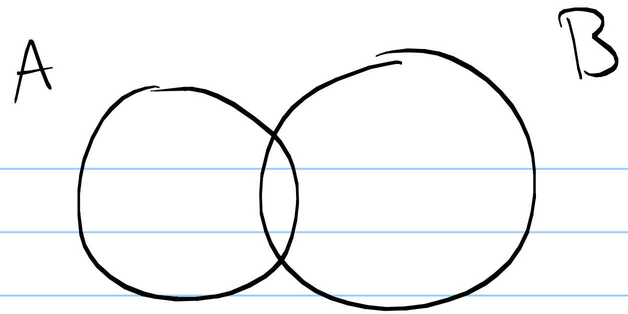
# Elementary operations with sets

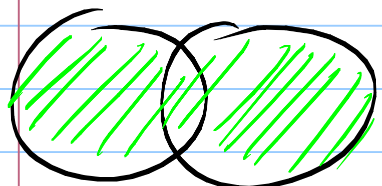
$A, B$ two sets

$$A \cup B = \text{"union"} = \left\{ \begin{array}{l} \text{elements which belong to } A \\ \text{or } B \text{ (or both} \end{array} \right\}$$

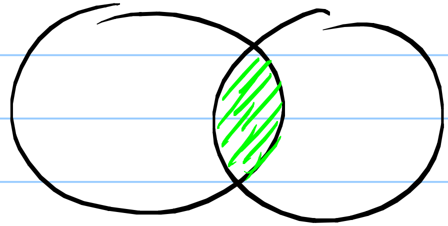
$$A \cap B = \text{"intersection"} = \left\{ \begin{array}{l} \text{elements which belong to} \\ \text{both } A \text{ and } B \end{array} \right\}$$

$$A \setminus B = \text{" difference"} = \left\{ \begin{array}{l} \text{elements which are in } A \\ \text{but NOT in } B \end{array} \right\}$$

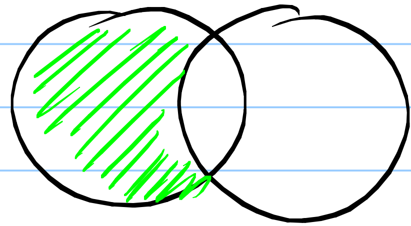


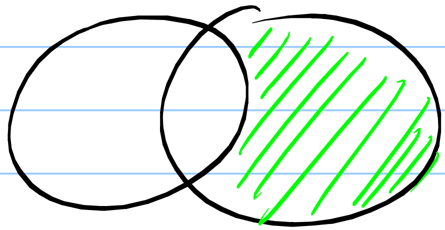
$A \cup B$



$A \cap B$



$A \setminus B$



$B \setminus A$

$$A \cup B = \overbrace{(A \setminus B)}^{\text{only in } A} \cup \overbrace{(B \setminus A)}^{\text{only in } B} \cup \overbrace{(A \cap B)}^{\text{in both } A \text{ and } B}$$

graphically

$A = B$ if every element in $A$ belongs also to $B$ and vice-versa.

Exercise

$A = \{1, 2, 3\}$ $B = \{2, 3, 7, 8\}$

$A \cup B = \{1, 2, 3, 7, 8\}$

$A \cap B = \{2, 3\}$

$A \setminus B = \{1\}$

$B \setminus A = \{7, 8\}$

$A = \{1, 2, 3\}$ $\qquad$ $B = \{\cancel{1}, 1, 2, 3, \star\} = \{1, 2, 3, \star\}$

$A \cup B = B = \{1, 2, 3, \star\}$

$A \cap B = A = \{1, 2, 3\}$

$A \setminus B = \emptyset$ $\leftarrow$ empty set

$B \setminus A = \{\star\}$

<u>Property</u>: $A \subseteq B$

- A is a subset of B
- A is contained in B
- B contains A

if eache element of A is also an element of B

$\forall a \in A$ we have $a \in B$

$\downarrow$

for each

$\forall$ -> for each

$\exists$ -> it exists

(existengial operators)

## Power of a set $A$

$$\mathcal{P}(A) = \{ B : B \text{ is a subset of } A \}$$

"power of $A$"

"l'insieme delle parti di $A$" $= \{$ collection of all the subsets of $A \}$

Ex. 1 $\quad A = \{0, 1\}$

since $\emptyset \subseteq A$ $\qquad$ since $A \subseteq A$

$$\mathcal{P}(A) = \{ \{1\}, \emptyset, \{0\}, \{0, 1\} \}$$

$$B = \{ \Delta, \circ, \star \}$$

$$\wp(B) = \{ \{\Delta\}, \{\circ\}, \{\star\}, \emptyset, \{\Delta, \circ\}, \{\circ, \star\},$$
$$\{\Delta, \star\}, \{\Delta, \circ, \star\} \}.$$

## CARTESIAN PRODUCT OF SETS

$$A \times B = \{ (a, b) \; ; \; a \in A, \; b \in B \}$$

↑ ordered couple

$\{1, 2\} = \{2, 1\}$ $\qquad$ $(1, 2) \neq (2, 1)$

**Ex.** $\quad A = \{0, 1, 2\} \qquad B = \{\sqrt{2}, \pi\}$

$$A \times B = \{ (0, \sqrt{2}), (1, \pi), (2, \sqrt{2}), (0, \pi), (1, \sqrt{2}), (2, \pi) \}$$

# CARDINALITY OF A SET

$|A| \in \mathbb{N}$

$\mathbb{N} \ni |A| =$ the number of elements of $A$

$$|\{0,1,2\}| = 3$$

$$\left|\left\{ p \in \mathbb{N} ; \begin{array}{l} p \text{ prime} \\ p \leq 10 \end{array} \right\}\right|$$

$$= |\{3, 2, 5, 7\}| = 4$$

**Ex.** $A, B$ two sets such that $|A| = 3$

$|B| = 10$. What is the cardinality of

$A \times B$? $30 = 3 \times 10$

$B$

| $A$ | | $b_1$ | $b_2$ | --- | $b_{10}$ |
|---|---|---|---|---|---|
| | $a_1$ | $(a_1, b_1)$ | $(a_1, b_2)$ | --- | $(a_1, b_{10})$ |
| | $a_2$ | $(a_2, b_1)$ | $(a, b_2)$ | --- | $(b_2, b_{10})$ |
| | $a_3$ | $(a_3, b_1)$ | --- | --- | $(a_3, b_{10})$ |

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{ (x, y) : x, y \in \mathbb{R} \}$$

$$A \times B \times C = \{ (a, b, c) \; ; \; a \in A,\ b \in B,\ c \in C \}.$$

**Ex.** $|A| = 10$ $\qquad |\wp(A)| = \, ?$

$A = \{1, 2, 3, \ldots, 10\}$

$$\wp(A) = \{ \{1\}, \{2\}, \ldots\ldots, \{10\}, \emptyset, A,$$

$$A \setminus \{1\}, A \setminus \{2\}, \ldots\ldots, A \setminus \{10\},$$

$$\{1,2\}, \{1,3\}, \{1,4\} \ldots\ldots, \{1,10\} \quad \leftarrow 9$$

$$+$$

$$\{2,3\}, \{2,4\}, \{2,5\}, \ldots\ldots, \{2,10\} \quad \leftarrow 8$$

$\vdots$

$\{8,9\}, \{9,10\}$ $\leftarrow 2$

$+$

$\{9,10\}$ $\leftarrow 1$

$=$

$45$

45 subsets of A, with cardinality 2

45 " " " 8

$$|\mathcal{P}(A)| = 2^{|A|}$$

$B \subseteq A$ then for each $x \in A$ : is $x \in B$?

$\{1, 2, 3, \ldots, 10\}$

Yes Yes ... Yes

No No ... No

$2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2 = 2^{|A|}$

$= 1024 = 2^{10}$

Let's define $C = \{ B \subseteq A : |B| = 3 \}$. $|C| = ?$

"How many subset of $A$ of cardinality 3 are there?"

$|C| = \binom{n}{3}$ $(n = |A|)$ binomial coefficients

$$\binom{n}{k} = \left( \begin{array}{l} \text{number of subsets of a set of } \# = n \\ \text{with } k \text{ elements} \end{array} \right)$$

$$= \frac{n!}{k!\,(n-k)!} = \frac{n(n-1)\cdots(n-k+1)}{1\cdot 2 \cdots k}$$

$n!$ = the product of the $n$ natural numbers up to $n$

$= 1\cdot 2\cdot 3 \cdots n$

$$\binom{n}{3} = \frac{n(n-1)(n-2)}{6}$$

$B \subseteq A$ $\qquad B = \{ b_1, b_2, b_3 \}$

in how many ways can I select $b_1$? 10

" " " " " " $b_2$? 9

"    "    "    " , "    $b_3$ ?    8

$10 \cdot 9 \cdot 8$ . But in this way I have selected many times $\{1, 2, 3\}$

| | | |
|---|---|---|
| $b_1 = 1$ | $b_1 = 2$ | 1, 2, 3 |
| $b_2 = 2$ | $b_2 = 1$ | 2, 1, 3 |
| $b_3 = 3$ | $b_3 = 3$ | 1, 3, 2 |
| | | 3, 1, 2 |
| | | 2, 3, 1 |
| | | 3, 2, 1 |

$\left.\begin{matrix} 1,2,3 \\ 2,1,3 \\ 1,3,2 \\ 3,1,2 \\ 2,3,1 \end{matrix}\right\} 6$

# subsets with 3 elmts $= \dfrac{10 \cdot \cancel{9}^{3} \cdot \cancel{8}^{4}}{\cancel{6}} = 120$ .

(binomial theorem)