Find $\int \sqrt{x^2+2x}\,dx = \int \sqrt{\underbrace{x^2+2x+1}_{(x+1)^2}-1}\,dx$

$$= \int \sqrt{(x+1)^2-1}\,dx$$

trig subst.

$\boxed{x+1 = \sec(t)}$

$\Rightarrow \boxed{dx = \sec(t)\cdot\tan(t)\,dt}$

$$= \int \sqrt{\sec^2(t)-1}\cdot\sec(t)\tan(t)\,dt$$

$$= \int \tan(t)\cdot\sec(t)\cdot\tan(t)\,dt$$

$$= \int \tan^2(t)\cdot\sec(t)\,dt$$

$$= \int \left(\sec^2(t)-1\right)\sec(t)\,dt$$

$$= \int \sec^3(t) - \sec(t)\,dt$$

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$$= \underbrace{\int \sec^{3}(t)\,dt} - \underbrace{\int \sec(t)\,dt}_{\text{direct}}$$

we know:

$n=3$

$$\int \sec^{n}(t)\,dt = \frac{\tan(t)\,\sec^{n-2}(t)}{n-1} + \frac{n-2}{n-1}\cdot\int \sec^{n-2}(t)\,dt, \quad n \neq 1.$$

$$= \frac{\tan(t)\cdot\sec(t)}{2} + \frac{1}{2}\cdot\int \sec(t)\,dt - \int \sec(t)\,dt$$

$$= \frac{\tan(t)\cdot\sec(t)}{2} - \frac{1}{2}\cdot\ln|\sec(t)+\tan(t)| + C = (*)$$

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But, $x+1 = \sec(t)$

and we need to find

$\tan(t)$!

we know,

$1+\tan^2(t) = \sec^2(t)$

$\Leftrightarrow \tan^2(t) = \sec^2(t) - 1$

$\Leftrightarrow \tan(t) = \sqrt{\sec^2(t) - 1}$

$\Leftrightarrow \tan(t) = \sqrt{(x+1)^2 - 1}$

$\Leftrightarrow \tan(t) = \sqrt{x^2+2x}$

Then,

$$\circledast = \frac{\sqrt{x^2+2x} \cdot (x+1)}{2} - \frac{1}{2}\ln\left|x+1+\sqrt{x^2+2x}\right| + C$$

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