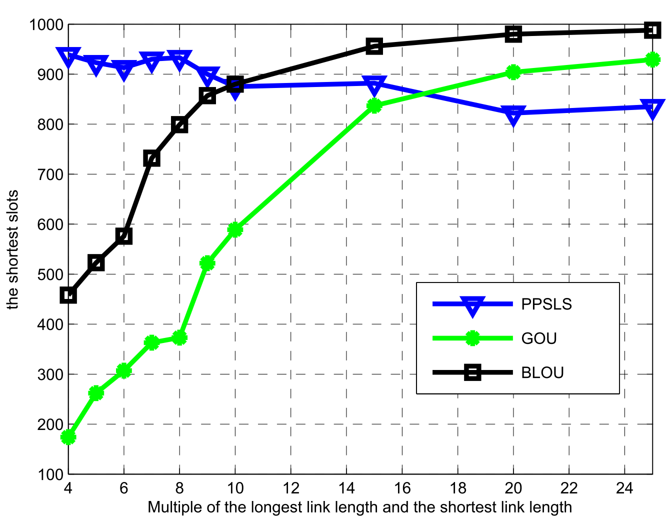

Fig. 8. Network area is 500*500, $l_{\min} = 1$, the number of links is 1000.

However, the distance between two small cells with the same serial number increases when ε decreases. Thus, the total number of links transmitting simultaneously decreases. That implies that we must select a proper value of ε to obtain a better result. In fact, the scheduling results are sensitive to many parameters, such as ε, α,

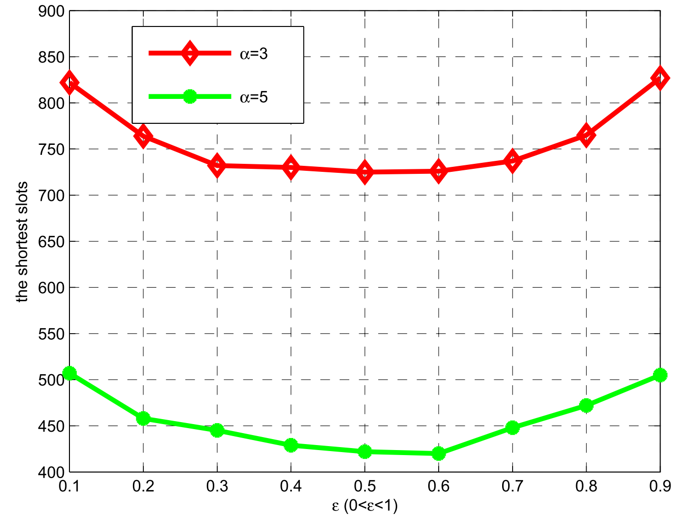

Fig. 9. ε influences the results.

and the scale of network, and our future research targets the investigation of their interdependence in link scheduling.

Note that *OneSlotScheduling* can be performed at many small cells concurrently, as only the local interference is considered. To design a low complexity distributed algorithm is attractive and challenging, especially in large scale wireless networks.

6. Conclusion and future research

In this paper, we consider the problem of shortest link scheduling in wireless networks subject to physical interference constraints. Due to the non-locality of the SINR model, it is difficult to design a SLS algorithm under SINR. We observe that the distance of links dominates the interference. That is, if two simultaneous transmission links are far away from each other, the interference of one on the other is insignificant. Moreover, if the difference of the link lengths is no more than twice of the shortest link, the set of links are easy to schedule. Therefore, we first classify the links based on their lengths and then partition each link class into disjoint local link sets with a certain distance away from each other. By this way we bound the interference from all the other local link sets with a constant so that independent scheduling inside each local link set is possible. What is more, the links that can transmit concurrently in a local link set are bounded under the oblivious power assignment. Note that it is crucial to compute a SINR-feasible set for each small cell in a time slot. As the further work, we will develop more challenging distributed link scheduling algorithms with low complexity for large scale wireless networks.

Acknowledgments

This work was partially supported by the NSF of China under Grants (61373027 and 61672321), NSF of Shandong Province under Grant ZR2012FM023, and STPU of Shandong Province under Grants (J14LN38 and J15LN05), STP of QFNU for contract xkj201409.

References

Blough, D.M., Resta, G., Santi, P., 2010. Approximation algorithms for wireless link scheduling with SINR-based interference. IEEE/ACM Trans. Netw. 18 (6), 1701–1712.

Deng, H., Yu, J., Yu, D., Li, G., Huang, B., 2015. Heuristic algorithms for one-slot link scheduling in wireless sensor networks under SINR. Int. J. Distrib. Sens. Net. 2015. http://dx.doi.org/10.1155/2015/806520

Goussevskaia, O., Oswald, Y.A., Wattenhofer, R., 2007. Complexity in geometric SINR. In: Proceeding of the ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc 2007), pp. 100–109.

Goussevskaia, O., Halldórsson, M.M., Wattenhofer, R., Welzl, E., 2009. Capacity of

arbitrary wireless networks. In: Proceeding of the IEEE International Conference on Computer Communications (IEEE INFOCOM 2009), pp. 1872–1880.
Goussevskaia, O., Halldrsson, M.M., Wattenhofer, R., 2014. Algorithms for wireless capacity. IEEE/ACM Trans. Netw. 22 (3), 745–755.
Gupta, P., Kumar, P.R., 2000. The capacity of wireless networks. IEEE Trans. Inf. Theory 46 (2), 388–404.
Halldórsson, M.M., Mitra, P., 2014. Wireless capacity with arbitrary gain matrix. Theor. Comput. Sci. 553, 57–63.
Halldórsson, M.M., Wattenhofer, R., 2009. Wireless communication is in APX. In: Proceeding of 36th International Colloquium on Automata, Languages and Programming (ICALP 2009), pp. 525–536.
Halldórsson, M.M., 2012. Wireless scheduling with power control. ACM Trans. Algorithms 9 (1).
Huang, B., Yu, J., Yu, D., Ma, C., 2014. SINR based maximum link scheduling with uniform power in wireless sensor networks. KSII Trans. Internet Inf. Syst. 8 (11), 4050–4067.
Joo, C., Lin, X., Ryu, J., Shroff, N.B., 2013. Distributed greedy approximation to maximum weighted independent set for scheduling with fading channels. In: Proceeding of the ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc 2013), pp. 89–98.
Le, L., Modiano, E., Joo, C., Shroff, N.B., 2010. Longest-queue-first scheduling under SINR interference model. In: Proceeding of the ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc 2010), pp. 41–50.
Li, D., Liu, Q., Hu, X., Jia, X., 2007. Energy efficient multicast routing in Ad Hoc wireless networks. Comput. Commun. 30 (December (18)), 3746–3756.
Moscibroda, T., Wattenhofer, R., 2006. The complexity of connectivity in wireless networks. In: Proceedings of the IEEE International Conference on Computer Communications (IEEE INFOCOM 2006), pp. 1–13.
Moscibroda, T., Wattenhofer, R., Zollinger, A., 2006. Topology control meets SINR: the scheduling complexity of arbitrary topologies. In: Proceedings of the ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc 2006), pp. 310–321.
Pei, G., Vullikanti, A., 2012. Low-complexity scheduling for wireless networks. In: Proceedings of the ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc 2012), pp. 35–44.
Wan, P., Xu, X., Frieder, O., 2010. Shortest link scheduling with power control under physical interference model. In: Proceedings of Sixth International Conference on Mobile Ad-hoc and Sensor Networks (MSN 2010), pp. 74–78.
Wan, P., Chen, D., Dai, G., Wang, Z., Yao, F., 2012. Maximizing capacity with power control under physical interference model in duplex model. In: Proceeding of the IEEE International Conference on Computer Communications (IEEE INFOCOM 2012), pp. 415–423.
Wan, P., Jia, X., Dai, G., Du, H., Frieder, O., 2014. Fast and simple approximation algorithms for maximum weighted independent set of links. Proceeding of the IEEE International Conference on Computer Communications (IEEE INFOCOM 2014), pp. 1653–1661.
Wang, C., Yu, J., Yu, D., Huang, B., Yu, S., 2015. An improved approximation algorithm for the shortest link scheduling in wireless networks under SINR and hypergraph models. J. Combinat. Optim.. http://dx.doi.org/10.1007/s10878-015-9908-4.
Xie, S., Wang, Y., 2014. Construction of tree network with limited delivery latency in homogeneous wireless sensor networks. Wirel. Pers. Commun. 78 (1), 231–246.
Xu, X., Tang, S., Wan, P., 2010. Maximum weighted independent set of links under physical interference model. In: Proceeding of International Conference on Wireless Algorithms, Systems, and Applications (WASA 2010), pp. 68–74.
Yu, J., Huang, B., Cheng, X., Atiquzzaman, M., 2016. Shortest link scheduling algorithms in wireless networks under SINR. IEEE Trans. Veh. Technol.. http://dx.doi.org/10.1109/TVT.2016.2580379.
Zhou, Z., Li, C., 2015. Approximation algorithms for maximum link scheduling under SINR-based interference model. Int. J. Distrib. Sens. Netw. http://dx.doi.org/10.1155/2015/120812
Zhou, Y., Li, X., Liu, M., Mao, X., Tang, S., Li, Z., 2014. Throughput optimizing localized link scheduling for multihop wireless networks under physical interference model. IEEE Trans. Parallel Distrib. Syst. 25 (10), 2708–2720.