Evaluate $\int_0^1 \frac{x-4}{x^2-5x+6}\,dx$

* All the conditions for Fundamental theorem of calculus are met.

since, $m<n$, the partial fractions should be obtained.

At first, factorize the denominator,

$$* \quad x^2-5x+6 = (x-3)(x-2)$$

quadratic formula can be used

→ Proceed to partial fractions

$$\frac{x-4}{x^2-5x+6} = \frac{A}{x-3} + \frac{B}{x-2}$$

$$\Leftrightarrow \quad x-4 = A(x-2) + B(x-3)$$

$$\Leftrightarrow \quad x-4 = Ax-2A + Bx-3B$$

$$\Leftrightarrow \quad x-4 = x(A+B) - 2A - 3B$$

comparing coefficients on left and right side

$$\begin{cases} A+B=1 \\ -2A-3B=-4 \end{cases} \Leftrightarrow \begin{cases} A=1-B \\ -2+2B-3B=-4 \end{cases}$$

$$\Leftrightarrow \begin{cases} A = 1-B \\ B = 2 \end{cases}$$

$$= \begin{cases} A=-1 \\ B=2 \end{cases} \quad //$$

So,

$$\frac{x-4}{x^2-5x+6} = \frac{-1}{x-3} + \frac{2}{x-2}$$

Now,

$$\int_0^1 \frac{x-4}{x^2-5x+6}\,dx = \int_0^1 \frac{-1}{x-3} + \frac{2}{x-2}\,dx$$

$$= \int_0^1 -\frac{1}{x-3}\,dx + 2\int_0^1 \frac{1}{x-2}\,dx$$

$$= \Big[-\ln|x-3|\Big]_0^1 + \Big[2\ln|x-2|\Big]_0^1$$

$$= (-\ln|-2| + \ln|-3|) + 2\ln|-1| - 2\ln|-2|$$

$$= -\ln(2) + \ln(3) + \overset{0}{2\ln(1)} - 2\ln(2)$$

$$= \ln(3) - \ln(2) - 2\ln(2)$$

$$= \ln(3) - 3\ln(2)$$

$$= \ln(3) - \ln(2^3)$$

$$= \ln(3) - \ln(8)$$

$$= \ln\left(\frac{3}{8}\right)$$ //