

Evaluate $\int_1^e \frac{\ln^3(x)}{x} dx$

At first, checking if the necessary conditions to use Fundamental theorem of calculus are fulfilled or not.
Conditions:

- $I = [1, e]$ is a closed interval.

- $f(x) = \frac{\ln^3(x)}{x}$ is continuous on I .

- $F(x) = \int f(x) dx = \int \frac{\ln^3(x)}{x} dx$

A.C.I

As we know,

$$\int f' f^n dx = \frac{f^{n+1}}{n+1} + c$$

In this case,

$$f = \ln(x)$$

$$f' = \frac{1}{x}$$

$$\begin{aligned} &\stackrel{\text{A.C.I}}{=} \int \underbrace{\frac{1}{x}}_{f'} \cdot \underbrace{\ln^3(x)}_{f^n} dz \\ &= \frac{\ln^{3+1}(x)}{3+1} + c \\ &= \frac{\ln^4(x)}{4} + c \end{aligned}$$

Now, using Fundamental theorem of calculus

$$\begin{aligned} \int_1^e \frac{\ln^3(x)}{x} dx &= \left[\frac{\ln^4(x)}{4} \right]_1^e \\ &= \frac{\ln^4(e)}{4} - \cancel{\frac{\ln^4(1)}{4}}^0 \\ &= \frac{1}{4} \end{aligned}$$