Evaluate $\int_0^1 \frac{x-2}{(x+1)(x+2)}\,dx$

* All the conditions for Fundamental theorem of calculus are met.

since, $m<n$, the partial fractions should be obtained.

For $I(x) = \int \frac{x-2}{(x+1)(x+2)}\,dx$, the partial fractions are,

A.C.I

$$\frac{x-2}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$\Leftrightarrow \quad x-2 = A(x+2) + B(x+1)$$

$$\Leftrightarrow \quad x-2 = Ax+2A + Bx + B$$

$$\Leftrightarrow \quad x-2 = x(A+B) + 2A + B$$

comparing coefficients of left and right hand side.

$$A + B = 1 \quad \ldots \text{(I)}$$

$$2A + B = -2 \quad \ldots \text{(II)}$$

solving eqn (I) and (II).

we get,

$$A = -3$$

$$B = 4$$

$$I(x) \overset{A.C.1}{=} \int \frac{-3}{x+1} + \frac{4}{x+2}\, dx$$

$$= -3\ln|x+1| + 4\ln|x+2| + C$$

Now,

$$\int_0^1 \frac{x-2}{(x+1)(x+2)}\, dx = \Big[I(x)\Big]_0^1$$

$$= \Big[-3\ln|x+1| + 4\ln|x+2|\Big]_0^1$$

$$= -3\ln|1+1| + 4\ln|1+2| - \left(-3\overset{0}{\ln|1|} + 4\ln|2|\right)$$

$$= -3\ln(2) + 4\ln(3) - 4\ln(2)$$

$$= 4\ln(3) - 7\ln(2)$$