Evaluate $\int_0^3 \frac{1}{(x+1)(x-2)}\, dx$

* All the conditions for Fundamental theorem of calculus are met.

Since, $m < n$, the partial fractions should be obtained.

For $I(x) = \int \frac{1}{(x+1)(x-2)}\, dx$, the partial fractions are,

A.C.I

$$\frac{1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$$

$$\Leftrightarrow \quad 1 = A(x-2) + B(x+1)$$

$$\Leftrightarrow \quad 1 = Ax - 2A + Bx + B$$

$$\Leftrightarrow \quad 1 = x(A+B) - 2A + B$$

$$\Leftrightarrow \quad 0 \cdot x + 1 = x(A+B) - 2A + B$$

comparing coefficients of left and right hand side.

$$A + B = 0 \quad \dots (I)$$

$$-2A + B = 1 \quad \dots (II)$$

Solving eqn (I) and (II)

we get,

$$A = \frac{-1}{3}$$

$$B = \frac{1}{3}$$

$$I(x) \overset{A.C.1}{=} \int \frac{-1}{3(x+1)} + \frac{1}{3(x-2)}\,dx$$

$$= -\frac{1}{3}\int \frac{1}{x+1}\,dx + \frac{1}{3}\int \frac{1}{x-2}\,dx$$

$$= -\frac{1}{3}\ln|x+1| + \frac{1}{3}\ln|x-2| + C$$

Now,

$$\int_0^3 \frac{1}{(x+1)(x-2)}\,dx = \Big[I(x)\Big]_0^3$$

$$= \left[-\frac{1}{3}\ln|x+1| + \frac{1}{3}\ln|x-2|\right]_0^3$$

$$= \left(-\frac{1}{3}\ln|4| + \frac{1}{3}\ln|1|\right)$$

$$- \left(-\frac{1}{3}\ln|1| + \frac{1}{3}\ln|-2|\right)$$

$$= -\frac{1}{3}\ln(4) + \frac{1}{3}\overset{0}{\cancel{\ln(1)}}$$

$$+ \frac{1}{3}\overset{0}{\cancel{\ln(1)}} - \frac{1}{3}\ln(2)$$

$$= -\frac{1}{3}\left(\ln(4) + \ln(2)\right)$$

$$= -\frac{1}{3}\ln(4\times 2)$$

$$= -\frac{1}{3}\ln(8)$$

$$= -\frac{1}{3}\ln(2^3)$$

$$= -\frac{3}{3}\ln(2)$$

$$= -\ln(2)$$ //