

\* Every definite integration problem begins with checking if the conditions for fundamental theorem of calculus are met or not.

Evaluate  $\int_{\sqrt{3}}^2 \frac{\sqrt{4-x^2}}{x} dx$

converting to form  $* R(x, \sqrt{1-f^2})$

So,

$$* I(x) = \int \frac{\sqrt{4-x^2}}{x} dx = \int \frac{\sqrt{\left(\frac{4-x^2}{4}\right) \times 4}}{x} dx$$

$$= 2 \int \frac{\sqrt{1-\left(\frac{x}{2}\right)^2}}{x} dx$$

A.C.I

Performing trigonometric substitution,

$$\frac{x}{2} = \sin(t)$$

$$\Rightarrow x = 2 \sin(t)$$

$$\Rightarrow dx = 2 \cos(t) dt$$

A.C.I

$$= 2 \int \frac{\sqrt{1-\sin^2(t)}}{2 \sin(t)} 2 \cos(t) dt$$

$$= 2 \int \frac{\sqrt{\cos^2(t)}}{\sin(t)} \cos(t) dt$$

$$= 2 \int \frac{\cos(t) \cdot \cos(t)}{\sin(t)} dt$$

$$= 2 \int \frac{\cos^2(t)}{\sin(t)} dt$$

$$= 2 \int \frac{1-\sin^2(t)}{\sin(t)} dt$$

$$= 2 \int \operatorname{cosec}(t) - \sin(t) dt$$

$$= 2 \int \operatorname{cosec}(t) dt - 2 \int \sin(t) dt$$

$$= 2 \ln |\operatorname{cosec}(t) - \cot(t)| + 2 \sin(t) + C$$

converting to function dependent on 'x'

A.C.L.

$$\sin(t) = \frac{x}{2}$$

$$\operatorname{cosec}(t) = \frac{2}{x}$$

$$\cos(t) = \sqrt{1 - \sin^2 t}$$

$$= \sqrt{1 - \frac{x^2}{4}}$$

$$= \frac{\sqrt{4-x^2}}{2}$$

$$\cot(t) = \frac{\cos(t)}{\sin(t)}$$

$$= \frac{\sqrt{4-x^2}}{x}$$

A.C.L.

$$= 2 \ln \left| \frac{2}{x} - \frac{\sqrt{4-x^2}}{x} \right| + \frac{\sqrt{4-x^2}}{2} + C$$

$$= 2 \ln \left| \frac{2 - \sqrt{4-x^2}}{x} \right| + \sqrt{4-x^2} + C$$

Now,

$$\begin{aligned}\int_{\sqrt{3}}^2 \frac{\sqrt{4-x^2}}{x} dx &= \left[ I(x) \right]_{\sqrt{3}}^2 \\&= \left[ 2 \ln \left| \frac{2-\sqrt{4-x^2}}{x} \right| + \sqrt{4-x^2} \right]_{\sqrt{3}}^2 \\&= \left( 2 \ln \left| \frac{2-\sqrt{4-4}}{2} \right| + \sqrt{4-4} \right) \\&\quad - \left( 2 \ln \left| \frac{2-\sqrt{4-3}}{\sqrt{3}} \right| + \sqrt{4-3} \right)\end{aligned}$$

$$= - \left( 2 \ln \left( \frac{1}{\sqrt{3}} \right) + 1 \right)$$

$$= -2 \ln \left( \frac{1}{\sqrt{3}} \right) - 1$$

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$$= -2 \ln(3)^{-1/2} - 1$$

$$= -2 \times \left( -\frac{1}{2} \right) \ln(3) - 1$$

$$= \ln(3) - 1$$

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