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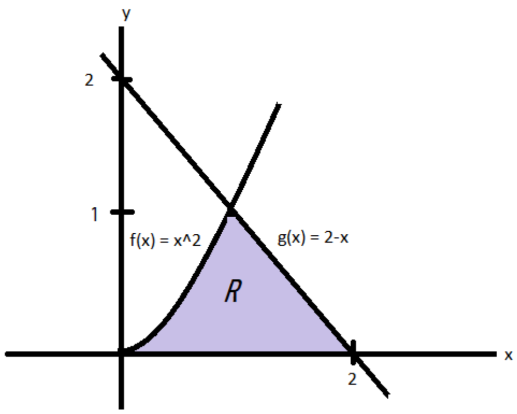
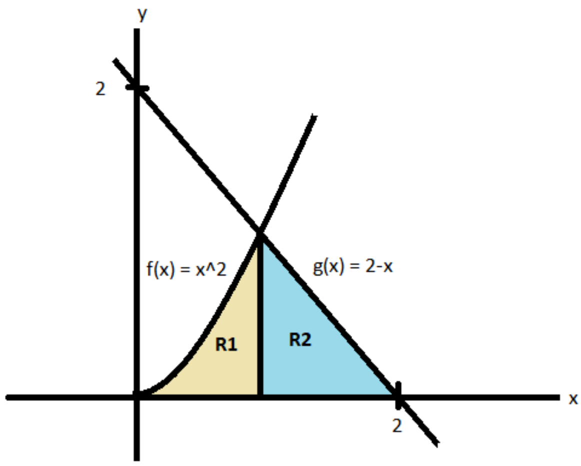


$y = x^2$ is a parabola opening upwards with vertex (0,0).

Remember that, Area bounded by the curves is given by,

Area $= \displaystyle\int_a^b f(x) - g(x)\, dx$, where $f(x)$ **is the upper curve** and $g(x)$ **is the lower curve** and $x \in [a, b]$.

In this case, there are two upper functions and one lower function. Therefore, it is necessary to split the region R into two regions ($R1$ and $R2$) such that there's only one upper function and only one lower function.



For $R1$, the upper function is $f(x) = x^2$ and lower function is $h(x) = 0$ and $x \in [0, 1]$.

$$
\begin{aligned}
A_1 &= \int_a^b f(x) - h(x)\,dx \\
&= \int_0^1 x^2\,dx \\
&= \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3} \text{ square units}
\end{aligned}
$$

For $R2$, the upper function is $g(x) = 2 - x$ and lower function is $h(x) = 0$ and $x \in [1, 2]$.

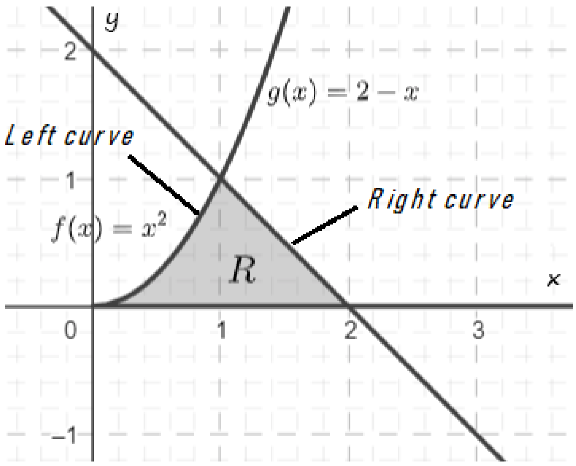
$$
\begin{aligned}
A_2 &= \int_a^b g(x) - h(x)\,dx \\
&= \int_1^2 2 - x\,dx \\
&= \left[2x - \frac{x^2}{2}\right]_1^2 = \frac{1}{2} \text{ square units}
\end{aligned}
$$

$$
\begin{aligned}
\text{The total area enclosed by region } R &= A_1 + A_2 \\
&= \frac{1}{3} + \frac{1}{2} \\
&= \frac{5}{6} \text{ square units}
\end{aligned}
$$

Alternate method (Integrating with respect to y)

When we take dx, we found that we need to divide the region R into 2 sub-regions because we had two different upper functions. However, when we take dy , i.e. treating x as a function of y, it fixes the problem. To understand further, click this link.

When integrating **with respect to y**, Area $= \displaystyle\int_a^b f(y) - g(y)\, dy$, where $f(y)$ **is the curve on the right side** and $g(x)$ **is the curve on the left side** and $y \in [a, b]$.



Now, rewrite the functions in function on y. We have $g(y) = 2 - y$ and $f(y) = \sqrt{y}$
(**Note that,** we don't need $f(y) = -\sqrt{y}$ here.)

In this case, the function on the right is $g(y) = 2 - y$ and function of the left is $f(y) = \sqrt{y}$ and $y \in [0, 1]$.

$$\begin{aligned}
\text{Area} &= \int_0^1 \text{right - left } dy \\
&= \int_0^1 2 - y - \sqrt{y}\, dx \\
&= \left[2y - \frac{y^2}{2} - \frac{2}{3}y^{\frac{3}{2}}\right]_0^1 \quad = \quad \frac{5}{6} \text{ square units}
\end{aligned}$$