MathE-project

First Order Homogeneous Differential Equation

Homogeneous Equations. First we recall some properties of a homogeneous function.

Definition 1.5 *A function $f(x, y)$ is called homogeneous of degree α if we have*

$$f(tx, ty) = t^{\alpha} f(x, y)$$

An important case is $\alpha = 0$ because the above relation is of the form $f(tx, ty) = f(x, y)$. This property is equivalent with

$$f(x, y) = f(1, \frac{y}{x}) = f(\frac{x}{y}, 1)$$

Definition 1.6 *A differential equation of the first order is called homogeneous zero degree if it is of the form*

$$y' = f(x, y), \qquad (1)$$

where f is a function of zero degree.

To integrate the equation (1) it is interesting to see that it can allways be transformed into separable equation by change of the dependent variable. More exactly we put y(x) = xu(x) where u(x) is a unknown function. Thus we have

$$y = xu \quad \Rightarrow \quad y' = xu' + u.$$

Replace y and y' in th eq. (1.12) we obtain

$$xu' + u = f(1, u) = g(u), \quad \Rightarrow \quad \frac{dx}{x} = \frac{du}{g(u) - u}, \quad g(u) \neq u.$$

The last equation is a separable equation and its solution it can be deduce.

Find the general solution of the equations

$$1. \quad x^2 dy = (x^2 + xy + y^2) dy \quad 2. \quad 2xydy = (x^2 + 3y^2) dx$$

$$3. \quad (2x - y) dy + (3x - 4y) dx = 0, \quad 4. \quad (2x + y) dy + (4x - 3y) dx = 0,$$

5. $(x+3y)dx-(x-y)dy=0$, 6.$(x^2+3xy+y^2)dx-(x^2-xy+2y^2)dy=0$,

$$7.\ (x^2-3y^2)dx=2xydy \quad 8.\ 2xydy=(3y^2-x^2)dx$$

$$9.\ xy'-y=\sqrt{x^2+y^2} \quad 10)\ xy'-y=\frac{x}{arctg\frac{y}{x}},$$

$$11)\ ydx+(2\sqrt{xy}-x)dx=0.$$

Reductible equations to the homogeneous equations.

A differentiable equation of the form

$$y'=f(\frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}) \qquad (2)$$

is called reductible equation to homogeneous equations.

For to find the general solution we can follow the next steps:

a) solve the next system

$$\begin{cases} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \end{cases} \qquad (3)$$

Suppose that x_0, y_0 is a unique solution of the system (3) Then if we put $y(x)-y_0=(x-x_0)u(x)$ where u(x) is a unknown function, the equation (2) come a separable equation.

If the system (3) is incompatible then $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$ the by substitution $u(x)=a_1x+b_1y+c_1$, the equation (2) become a separable equation.

Solve the next differentiable homogeneous equations

1) $2x+2y+1+(x+2y-1)y'=0$, 2) $2x+2y-1+(x-2y+3)y'=0$,

3) $(2x-y+5)dx+(2x-y+4)dy=0$, 4) $(x+y-1)dx+(-x+y+1)dy=0$,

5) $(2x+2y+5)dx+(2x+2y-6)dy=0$ 6) $(3x-4y+10)dx+(x+y+2)dy=0$.