Manipulation of Algebraic Expressions

Indices

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Roz Whelan

Indices

- For the number 2^3, 2 is the base and 3 is the index.
- The index tells us how many times the number is multiplied by itself.
- In the above case 2^3 means 2 is multiplied by itself 3 times.
- 2 x 2 x 2 =8 i.e. $2^3 = 8$
- Every number has an index, for example, $5 = 5^1$; $16 = 16^1$
- There are Laws of Indices which can be applied but only where the bases are the same.
- If index is an integer it is called a power.

Indices

Law 1: When **multiplying numbers** with the same base **add the indices:**

$$\mathbf{a^m \times a^n = a^{m+n}}$$

$$3^2 \times 3^4 = 3^{2+4} = 3^6$$

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Law 2: When **dividing numbers** with the same base **subtract the indices.**

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{3^5}{3^2} = 3^{5-2} = 3^3$$

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Law 3: When a number which is raised to a power is raised to a **further power**, the indices are **multiplied**. Thus:

$$(a^m)^n = a^{mxn}$$

$$(3^5)^2 = 3^{5x2} = 3^{10}$$

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Law 4: When a number has an **index of 0** its **value is 1** thus:

$$\mathbf{a^0 = 1}$$

$$3^0 = 1$$

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Law 5: A number raised to a **negative power** is the **reciprocal** of that number raised to a positive power. Thus:

$$\mathbf{a^{-n} = \frac{1}{a^n}}$$

$$3^{-4} = \frac{1}{3^4}$$

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Law 6: When a number is raised to a fractional power the **denominator of the fraction is the root of the number and the numerator is the power.** Thus:

$$\mathbf{a}^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$4^{\frac{2}{3}} = \sqrt[3]{4^2} = 2.52$$

Indices

Often used applications of Laws of Indices:

$$\sqrt{x} = x^{\frac{1}{2}}$$

Also $\frac{1}{\sqrt{x}} = x^{\frac{-1}{2}}$