Find

$$\int \ln(\sqrt{x})\,dx =$$

$$= \int \underbrace{\ln(t)}_{g} \cdot \underbrace{2t}_{f}\, dt$$

$$\overset{\text{by parts}}{=} t^2 \cdot \ln(t) - \int t^2 \cdot \frac{1}{t}\, dt$$

$$= t^2 \cdot \ln(t) - \int t\, dt$$

$$= t^2 \cdot \ln(t) - \frac{t^2}{2} + C$$

$$= x \cdot \ln(\sqrt{x}) - \frac{x}{2} + C$$

substitution:

$$\boxed{x^{1/2} = t}$$

$$\Rightarrow \frac{1}{2} x^{-1/2}\, dx = dt$$

$$\Leftrightarrow \frac{1}{2x^{1/2}}\, dx = dt$$

$$\Leftrightarrow \boxed{dx = 2t\, dt}$$

- $f(t) = 2t \Rightarrow \int 2t\, dt = t^2 + C$
- $g(t) = \ln(t) \Rightarrow g'(t) = \frac{1}{t}$