1. Scenario

Consider a fishery resource shared by two countries. Each country could act to deplete the resource or conserve it. If the two countries follow the same strategies, they will share the economic returns from the fishery. If the first country chooses to deplete the resource, while the second country chooses to conserve, you can assume that the first country can deplete the resource so quickly that the second country receives nothing.

2. Game Theory Approach

2.1. Game Definition and Assumptions

The players of the game are country A and country B. Each country could act to deplete the fishery resource or conserve it. If the two countries choose to deplete, they will share the economic returns from the fishery. However, in the long run, the resource will be in serious decline due to inadequate conservation measures, threatening the fish food security and the livelihood of local communities. Both depleting countries will also share this cost. Conversely, if the first country chooses to deplete the resource, while the second country chooses to conserve, the first country can deplete the resource so quickly that the second country receives nothing. In this instance, the conservation efforts of the second country will not yield any results, while the depletion of the fishery will affect both countries in the long run. Lastly, if the two countries choose to conserve, they will share the returns of protecting the environment and its natural resources by creating a sustainable, productive fishery to meet food demands for future generations.

Assumptions

1. Each country ["Player"] has two well-specified choices (called "Plays") of either depletion or conservation.
2. The game considers each country as a single, indivisible unit. Such an assumption is very simplistic since it ignores a fundamental conflict, namely the tragedy of the commons. While the government is interested in sustaining the fisheries that provide profits and feeds it the population, but individual fishers have an incentive to take as much as they can as quickly as they can.
3. Every possible combination of plays leads to a well-defined end-state of payoffs that terminates the game.
4. Each country has perfect knowledge of the game and their opposition; that is, each country knows the rules of the game and the payoffs for the other country completely.
5. Although fishery resources are freely accessible, if a country chooses conservation, no individual from the country will try to fish in the resource.
6. All national agents are rational; that is, each country, given two alternatives, will select the one that yields the greater payoff.

2.2. Pavoff Matrix

Assuming there is no cooperation between the two countries and there is no communication among the resource managers, this is a non-cooperative game that resembles the Prisoner's Dilemma (PD).

A PD constitutes a two-agent situation in which:

1. Country A stands to gain more in a transaction by acting selfishly towards Player B if B offers to conserve, and vice-versa; yet
2. Country A and Country B together could gain a higher total reward by cooperating than by both depleting.

If both players conserve, they both receive an overall payoff (considering both economic, social and ecological benefits in arbitrary units) of 3 points. If one player depletes while the other conserves, then the former player receives all the short-term economic benefits (5 points), and has to incur the long-term costs of depletion (-1 point). The total payoff is 4 points, while the latter country incurs a -1 point due to the cost of environmental degradation since the fishery is a shared resource. Finally, if both plays deplete, each receives fewer economic benefits because they're competing for a scarce resource in the short run (2) and will also incur the long-term costs for loss of ecological balance and fishery sources (-1).

In [1]:
```
from IPython.display import Image

Image("1.png")
```

Out[1]:

Country A/Country B	Conserve	Deplete
Conserve	3,3	-1,4
Deplete	4, -1	1,1

2.3. Nash Equilibria

Qualitatively, for each player, attempting to conserve exposes that player to the risk of depletion by the other, and in this dilemma, being depleted against while offering to conserve is worse than mutual depletion. As a result, though the ideal outcome is mutual conservations, without mechanisms for both coordination and trust, the equilibrium state is mutual depletion.

Quantitatively, in each row (i.e., for each possible choice by Country A) Country B's best outcome is in the Deplete column. In each column (i.e., for each possible choice by Country B) Country A's best outcome is in the Deplete row. So, mutual depletion is the Nash equilibrium for the fishery game. And yet, both countries do much better if they both cooperate than if they both defect.

2.4. Evolutionary Stable Strategies

Let us consider the Fishery game. Suppose both countries are hard-wired to play conservation, and a small mutation to play depletion is introduced. The population is now composed of 1-n conservers and n depletors. Each conserver and depletor will be randomly paired with another country, so each will have a 1-n chance of being paired with conserver and an n-chance of being paired with a depletor.

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In [25]: Image("3.png")
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Out[25]:

The average payoff to the incumbent conserver	The average payoff to the mutant depletor
$(1-n)\times(3)+n\times(-1)$	$(1-n)\times(4)+n\times(1)$

The depletors do better (on average) than the conserving incumbents. This mutation will not die out. Thus, a population that consists 100% of conservers is not evolutionarily stable.

Conversely, suppose all nations the population are hard-wired to play depletion. Now, a small mutation to play conservation is introduced. The population mix is then 1-n depletors and n conservers. Each conserver and each depletor will be randomly paired with another country, so each will have a 1-n-chance of being paired with depletor and an n-chance of being paired with a conserver.

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In [26]: Image("4.png")
```

Out[26]:

The average payoff to the incumbent depletor	The average payoff to the mutant conserver
$(1-n)\times(1)+n\times(4)$	$(1-n)\times(-1)+n\times(3)$

The conserving mutants do worse (on average) than the depleting incumbents. This mutation will die out. Thus, a population that consists of 100% of depletors is evolutionarily stable i.e. if all members of the population adopt it, then any small mutation playing a different strategy would die out.

Evolutionary stability does not imply nice or good or efficient. Also, strictly dominated strategies cannot be evolutionarily stable. In addition, although evolutionary stability implies Nash Equilibrium (NE), the converse isn't true, i.e. NE doesn't imply evolutionary stability. Only if a strategy is a strict NE, e.g. mutual depletion, then the strategy is evolutionarily stable.

2.5. [Optional] Generic Payoff Matrix

Consider the following payoff points:

Country A: A>B>C>D

Country B: a>b>c>d

In [6]:
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Image("4.png")
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Out[6]:

Country A/Country B	Conserve	Deplete
Conserve	B, b	D, a
Deplete	A, d	C, c

If Country B conserves, then Country A will want to deplete as A is greater than B. If Country B depletes, then Country A will still want to deplete as they prefer C to D. Therefore, in equilibrium, country B depletes.

If country A conserves, then Country B will want to deplete as a is greater than b. Conversely, if country A depletes, then country B will still want to deplete as c is greater than d. Therefore, for all forms of this game, the only sensible payoff is of mutual depletion.

In [22]:
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Image("5.png")
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Out[22]:

Solving for Country A's Mixed Strategy

Step 1: Let utility of conserving for Country B as a function of the mixed strategy of Country A: $[U_c]$

Some percentage (p) of the time, Country B gets b. The rest of the time (1-p), it gets d.
Mathematically,

$$[U_c] = p \cdot b + (1 - p) \cdot d$$

Step 2: Let utility of defecting for Country B as a function of the mixed strategy of Country A: $[U_d]$
Some percentage (p) of the time, Country B gets a. The rest of the time (1-p), it gets c.
Mathematically,

$$[U_d] = p \cdot a + (1 - p) \cdot c$$

Step 3: A mixed strategy is supposed to make the opponent indifferent between their pure strategies. Therefore, the utility of playing conserve or defect must be equal.

$$[U_c] = [U_d]$$
$$p \cdot b + (1 - p) \cdot d = p \cdot a + (1 - p) \cdot c$$
$$p = \frac{c-d}{b+c-a-d}$$

Step 4: If a Mixed Strategy Nash Equilibrium (MSNE) violates any of the following rules, that alleged MSNE is not an MSNE:

- No events can occur with negative probability.

$$\frac{c-d}{b+c-a-d} \geq 0$$

The numerator is positive because we asserted earlier that c is greater than d. Therefore, the denominator also must be positive. As a result, a valid MSNE will exist only if the following is true:

$$b + c - a - d > 0$$
$$b + c > a + d$$

- No events can occur with a probability greater than 1.

$$\frac{c-d}{b+c-a-d} \leq 1$$

To meet this condition:

$$c - d \leq b + c - a - d$$
$$0 \leq b - a$$
$$a \leq b$$

This violates the rule a>b>c>d, and the mixed strategy is invalid. Thus, we've proven that there is no MSNE to the game and the mutual depletion is a unique solution to the Fishery game.

Note: The symbols to evaluate mixed strategy were created and pasted from an online LaTex Equation Editor.

3. Decision Tree Approach

3.1. Tree Construction

In [9]:
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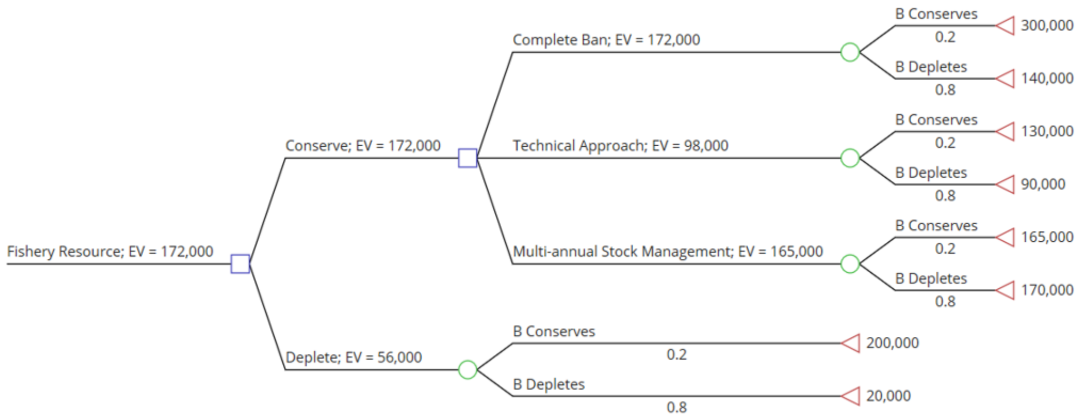
Out[9]:



There are two major decisions in the Fishery Resource problem. First, country A must decide whether to conserve or deplete the fishery resource. Second, if it decides to conserve, the National Fisheries Resource Conservation Program must decide which measure to take amongst:

• A complete ban on fishing,

• Technical approaches involving mandatory selective fishing gear and bycatch reduction devices

• Long-term stock management process to set annual catch limits, protect spawning areas through zones and periods to protect spawning areas and rebuild fish stocks

For this decision tree, we will assume that the country has very limited financial and administrative resources and can only undertake one of the above measures. Each terminal node has an associated terminal value, sometimes called a payoff value representing the number of fish lives saved. Each terminal value measures the result of a scenario: the sequence of decisions and events on a unique path leading from the initial decision node to a specific terminal node. Since two countries share the fishery resource, the payoffs for each alternative depends on whether country B decides to deplete or conserve. The game theoretic analysis above establishes that for each player, attempting to cooperate exposes that player to the risk of defection by the other, there a higher probability is assigned to country B's depletion.

In [11]:
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Image("7.png")
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Out[11]:

Alternatives	Country B Conserves (S)	Country B Depletes (S)
Complete Ban	300,000	140,000
Technical Approach	130,000	90,000
Multi-annual Stock Management	165,000	170,000
Deplete	200,000	20,000

Over a 100-year time scale, a complete ban with proper implementation and regular compliance monitoring will save the greatest number of fish if both countries commit to conservation approaches. While a technical approach will also reduce the exploitation of vulnerable fish species, comprehensive stock management is more multi-dimensional and sustainable, and will also save greater fish lives even if country B decides to deplete the resource. Conversely, if country A decides to deplete, country B's conservation efforts is predicted to save at least 200,000 fish lives. If mutual depletion occurs, an unprecedented amount of fish slaughter will only leave 20,000 fish alive in the resource.

Note that the units of the payoffs are in US dollars. Therefore, the economic return from saving one fish is equal to $1 respectively.

Note also that the decision tree above includes rollback values (initialized by EV), which are calculated using a backward induction method (Middleton, 2003). Rollback is a process of successively calculating expected values by beginning at an endpoint and calculating subsequent expected values back towards the root node.

After the rollback method has determined certain equivalents for each node, the optimal strategy can be identified by working forward through the tree. At the initial decision node, the 172,000 fish lives rollback value equals the rollback value of the Conserve branch, indicating the alternative that should be chosen. A subsequent decision involves choosing the conservation method, which corresponds to the rollback value of a complete ban, indicating it the alternative that should be chosen. Expected values take uncertainty into account by considering the probability of each possible outcome. However, its usefulness is limited to the accuracy of the probabilities and may not correspond to any of the actual potential outcomes for a one-off project.

3.2. [Optional] Expected Value of Sample Information

Suppose there is an intelligence analyst who can provide additional information about the state of nature, i.e. how likely country B is to either conserve or deplete the fishery resource. The analyst has an accuracy history as follows:

• Correctly relayed conservation intentions of country B 45% of the time • Correctly relayed depletion intentions of country B 90% of the time

The decision to hire a consultant depends on two factors, how successful the consultant has been in the past, and how much does it cost to hire the consultant.

Let,

pC: predicted conservation pD: predicted depletion C: Actual conservation D: Actual depletion

Therefore,

• P(C) = 0.2

• P(D) = 0.8

• P (pC | C) = 0.45

• P (pD |C) = 0.55

• P (pD | D) = 0.9

- P (pC| D) = 0.1

Scenario 1: Analyst reports Conservation

P(pC) = P(C&pC) + P(D&pC) = 0.09 + 0.08 = 0.17

In [12]:
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Image("8.png")
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Out[12]:

	Prior	Conditional	Joint	Posterior
Conservation	P(C) = 0.2	P (pC | C) = 0.45	P(C&pC) = 0.2 * 0.45 = 0.09	P(C|pC) = P(C&pC)/P(pC) = 0.09/0.17=0.5294
Depletion	P(D) = 0.8	P (pC| D) = 0.1	P(D&pC) = 0.8 * 0.1 = 0.08	P(D|pC) = P(D&pC)/P(pC) = 0.08/0.17=0.4706

Scenario 2: Analyst reports Depletion

P(pD) = P(D&pD) + P(C&pD) = 0.11 + 0.36 = 0.47

In [13]:
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Image("9.png")
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Out[13]:

	Prior	Conditional	Joint	Posterior
Conservation	P(C) = 0.2	P (pD | C) = 0.55	P(C&pD) = 0.55 * 0.2 = 0.11	P(C|pD) = P(C&pD)/P(pD) = 0.11/0.47 = 0.234
Depletion	P(D) = 0.8	P (pD| D) = 0.45	P(D&pD) = 0.8 * 0.45 = 0.36	P(D|pD) = P(D&pD)/P(pD) = 0.36/0.47=0.766

Our new tree, incorporating the analyst's contribution is:

In [14]:
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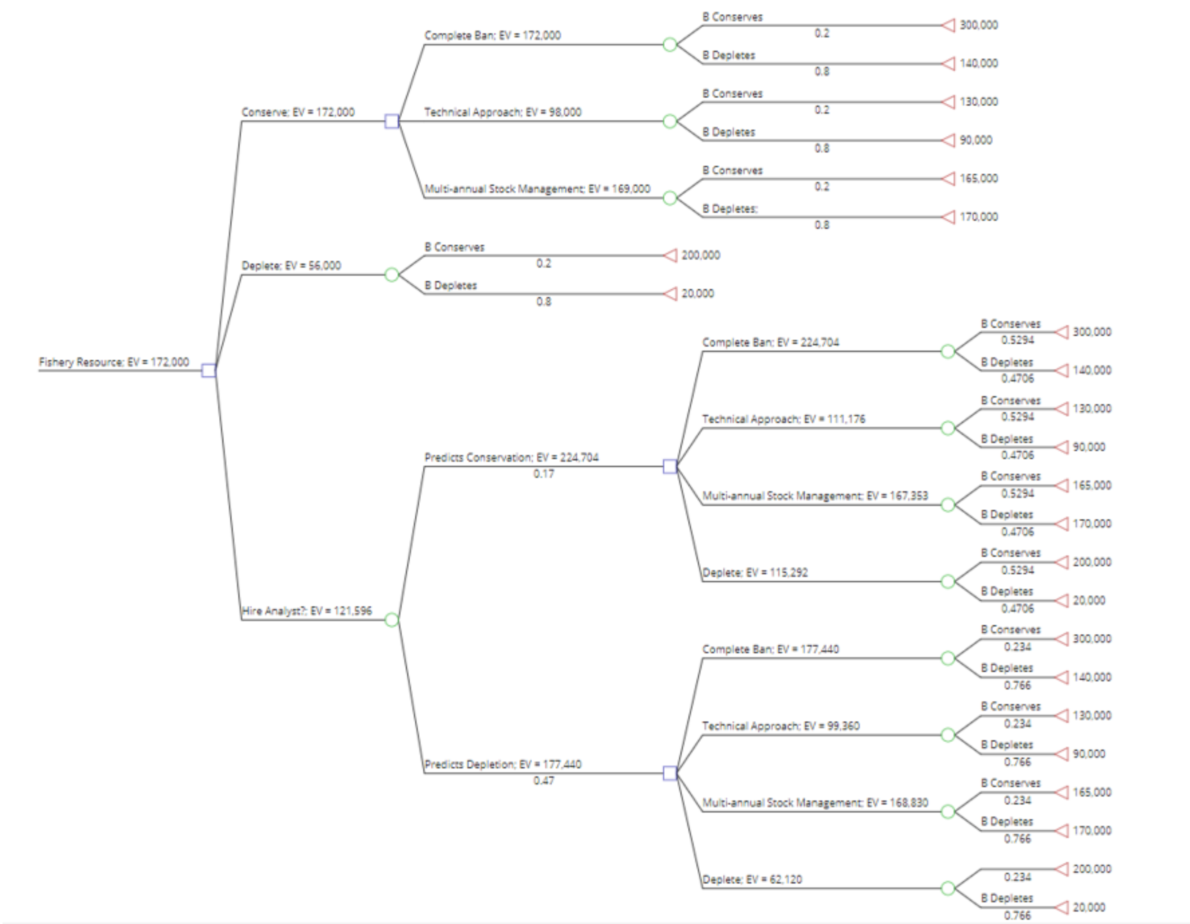
Out[14]:



The expected value of sample information (EVSI) estimates the value of information supplied by the analyst.

EVSI = EV with SI (Expected payoff if the analyst is hired) - EV without SI (Best expected payoff if the analyst isn't hired)

EVSI = 224,704 - 172,000 = 52,704

Therefore, if the cost of hiring an analyst is greater than the EVSI (which is 52,704), country A will stick with its original decision to implement a complete ban and choose not to hire the analyst. However, if the cost of hiring an analyst is less than EVSI, country A's strategy will be to hire the analyst. If the analyst predicts conservation, implement a complete ban. Conversely, if the consultant's report is depletion, also implement a complete ban.

However, note that before the additional information, expected value calculations indicated a complete ban is the optimal decision. This also applies when considering hiring an analyst . Therefore, the EVSI is not very useful in determining the best decision strategy in this scenario. However, conducting such a thorough calculation is still beneficial in validating policy decision, as this will allow political leaders to have greater confidence in their actions.

3.3. Comparing Different Strategies

If the future were certain, decision making proceeds towards the alternative that promises the highest economic returns or the lowest costs, e.g. Maximax or Maximin. With uncertainty, alternatives with the highest expected monetary value or lowest expected opportunity cost are pursued.

Decision Making Under Certainty

Maximax Approach (Optimistic)

The Maximax approach maximizes the maximum payoff (Best of Best). The criteria ignores all information except the maximum value as it assumes that the most favorable state of nature will occur, and is suitable for an optimist, or 'risk-seeking' country. A maximax criterion would implement a complete ban, among all possible alternatives been conservation and depletion.

In [15]:
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Image("11.png")
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Out[15]:

Alternatives	Country B Conserves (S)	Country B Depletes (S)	Best (S)
Complete Ban	300,000	140,000	**300,000**
Technical Approach	130,000	90,000	130,000
Multi-annual Stock Management	165,000	170,000	170,000
Deplete	200,000	20,000	200,000

Maximin Approach (Pessimistic)

The maximin approach maximizes the minimum payoff of each alternative (Best of Worst). This criterion ignores all information except the minimum payoff for each action. Therefore it is pessimistic. The approach is appropriate for a country which seeks to achieve the best results if the worst happens, in the process losing out on the opportunity to make big profits. A maximin criterion would implement multi-annual stock management.

In [16]:
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Image("12.png")
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Out[16]:

Alternatives	Country B Conserves (S)	Country B Depletes (S)	Worst (S)
Complete Ban	300,000	140,000	140,000
Technical Approach	130,000	90,000	90,000
Multi-annual Stock Management	165,000	170,000	**165,000**
Deplete	200,000	20,000	20,000

Minimax Regret Approach

Nations are also concerned with forgone opportunities. The minimax regret strategy is one that minimises the maximum regret and is appropriate for the risk-neutral decision maker. 'Regret' in this context refers to the difference between the optimal profit or pay-off and the actual pay-off received, i.e., it is the amount lost by not picking the best alternative. When a decision with a less-than expected payoff is made, this criterion encourages the avoidance of regret. The minimax regret criterion selects a complete ban.

Regret Table

In [17]:
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Image("13.png")
```

Out[17]:

Alternatives	Country B Conserves (S)	Country B Depletes (S)	Maximum (S)
Complete Ban	0	30,000	**30,000**
Technical Approach	170,000	80,000	80,000
Multi-annual Stock Management	135,000	0	135,000
Deplete	100,000	150,000	150,000

Decision Making Under Risk

Expected Monetary Value (EMV) Approach

The EMV is calculated by multiplying the value of each possible results by its probability of occurring and adding the weighted probability values of the result. This is an important risk management strategy in choosing between alternative strategies, but the reliability of the analysis is based on the data provided as input to the technique and the risk attitude of the decision maker. For an objective analysis, the country should be risk-neutral and seek to maximize their EMV instead of their subjective utility. However, note thatEMV is an average and therefore only applicable when there are repeated trials or decisions. In the decision scenario, the EMV criterion selects a complete ban.

In [18]:
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Image("14.png")
```

Out[18]:

Alternatives	Country B Conserves (S)	Country B Depletes (S)	EMV (S)
Complete Ban	300,000	140,000	**172,000**
Technical Approach	130,000	90,000	98,000
Multi-annual Stock Management	165,000	170,000	169,000
Deplete	200,000	20,000	56,000
Probability	0.2	0.8	

Expected Opportunity Loss (EOL) Approach

The EOL also computes a regret table and multiplies the respective probabilities with each regret. Then, the EMV selects the minimal loss. Considering both the absolute payoffs and their associated probabilities confers the benefit of a more informed decision. The EMV decision is to implement a complete ban.

Regret Table

In [19]:
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Image("15.png")
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Out[19]:

Alternatives	Country B Conserves ($)	Country B Depletes ($)	EOL ($)
Complete Ban	0	30,000	**24,000**
Technical Approach	170,000	80,000	98,000
Multi-annual Stock Management	135,000	0	27,000
Deplete	100,000	150,000	140,000
Probability	0.2	0.8	

3.3. Risk and Uncertainty

Both risk and uncertainty share a lack of knowledge about a future outcome, but the concepts differ in the following sense:

• Risk: Full knowledge about the probabilistic characteristics of the possible outcomes (measurable-"known unknowns")

• Uncertainty: Outcome not associated with a specific probabilistic function (not measurable - "unknown unknowns"). Not all outcomes may be accurately foreseen, and the probabilities cannot be deduced or based on empirical data. Assignment of probabilities is on a subjective basis, using experience or expert judgment.

The three sources of uncertainty in the Fishery Resource problem are:

• Over the economic returns of fish and impacts of environmental damage, and

• Over the selection of conservation/depletion strategy by country A

• Over the conservation/depletion strategy of country B, which are subjectively estimated as 0.2 and 0.8 respectively

In the tree, decision nodes and branches represent the controllable factors in a decision problem, i.e. the risks, while event nodes and branches represent uncontrollable factors, e.g. uncertainty. While making many decisions is difficult, the difficulty of making uncertain decisions is that the results of choosing the alternatives available may be variable, ambiguous, unknown or unknowable.

In the domain of uncertainty, decisions related to natural resource management are made at every level of government. Community-level allocation of natural resources can give rise to local clashes. National-level decisions, such as policies on landownership or large-scale concessions, can lead to more widespread tensions and conflict. Fishery sources provide the foundation for the socioeconomic development of a country, and conflicts at multiple levels are diverse in scale, varying from one context to another.

Further, countries recovering from resource-related conflicts are more likely to relapse into conflict, and relapse twice as quickly as countries recovering from other types of conflict ('From Fragility to Resilience,' 2016). Wealth in the form of natural resources spurs frequent conflict, poor governance, and corruption, in other words, a non-linear ‘resource curse.’ Although effects are irreversible, the effects of impacts are usually localized to the geographical location. Since the event doesn't lead to systemic, widespread harm, according to Taleb et al. (2014) the precautionary principle (PP) shouldn't be evoked. Nevertheless, it is important to exercise safe and rigorous risk management.

3.3. Cognitive Biases

A utility function often exists that reflects the agent's degree of aversion to large losses. Many, if not most, agents are cautious in situations where they think they might be vulnerable to large losses. If a nation is loss averse, its government will tend to use Maximin strategy more often to minimize the losses and avoid taking any risk. Decisions made by risk-averse agents tend to maximize their expected utility rather than expected value, and that utility may give serious (negative) weight to the possibility of large losses. For instance, many governments shy away from project decisions which, if they were to fail, would expose the party to the probability of failure in the next elections, even if such project decisions might also offer a possibility of large gains associated with success.

Conversely if the national agent is risk-seeking, a Maximax strategy may be preferred. This behavior might be called "risk-averse". To avoid the effects of subjective utilities, risk neutral approaches liked EMV or EOL are ideal. However, this assumes that country A is a rational agent which wants to choose the alternative that maximizes its expected monetary value or minimize its expected cost. This "risk-neutral" behavior may represent a nation that has many projects and can thrive if it succeeds "on the average."

Since we are choosing future payoffs, temporal discounting is also an important consideration. Assume that if country B depletes, country A's payoffs for the technical approach and multi-annual stock management approach take 60 and 75 years respectively. Also, the payoffs for the latter approach is higher and is depicted as the bar graph farther to the right in Figure 1.

In [20]:
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Image("16.png")
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Out[20]:

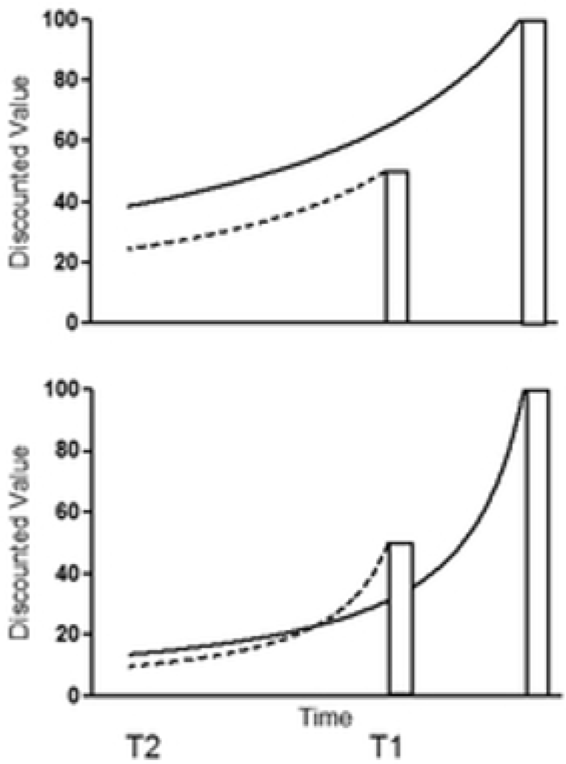

Figure 1. Comparison of intertemporal choice as predicted by exponential discounting (top) and hyperbolic discounting (bottom) ("PsycNET," n.d.).

Figure 1 depicts that exponential discounting leads to consistency. Since the stock management approach is better in value, the preference for the alternative remains higher for both longer and shorter time intervals i.e., for both T1 and T2. However, in the hyperbolic discounting scenario, at T1, the preference may be for the near-term benefit, i.e. technical measures. However, a preference reversal occurs for longer time interval, i.e. in T2, whereby the stock management payoffs that are farther in the future but in higher in returns are more valued.

The government has an interest in sustaining the fisheries that yield profits and feeds both the current and unborn population in the long run, modelled by exponential discounting. However, individual fishers have an incentive to take as much as they can as quickly as they can, modelled through hyperbolic discounting. For the latter, rewards that are very close in time are more valuable. Differences in time preferences between fishers and governments should be considered when developing conservation strategies. If fishers are present-biased and greatly discount the future, vocational programs to teach new skills and bring other forms of income are useful. Also, community-wide education programs to teach common pitfalls in gut reasoning and better decision-making strategies can also mitigate its effects.

3.3. Reflection

While the game theory approach shows that deplete is the optimal option, various decision tree strategies show that conservation by implementing a complete ban is more desirable. The reason for this discrepency highlights a major caveat of decision trees i.e. the values of outcomes, subsequent decisions and payoffs is largely subjective. In addition, large trees that include dozens of decision nodes (spots where new decisions are made) can be convoluted and may have limited value. Regardless, decision trees allow the consideration of a range of possible outcomes and subsequent decisions made after an initial decision, along with their associated probabilities.

Alternatively, game theory provides a useful and quantitative framework for decision making. However, the sheer complexity of resource management along with the assumptions about players and their level of knowledge about their own pay-offs (and that of others') limits its practical applications. In addition, national agents do not always act rationally.

Furthermore, if the future were certain, decision making deals with payoff values to either maximize returns or lower costs and/or regret, e.g. Maximax, Maximin or Minimax Regret. With uncertainty, alternatives that account for both payoffs and their respective probabilities are used e.g. highest Expected Monetary Value or lowest Expected Opportunity Loss. In the fishing scenario, a complete ban is desirable under all decision strategies except the Maximin scenario. If country A is particularly loss averse, a Maximin strategy should be evoked since it seeks out the decision that yields the smallest loss. Otherwise a complete ban is the most reasonable policy to implement.

Lastly, in the real world, different individuals have different risk preferences and temporal discounting factors, and making optimal decisions for irrational agents requires us to incorporate such preferences in the decision making process. Subjective preferences can be investigated by filling out a questionnaire, which consists of a series of questions concerning different situations involving risk or asking some questions about how the agent trades off risk and payoffs. This way, we can determine whether a policy that makes less money and has less risk would be preferred to one that makes more money but with more risk.

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