Shortest Link Scheduling Algorithms in Wireless Networks Under the SINR model

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***Abstract*—This paper considers the shortest link scheduling (SLS) problem in wireless networks under the Signal-to-Interference-plus-Noise-Ratio (SINR) model. We propose an $O(\log(l_{\max}/l_{\min}))$ approximation algorithm SLSPC (Shortest Link Scheduling with Power Control) with oblivious power assignment and an $O(\log_{1+\varphi}(l_{\max}/l_{\min})$ approximation algorithm SLSUM (Shortest Link Scheduling with Uniform or Mean power assignment) with uniform or mean power control, where $\varphi > 0$ is a constant serving as a regulatory factor for slight transmit power adjustment, and $l_{\max}$ and $l_{\min}$ denote the lengths of the longest and the shortest links, respectively. We conduct rigorous theoretical performance analysis to analyze the feasibility and approximation factors of the proposed algorithms. We also carry out an extensive comparison-based simulation study, whose results indicate that the performances of SLSPC and SLSUM are superior over the state-of-the-art as the set of the so-called "black and gray" links, which are difficult to schedule and should be scheduled sequentially, are completely removed by adjusting the transmit power appropriately via φ. Our numerical analysis demonstrates that the approximation ratios of our algorithms are tighter than the best known ones.**

Keywords*—wireless networks; shortest link scheduling; physical interference model; SINR; power control; oblivious power assignment

I. Introduction

As a fundamental problem, link scheduling plays a crucial role in improving the networking performances via maximizing throughput and improving fairness properties, especially for wireless networks with stringent quality of service restrictions. There exist two major versions of the link scheduling problem: the *maximum link scheduling (MLS) problem*, and the *shortest link scheduling (SLS)* (also known as the *delay minimization problem*). MLS (e.g. [1], [2], [3], [4]) intends to find a maximum subset of links to be scheduled simultaneously in one time slot, given a set of communication links with each having a unit traffic demand. A variation of MLS with throughput maximization is the *maximum weighted link scheduling (MWLS) problem* (e.g. [5], [6], [7]). SLS (e.g. [8], [9], [10], [11]), on the other hand, intends to find the minimum number of time-slots needed to successfully schedule all the given links. In this paper, we focus on the SLS problem.

It is well-known that interference is the most critical challenges in wireless scheduling, as simultaneous transmissions sharing the same channel may corrupt each other and bring down the network throughput. Moreover, a transmission schedule must be applicable in practical scenarios, which requires accurate radio signal propagation and realistic interference models for the scheduling algorithm design. In recent years, many interference models have been adopted in link scheduling (e.g. [12], [13], [14], [15], [16], [17], [18]); but unfortunately, most have mainly revolved around various graph-based models, where interference is treated as a pairwise constraint. Nevertheless, graph-based models, despite being a useful abstraction, are too simplistic, which fail to capture many essential characteristics of wireless communications such as the many-to-many relationship underlying the wireless interference and the gradual signal attenuation with distance.

In this paper, we employ the physical interference model [19], in which a signal is received successfully if and only if the Signal-to-Interference-plus-Noise-Ratio (SINR) at the receiver is above a threshold depending on hardware and physical layer technologies. The SINR model considers the cumulative interference on a link l, i.e. the combined interference on l caused by all other links concurrently transmitting with l, even though the interference from some link is tiny. In other words, the SINR model considers the global interference and is more related to physical layer constraints. It is a challenging issue to design algorithms and protocols under the SINR model. Note that MLS and SLS under the SINR model are both NP-hard [20], [21], [22].

Power control is another important factor that affects the link scheduling performance. It is one of the most efficient means to increase the capacity of a wireless network. Large power of a link increases the transmission ability of the link but causes large interference to other simultaneously transmitting links. Quite a few link scheduling algorithms with power control were proposed recently (e.g. [23], [8], [24], [25]). To achieve the best theoretical results, these algorithms rely on the solutions to complex optimization problems, in which the transmit power of one node potentially depends on the transmit powers of all others [26]. But in practice, the transmit power of a link can not be set according to those of the other concurrent transmissions, leaving the distance between its transmitter and receiver the only factor that should be considered for power assignment. This is known as the *oblivious power control*,

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which includes the *uniform power assignment,* the *mean power assignment*, and the *linear power assignment*, as special cases.

In this paper, we study the SLS problem in wireless networks under the SINR model with power control. The main contributions can be presented as follows.

(1) We consider a more realistic and accurate SINR interference model, and propose an $O(\log(l_{\max}/l_{\min}))$ approximation algorithm SLSPC (Shortest Link Scheduling with Power Control) with oblivious power assignment, and an $O(\log_{1+\varphi}(l_{\max}/l_{\min}))$ approximation algorithm SLSUM (Shortest Link Scheduling with Uniform or Mean power assignment) with uniform or mean power assignment, where $l_{\max}$ and $l_{\min}$ respectively denote the length of the longest and shortest link, and $\varphi > 0$ is a small constant. Our numerical analysis and simulation study demonstrate the efficiency of SLSPC and SLSUM compared to the state-of-the-art algorithms. Moreover, *under the assumption of bounded transmit power or bounded transmit range, the approximation ratios of our algorithms are constants.*

(2) Our plane partition based algorithms are simple and efficient for realizing spatial reuse. They are motivated by cellular networks, in which each cell uses a set of frequencies that are different from the neighboring cells to avoid interference and provide guaranteed bandwidth. We partition the network region into regular hexagons and two arbitrary stations in non-adjacent hexagons can receive messages simultaneously. Simulations demonstrate that our hexagon-partition is more efficient than the square-partition employed by [22] and [27], partially because the number of hexagons that can be simultaneously scheduled is larger than that of the simultaneously scheduled squares under the same network settings.

(3) When the lengths of the links are approximately equal, link scheduling is tractable [28]. Therefore we partition the link set into disjoint subsets, in which the links in each subset have roughly the same length, and consider each subset independently. The transmit power of a link is determined by its length and length class. A link is called "black" (or "gray") if the SINR at its receiver in absence of interference is β (or $\beta + \epsilon$, for some arbitrarily small constant $\epsilon > 0$). The "black and gray" links are difficult to schedule according to [27]. To overcome this challenge, we slightly increase the transmit power such that the SINRs of the "black and gray" links exceed $\beta + \epsilon$. In this paper, we define a *regulatory factor*, which controls the transmit power to make it just larger than the "exact" power assigned to the "black and gray" links in [27]. Simulation results show that this operation can significantly decrease the latency of the schedule with a slight increase of energy consumption.

The rest of the paper is organized as follows. Section II reviews the most related work. In section III, we detail the network model and the SINR model, and formally define the SLS problem under our consideration. Section IV proposes the two approximation algorithms SLSPC and SLSUM for the SLS problem with oblivious and uniform/mean power assignment under the SINR model, and presents a rigorous analysis on the feasibility and approximation ratio of the algorithms. A numerical analysis and simulation study based performance analysis is conducted in Section V. Section VI concludes the paper and outlines our future research.

II. Related Work

Many research on wireless link scheduling considering SINR model has been proposed. Goussevskaia *et al.* [22] proved that both MLS and SLS are NP-hard under the SINR model, and proposed an $O(\log(l_{\max}/l_{\min}))$ approximation factor algorithm called GOW for the SLS problem. GOW consists of two steps: partitioning the problem instance into disjoint link length classes and then constructing a feasible schedule for each length class using a greedy strategy. The interference model used in [22] is an approximation of the SINR model, in which the effect of the ambient noise is neglected. In this case, the SINR model is simplified to SIR (Signal to Interference Ratio) model, in which the transmission range of a link can be infinite; thus the possible number of link length classes can be infinite as well. In [2], Goussevskaia *et al.* proposed a constant approximation factor algorithm for the MLS problem; by applying the MLS subroutine repeatedly, the authors of [2] realized an $O(\log n)$-approximation for the problem of SLS, where n is the number of communication links. Note that the algorithms in [22] and [2] were proposed for uniform power assignment, and can not be easily extended to non-uniform power control.

Considering the exact SINR model that takes into account both the ambient noise and the accumulated interference at a receiver, Blough *et al.* [27] proposed an SLS algorithm called GOW* with uniform power assignment. A class of links called "black and gray" links were specially considered in [27], whose length is equal or near to the maximum transmission range of the sender. Compared with GOW, GOW* has the following two significant features: (1) links are grouped based on SNR (Signal to Noise Ratio) instead of distance, to emphasize that the more relevant factor of link scheduling is signal strength instead of link length; (2) links with small SNR values, i.e., the "black and gray" links, are treated separately as they are hard to schedule.

The approximation bound of GOW* is heavily affected by the number of "black and gray" links. The authors of [27] claimed that if few or no such links appear, GOW* has a constant approximation bound, which is true only if the number of link classes and the maximum number of receivers in a cell are treated as constants. More details are given in Subsection IV-B. Generally speaking, the approximation bound increases when more number of "black and gray" links present in the network. In the extreme case when all the to-be-scheduled links are "black and gray", the approximation bound is $O(n)$, with n being the total number of links.

Xu *et al.* [5] studied the MWLS problem under the SINR model with oblivious power assignment in wireless networks. The main idea of the algorithm in [5] utilizes partition and shifting strategies to find multiple sets of well-separated links and then select the one with the largest weight. In the case of linear power assignment, the approximation factor of the algorithm is a constant. In [29], Xu *et al.* studied the SLS problem under linear power control, uniform power control, and arbitrary power control. The network plane is partitioned into squares with a side length of r, and the links whose senders lie in the squares separated by a distance of R can transmit simultaneously. Note that the algorithms in [5] and [29] do not consider link diversity when determining the size of the squares, leading to large approximation factors. Obviously, a long link can tolerate a small interference while a short link can tolerate a strong interference. Therefore, parameters such as r should be changeable with different link lengths rather than independent of the link lengths.

In [30], Halldórsson and Mitra studied the MLS problem with oblivious power assignment. They gave a constant-factor approximation ratio for any length-monotonic link under the sub-linear power assignment, and showed that the mean power assignment is optimal for the MLS problem. They also claimed that by using mean power, one can get an approximate capacity maximization with respect to arbitrary power control within a factor of $O(\log n + \log\log\Delta)$, where n is the number of links and Δ is the length ratio of the longest link and the shortest link. In [31], Wan *et al.* assumed that all nodes can adjust their transmit powers to any value in a given set P_{set}, and proposed a polynomial $O(\chi)$-approximation algorithm for MLS, with χ being the cardinality of the maximum link set. By directly applying the MLS algorithm in a greedy fashion, Wan *et al.* [31] achieved a polynomial $O(\Lambda \ln \chi)$-approximation algorithm for SLS, where Λ is the power diversity, i.e, $\Lambda = \log(P_{\max}/P_{\min})$, with $P_{\max}$ and $P_{\min}$ being respectively the maximum and minimum power.

Note that the algorithms mentioned above are all centralized. Due to the global characteristic of the SINR model, designing a distributed link scheduling algorithm with low complexity is a challenging problem. Next we summarize the most related research in distributed link scheduling.

Greedy algorithms for distributed maximal link scheduling under interference localization and uniform power assignment were proposed in [32] and [33], in which an "interference neighborhood" of a link l is defined and the SINR interference is successfully localized. A link l only needs to perform scheduling coordination inside its "interference neighborhood". The interferences of the simultaneously transmitting links inside and outside of the "interference neighborhood" on a link l are at most $(1-\zeta)I_{\max}(l)$ and restricted to $\zeta I_{\max}(l)$, respectively, where $I_{\max}(l)$ is the maximum interference that l can tolerate and $0 < \zeta < 1$ is a constant. These algorithms sound promising, but the procedure for determining the interference neighborhood is centralized, and links need to calculate their cumulative interference using an iterated procedure, which is impractical for large scale networks.

By combining the partition and shifting strategies with a pick-and-compare scheme, Zhou *et al.* [7] presented a class of localized scheduling algorithms with a provable throughput guarantee under the SINR constraint. The algorithm under the linear power setting is the first localized one that achieves at least a constant fraction of the optimal capacity region subject to the SINR constraint; while the algorithm with the uniform power setting is the first localized one with a logarithmic approximation ratio to the optimal solution. The basic idea of the algorithms in [7] is to create a set of disjoint local link sets in which the scheduling can be done independently without violating the global interference constraints. The distance of two cells is determined by the longest link, resulting in a loose approximation factor. Moreover, the links in the feasible set S are picked from "sub-squares"; nevertheless, some links in the "super-subsquares" but outside of the "sub-squares" that can transmit concurrently with S are not picked, which also contributes to the loose approximation factor.

The main differences of our algorithms SLSPC and SLSUM compared with the most related ones mentioned above are summarized as follows.

(1) In [22] and [27], the plane is partitioned into squares that are 4-colored to ensure that no two adjacent ones have the same color. In our design, the plane is partitioned into hexagons that are 3-colored such that the neighboring ones have different colors. As all the scheduling algorithms select links for each time slot from the cells with the same color, more number of cells can be considered simultaneously with a hexagon partition, which implies that more links can be scheduled in one time slot, resulting in a better performance, as indicated in Section V.

(2) In [5] and [7], the side length of a square is fixed for all links. But generally speaking, a short link can tolerate a strong interference while a long link can tolerate a weak interference. Therefore in our design, we partition the link set into disjoint subsets such that the links within the same subset have approximately equal length. Thus the side length of a hexagon is small for a short link length class and is large for a long link length class.

(3) In [22] and [27], the transmit power is uniform, which is simple but may result in a low efficient power assignment as all senders are assigned to the power that is large enough for the longest link, causing a significant power wastage at the short links and a strong interference in the whole network. In our design, we adopt the oblivious power assignment, which depends not only on the length of a link but also on the length class the link belongs to. Our algorithm is economical with respect to every consumption.

(4) In [27], the authors considered a special link class consisting of "black and gray" links, which are difficult to schedule. The approximation factor is a linear function of the number of "black and gray" links. By properly and slightly increasing the transmit power, we can successfully remove all the "black and gray" links and significantly decrease the approximation factor.

III. Models and Preliminaries

A. Network model

A wireless network can be modeled as a graph $G = (V, E)$ with V being the set of nodes and E the set of links. Assume that all nodes are deployed in a Euclidean plane. A link $l = (s, r) \in E$ if and only if the signals received at the receiver r can be decoded successfully when no other link transmits concurrently. The Euclidean distance between a sender s and a receiver r represents the length of l, i.e., $d(l) = d(s, r)$. Sometime we use l to denote the length of link l if there is no ambiguity from context. The asymmetric distance from link $l' = (s', r')$ to $l = (s, r)$ is the distance from s' to r, denoted by $d(l'l) = d(s', r)$. We assume that each link has a unit-traffic demand, and model the case of non-unit-traffic demands by replicating the links. Let l_{min} and l_{max} be respectively the minimum and maximum link lengths in E. For simplicity, we assume that $l_{min} \geq 1$.

A link l transmits signals with power P_l. When the power assignment is a part of the problem on link scheduling, the problem is called *link scheduling with power control*. We adopt the so-called *oblivious power* assignment, that is, the transmit power of l depends only on the length of l, which is defined by $P_l = cl^{k\alpha}$, where $c > 0$ and $0 \leq k \leq 1$ are constants. Particularly, if $k = 0$, all the links employ the same power, which is called the *uniform* (or *fixed*) *power assignment*; when $k = 1/2$ or $k = 1$, the oblivious assignment is called *mean power assignment* or *linear power assignment*, respectively.

B. SINR model

We employ the path loss radio propagation model for the reception of signals, where the signal received at r is P_l/l^α, with $\alpha > 2$ denoting the path loss exponent whose exact value depends on the external conditions of the medium (humidity, obstacles etc.). Such a transmission is successful if the strength of the desired signal is at least a factor β higher than that of the combined simultaneous transmissions plus ambient noise. This model is called the *Signal-to-Interference-plus-Noise-Ratio* (SINR) model. More formally, a communication over a link l succeeds if and only if the following condition holds.

$$\frac{P_l/l^\alpha}{N + \sum_{l' \in S \setminus \{l\}} P_{l'}/d^\alpha(l'l)} \geq \beta. \tag{1}$$

where β denotes the minimum SINR value required for a message to be successfully received, N is the ambient noise, and S is the set of concurrently scheduled links. If each link in S can satisfy (1), then S is called an *SINR-feasible link set*. If links $l = (s, r)$ and $l' = (s', r')$ can transmit concurrently, the interference of l' on l is defined as $I_{l'}(l) = P_{l'}/d^\alpha(l'l)$.

C. The SLS problem

Given a set $L = \{l_1, l_2, \cdots, l_n\}$ of communication links, the *shortest link scheduling* (SLS) problem under the SINR model is represented by a schedule $S = (S_1, S_2, \cdots, S_T)$ with a shortest *length* or *latency* T, where S_t $(1 \leq t \leq T)$ denotes a SINR-feasible link set of L, designated to time slot t. In other words, the SLS problem seeks a partition of the link set into a minimum number of subsets, such that the links in each subset satisfy the SINR-constraints, and thus can be scheduled in one time slot.

IV. Algorithm Description and Analysis

In this section, we first analyze two existing algorithms that are closely related to ours. Then we present our SLS algorithms SLSPC (Shortest Link Scheduling with Power Control) and SLSUM (Shortest Link Scheduling with Uniform or Mean power assignment), and prove their correctness and effectiveness by theoretical analysis and simulation study.

A. Motivation

SINR is a *global* interference model - even a tiny amount of interference should not be neglected. However, it is observed that the interference between two links, which is determined by their distance, is a key factor to hinder them from being scheduled concurrently. As low interference is the necessary condition for link coexistence, concurrent links are required to be sufficiently separated in a geometric space. Therefore space partition techniques are popular for link scheduling (e.g. [7], [22], [27], [29]). In these methods, the network region is partitioned into cells such that the links are divided into disjoint local link sets where scheduling can be done independently within the cells that are sufficiently separated without violating the global interference constraint. These approaches are all based on the following fundamental observation: if a transmitting link is placed a certain distance away from all other transmitting links, the total interference it receives may get bounded.

The main ideas of the algorithms in [22], [27], [29] are similar. All the links that can transmit concurrently at the same time slot are picked from different cells - at most one link whose receiver [22], [27] or sender [29] lies in one cell is picked for each slot. The correctness of these algorithms is guaranteed by the distance of two cells, i.e., the value of R, which is a function of the link length and is equal to r in [22], [27] with r being the side length of a cell, or a constant independent of the link length as in [29]. A sufficient condition for a set S of links to be SINR-feasible is that R is greater than a threshold.

B. Analysis on Algorithms GOW and GOW*

The main ideas of our algorithms are motivated by GOW [22] and GOW* [27]. Therefore, in order to clearly outline our ideas and address the differences between our algorithms and GOW/GOW*, we first analyze GOW and GOW*.

In [22], the communication link set $L = \{l_1, l_2, \cdots, l_n\}$ is partitioned into subsets $C_0, C_1, \cdots, C_{\log(l_{\max})}$, where $l_i \in C_k$ if and only if $2^k \leq d(l_i) < 2^{k+1}$. Each C_k is a unique link class and $\lfloor l_{\max} \rfloor$ is called the *link length diversity*. Such a partition has the following property: for any $l_i, l_j \in C_k$, $1/2 \leq l_i/l_j < 2$. For each link length class, the network region is partitioned into square cells that are 4-colored such that no two adjacent squares have the same color. A fundamental property of algorithm GOW is that if the side length of the squares is chosen properly, links whose receivers are in different squares with the same color can be scheduled in the same time slot without corrupting each other's transmissions. The proofs of correctness and approximation-factor of GOW are based on an inexact SINR model in which the effect of the ambient noise is ignored. In such a case, the maximum transmission range can be infinite, i.e., $l_{\max} \to \infty$, which is equivalent to infinite length diversity. The side length of a square cell can be infinite as well, since it depends on the length of a link class.

In [27], an approximation algorithm called GOW* based on the exact-SINR model under the uniform power assignment was proposed. Instead of distance-based, the links are grouped into $C_0, C_1, \cdots, C_{\bar{\gamma}}$ according to a SNR-based criterion,

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where $\bar{\gamma} = \lfloor \log_{1+\epsilon}(P/\beta N) \rfloor$ is called the *SNR diversity* of the link set L, P is the uniform transmit power that must be large enough for the longest link, i.e, $P = \beta N l_{\max}^{\alpha}$, and $\epsilon \geq 1/7$ is a constant. The length of a link in the ith SNR class C_i must satisfy the following inequality:

$$\left(\frac{P}{(1+\epsilon)^{i+1}\beta N}\right)^{1/\alpha} < L_i \leq \left(\frac{P}{(1+\epsilon)^{i}\beta N}\right)^{1/\alpha}$$

Note that when $i = 0$, $(1+\epsilon)^{-1/\alpha} l_{\max} < L_0 \leq l_{\max}$. These links forming C_0 are called "black and gray" links, whose SNR lies in $[\beta, (1+\epsilon)\beta)$, which are difficult to schedule and must be scheduled sequentially [27]. The approximation ratio of GOW* depends on the number of "black and gray" links, $\bar{\gamma}$, and $\Delta_{\max}$, where $\Delta_{\max}$ is the maximum number of receivers in a cell of class C_{γ}, and $1 \leq \gamma \leq \bar{\gamma}$. In fact, the length of a schedule computed by GOW* is upper-bounded by $|C_0| + 4\bar{\gamma}\Delta_{\max}$ (See Theorem 2 in [27]), which implies that the approximation ratio can not be $O(1)$ even when no or few "black and gray" links appear ([27] claims that the approximation bound of GOW* is $O(1)$ when no or few "black and gray" links appear), as $\bar{\gamma}$ and $\Delta_{\max}$ are variables depending on the input link lengths.

We observe that the "black and gray" links can be removed by enlarging their transmit powers. On one hand, the received power of each link is improved, thus improving the success rate of correct decoding at the receivers; on the other hand, these links are scheduled as other regular ones, which may decrease the approximation factor as no sequential scheduling is needed. But increasing the transmit powers implies the increase of interferences on other nodes; therefore it is important to ensure that the cost of enlarging transmit powers is worthy of the improvement of the network performance.

C. Basic Idea of SLSPC

The basic idea of SLSPC is presented as follows. First, the request link set $L = \{l_1, l_2, \cdots, l_n\}$ is partitioned into disjoint subsets $C_0, C_1, \cdots, C_{\bar{k}}$, where $\bar{k} = \lfloor \log(l_{\max}/l_{\min}) \rfloor$, and $C_i = \{l | 2^i l_{\min} \leq l < 2^{i+1} l_{\min}\}$ for $i = 0, \cdots, \bar{k}$. The transmit power of $l \in C_i$, P_l, equals $(1+\varepsilon)\beta N (2^{i+1} l_{\min})^{(1-k)\alpha} l^{k\alpha}$, where $\varepsilon > 0$ is called a *regulatory factor*. The maximum transmit power assigned to the class C_i is denoted by $P_i^{\max}$. Formally,

$$P_i^{\max} = (1+\varepsilon)\beta N (2^{i+1} l_{\min})^{\alpha}.$$

The SNR of the link class C_i is denoted by SNR_i. Obviously,

$$(1+\varepsilon)\beta < \mathrm{SNR}_i \leq 2^{(1-k)\alpha}(1+\varepsilon)\beta,$$

which implies that there is no "black and gray" link in any link class C_i.

Second, for each link length class, the network region is partitioned into hexagons, with the side length of each hexagon being $\mu 2^{i+1} l_{\min}$, where μ is to be determined later. Then we 3-color all the hexagons such that no two adjacent ones have the same color. By this way all the hexagons are grouped into three different sets, with each having the same color. Fig. 1 illustrates the partition of a plane and the 3-coloring of the hexagons, where 1, 2, and 3 are used to denote the three different colors. A link belongs to a hexagon if its receiver resides in the hexagon. Next, we consider the hexagons group by group and select the links for a time slot t by choosing one from each hexagon in the same group. This procedure is repeated until all the links belonging to the same group are scheduled. Then we consider other groups until all links in the link length class under consideration are scheduled. The pseudo-code of SLSPC algorithm is presented in Algorithm 1.

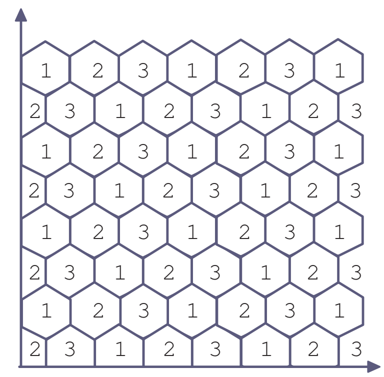

Fig. 1: Plane partition & 3-coloring.

Algorithm 1 SLSPC (Shortest Link Scheduling with Power Control)

begin

Input: **A set** L **of links located arbitrarily in the Euclidean plane**

Output: $S_1, S_2, \cdots, S_T$**, such that all links in each feasible set** S_t **can successfully transmit concurrently**

Let $L = C_0 \cup \cdots \cup C_{\bar{k}}$**, such that** $C_i = \{l | 2^i l_{\min} \leq d(l) < 2^{i+1} l_{\min}\}$, $i = 0, \cdots, \bar{k}$, $\bar{k} = \lfloor \log(l_{\max}/l_{\min}) \rfloor$;
$t = 0$;
for each $C_i \neq \emptyset$ **do**
  Partition the network region into hexagons with a side length of $\mu 2^{i+1} l_{\min}$;
  3-color the hexagons such that no two adjacent ones have the same color;
  for $j = 1$ **to 3 do**
    repeat
      $t = t + 1$, $S_t = \emptyset$;
      for each hexagon A **of color** j **do**
        pick one link $l_u \in C_i$ **with receiver** r_u **in** A;
        assign l_u **to time slot** t**, i.e.,** $S_t = S_t \cup \{l_u\}$;
        Assign transmit power $(1+\varepsilon)\beta N (2^{i+1} l_{\min})^{(1-k)\alpha} l_u^{k\alpha}$ **to link** l_u;
        $C_i = C_i \setminus \{l_u\}$;
      end for
    until all links of C_i **in the hexagons with color** j **are scheduled**
  end for
end for
return $S_1, S_2, \cdots, S_t$.

end

In the following, we prove that our algorithm is feasible and has a $O(\bar{k})$ approximation ratio to the optimal solution.

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Theorem 1. *Let*

$$\mu = \left(\left(1+\frac{1}{\varepsilon}\right)6\beta\left(2^\alpha + \frac{4^\alpha}{(3\sqrt{3})^\alpha}\left(\frac{\alpha-1}{\alpha-2}+\frac{\alpha}{\alpha-1}\right)\right)\right)^{1/\alpha}.$$

Then the schedule computed by SLSPC is feasible under the SINR model. That is, the links in S_t, where $t = 1, 2, \cdots, T$, can transmit simultaneously.

Proof: Without loss of generality, assume that $S_t \subseteq C_i$, where $0 \le i \le \bar{k}$. Let $l_v = (s_v, r_v) \in S_t$ and r_v be in hexagon A (see Fig. 2). First, we derive the upper-bound of the interference accumulated at l_v caused by all links other than l_v in S_t. According to the SLSPC algorithm, the receivers of the links in S_t lie in the hexagons with the same color as that of A and these hexagons form hexagon-shaped belts, as shown in Fig. 2. The first belt, i.e., the inner belt, contains 6 hexagons (black hexagons in Fig. 2), and the distance of the links in these 6 hexagons to l_v is at least $(\mu-1)2^{i+1}l_{\min}$. More generally, the hth belt contains $6h$ hexagons. When h is even, the distance of links in the $6h$ hexagons to l_v is at least $\left(\frac{(3h-2)\sqrt{3}\mu}{2}-1\right)2^{i+1}l_{\min}$; when h is odd, the distance of the links in the $6h$ hexagons to l_v is at least $\left(\frac{\sqrt{27h^2-36h+13}}{2}\mu-1\right)2^{i+1}l_{\min}$. Since $(3h-2)\sqrt{3} = \sqrt{27h^2-36h+12} < \sqrt{27h^2-36h+13}$, the total interference I_{r_v} experienced by r_v can be upper-bounded by

$$\begin{aligned}
I_{r_v} &\le \frac{6P_i^{\max}}{((\mu-1)2^{i+1}l_{\min})^\alpha} + \sum_{h=2}^{\infty}\frac{6hP_i^{\max}}{\left(\left(\frac{(3h-2)\sqrt{3}\mu}{2}-1\right)2^{i+1}l_{\min}\right)^\alpha} \\
&= \frac{6P_i^{\max}}{(2^{i+1}l_{\min})^\alpha}\left(\frac{1}{(\mu-1)^\alpha} + \sum_{h=2}^{\infty}\frac{h}{\left(\frac{(3h-2)\sqrt{3}\mu}{2}-1\right)^\alpha}\right) \\
&\le \frac{6P_i^{\max}}{(2^{i+1}l_{\min})^\alpha}\left(\frac{1}{(\mu-1)^\alpha} + \sum_{h=2}^{\infty}\frac{h}{\left(\frac{3\sqrt{3}\mu}{2}(h-1)-1\right)^\alpha}\right) \\
&\le \frac{6P_i^{\max}}{(2^{i+1}l_{\min})^\alpha}\left(\frac{2^\alpha}{\mu^\alpha} + \frac{4^\alpha}{(3\sqrt{3})^\alpha\mu^\alpha}\sum_{h=2}^{\infty}\frac{h}{(h-1)^\alpha}\right) \qquad (2) \\
&= \frac{6P_i^{\max}}{(2^{i+1}l_{\min})^\alpha}\left(2^\alpha + \frac{4^\alpha}{(3\sqrt{3})^\alpha}\left(\frac{\alpha-1}{\alpha-2}+\frac{\alpha}{\alpha-1}\right)\right)\frac{1}{\mu^\alpha}.
\end{aligned}$$

where (2) follows because $x-1 \ge x/2$ for $x \ge 2$, and indeed $\mu \ge 2$ holds.

Therefore, we can bound the SINR of r_v as:

$$\begin{aligned}
\mathrm{SINR}(r_v) &= \frac{P_{l_v}/d^\alpha(l_v)}{N+I_{r_v}} \ge \frac{P_i^{\max}/(2^{i+1}l_{\min})^\alpha}{N+I_{r_v}} \\
&\ge \frac{P_i^{\max}/(2^{i+1}l_{\min})^\alpha}{N+\frac{6P_i^{\max}}{(2^{i+1}l_{\min})^\alpha}(2^\alpha+\frac{4^\alpha}{(3\sqrt{3})^\alpha}(\frac{\alpha-1}{\alpha-2}+\frac{\alpha}{\alpha-1}))\frac{1}{\mu^\alpha}}.
\end{aligned}$$

By setting the righthand side of the inequality in $\mathrm{SINR}(r_v)$ to β we obtain

$$\mu = \left(\left(1+\frac{1}{\varepsilon}\right)6\beta\left(2^\alpha + \frac{4^\alpha}{(3\sqrt{3})^\alpha}\left(\frac{\alpha-1}{\alpha-2}+\frac{\alpha}{\alpha-1}\right)\right)\right)^{1/\alpha},$$

which can guarantee that $\mathrm{SINR}(r_v) \ge \beta$. Therefore, the theorem holds. ■

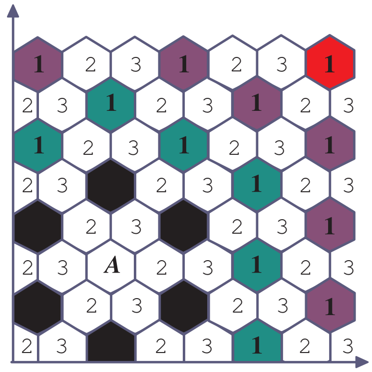

Fig. 2: Links with receivers in the colored hexagons can transmit concurrently; Hexagons with same color form hexagon-shaped belts.

Theorem 2. *The approximation ratio of the algorithm SLSPC under the oblivious power assignment is $O(\bar{k})$, where $\bar{k} = \lfloor\log(l_{\max}/l_{\min})\rfloor$ denotes the link length diversity.*

Proof: Without loss of generality, we consider the link class C_i and give the upper bound on the number of links whose receivers belong to the same hexagon but can be scheduled simultaneously. Assume that $l_v = (s_v, r_v) \in C_i$ and there are at most $\triangle_{max}$ receivers belonging to the same hexagon as r_v and at most x of them can be scheduled simultaneously. Next, we prove that

$$x \le \frac{(2\mu+1)^\alpha(2^\alpha-(1+\varepsilon))}{\beta}.$$

For contradiction we assume that there are more than $\frac{(2\mu+1)^\alpha(2^\alpha-(1+\varepsilon))}{\beta}$ receivers in a hexagon that can be scheduled simultaneously. According to the SINR constraints, we have

$$\begin{aligned}
\mathrm{SINR}(r_v) &\le \frac{\frac{P_i^{\max}}{(2^i l_{\min})^\alpha}}{N + x\frac{P_i^{\max}}{(2\mu 2^{i+1}l_{\min}+2^{i+1}l_{\min})^\alpha}} \\
&< \frac{\frac{P_i^{\max}}{(2^i l_{\min})^\alpha}}{N + \frac{(2\mu+1)^\alpha(2^\alpha-(1+\varepsilon))}{\beta}\frac{P_i^{\max}}{(2\mu 2^{i+1}l_{\min}+2^{i+1}l_{\min})^\alpha}} \\
&= \beta.
\end{aligned}$$

Thus our assumption is incorrect as the SINR experienced by r_v must be at least β; therefore, there are at most x receivers that can be scheduled simultaneously.

Next we set $x = \frac{(2\mu+1)^\alpha(2^\alpha-(1+\varepsilon))}{\beta}$. Let OPT denote an optimal link scheduling algorithm. To schedule all the $\triangle_{\max}$ links, the number of slots needed by OPT, denoted by T_{OPT}, must satisfy

$$T_{\mathrm{OPT}} \ge \frac{\Delta_{\max}}{x}. \qquad (3)$$

On the other hand, our algorithm SLSPC schedules all the links in T_{SLSPC} number of time slots, and we have

$$T_{\mathrm{SLSPC}} \le 3\Delta_{\max}\bar{k}. \qquad (4)$$

Following from (3) and (4) we obtain $\frac{T_{\mathrm{SLSPC}}}{T_{\mathrm{OPT}}} \le 3x\bar{k}$, where $O(\bar{k}) = O(\log(l_{\max}/l_{\min}))$, as x is a constant. ■

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D. SLSUM with Uniform Power Assignment

In this section, we present a SLS algorithm SLSUM, which stands for Shortest Link Scheduling with Uniform or Mean power assignment. For easy elaboration, we first detail SLSUM in the context of uniform power assignment. Let $P = (1+\varphi)\beta N l_{\max}^{\alpha}$ be the fixed transmit power, where $\varphi > 0$ is a constant.

First, we partition the set of links L into disjoint SNR classes instead of link length classes. Let $K_i = (P/((1+\varphi)^i \beta N))^{1/\alpha}$. Then we define a SNR link class as $C_i = \{l \mid K_{i+1} < l \le K_i\}$. where $i = 1, ..., \bar{k}$ and $\bar{k} = \lceil \alpha \log_{1+\varphi}(l_{\max}/l_{\min}) \rceil$. Note that the SNR of the links in C_i is in $[(1+\varphi)^i\beta, (1+\varphi)^{i+1}\beta)$, which implies that no "black and gray" link exists in L under our uniform power assignment.

Second, we partition the network plane into hexagons with a side length of μK_{i+1} for each SNR class C_i, where

$$\mu = \left(\frac{6\beta(1+\varphi)^{i+1}}{(1+\varphi)^i - 1}\left(2^{\alpha} + \left(\frac{4}{3\sqrt{3}}\right)^{\alpha}\left(\frac{\alpha-1}{\alpha-2} + \frac{\alpha}{\alpha-1}\right)\right)\right)^{1/\alpha},$$

which is to be explained later.

Next, we 3-color all the hexagons such that no two adjacent ones have the same color, as in Algorithm 1. Then we perform the same procedure as in Algorithm 1, except that we do not need to assign a transmit power to each link when we schedule all links in L based on the hexagons they reside and the colors of the hexagons. The pseudo-code is given in Algorithm 2.

Algorithm 2 SLSUM (Shortest Link Scheduling with Uniform or Mean power assignment)

begin

Input: A set L of links located arbitrarily in the Euclidean plane

Output: $S_1, S_2, \cdots, S_T$, **such that all links in each** S_t **can simultaneously transmit successfully**

1: **Partition** L **into** $C_1, C_2 \cdots, C_{\bar{k}}$ **such that** $C_i = \{l | K_{i+1} < l \le K_i, l \in L\}, i = 1, \cdots, \bar{k}$**, and** $K_i = \left(P/((1+\varphi)^i\beta N)\right)^{1/\alpha}$;
2: $t = 0$;
3: **for each** $C_i \neq \emptyset$ **do**
4: **Partition the plane into hexagons with a side length of** μK_{i+1};
5: **3-color the hexagons such that no two adjacent ones have the same color;**
6: **for** $j = 1$ **to 3 do**
7: **repeat**
8: $t = t+1$, $S_t = \emptyset$;
9: **for each hexagon** A **of color** j **do**
10: **pick an unscheduled link** $l_i \in C_i$ **with receiver** r_i **in** A;
11: **assign** l_i **to time slot** t**, i.e.,** $S_t = S_t \cup \{l_i\}$;
12: **end for**
13: **until all links of** C_i **in the hexagons of color** j **are scheduled**
14: **end for**
15: **end for**
16: **return** $S_1, S_2, \cdots, S_t$.

end

Theorem 3. *The schedule computed by SLSUM is feasible under the SINR model with uniform power assignment.*

Proof: Consider S_t, the set of links scheduled for the time slot t. Without loss of generality, assume that $S_t \subseteq C_i$, where $1 \le i \le \bar{k}$. Let $l_k = (s_k, r_k) \in S_t$. Assume that r_k lies in hexagon A (see Fig. 2). Now, we derive the upper-bound of the interference experienced at r_k caused by other links in S_t. With a similar analysis as that in the proof of Theorem 1, we conclude that the distance of the links in the 6 hexagons of the inner/first hexagon-shaped belt to l_k is at least $\mu K_{i+1} - K_i$, and that of the links from the $6h$ hexagons in the hth hexagon-shaped belt to l_k is at least $\frac{(3h-2)\sqrt{3}}{2}\mu K_{i+1} - K_i$. Then the total interference experienced by r_k can be upper-bounded by

$$\begin{aligned}
I_{r_k} &\le \frac{6P}{(\mu K_{i+1} - K_i)^{\alpha}} + \sum_{h=2}^{\infty} \frac{6hP}{\left(\frac{(3h-2)\sqrt{3}}{2}\mu K_{i+1} - K_i\right)^{\alpha}} \\
&= \frac{6P}{K_i^{\alpha}}\left(\frac{1}{(\mu(1+\varphi)^{-1/\alpha} - 1)^{\alpha}} + \right. \\
&\left. \sum_{h=2}^{\infty} \frac{h}{\left(\frac{(3h-2)\sqrt{3}}{2}\mu(1+\varphi)^{-1/\alpha} - 1\right)^{\alpha}}\right) \\
&\le \frac{6P}{K_i^{\alpha}}\left(\frac{1}{(\mu(1+\varphi)^{-1/\alpha} - 1)^{\alpha}} + \right. \\
&\left. \sum_{h=2}^{\infty} \frac{h}{\left(\frac{3\sqrt{3}}{2}\mu(1+\varphi)^{-1/\alpha}(h-1) - 1\right)^{\alpha}}\right) \\
&\le \frac{6P}{K_i^{\alpha}}\left(\frac{1}{(\frac{\mu(1+\varphi)^{-1/\alpha}}{2})^{\alpha}} + \right. \\
&\left. \sum_{h=2}^{\infty} \frac{h}{\left(\frac{3\sqrt{3}}{4}\mu(1+\varphi)^{-1/\alpha}(h-1)\right)^{\alpha}}\right) \qquad (5) \\
&= \frac{6P(1+\varphi)}{K_i{}^{\alpha}}\left(2^{\alpha} + \left(\frac{4}{3\sqrt{3}}\right)^{\alpha}\left(\frac{\alpha-1}{\alpha-2} + \frac{\alpha}{\alpha-1}\right)\right)\frac{1}{\mu^{\alpha}}.
\end{aligned}$$

where (5) follows because $x - 1 \ge x/2$ for $x \ge 2$, and indeed $\mu(1+\varphi)^{-1/\alpha} \ge 2$ holds. Therefore, we can bound the SINR of r_k as:

$$\begin{aligned}
\mathrm{SINR}(r_k) &= \frac{P/l_k{}^{\alpha}}{N + I_{r_k}} \\
&\ge \frac{P/K_i{}^{\alpha}}{N + \frac{6P(1+\varphi)}{K_i{}^{\alpha}}\left(2^{\alpha} + \left(\frac{4}{3\sqrt{3}}\right)^{\alpha}\left(\frac{\alpha-1}{\alpha-2} + \frac{\alpha}{\alpha-1}\right)\right)\frac{1}{\mu^{\alpha}}}
\end{aligned}$$

By setting the righthand side of the inequality in $\mathrm{SINR}(r_k)$ to β we obtain $\mu = \left(\frac{6\beta(1+\varphi)^{i+1}}{(1+\varphi)^i - 1}\left(2^{\alpha} + \left(\frac{4}{3\sqrt{3}}\right)^{\alpha}\left(\frac{\alpha-1}{\alpha-2} + \frac{\alpha}{\alpha-1}\right)\right)\right)^{1/\alpha}$, under which we have $\mathrm{SINR}(r_k) \ge \beta$.

Therefore, the theorem holds. ∎

Theorem 4. *The approximation ratio of SLSUM with uniform power assignment is* $O(\log_{1+\varphi}(l_{\max}/l_{\min}))$.

Proof: Without loss of generality, we consider the SNR class C_i. Assume that there are at most $\triangle_{\max}$ receivers belonging to the same hexagon and at most x receivers can be scheduled simultaneously. According to the SINR constraints, we have

$$\frac{P/K_{i+1}^{\alpha}}{N + xP/(2\mu K_{i+1} + K_i)^{\alpha}} \geq \beta,$$

which leads to

$$x \leq \left(1 - \frac{1}{(1+\varphi)^{i+1}}\right) \frac{\left(2\mu + (1+\varphi)^{1/\alpha}\right)^{\alpha}}{\beta}.$$

Let $q = \frac{\left(2\mu+(1+\varphi)^{1/\alpha}\right)^{\alpha}}{\beta}$. The maximum number of links that can be scheduled simultaneously by an optimal link scheduling algorithm in the same hexagon is not larger than q. Therefore, to schedule all the $\triangle_{\max}$ links, the optimal link scheduling algorithm needs at least $\frac{\triangle_{\max}}{q}$ time slots. That is, $T_{\text{OPT}} \geq \frac{\triangle_{\max}}{q}$. On the other hand, SLSUM schedules all links in $3\triangle_{\max}\bar{k}$ time slots. That is, $T_{\text{SLSUM}} \leq 3\triangle_{\max}\bar{k}$, which leads to $\frac{T_{\text{SLSUM}}}{T_{\text{OPT}}} \leq 3q\bar{k}$. Note that, q is a constant and $\bar{k} = \lceil \alpha \log_{1+\varphi}(l_{\max}/l_{\min}) \rceil$. Therefore, the approximation ratio of the algorithm SLSUM is $O(\log_{1+\varphi}(l_{\max}/l_{\min}))$. ■

E. SLSUM with Mean Power Control

The proposed SLSUM algorithm 2 can handle the mean power assignment. In this case, the power of each link l is set to $P_l = cl^{\alpha/2}$, where $c = (1+\varphi)^2 \beta N l_{\max}^{\alpha/2}$ is a constant once $l_{\max}$ is determined. Let $K_i = \left(c/\left((1+\varphi)^{i+1}\beta N\right)\right)^{2/\alpha}$. We define a SNR link class as $C_i = \{l \mid K_{i+1} < l \leq K_i\}$, where, $i = 1, ..., \bar{k}$, and $\bar{k} = \lceil \alpha/2 \cdot \log_{1+\varphi}(l_{\max}/l_{\min}) \rceil$. As the SNRs of the links in C_1 are greater than $(1+\varphi)^2\beta$ and lower than $(1+\varphi)^3\beta$, the "black and gray" links are successfully eliminated under our mean power assignment.

With similar proofs as those in Theorems 3 and 4, we obtain the following results.

Theorem 5. *Let*

$$\mu = \left(\frac{6\beta(1+\varphi)^{i+3}}{(1+\varphi)^{i}-1}\left(2^{\alpha}+\left(\frac{4}{3\sqrt{3}}\right)^{\alpha}\left(\frac{\alpha-1}{\alpha-2}+\frac{\alpha}{\alpha-1}\right)\right)\right)^{1/\alpha}.$$

For any $\varphi > 0$, the schedule computed by SLSUM is feasible under the SINR model with the mean power assignment.

Theorem 6. *The approximation ratio of algorithm SLSUM under the mean power assignment is $O(\log_{1+\varphi}(l_{\max}/l_{\min}))$.*

V. Performance Analysis and Discussions

To show the efficiency of our algorithms SLSPC and SLSUM, we compare them with GOW [22], GOW*[27], and MLLS [29] in this section. Note that GOW and GOW* are briefly introduced in Section IV-B. In MLLS [29], the plane is partitioned into small squares with a side length of $l_{\max}/\sqrt{2}$. A large-square consists of $(K+1) \times (K+1)$ small squares. A SINR feasible set is formed by picking up at most one link from each small cell lying in the same relative location within every large-block; this is, at most one link is picked from each large-block to transmit in one time slot, and the links for the same time slot reside at the same relative position in a large cell. The approximation ratio of MLLS is at most $(K+1)^2\omega$, where K and ω are defined later in this section.

A. Numerical Analysis on the Approximation Factor

In this subsection, we show that SLSPC and SLSUM are efficient by numerical analysis. In the following we first deduce that the approximation factor of MLLS is $O(l_{\max}^{\alpha})$, which is larger than $O(\log(l_{\max}/l_{\min}))$.

Lemma 1. *The transmit power of l in MLLS [29] is at least $\beta N l_{\max}^{(1-k)\alpha} l^{k\alpha}$, and the approximation factor of MLLS is $O(l_{\max}^{\alpha})$.*

Proof: According to the Theorem 2 in [29], the approximation factor of MLLS is $(K+1)^2\omega$, where

$$K = \lceil \sqrt{2}((4\tau)^{-1}(\beta^{-1} - N(c\eta)^{-1} l_{\max}^{(1-k)\alpha}))^{-1/\alpha} + \sqrt{2} \rceil,$$

$\tau = \frac{\alpha(1+2^{-2/\alpha})}{\alpha-1} + \frac{\pi 2^{-2/\alpha}}{2(\alpha-2)}$, $\omega = \lceil \frac{2^{\alpha}P}{\beta^2 N} + 1 \rceil$, and $0 < \eta < 1$ is the reference loss factor. Since $K > 0$, we have

$$\beta^{-1} > N(c\eta)^{-1} l_{\max}^{(1-k)\alpha}.$$

That is,

$$c > \frac{\beta N l_{\max}^{(1-k)\alpha}}{\eta}.$$

Therefore,

$$P_l = cl^{k\alpha} > \frac{\beta N}{\eta} l_{\max}^{(1-k)\alpha} l^{k\alpha} > \beta N l_{\max}^{(1-k)\alpha} l^{k\alpha},$$

where the last inequality holds since $0 < \eta < 1$. Then we have,

$$\begin{aligned}\omega &= \lceil \frac{2^{\alpha}P}{\beta^2 N} + 1 \rceil \\ &> \frac{2^{\alpha}\beta N l_{\max}^{(1-k)\alpha} l^{k\alpha}}{\beta^2 N} + 1 = \frac{2^{\alpha} l_{\max}^{(1-k)\alpha} l^{k\alpha}}{\beta} + 1.\end{aligned}$$

Thus the approximation factor of MLLS is $(K+1)^2(\frac{2^{\alpha} l_{\max}^{(1-k)\alpha} l^{k\alpha}}{\beta} + 1)$, which can be denoted by $O(l_{\max}^{\alpha})$. ■

Compared with the two existing algorithms GOW and GOW*, SLSPC and SLSUM have tighter approximation factors. We use GOW_f, $\text{GOW}^*{}_f$, and SLSPC_f to denote the approximation factors of GOW, GOW*, and SLSPC, respectively, which are listed in Table I, where $D_1 = \log_2(l_{\max}/l_{\min})$, $D_2 = \log_{1+\epsilon}(l_{\max}/l_{\min})$, and $D_3 = \log_{1+\varphi}(l_{\max}/l_{\min})$. Furthermore, we use $\frac{\text{GOW}}{\text{SLSPC}}$ to denote the ratio of GOW_f to SLSPC_f, and use $\frac{\text{GOW}^*}{\text{SLSPC}}$ to denote the ratio of $\text{GOW}^*{}_f$ to SLSPC_f.

Obviously, the path loss exponent α and the SINR threshold β significantly influence the approximation factors. When α is close to 2, the side length of the squares in [22], [27] is approaching infinite, and the squares are then 4-colored in such a way that no two adjacent ones have the same color. Therefore, the approximation factors of GOW and GOW* are approaching $O(n)$. However, we can appropriately set φ such that SLSP_f is a constant. For example, when $\alpha = 2.0001$,

TABLE I: Approximation Factors for Different Algorithms

Algorithm	Appr. factor	q	μ
GOW	$4qD_1$	$\frac{2^\alpha(\sqrt{2}\mu+1)^\alpha}{\beta}$	$4\left(8\beta\frac{\alpha-1}{\alpha-2}\right)^{1/\alpha}$
GOW*	$\|C_0\|+4q\alpha D_2$	$\frac{\left((1+\epsilon)^{1/\alpha}+\sqrt{2}\mu\right)^\alpha}{\beta}$	$2\left(63\beta(1+\epsilon)\frac{\alpha-1}{\alpha-2}\right)^{1/\alpha}$
SLSPC	$3qD_1$	$\frac{(2\mu+1)^\alpha(2^\alpha-(1+\varepsilon))}{\beta}$	$\left(\frac{6\beta(1+\varepsilon)}{\varepsilon}\left(2^\alpha+\left(\frac{4}{3\sqrt{3}}\right)^\alpha\left(\frac{\alpha-1}{\alpha-2}+\frac{\alpha}{\alpha-1}\right)\right)\right)^{1/\alpha}$
SLSUM(Uni)	$3q\alpha D_3$	$\frac{\left((1+\varphi)^{1/\alpha}+2\mu\right)^\alpha}{\beta}$	$\left(\frac{6\beta(1+\varphi)^2}{\varphi}\left(2^\alpha+\left(\frac{4}{3\sqrt{3}}\right)^\alpha\left(\frac{\alpha-1}{\alpha-2}+\frac{\alpha}{\alpha-1}\right)\right)\right)^{1/\alpha}$
SLSUM(Mean)	$\frac{3}{2}q\alpha D_3$	$\frac{\left((1+\varphi)^{2/\alpha}+2\mu\right)^\alpha}{\beta}$	$\left(\frac{6\beta(1+\varphi)^4}{\varphi}\left(2^\alpha+\left(\frac{4}{3\sqrt{3}}\right)^\alpha\left(\frac{\alpha-1}{\alpha-2}+\frac{\alpha}{\alpha-1}\right)\right)\right)^{1/\alpha}$

$\beta = 10$, $\epsilon = 1/7$ [27], $\varepsilon = 3$, we have $\frac{\text{GOW}}{\text{SLSPC}}$=$2.6 \times 10^5$ and $\frac{\text{GOW}^*}{\text{SLSPC}}$= 1.5×10^6; when $\alpha = 5.9999$ (other parameters are unchanged), we have $\frac{\text{GOW}}{\text{SLSPC}}$=11 and $\frac{\text{GOW}^*}{\text{SLSPC}}$=1; if we consider the typical settings of $\alpha = 3$, $\beta = 10$, $\epsilon = 1/2$ [27], and $\varepsilon = 3$, we have $\frac{\text{GOW}}{\text{SLSPC}}$=11.8 and $\frac{\text{GOW}^*}{\text{SLSPC}}$=11.3. Therefore, we claim that SLSPC has a tighter approximation factor than GOW and GOW*.

For SLSUM, we set $\epsilon = \varphi = 1$ and compare the approximation factors with those of GOW and GOW*. The ratio of the approximation ratios of GOW to SLSUM(Uniform), GOW* to SLSUM(Uniform), GOW to SLSUM(Mean), and GOW* to SLSUM(Mean) are respectively denoted as $\frac{\text{GOW}}{\text{SLSUM}_{\text{uniform}}}$, $\frac{\text{GOW}^*}{\text{SLSUM}_{\text{uniform}}}$, $\frac{\text{GOW}}{\text{SLSUM}_{\text{mean}}}$, and $\frac{\text{GOW}^*}{\text{SLSUM}_{\text{mean}}}$. For the purpose of showing the advantage of SLSUM over GOW and GOW*, we give the ratios with different values of α and β in Table II. For specific α and β, the numbers in each entry of the table denote $\frac{\text{GOW}}{\text{SLSUM}_{\text{uniform}}}$, $\frac{\text{GOW}^*}{\text{SLSUM}_{\text{uniform}}}$, $\frac{\text{GOW}}{\text{SLSUM}_{\text{mean}}}$, and $\frac{\text{GOW}^*}{\text{SLSUM}_{\text{mean}}}$, from left to right.

From table II one can conclude that SLSUM has a great advantage over GOW even though GOW neglects the ambient noise. On the other hand, SLSUM has a small advantage over GOW* when $\alpha = 6$ and $\beta \leq 10$. But we want to point out that we do not consider the contribution of C_0 to the approximation factor of GOW* in our comparison study since C_0 is strongly related to the input instances. If we consider the typical settings of $\alpha = 3$ and $\beta = 10$, the ratio of GOW to SLSUM(Uniform), GOW* to SLSUM(Uniform), GOW to SLSUM(Mean), and GOW* to SLSUM(Mean) are respectively 46, 34, 24, and 18.

B. Discussion on Power Consumption

Here, we compare the lowest power assigned to the link set L with different power assignment, including uniform power assignment ($\text{P}_{\text{uniform}}$), mean power assignment (P_{mean}), and linear power assignment (P_{linear}). To ensure the successful transmissions of the longest link, we define the power assignment to l as follows:

$$\text{P}_{\text{uniform}} = \beta N l_{max}^{\alpha},$$

$$\text{P}_{\text{mean}} = \beta N l_{max}^{\alpha/2} l^{\alpha/2},$$

$$\text{P}_{\text{linear}} = \beta N l^{\alpha}.$$

GOW and GOW* adopt the uniform power assignment, while MLLS employs linear power control, uniform power control, and arbitrary power control. We can deduce that the transmit power of l in MLLS is at least $\beta N l_{\text{max}}^{(1-k)\alpha} l^{k\alpha}$ (see Lemma 1), which indicates that when $k = 0, 1/2$, and 1, the corresponding power assignments are equal to $\text{P}_{\text{uniform}}, \text{P}_{\text{mean}}$, and P_{linear}, respectively.

For SLSPC, the power assigned to $l \in C_i$ is $(1 + \varepsilon)\beta N(2^{i+1} l_{\text{min}})^{(1-k)\alpha} l^{k\alpha}$. When $k = 0, 1/2$, and 1, the power assignments are respectively denoted by *quasi-uniform* power assignment ($\text{QP}_{\text{uniform}}$), *quasi-mean* power assignment (QP_{mean}), and *quasi-linear* power assignment ($\text{QP}_{\text{linear}}$), with the following settings:

$$\text{QP}_{\text{uniform}} = (1 + \varepsilon)\beta N(2^{i+1} l_{\text{min}})^{\alpha},$$

$$\text{QP}_{\text{mean}} = (1 + \varepsilon)\beta N(2^{i+1} l_{\text{min}})^{\alpha/2} l^{\alpha/2},$$

$$\text{QP}_{\text{linear}} = (1 + \varepsilon)\beta N l^{\alpha},$$

where $\varepsilon > 0$ is the *regulatory factor*. When ε is small, $\text{QP}_{\text{uniform}}$ and QP_{mean} are more economical than the corresponding $\text{P}_{\text{uniform}}$ and P_{mean}. For SLSUM with uniform power assignment and mean power assignment, the powers of l are $(1 + \varphi)\beta N l_{\text{max}}^{\alpha}$ and $(1 + \varphi)\beta N l_{\text{max}}^{\alpha/2} l^{\alpha/2}$, respectively. Here, φ is defined as a "regulatory factor" too; and a large φ means a large energy consumption, which implies a low latency.

Consider a wireless network shown in Fig. 3, where most of the active nodes are concentrated in a certain region (region 2 of Fig. 3) and few nodes are close to or far away from the base station (region 1 and region 3 of Fig. 3). In this setting, $\text{P}_{\text{uniform}}$ consumes more power than $\text{QP}_{\text{uniform}}$, since a short link must be assigned a large power. For the linear power assignment, if the links are assigned an *exact* power, that is, $P_l = \beta N l^{\alpha}$, all the links are "black and gray" and they must be scheduled in sequence. However, we can eliminate the "black and gray" links by increasing the transmit power with $(1+\varepsilon)$. Therefore, it is necessary to have $\text{QP}_{\text{linear}}$ larger than P_{linear}.

C. Simulations and Discussions

In this subsection, we demonstrate that our algorithms SLSPC and SLSUM are indeed efficient via simulation study.

TABLE II: The Ratios of the Four Approximation Factors

Ratio β / α	5	10	20	50	100
2.5	22,27,12,14	43,54,23,28	86,107,45,56	212,256,111,138	422,526,221,275
3	24,18,12,9	**46,34,24,18**	90,67,48,36	221,165,117,88	436,327,231,173
4	39,10,21,6	75,20,41,11	146,38,79,20	351,90,189,49	688,176,371,95
6	160,5,88,3	301,9,166,5	575,16,315,9	1355,36,743,20	2607,70,1431,38

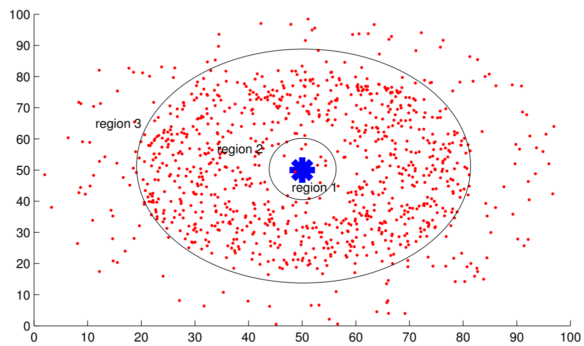

Fig. 3: Node distribution, where the central blue point denotes the base station and the red points denote the active nodes. A link is from a red point to the blue base station.

Assume that all nodes are deployed in a large network region with an area of $2000m \times 2000m$. The number of links varies from 200 to 1000; $l^\star_{\min} = 1$ and $l^\star_{\max} = 30$. The background noise N is fixed to $-70db$. We also set $\alpha = 3$, $\beta = 10db$, $\epsilon = 0.4$ (for GOW* in [27]), and $\eta = 0.8$ (for MLLS in [29]), unless the impact of a parameter is under consideration, in which case we vary the corresponding parameter in a reasonable value range. Note that $0 < \epsilon < 1$ and $0 < \eta < 1$ according to [27] and [29], respectively. We choose $\epsilon = 0.4$ and $\eta = 0.8$ as these settings yield the best performance in latency based on our simulations on GOW* and MLLS. We further assume that the link lengths are arbitrarily distributed; thus we randomly pick up two points forming the link $l_i = (s_i, r_i)$ in the network region satisfying the following three conditions:

(1) $d(s_i, r_i) \le l^\star_{\max}$,

(2) $d(s_i, r_i) \ge l^\star_{\min}$, and

(3) the distance from s_i or r_i to any other previously generated node (point) in the region is at least $l_{\min}$.

Some important parameters are listed in Table III.

In our design, we increase the transmit power to avoid scheduling the so-called "black and gray" links. Therefore in this simulation study, we illustrate the advantages of our algorithms compared to the-state-of-the-art from two aspects for a given link set L: when the total transmit powers assigned to the links in L in the computed schedules are approximately equal, our algorithms yield schedules with lower latency; when the schedules computed by all the algorithms have approximately the same latency, the schedules computed by our algorithms require less total transmit power. Note that in order to test these scenarios, we have to select appropriate values for the involved parameters. Fortunately this is not difficult as many parameter settings satisfy the requirements and the results demonstrate the same trends. Therefore in the following we report the simulation results for one set of the parameter settings. Also note that all the results are averaged over 30 runs.

TABLE III: Some important parameter settings.

Parameter	Value	Parameter	Value
$\#links$	200∼400	N	$-70db$
$l^\star_{\max}$	30	$l^\star_{\min}$	1
α	2∼6	β	> 1
ε	> 0	φ	> 0

Consider the algorithms GOW, GOW*, MLLS, SLSUM, and SLSPC under the parameter settings listed in Table VI. The total transmit powers of the schedules yielded by these algorithms are reported in Table IV, and the corresponding schedules for uniform, mean, and linear power assignments are illustrated in Fig. 4, 5, and 6, respectively. Note that the total transmit powers differ slightly as it is impossible to make the total transmit powers of the five algorithms exactly equal by adjusting the parameters. We set the reference total transmit power to be the minimum amount needed by GOW and GOW* with uniform power assignment as GOW and GOW* were designed for uniform power control. Also note that MLLS and SLSUM take the uniform power assignment while SLSPC adopts the quasi-uniform power assignment when $k = 0$. In all cases we notice that MLLS and SLSUM consume a little bit more power than GOW and GOW* because $\mathrm{P}_{\mathrm{MLLS}} = cl^{k\alpha}$, $\mathrm{P}_{\mathrm{SLSUM}} = (1+\varphi)\beta N l^\alpha_{\max}$, and $c > \frac{\beta N l_{\max}^{(1-k)\alpha}}{\eta}$.

A careful study on Fig. 4 reveals the following observations for the case of uniform power assignment: 1) GOW has the lowest latency as it does not consider the ambient noise. This is not reasonable as the ambient noise can not be ignored especially for low power assignments such as in sensor networks. 2) MLLS incurs the highest latency. This is because the parameter c defined in [29] must be small in response to the low power consumption, which results in a

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TABLE IV: The total transmit power assigned to the link set L for different algorithms in Figs. 4, 5, and 6.

Total Power / $\#links$	Fig. 4				Fig. 5			Fig. 6	
	P_{uniform}	P_{MLLS}	P_{SLSUM}(uniform)	P_{SLSPC}	P_{MLLS}	P_{SLSUM}(mean)	P_{SLSPC}	P_{MLLS}	P_{SLSPC}
200	5.40	6.76	5.94	5.32	5.58	5.16	5.39	5.32	5.32
400	10.80	13.53	11.88	10.56	11.21	10.38	10.87	10.58	10.58
600	16.20	20.29	17.82	15.90	16.80	15.55	16.30	16.29	16.29
800	21.60	27.06	23.76	21.08	22.48	20.81	21.80	21.66	21.66
1000	27.00	33.82	29.70	26.24	27.89	25.81	26.94	26.76	26.76

large side length of a large-block, causing a small number of large-blocks. In such a case, the number of links that can transmit simultaneously in the same slot becomes small. 3) Our algorithm SLSUM performs better than GOW* as the number of simultaneously scheduled hexagons of SLSUM is larger than that of the simultaneously scheduled squares of GOW* for the corresponding class, and GOW* schedules "black and gray" links sequentially while SLSUM schedules them as other links by slightly increasing their transmitting powers. We also observe that SLSPC is slightly better than SLSUM due to the difference of the side lengths of the hexagons. More specifically, the side length of a hexagon in SLSPC is smaller than that of the hexagon in SLSUM; therefore the number of hexagons partitioned in SLSPC is larger than that of those partitioned in SLSUM, resulting in the smaller latency of SLSPC compared to SLSUM.

For the cases of mean power assignment where $k = 0.5$ (Fig. 5) and linear power assignment where $k = 1$ (Fig. 6), we observe that our algorithms perform better for sparser networks with smaller number of links while MLLS performs better in denser networks with more number of links. This observation can be justified from the following two aspects. First, the side length of the small square in MLLS is $l_{\max}/\sqrt{2}$, which is smaller than that of a hexagon. That is, the number of small squares is larger than that of hexagons. Second, when the network region is fixed, more small squares that do not contain any receiver participate in the scheduling, which results in a relatively larger latency in MLLS compared to our algorithms. On the other hand, the probability that small squares contain receivers is larger when more number of links exist, making MLLS more efficient in scheduling. In mean power assignment, SLSPC performs better than SLSUM due to the same reasons mentioned before.

TABLE VI: Parameter settings in Fig. 4, 5 and 6

Parameter	Fig.4	Fig.5	Fig.6
k	0	0.5	1
ε	0.3	0.9	1.5
φ	0.1	0.3	0.5
c	3.382×10^{-2}	3×10^{-4}	2.5×10^{-6}

Now we consider each of MLLS, SLSPC, and SLSUM under oblivious power assignment when the total transmit power is approximately equal (see Table V). The results are reported in Fig. 7, 8, and 9 for MLLS, SLSPC, and SLSUM,

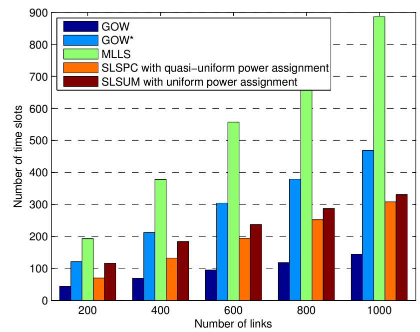

Fig. 4: Latency comparison with uniform power assignment.

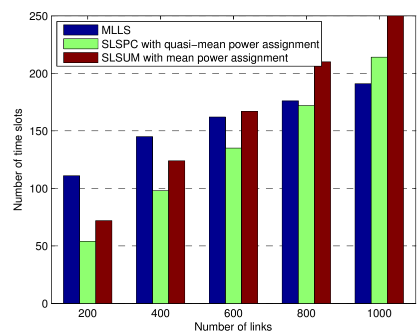

Fig. 5: Latency comparison with mean power assignment.

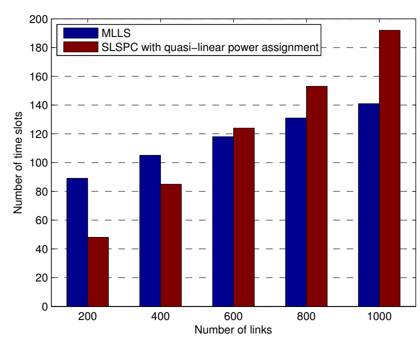

Fig. 6: Latency comparison with linear power assignment.

TABLE V: The total transmit power assigned to the link set L for the algorithms in Fig. 7, 8, 9.

Total Power / $\#links$	Fig. 7 (P_{MLLS})			Fig. 8 (P_{SLSPC})			Fig. 9 (P_{SLSUM})	
	with uniform	with mean	with linear	with quasi-uniform	with quasi-mean	with quasi-linear	with uniform	with mean
200	6.76	5.58	5.32	5.32	5.39	5.32	5.94	5.16
400	13.53	11.21	10.58	10.56	10.87	10.58	11.88	10.38
600	20.29	16.80	16.29	15.90	16.30	16.29	17.82	15.55
800	27.06	22.48	21.66	21.08	21.80	21.66	23.76	20.81
1000	33.82	27.89	26.76	26.24	26.94	26.76	29.70	25.81

respectively, for the parameter settings listed in Table VII. We notice that the latency decreases for all the three algorithms. In MLLS, the average number of links scheduled simultaneously in a slot increases with the increase of k (Fig. 7). More specifically, the K parameter in MLLS is inversely proportional to k when other parameters in the calculation of K are fixed. That is, the larger the k, the smaller the K, and the larger number of links scheduled simultaneously in a time slot. We also observe that K decreases sharply with the increase of k when k is close to 0, and changes very slightly when $k > 0$. K is also affected by c, but the impact of c on K is small when k is close to 0. This explains why the performance of MLLS improves significantly when k changes from 0 to 0.5 but the improvement is relatively small when k changes from 0.5 to 1.0 in Fig. 7. The latency decrease of SLSPC and SLSUM is not because of the increase of k, but because of the increases of other parameters (ε and φ) in Table VII. This is because the side lengths of SLSPC and SLSUM are independent of k, but they are inversely proportional to ε and φ in SLSPC and SLSUM, respectively. Therefore, with the increase of the values of ε and φ, as shown in Table VII, the latencies of SLSPC and SLSUM decrease (Figs. 8 and 9). Next we consider the impact of key parameters on the scheduling latency. The function of parameter c defined in [29] is similar to those of ε and φ in our algorithms SLSPC and SLSUM, i.e., c controls the transmit power and the performance of MLLS. However, the claim that $c > 0$ in [29] is not accurate. In fact, $c > \frac{\beta N}{\eta} l_{max}^{(1-k)\alpha}$ according to Lemma 1. On the other hand, the scheduling results of our algorithms do not change when k varies, as the plane partition is independent of k. An example is shown in Table VIII, which indicates that about 85 slots (the variation is caused by the 30 runs for average computation) are needed for 400 links when $\alpha = 3$, $\beta = 10$, $l_{max} = 30$, and $\eta = 0.8$. We also notice that the scheduling performance of MLLS increases sharply when k changes from 0 to 0.5, which is consistent with our observation in Fig. 7. However, as indicated in Table IX, the total transmit power of MLLS increases significantly, which implies that MLLS achieves a

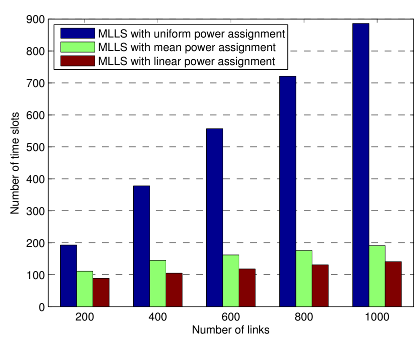

Fig. 7: MLLS has shorter latency with the increase of k.

TABLE VII: Parameter settings in Fig. 7, 8 and 9

Parameter	$k = 0$	$k = 0.5$	$k = 1$
ε	0.3	0.9	1.5
φ	0.1	0.3	0.5
c	3.382×10^{-2}	3×10^{-4}	2.5×10^{-6}

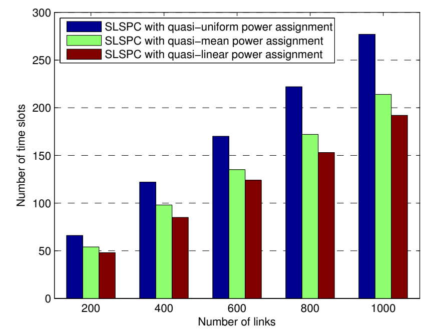

Fig. 8: SLSPC has a shorter latency with the increase of k.

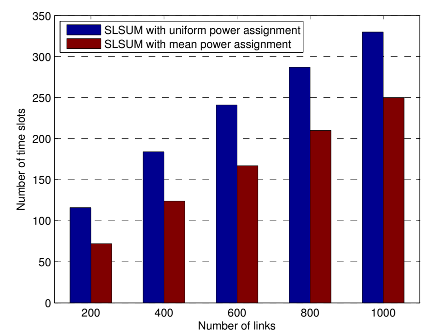

Fig. 9: SLSUM has a shorter latency with the increase of k.

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short schedule at the cost of high transmit power. Comparing Table VIII and Table IX, we observe that when k changes from 0.5 to 1, the scheduling result of MLLS does not change much even though the total transmit power is significantly increased, as K is approximately equal when $k = 0.5$ and $k = 1$. We conclude that SLSPC is more economical and suitable for arbitrary power assignment than MLLS in wireless networks.

TABLE VIII: SLSPC and SLSUM yield stable scheduling results with different k for $\varepsilon = 2, \varphi = 1, c = 0.04, \#links = 400$.

Slots \ Algorithm (k)	MLLS	SLSPC	SLSUM
0	178	85	87
0.5	89	82	88
1	90	85	-

TABLE IX: Total Power $\varepsilon = 2, \varphi = 1, c = 0.04, \#links = 400$.

Algorithms	$k = 0$	$k = 0.5$	$k = 1$
MLLS(mW)	16	1487	168601
SLSPC(mW)	25	17	13
SLSUM(mW)	22	24	-

In GOW*, no matter how large the network region is, the “black and gray” links are scheduled sequentially. Nevertheless, intuitively, if two “black and gray” links are far away from each other and are assigned appropriate transmit powers, they should be able to transmit concurrently. This consideration motivates SLSPC and SLSUM to remove the “black and gray” links by enlarging the transmit power so that the “black and gray” links can be scheduled as regular ones in less number of time slots. This is demonstrated by Fig. 10 for the parameter settings listed in Table X, in which case all the links are deemed as “black and gray”. We observe that by increasing ε and φ, we increase the total transmit powers of SLSPC and SLSUM, and the schedule latencies decrease accordingly for both SLSPC and SLSUM.

TABLE X: Parameter settings in Fig. 10

k	$l_{\min}$	$l_{\max}$	$\#links$
0	27	30	600

The impact of β on SLSPC and SLSUM is illustrated in Fig. 11, which reports the scheduling results for the parameter settings listed in Table XI. We notice that more time slots are needed when the threshold β of SINR increases. This is because the side length of a hexagon is proportional to β under the SINR model: the smaller the β, the smaller the side length, the more the number of links that can be scheduled to one time

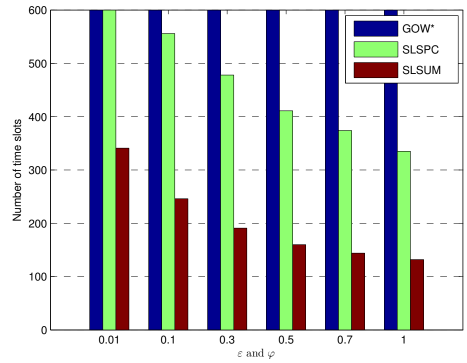

Fig. 10: The latencies of SLSPC and SLSUM decrease with the increase of ε and φ.

slot, and the less the number of time slots. When β increases, a link l must tolerate less interference in order to successfully transmit. This implies that the links transmitting concurrently with l must be farther away from l; therefore the side length of a hexagon must be enlarged.

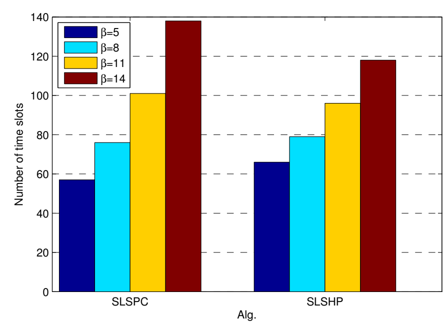

Fig. 11: The threshold β influences SLSPC and SLSUM.

TABLE XI: Parameter settings in Fig. 11

k	ε	φ	$\#links$
0	1	1	400

Note that in this paper, we define $\varepsilon > 0$ and $\varphi > 0$ as the *regulatory factor* for SLSPC and SLSUM, respectively, which are closely related to transmit power assignment and “link diversity”. When ε and φ are small, the transmit power is low, which is economic for wireless networks. However, the side length of a partitioned hexagon is related to the *regulatory factor*, and so do the scheduling results. Our simulation results are reported in Figs. 12 and 13 for the parameter settings listed in Table XII. Particularly, for SLSPC and SLSUM with uniform power assignment, the larger the *regulatory factor*, the lower the latency. Meanwhile, a large *regulatory factor* implies a large transmit power. However, SLSUM with mean power assignment has the lowest latency when $\varphi = 1$ and the latency increases when $\varphi > 1$ (see Fig. 13). This can be justified as follows. The side length of the hexagons in SLSPC only depends on ε when α and β are fixed, and it decreases with the increase of ε, leading to the decrease of the latency (see Fig. 12). However SLSUM is more complicated because φ influences both μ and the SNR classification, which together determine the side length of the hexagons in SLSUM, causing the nonlinear relationship between the schedule latency and φ.

TABLE XII: Parameter settings in Fig. 12 and 13

k	α	β	$\#links$
0	3	10	400

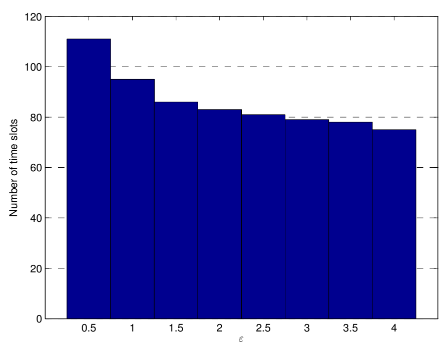

Fig. 12: ε influences SLSPC.

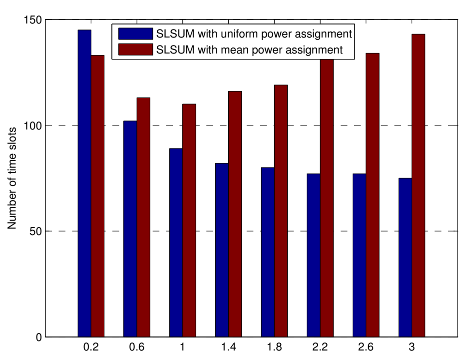

Fig. 13: φ influences SLSUM.

VI. Conclusion and Future Work

In this paper, we propose two algorithms SLSPC and SLSUM for shortest link scheduling under the SINR model with oblivious power assignment. Different from previous works, we partition the network area into hexagons and color them with 3 different colors such that any two adjacent hexagons have different colors. Our algorithms ensure that all the links selected from different hexagons with the same color can transmit concurrently. By adjusting the transmit power appropriately, the "black and gray" links are removed such that the performance of our algorithms are significantly enhanced. Compared with other algorithms for oblivious power assignment, SLSPC is more economical and more suitable for wireless networks. Our future research will proceed along the following two directions: first, we will focus on the development of distributed link scheduling algorithms targeting large-scale wireless networks; second, as whether or not the shortest link scheduling problem under SINR possesses a small constant approximation ratio algorithm is still open, we will make effort towards this challenge.

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