LINEAR PROGRAMMING (LP): Formulation example

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Example of LP model:

A company produces two products (A and B), using the resources R1 and R2. The time required for the production of each product is shown in the table (in hours):

	R1	R2
Product A	2	4
Product B	4	4

The capacity (monthly availability) of the resources R1 and R2 is 720 hours and 880 hours, respectively. It is known that the maximum sales of the product A are limited to 160 monthly units.

The profit obtained from the sales of these products is 6€ per unit of product A and 3€ per unit of product B.

Determine the monthly production plan in order to maximize the profit obtained.

Formulation of LP model

1st: Choice the decision variables: The company produces products A and B. How many units of these products the company should produce per month to maximize the profit?

- ▶ Number of units of product A to be produced per month: x_1
- ▶ Number of units of product B to be produced per month: x_2

2nd: Nonnegativity constraints: $x_1, x_2 \geq 0$.

2nd: Functional constraints:

- Resource 1: $2x_1 + 4x_2 \leq 720$, where:
  - $2x_1$ is the number of hours used to produce the product A in the resource R1;
  - $4x_2$ is the number of hours used to produce the product B in the resource R1;
  - 720 is the number of hours available in the resource R1 per month.
- Resource 2: $4x_1 + 4x_2 \leq 880$, where:
  - $4x_1$ is the number of hours used to produce the product A in the resource R2;
  - $4x_2$ is the number of hours used to produce the product B in the resource R2;
  - 880 is the number of hours available in the resource R2 per month.
- Market: $x_1 \leq 160$ represents the number of units of the product A that are absorbed by the market.

3rd: Objective function=Z: It is necessary to know the company's monthly profit to solve a maximization problem.

- $Z = 6x_1 + 3x_2$, where:
  - $6x_1$ is the profit (in €) obtained from the sale of x_1 units of product A;
  - $3x_2$ is the profit (in €) obtained from the sale of x_2 units of product B;

The standard formulation of this LP model is the following:

$$\text{Max } Z = 6x_1 + 3x_2$$
$$\text{subject to}$$
$$2x_1 + 4x_2 \leq 720$$
$$4x_1 + 4x_2 \leq 880$$
$$x_1 \leq 160$$
$$x_1, x_2 \geq 0.$$

Reference

Hillier F. S., & Lieberman G.R. (2010). Introduction to operations research (9th ed.). New York: McGraw-Hill.