



Example 3 (Conditional probability, intersection, tree diagram)

A cautious individual usually carries his umbrella around, approximately in 80% of his daily errands. Curiously, when he carries his umbrella, he estimates that the probability of raining is about 50%, whereas when he forgets his umbrella at home, it rains in approximately 60% of those occasions.

It started pouring a while ago. Calculate the probability the individual left the house carrying his umbrella.

Answer:

Consider the following events:

U: the individual carries his umbrella; R: it is raining

The situation mentioned above can be represented with the help of the tree diagram shown.

We need to calculate the probability the individual brought the umbrella, given the fact we already

know it is raining, $P(U|R) = \frac{P(U \cap R)}{P(R)}$

As $P(U \cap R) = P(R \cap U)$ and $P(R \cap U) = P(R|U) \times P(U) = 50\% \times 80\% = 40\%$

$P(R) = P(R|U) \times P(U) + P(R|\bar{U}) \times P(\bar{U}) = 50\% \times 80\% + 60\% \times 20\% = 52\%$

So, $P(U|R) = \frac{P(U \cap R)}{P(R)} = \frac{40\%}{52\%} = 76.92\%$

