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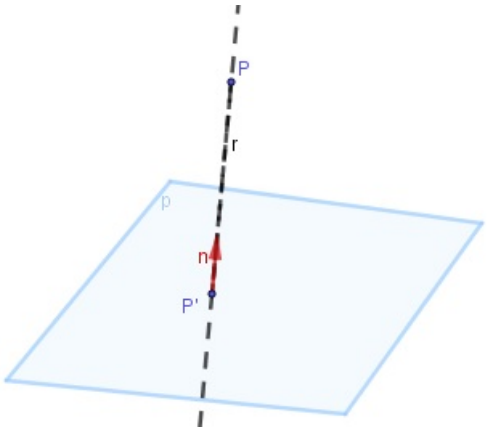
Distance from a point to a plane, from a line to a parallel plane or between two parallel planes

Distance from a point to a plane:

We can determine the distance from point P to plane p, by performing:

- Calculate the line r that contains the point P and is normal to the plane p;
- Calculate $P' = r \cap p$;
- Determine the distance from P to P'.



Example: To calculate the distance from $P = (1, 2, -1)$ to the plane $p : x - y + z = 0$, we can take $n = (2, -2, 1) \perp p$ and the line r that contains P and is normal to the plane p is $r : \dfrac{x-1}{2} = \dfrac{y-2}{-2} = z + 1$.

$$P' = r \cap p = \begin{cases} x - 1 = -y + 2 \\ -y + 2 = z + 1 \\ x - y + z = 1 \end{cases} \Leftrightarrow \begin{cases} x = 2 \\ y = 0 \\ z = 1 \end{cases}$$

Then, $d(P, p) = d(P, P') = \sqrt{(1-2)^2 + (2-0)^2 + (-1-1)^2} = 3$

Distance from a straight line to a parallel plane:

Given a line r parallel to a plane p, the distance d from the line r is the distance from any point p on the line to the plane, that is,

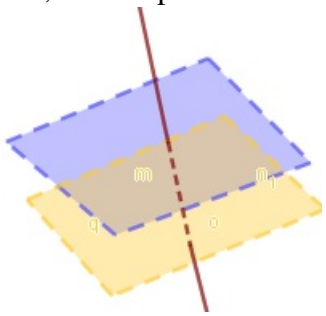
$$d(r, p) = d(P, p)$$

Example: To calculate the distance from $r : \dfrac{x-1}{2} = -y = \dfrac{z+1}{-3}$ to the plane $p : x - y + z = 0$ is the distance from $P = (1, 0, -1) \in r$ to the plane p.

Distance between two parallel planes:

To calculate the distance between two planes α and β parallel to each other, we can perform:

- Calculate the line r that is normal to the planes α and β;
- Calculate $A = r \cap \alpha$;
- Calculate $B = r \cap \beta$;
- Determine the distance from A to B.



Example: To calculate the distance from $p_1 : x - 2y + z + 6 = 0$ to the plane $p_2 : 2x - 4y + 2z + 6 = 0$, we consider:

- $r : x = -\dfrac{y}{2} = z$ that is normal p_1 and p_2;
- $A = r \cap p_1 : \left\{ \begin{array}{r} x = -\dfrac{y}{2} \\ x = z \\ x - 2y + z + 6 = 0 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{r} y = -2x \\ z = x \\ x + 4x + x + 6 = 0 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{r} y = \dfrac{1}{2} \\ z = -1 \\ x = -1 \end{array} \right. ;$
- $B = r \cap p_2 : \left\{ \begin{array}{r} x = -\dfrac{y}{2} \\ x = z \\ 2x - 4y + 2z + 5 = 0 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{r} y = -2x \\ z = x \\ 2x + 8x + 2x + 6 = 0 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{r} y = 1 \\ z = -\dfrac{1}{2} \\ x = -\dfrac{1}{2} \end{array} \right. .$
- Then, $A = (-1, \frac{1}{2}, -1)$, $B = (-\frac{1}{2}, 1, -\frac{1}{2})$ and

$$d(p_1, p_2) = \overline{AB} = \sqrt{(-\frac{1}{2} + 1)^2 + (1 - \frac{1}{2})^2 + (-\frac{1}{2} + 1)^2} = \frac{\sqrt{3}}{2}.$$