Integration by Parts

Let consider the product rule for derivatives:

$$
\begin{aligned}
& (F \cdot g)' = F' \cdot g + F \cdot g' \\
\Leftrightarrow \quad & \int (F \cdot g)' \, dx = \int F' \cdot g + F \cdot g' \, dx \\
\Leftrightarrow \quad & F \cdot g = \int F' \cdot g \, dx + \int F \cdot g' \, dx \\
\Leftrightarrow \quad & F \cdot g = \int f \cdot g \, dx + \int F \cdot g' \, dx, \quad \text{if } F(x) = \int f \, dx \\
\Leftrightarrow \quad & \int f(x) g(x) \, dx = F(x) g(x) - \int F(x) g'(x) \, dx \quad \text{if } F(x) = \int f \, dx
\end{aligned}
$$

Then the formula to integrate by parts is

Integration by Parts

$$\int f(x)g(x)\,\mathrm{d}x = F(x)g(x) - \int F(x)g'(x)\,\mathrm{d}x,$$

such that, $F(x) = \int f(x)\,\mathrm{d}x$

Remarks:

- Useful to integrate products which involve:
  - polynomial and exponential functions;
  - trigonometric and exponential functions or trigonometric and polynomial functions;
  - inverse trigonometric functions;
  - logarithmic functions;.
- Sometimes it is necessary to apply the method of integration by parts multiple times before a result is obtained.

Integration by Parts

Examples

- $\displaystyle\int \underbrace{x}_{g}\underbrace{(x-1)^9}_{f}\,\mathrm{d}x$
- $\displaystyle\int \underbrace{\mathrm{e}^x}_{f}\underbrace{(x+1)}_{g}\,\mathrm{d}x$
- $\displaystyle\int \ln(x)\,\mathrm{d}x = \int \underbrace{1}_{f}\cdot\underbrace{\ln(x)}_{g}\,\mathrm{d}x$
- $\displaystyle\int \operatorname{arctg}(x)\,\mathrm{d}x = \int \underbrace{1}_{f}\cdot\underbrace{\operatorname{arctg}(x)}_{g}\,\mathrm{d}x$
- $\displaystyle\int \mathrm{e}^x\cdot\cos(x)\,\mathrm{d}x$