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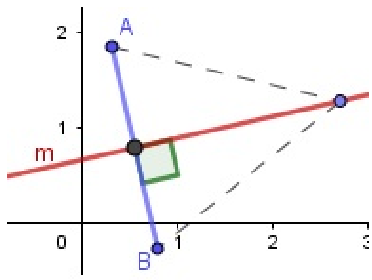
# Mediatrix line and mediating plane

## Mediatrix line of a line segment in $\mathbb{R}^2$

Given two points $A, B \in \mathbb{R}^2$, the locus of points that are equidistant from $A$ and $B$ is a line called mediatrix (or the perpendicular bissector) of $[AB]$.




**Note:** The mediatrix $m$ of $[AB]$ checks the following:

- is orthogonal to $[AB]$;
- contains the midpoint of $[AB]$.

**Example:** Considere the points $A = (-2, 1)$ and $B = (-1, 0)$. The set of the points $P = (x, y)$ equidistant from $A$ and $B$, are such that:

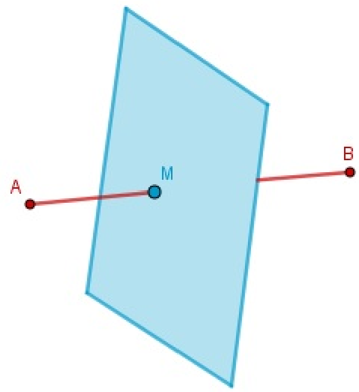
$$\overline{AP} = \overline{BP} \Leftrightarrow \sqrt{(x+2)^2 + (y-1)^2} = \sqrt{(x+1)^2 + (y)^2} \Leftrightarrow 4x + 4 - 2y + 1 = 2x + 1 \Leftrightarrow x - y + 2 = 0.$$

Thus we have the equation of the mediatrix $m : x - y + 2 = 0$, whose director vector $\vec{v} = (1, 1)$ is perpendicular to $\overrightarrow{AB} = B - A = (1, -1)$. De facto, $\overrightarrow{AB} \cdot \vec{v} = 0$.

Notice also that $M = (\frac{-1-2}{2}, \frac{1}{2}) \in m$, because $-\frac{3}{2} - \frac{1}{2} + 2 = 0$.

## Mediating plane of a line segment in $\mathbb{R}^3$

Given two points $A, B \in \mathbb{R}^3$, the locus of points that are equidistant from $A$ and $B$ is a plane, called mediating plane (or the perpendicular bissector) of $[AB]$.




**Note:** The mediating plane $m$ of $[AB]$ checks the following:

- is orthogonal to $[AB]$;
- contains the midpoint of $[AB]$.

**Example:** Considere the points $A = (2, 3, 1)$ and $B = (-1, 1, 0)$ of $\mathbb{R}^3$.
The set of the points $P = (x, y, z)$ equidistant from $A$ and $B$, are such that $\overline{AP} = \overline{BP} \Leftrightarrow$

$$\sqrt{(x-2)^2 + (y-3)^2 + (z-1)^2} = \sqrt{(x+1)^2 + (y-1)^2 + z^2} \Leftrightarrow -4x+4-6y+9-2z+1 = 2x+1-2y+1.$$

Thus, we have the equation of the mediating plane $\pi : -6x - 4y - 2z + 12 = 0$, is orthogonal to the vector $\vec{v} = (-6, -4, -2)$, which is collinear with $\overrightarrow{AB} = B - A = (-3, -2, -1)$.