MathE Training Program

No Author Given

No Institute Given

1 E-Learning Tools and Strategies Group

This section will look at publications and resources related to E-learning of Mathematics.

1.1 E-learning Research

This section will review the literature - summarising scientific publications related to e-learning tools in mathematics.

The term open learn, distance learn appeared at the beginning of this century and was considered as a significant trend by UNESCO [37].

This term and hereafter rephrased as e-learning, was used to describe the digital transformation of education systems in all levels [39] and was firstly defined as "the interaction in any course that is delivered to students who are not physically present in the same room within themselves or between students and instructor" Also, it was used to describe "the use of electronic technology to deliver Education and training applications, monitor learners' performance and report learners progress".

A systematic review over e-learning in general or, particularly, the special case of Mathematics can be seen in [3, 6, 29, 31, 39]. In a recent work [6], the authors identified five sub-areas of research and important trends of development: mobile technologies, massive open online courses (MOOCs), digital libraries and designing learning objects, collaborative learning using digital technology, and teacher training using blended learning.

The research on that topic is nowadays considerable and an effort is done taking into account the COVID-19 pandemic. With this worldwide problem many institutions, all over the world, used online platforms and online tools to deliver knowledge to their students. In this process, the engagement of all the participants is really important. In this way, there are some works trying to measure the engagement and the performance of the students when using online tools [13], for instance. In [36] the authors analysed the challenges of blended e-Learning tools in Mathematics (higher education level). Another recent book [31] is also devoted to e-learning in mathematics in higher education. They organised the book into 4 main section:the first one related to the incorporation of new technologies into mathematics classrooms through the construction or use of digital teaching and learning platforms; the second section presents a wide range of perspectives on the study and implementation of different tutoring systems and/or

computer assisted math instruction; four new innovations in mathematics learning and/or mathematics teacher education that involve the development of novel interfaces' for communicating mathematical ideas and analyzing student thinking and student work are presented in the third section and, at last, the fourth section presents the latest work on the construction and implementation of new MOOCs and rich media platforms developed to carry out specialized mathematics teacher education.

Moreover, the scientific works found, study different aspects of this topic. For instances, some works deepen the platforms used between students and teachers to communicate, like Zoom, Teams, WhatsApp, mobile communications, among others [6].

Other authors explore the media, videos and all the material prepared by teachers to their students as well as the MOOC's that exist in the educational institutions.

It is also possible to find work related to the design and conception of learning educational systems. For example, in [6] (and the references therein) the authors reported developments related to mobile technologies, MOOCs, digital libraries and designing learning objects, collaborative learning using digital technology, and teacher training using blended learning.

The generalised access to the internet by "almost" all the people allows education to everyone, and in some cases, at their own pace. This access also allows collaborative work (learning and teaching) between teachers, between teachers and students, and between students supported by digital libraries and management systems learning (MSL).

Finally it is also possible to find research related to the tools used to teach, learn and assess some subject, in particular, mathematics .

To show how many research it has been doing along time, some experiments are here presented; if a search is performed in the bibliographic database SCOPUS with the words "higher education" and "e-learning tools" it is retrieved 3053 documents, being almost 2000 from conferences. If the word "Mathematics" is add to the search, 119 documents appear as the result of this new search. On the other side, if the keywords "higher education" are removed from the previous search it is obtained 538 documents, meaning that the use of e-learning tools in mathematics is more developed for primary and secondory education than for higher education. Nevertheless, the 119 documents found, related to elearning tools + mathematics + higher education show that there are researchers devoted to this field and this research is growing.

The e-learning tools, as previously said, are closely related to the methodologies to teach, to learn and to assess/evaluate mathematics using online environments, but these topics will be deepen in the next sections.

1.2 E-learning Platforms

This section will provide information on platforms and tools, for E-learning in general and for mathematics specifically. It will summarise some of the main platforms for teaching, learning and collaboration. E-learning platforms are being

referred to by many terms in education, Virtual Learning Environments (VLEs), Learning Management Systems (LMS) or Collaboration suites. Whichever term is applied they all have a number of attributes in common.

- They exist in an electronic form
- They support document exchanges
- They support two way communication either text base, video or both.

Table 1 includes some of the main platforms in use and each will be discussed from the perspective of their application to teaching and more specifically in the higher education environment and the teaching of mathematics.

Table 1: E-learning Platforms.

Platform	Description
Blackboard	is a virtual learning environment that provides a medium for the exchange of teaching and learning materials. This included learning activities, assessments and grade feedback
Google Classroom	Google Classroom sits between you and Google Drive and provides a teacher/student-friendly way of managing classroom documents.
MS Teams	This is an all-in-one collaboration suite that supports online chat, video conferencing and video calls. This platform facilitates file exchanges and material uploads.
Moodle	This is a free open source learning management system. It is used for blended learning, distance education, flipped classroom and other e-learning projects in schools, universities and workplaces.
Zoom	This is a cloud platform for video and audio conferencing, chat, and webinars across mobile, desktop, and room systems. Zoom has both free and subscription options.

Blackboard With the dual developments of technology enhancements and its wider availability and the challenges posed by continuing education during a global pandemic VLE's have needed to be embraced by educators and students alike. Alkhaldi and Abualkishik's paper on The mobile blackboard system in higher education identified a number of benefits and challenges facing students when using VLE's. The majority of the students surveyed, 73%, found the using Blackboard helped them to learn. Additionally 66% of the students either agree or strongly agree that the Blackboard system has enhanced their effectiveness in their educational process. When looking at productivity the study found that 56% of the students believe that the Blackboard system has increased their productivity. Despite these strong benefits several challenges were faced by students. Many students were using the Blackboard mobile app and hence many

of the challenges were centred around screen size, cost of mobile data and Wi-Fi connections and not directly to Blackboards functionality [2].

When looking at the use of Blackboard specifically around problem based learning (PBL) all of the students surveyed in [1] work cited benefits in using Blackboard. The students found that Blackboard allowed them to take advantage of technology and develop their skills to function in an electronic learning environment. The students also stated that Blackboard facilitated learning with other student by helping them to share ideas and engage in group discussions. Similar challenges were identified as with [2] paper. Poor network connections were cited which resulted in difficulties in updating the package and in downloading content. Students identified a weakness in the system that it did not have a notification service to students.

Google Classroom This platform has many application but we are particularly interested in its application at higher or third level education. Research conducted by [17] showed increased student participation with the use of Google Classroom. This platform also enhanced learning and improved the classroom dynamics.From [17] research it can be seen that 91.7% of students rated Google Classroom in the good or very good categories. As this cohort of students were final year primary teachers education students they were also asked if they would use Google Classroom in their own classrooms and 87.5% confirmed they world if possible.Naturally there were some issues around usability. The stream or feed of discussions was set that the newest message was first so accessing older information was time consuming. [17] noted that when deciding to use a VLE consideration should be given to 4 key concepts;

- Ease of Access
- Collaboration
- Student Voice/Agency
- Pace

[35], research compared a number of E-learning Platforms including MS Teams, Zoom and Google Meet. MS Teams was determined the most feasible platform. The study assessed 7 platforms and assessed each against 5 criteria and received feedback from 364 Engineering students.

- Features
- Overview
- Pricing
- Reviews
- Integration's

Despite the acknowledged benefits of e-learning concerns were again highlighted. These concerns included:

- Security Concerns: with high numbers attending online there is greater opportunities for cybercriminals to gain access

- Online engagement: Challenges around Proctor mode which requires students to turn on their video which consumes greater amounts of data. In Lecture mode the opportunity arises for misbehaviour by some student causing disruptions.
- Introduction of new features: Line polling, inbuilt note pads and live quizzes were proposed as potential useful additions to some of the platforms.

Moodle Similar to the other E-learning platforms discussed Moodle has a wide range of functions available for both teachers and students to use. One function, in particular, has been used widely for supporting mathematics learning. The quiz function in Moodle has been researcher from a variety of aspects to determine its benefits or challenges in teaching mathematics. [4] research looked at engineering students, some of whom, had a lack of basic knowledge and engagement with topics of foundation mathematics. The use of Moodle quizzes with multiple-choice questions were developed to support the weaker students for exam revision and as a support to their independent learning. [4] research showed more than 90% of the students felt positive about the use of these quizzes in their learning. These quizzes facilitated multiple attempts so students could learn from their mistakes and work on the areas they were weak in. Students also commented that this was an enjoyable way to learn mathematics. [5] work had previously identified the benefits of using Moodle quizzes to support students learning of mathematics. The quizzes were seen as a welcome alternative to pen and paper assessments. [5] found that Moodle quizzes were a reliable tool to inform students of their progress throughout the year. The flexibility of Moodle quizzes allows them to be used as both continuous and formative assessments in teaching mathematics.

Zoom [30] investigated the student experience in 31 universities. The students attitudes and perceptions of their learning and engagement were investigated to determine their opinions on Zoom versus face to face teaching.Due to the rushed nature of many universities going on line in 2020 students found some of the experiences less than satisfactory. Almost 79% of students noted the greater flexibility of learning in the online mode. But 42% of students felt there were more distractions to learning by not being in a classroom environment. Worryingly almost 37% found the quality of interaction was reduced. However it was accepted that the rushed nature of the move and the lack of experience in teaching or learning in this way are challenges that can be addressed. Many of the disadvantages notes did not refer directly to the Zoom application but the the study environment students found themselves in. [33] research compared online first students previous experience of online lecturing methods. 6 applications were compared:

- Google Classroom 12%
- YouTube 17%
- WhatsApp 15%

- Instagram 14%
- Edmodo 8%
- Zoom 31%

The largest prior experience had been with Zoom. The study enphised the importance of developing of distance lecture design. The roles participants play (Lecturer and Students) are vital to the success of the teaching and learning.

1.3 E-learning Tools and Applications

Table 2 lists a range of tools and applications for use in mathematics learning and teaching scenarios. Some of these are intended for teacher or classroom use and others are supports for student use, that is they provide opportunities to augment and scaffold learning. E-learning has been classified as a sub class of Resource-based Learning (RBL). A recent study by Yaniawati *et al* suggests that integration of E-learning and RBL improves the self confidence and critical thinking ability of students in Mathematics [42]. The researchers observed the differences in skill and confidence development in three groups of students: the first taught conventionally, the second taught implementing a scientific methodology and the third taught using the scientific method augmented with RBL. The outcome for both self-confidence and critical thinking skills was best in the third group.

The COVID-19 pandemic and the rapid transfer to virtual teaching and learning for Higher Education Institutions (HEIs) was a test for both teachers and students. The E-learning platforms were of prime importance for continuity of contact and communication. In one sense the pandemic provided an opportunity for deeper engagement with and utilisation of the capabilities of the VLEs like Moodle. Several studies observing the transition from face-to-face to distance learning using VLEs have emerged. Lopes & Soares investigated the impact of the use of interactive E-learning materials for students' engagement in a Mathematics course that was transferred to on-line because of the pandemic [23]. While they drew on previous experience from projects on E-learning, they acknowledge the challenge for educators today: they "must apply digital technology, need to have an open mind-set to innovation, to new proposals and suggestions, must be flexible, creative and adaptable to new challenges". Their study reported student responses to the on-line delivery that combined Zoom, Moodle, interactive videos and quizzes. Eighteen students responded to a survey, largely positive on areas like increased autonomy, interest, problem-solving ability, understanding and reinforcement of previous knowledge. However, while 100% of students rated the model as appropriate, only 61.6 % agreed that it increased their motivation and aptitude.

Table 2: E-learning tools and applications.

Tool	Source	Description
Brilliant	https://brilliant.org/	Offers interactive courses in mathematics.
Can Figure It Geometry	https://www.canfigureit.com/	Highly interactive tool provides an easy-to-use workspace where students can practice proofs while developing their deductive reasoning skills. Features: An interactive canvas to make and prove claims; A continuously updated diagram indicating where students are in the proof; An embedded glossary with visuals and templates demonstrating the structure of theorems; A teacher dashboard that provides performance reports at the student and classroom level.
Desmos	https://www.desmos.com	Maths platforms including graphics processor, graphing calculator, scientific calculator
Differentiation	https://www.derivative-calculator.net/	The Derivative Calculator lets you calculate derivatives of functions online for free. The calculator allows you to check solutions to calculus exercises. It helps you practice by showing you the full working (step by step differentiation).
Doceri	https://doceri.com	A professional iPad interactive whiteboard and screencast recorder with sophisticated tools for hand-drawn graphics and built-in remote desktop control. It allows live problem-solving demonstrations, but these can be shared by projector and also recorded for later viewing.
FluidMath	https://fluidmath.net/	Handwriting-based educational math app designed for teachers and students in middle school and high school, and applicable to pre-algebra through calculus courses. For students, it is designed to help explore and understand concepts in mathematics and science.
GeoGebra	https://www.geogebra.org/	Free digital tools for teaching activities, graphing, geometry, collaborative digital whiteboard and more.
Khan Academy	https://www.khanacademy.org	Completely free personalized learning resource with online courses, videos, and exercises. Students can complete daily reviews and keep track of their progress within the platform's learning dashboard. The math tutorials are categorized by subject and by grade level for easy navigation and utilize specialized content to bring the lessons to life.

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Table 2 – continued from previous page

Tool	Source	Description
LaTeX	https://www.sharelatex.com/	Now available in combination with OverLeaf:The feature that makes LaTeX the right editing tool for scientific documents is the ability to render complex mathematical expressions. LaTeX allows two writing modes for mathematical expressions: the inline mode and the display mode. The first one is used to write formulas that are part of a text. The second one is used to write expressions that are not part of a text or paragraph, and are therefore put on separate lines.
Maple Learn	https://maplesoft.com	Show exactly the level of detail you want in a calculation, by working through problems using a combination of manual steps and computations performed by Maple Learn. Provide illuminating graphs, computations, explanations, parameterisations and interactive explorations all together in a single online document. Easily share documents and applications with a class to explore on their own
MathE	http://mathe.pixel-online.org/	A virtual place to exchange maths teaching and learning experiences between teachers and students. Has question banks for self and final assessments and a multimedia resource library. It has also a community of practice.
Mathway	https://www.mathway.com	Mathway is a math calculator for algebra, graphing, calculus and more. Simply point your camera and snap a photo or type your math homework question for step-by-step answers.
Microsoft Math Solver	https://math.microsoft.com/en	Step by step solutions to mathematical problems. Free application for iOS and Android devices that assists users in solving advanced mathematical equations involving arithmetic, algebra, trigonometry, calculus, statistics, and more.
MIT Open courseware	https://ocw.mit.edu/courses/mathematics	MIT has several short courses freely available for students to complete in their own time. These include various topics in Calculus.

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Tool	Source	Description
MyMathLab	https://www.mathxl.com	Series of online courses that align to publisher Pearson's programs in mathematics and statistics. Powered by MathXL for School, this online program offers students personalized instruction and practice and provides teachers with the tools they need to deliver all or a portion of their course online. MyMathLab for School comes complete with resources to ensure students are successful in their mathematics course.
onlinemath tools	https://onlinemathtools.com	A collection of useful mathematics utilities. All math tools are simple, free and easy to use and work right in ther browser. All utilities work exactly the same way — load variables, get result.
PhET Simulations	https://phet.colorado.edu	The PhET Interactive Simulations project at the University of Colorado Boulder creates free interactive math and science simulations. PhET sims are based on extensive education research and engage students through an intuitive, game-like environment where students learn through exploration and discovery. There is a suite of simulations on maths topics.
photomath	https://photomath.com/en/teachers	Photomath is a mobile application described as a "calculator camera", which uses a phone's camera to recognize mathematical operations and to display the solution directly on the screen. It is available for free on Google Android and iOS.
Symbolab	https://www.symbolab.com	Symbolab provides guided answers and numerous practice opportunities to a wide range of topics including Algebra, Calculus, Functions, Matrices, Vectors, Geometry, Statistics, Physics and Chemistry. Once you learn the topic, it is important to go a few problems step by step to understand what is going on. The more you practice the better you get.
Virtual Nerd	https://www.virtualnerd.com	Repository with many video tutorials that are high-quality and easy to follow; they cover a wide range of math topics.

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Table 2 – continued from previous page

Tool	Source	Description
Wacom	https://www.wacom.com/en-gb/products/pen-tablets	Wacom creative pen displays and pen tablets combined with inking apps make it easy to explain concepts visually, encourage collaboration and express your creativity. Convert handwritten notes to text, stimulate hands-on collaboration and work on a real-time canvas simultaneously with students
Wolfram Alpha Mathworld	https://mathworld.wolfram.com	Includes interactive GIFs and demonstrations, downloadable notebooks, and "capsule summaries" for various maths terms. Students can explore the more than 13,000 entries to strengthen their math foundation and build up their understanding.
YouTube	https://www.youtube.com	YouTube can be a source of some excellent supplementary material on many maths topics. Among recommended channels are "My Secret Math Tutor", "Patrick JMT" and "MathE Project".

2 Methodologies to Teach Mathematics

Although lecturing has long been the most widely used form of teaching at all levels of the educational system, mathematics lectures are challenging for many students to follow. Therefore, after some time, say 15 minutes of lecture, the student's attention typically drops. Therefore, the use of interactive activities could be used to keep students engaged in learning more cognitively and emotionally. In this section, creative and alternative approaches liable to attract higher education students to mathematics classes and engage them into learning, will be described and related to the theoretical framework portrayed in research. The paper [22] presents a systematic literature review that identifies innovative strategies for the improvement of student outcomes in higher education unit courses relying on a strong mathematical content. The studies it covers highlight the importance of understanding how thinking and learning processes work (see the seminal book [27] on this topic) as well as the necessity to update the way mathematics is taught, its communication and the use of technology in classes.

One of such interactive tools for teaching Math and other technical disciplines is games. Knowing the fact that these days students have grown up surrounded by a wide variety of interactive games and equipment it is natural that more and more of these methods and techniques should appear in the teaching process.

Another teaching method including the Mind Mapping (MM) can be useful to explain more clear and in a comprehensive way theoretical notions but also algorithms of solving problems.

2.1 Teaching Mathematics using MM (Mind Mapping)

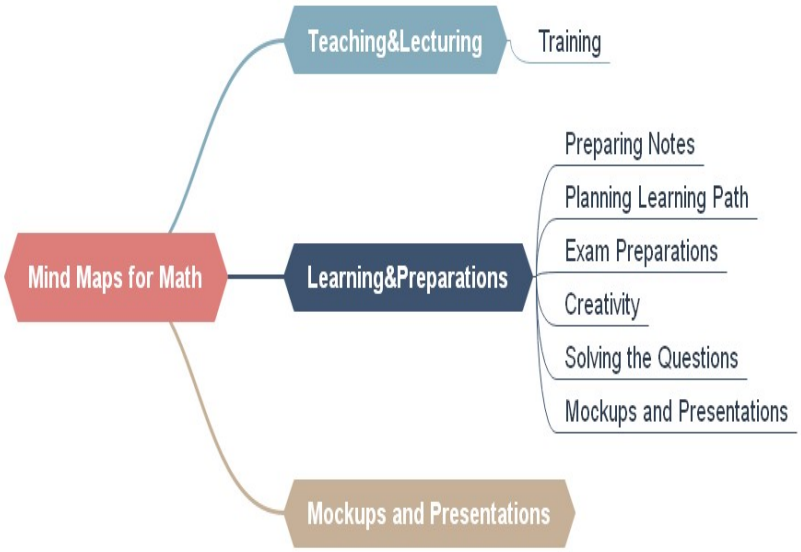
Marcel Roman and Ariadna Pletea
"Gheorghe Asachi" Technical University of Iasi (TUIASI)

1) About MM

Mind mapping is a process which is done via graphical illustrations, typically using a well-labeled diagram. A mind map comprises a box that represents the main topic of the plan or strategy. This box can then be further extended with the help of several small boxes that represent categories or parts, and also with even smaller boxes to denote subcategories, and so on. Also, using different colors and a tree configuration, MM activates both brain hemispheres!

It is easy to learn through the diagrams because they require less brainpower to memorize things as compared to the traditional method of preparing text documentations, notes, etc., and then reading and remembering them. Therefore, the mind map approach for brainstorming is now being adopted by many organizations and educational institutions.

As mentioned earlier, mind maps are pretty helpful in academics as well, and math is not any exception. With mind maps, math can be easy to understand. Below are a couple of ways mind maps can be useful in studying mathematics:

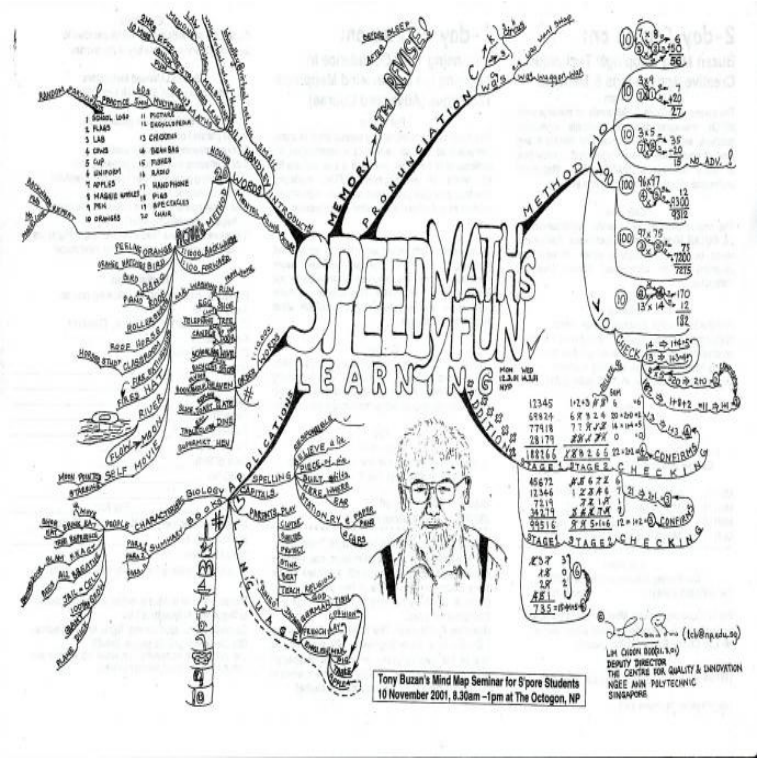


From the trainers' perspective, mind maps offer some flexibility and benefits while teaching math, and the course can a great fun. As it could be tedious to dictate notes to the students and wait for the weakest one to write what you

have dictated, educating them about mind maps and its benefits once helps the scholars take down the notes comparatively faster. As a result, the pace of your training and their learning could be remarkably increased.

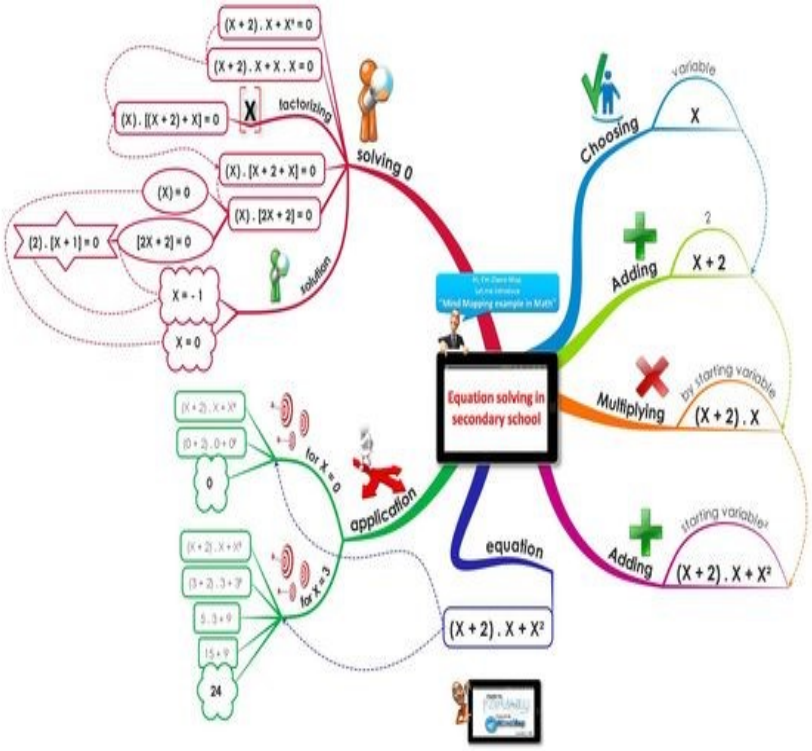
While studying, it could be a bit challenging to listen to the lecturer and jot down the notes word-by-word. Not because doing so is complicated, but because it consumes more time. An easier approach could be to use lines and rectangles and label them accordingly instead. These shapes can be drawn quickly using any efficient mind mapping solution like MindMaster (https://www.mindmaster.io/), and labeling them with the proper names would be less tedious.

Mathematics can be overwhelming at times due to all the alien names and equations the syllabus is populated with. Therefore, sometimes it could be daunting to decide which chapter to study first and how to study it. With mind maps, you can draw a diagrammatic timetable for yourself that would contain a sequential list of the lessons you should learn and practice. If you have used mind maps to prepare notes while studying math, revising the lessons and chapters at the time examinations would be extremely easy and fun. This is something you won't enjoy if you have followed the text notes approach during the class lectures.



Although math equations can be solved using some particular formulas and principles, the way of writing notes still varies from students to students. Similar is the case with mind maps. The approach you take to create your mind map may differ from that drawn by your class fellow. This makes you more creative and helps you diagnose the problems thoroughly. This further enables you to solve the questions quickly and easily. Solving a math question requires several steps, and a complex problem could be solved in multiple phases where, in some cases, each phase needs you to apply a different formula.

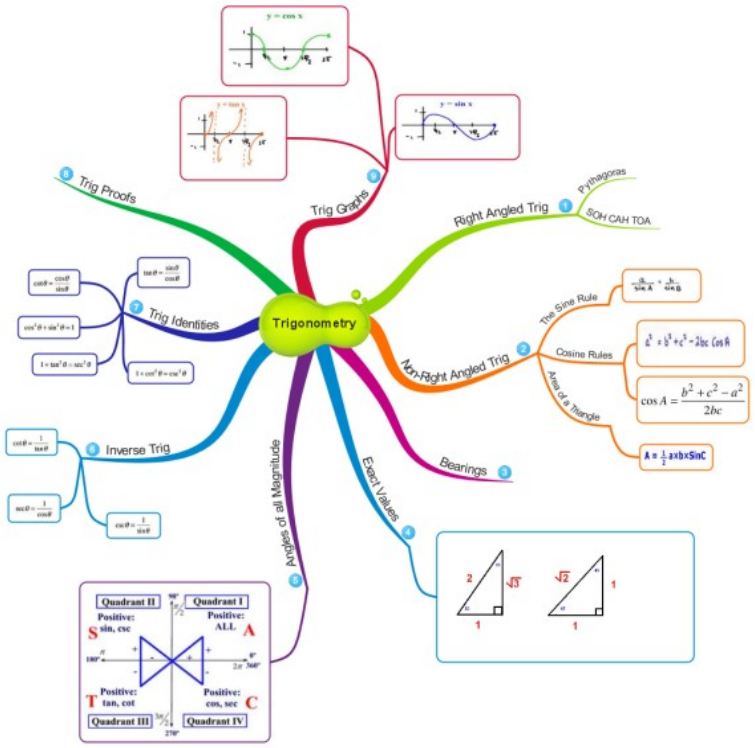
Since the entire process could be quite confusing, mind maps prove to be a savior in a way that you can draw a diagram illustrating the number of phases involved in the solution, and what formula or logic should be applied in which phase to get to the final result.



Also, the basic trigonometry can be summarize in a nice way:

Source: https://www.biggerplate.com/mindmaps/ONgiO86N/trigonometry

To summarize, with mind maps, math training and learning could be great fun, easy to understand, and hard to forget. While the traditional approaches are useful and help the students gain knowledge, mind maps are less time consuming



and make the entire topic pretty entertaining, regardless of all the complexities that it may have.

How to draw a mind map:

• **Use colors, drawings and symbols copiously.** Be as visual as you can, and your brain will thank you. I've met many people who don't even try, with the excuse they're "not artists". Don't let that keep you from trying it out!

• **Keep the topics labels as short as possible**, keeping them to a single word – or, better yet, to only a picture. Especially in your first mind maps, the temptation to write a complete phrase is enormous, but always look for opportunities to shorten it to a single word or figure – your mind map will be much more effective that way.

• **Vary text size, color and alignment.** Vary the thickness and length of the lines. Provide as many visual cues as you can to emphasize important points. Every little bit helps engaging your brain.

• **Group the content in classes of structure.** It is very important to have a very well structure of the information such that the essential notion to be emphasized.

2) TUIASI experience in applying MM in Analytic Geometry

After a discussion with one of our colleague from Department of Pedagogy at our university, prof. Tudor Stanciu, I learned how to prepare a mind map in order to summarize the theoretical notions of two chapters, Conics and Quadrics in only one mind map. I was surprised by the impact of the created material among my students, they managed to learn much more easily the whole structure of these chapters of curves and, surfaces of order two and consequently, they performed better at the final exam. After this experience, I continued to explain and to summarize the content of each chapter in a mind map.

Also, I have encouraged my students to prepare the exams, not only for Mathematics, using the mind map. To create maps of each chapter in order to have a complete frame of what they have to know. The feedback was excellent and I realized that new methods of teaching and learning Math could be applied with great results. I shared this experience with my colleagues from the Department of Mathematics and some of them starts to use MM in teaching Mathematics.

Two mind maps are presented below: for Conics and Quadrics (Fig.1) and for Change of Coordinates (Fig.2) and I will explain how was made the first mind map (Fig.1).

It is divided in two parts:

• in the lower part the conics are presented with the three families, each standard equation being acompanied by the image and followed by the degenerations of the corresponding family.

• in the upper part the Quadrics are presented also with the corresponding three families in the same manner, each standard equation with image, emphasizing the rulled surfaces (the one-sheet hyperboloid and the hyperbolic paraboloid) and the degenerations of the quadrics.

I used different colors for each family and on vertical representation we have the same family in plane or in 3D-space (conics or quadrics).

There are free softwares on internet which help you, providing templates to draw mind maps:

http://www.mindmapper.com
http://www.mindmaster.com
http://www.lucidchart.com
http://www.simplemind.eu
http://www.imdevin.com/top-10-totally-free-mind-mapping-software-tools/

Recommended bibliography:

1. Brian Holland; Lynda Holland; Jenny Davies (2004). *"An investigation into the concept of mind mapping and the use of mind mapping software to support and improve student academic performance"*.

2. Joeran Beel, Stefan Langer (2011). *"An Exploratory Analysis of Mind Maps"* (PDF). Proceedings of the 11th ACM Symposium on Document Engineering (DocEng'11). ACM.

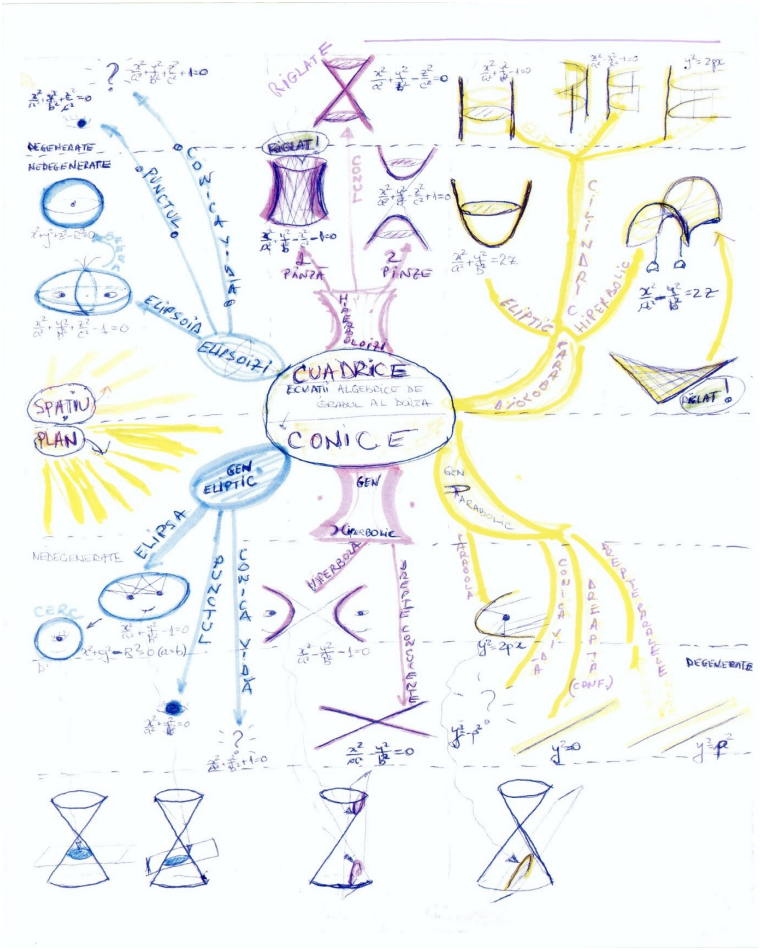


Fig. 1: Mind map: Conics and Quadrics (M.Roman)

2.2 Gamification in Mathematics Education

Nowadays, a gamification is seen as an interactive tool that could be used in teaching mathematics more effective. With the help of information technologies, it makes sense to consider the integration of mobile devices and digital media allowing students actively participate in the lecture. Usually, students do this very willingly. Thus, the teachers have an additional and, as some researches show, very significant opportunity to diversify their teaching methodology. Despite the lecturer's desire to make lectures more attractive, it also requires the additional professional development and the intention to learn the new approaches of teaching.

Many investigations have been done since technologies came into the teaching process showing that computer games offer great advantages in teaching the

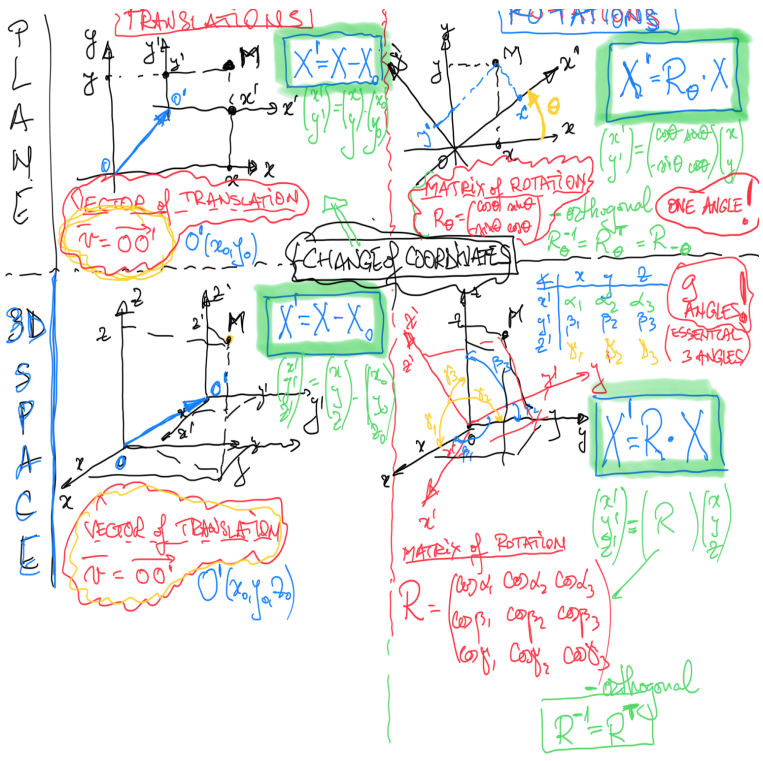


Fig. 2: Mind map: Change of coordinates (M.Roman)

sciences. Kebritchi et al. [19] in the early studies revealed that games in math lectures for high school students significantly increase their final grade. Continuing on this advantage, Nejem and Muhanna [25] made a special experiment to show that the number sense on post-test achievements of students who studied with the help of computer games was statistically significantly better than that of those who studied in the usual way. Comparatively, Demirbileck and Tamer [10] review the main problems faced by teachers when using computer games in the classroom. First, it is a lack of computers in the classes, the absence of games in the native language as well as a lack of games for the specific topics. It also refines the main benefits of the teaching process: "improve students' creativity, allow students to participate lessons actively, develop Math vocabulary, and comprehend the Math concepts easily, increases the span of keeping subject in mind." The latter authors also agree with the idea that the education of the mathematics must change and it could be done with the help of computer games. On the other hand, Ertmer et al. [12] emphasize that one of the main reason which hinders teachers' desire to include games in lectures is their outdated attitude and lack of knowledge about the latest technological solutions. Callaghan et al. [7] research shows that professional development, with is crucial for using

new technologies in the field, encourages teachers to use educational computer games. Beside this, the study also reveals a link between the use of computer games in lectures and better learning scores.

So, the question is what distinguish educational games from other interactive tools such as quizzes, polls or surveys that may be used for teaching. In fact, we observe the same elements or building blocks that are used to develop computer games, but the topic and question related with mathematics. This includes:

- reward and penalty points;
- competition within the classroom;
- levels, checkpoints, milestones and other approaches that show a progress individually;
- animated progress bars that compare and reflect the performance within some group of students or classroom;
- rewards such as badge, belt or other "certificates" to identify unique achievements;
- taking a risk (a challenging questions with limited time) that remarkably rewards or grades backward;
- lives or help options (ask help for successors or teacher);
- individualized or team gaming.

There exist many platforms and tools developed to suggest learning in a more interactive and stimulating environment. For example, Kahoot! is a popular online gaming platform that could be used in higher education to enhance motivation, engagement and active learning [41]. The concept of this tool is to combine a student response system, the technical infrastructure (digital devices), social networking, and gaming into one system, which results in positive findings and improved learning results [40]. It reflects the functionality provided by many similar testing and quiz makers tools. Another example is comparatively fresh – Educational Escape Rooms, like EscapED [38], are developed based on the concept of physical adventure game – escape room – and adopted for the use in Education. It includes not only solving mathematical puzzles within the allotted time, but also coordinate the team, solidify your objectives, communicate and help each other. Also, there exist methodologies and recommendations published how to create and guide on Virtual Escape Rooms using online platforms such as MS Teams and Zoom [20]. Below we provide the experience gained using Mentimeter.

Practical example with menti.com

As an example of using interactive games with mobile devices, we present the example of using *menti.com* (for lecturers *www.mentimeter.com*). At the end of the lecture we give students the opportunity to check their knowledge of the topic with the help of this game. The process can be listed in the following way:

- short specific questions are presented to students;
- the student chooses the answer he/she thinks is correct;
- if the majority of students answer the question incorrectly, the teacher explains the question or revise the topic related to the question immediately after the game;

– the top ten students can collect extra points or bonuses that would could be later added to the final score.

In Figure 3 we present a small illustration of this game which was given in the lecture of Probability Theory and Statistics. In the last slide we can see the students with the maximum points.

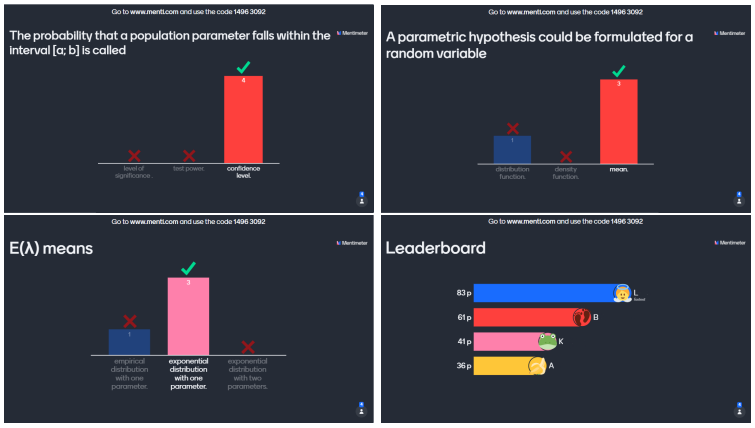


Fig. 3: Example with menti.com

The advantages of including this method into theoretical lectures that we noticed can be summarized in a few aspects:

– during the lecture a student figures out what he/she did not learn or did not understand;
– the teacher find out which questions need to be revised or explained in a much more detailed manner;
– student activity in lectures increases.

2.3 Collaborative Learning in Linear Algebra and Analytic Geometry

This section presents an experiment that was carried out in a Linear Algebra and Analytic Geometric course for first year (first semester) students of Civil Engineering. Research from the last years shows that while university students usually lack the necessary skills to follow the mathematical contents they are supposed to apprehend, they do not have the opportunity to participate in an open and rigorous exchange of ideas, that would allow them to formulate ideias and acquire knowledge by defending opinions and considering the alternative perspectives offered by their peers. The approach in many classrooms is of conventional, non-interactive lectures, where the teacher talks and the students

passively listen. That type of classroom dynamic does not seem to be effective in supporting students to develop mathematical and scientific competence nor, on the other hand, in preparing them to develop full citizenship and participation in today's globalized, and rapidly-changing society with its multiple challenges [26].

Following long term reflection about the continued use of conventional lecturing in higher education mathematics classes and being aware that the major difficulty they face is the lack of students' motivation, the authors committed to the implementation of alternative teaching and learning methodologies to improve the learning outcomes of the course. Students have changed throughout time and, therefore, teaching methodologies require refreshment and adaptation. Research about the use of active, collaborative methodologies in classroom dynamics supports the approach that was followed: there is an overall improvement of the results as well as a boost in the students' engagement and autonomy when they are demanded to be active agents in their learning process [14, 18, 32]. In order to acquire a thorough understanding of a subject, learners must follow an established sequence of stages: acquisition of skills, their practice and integration and knowledge of when and how to apply what has been learnt [24].

The design of the methodology herein described focused on the articulation of the program topics to be covered, the activities that students were involved into and the chosen assessment methods. Students opinions were heard and taken into account in every step of the procedure - for example, through co-construction of tasks and assessment - and strong emphasis has been placed on collaborative group work. The syllabuses and other educational resources were explored by the students , with the aim of responding to 'real life' problems in the scientific area of their course (Civil Engineering) Theoretical concepts emerged naturally from the need to understand and model the problems, evolving from the concrete to the abstract: all theoretical knowledge was acquired in a problem-based-learning context and applied to the resolution of previously selected real problems.

Collaborative, active teaching promotes communication skills – both oral and written - and effective articulation of thoughts and ideas as well as the ability to effectively listen, respect, collaborate and work with other people and. This type of teaching methodology cultivates and develops flexibility and willingness to cede in order to accomplish common targets ([14]) and, therefore, activities that put students as instructional resources to each other were encouraged.

The 25 students in the class worked in weekly in-classroom blocks of 4 hours, organized into groups of 3 or 4, under the permanent supervision of two teachers; in the first class of the semester, both the teachers and the students informally introduced themselves through an ice-breaker activity (for example, asking each person to share 2 things they liked and 2 things they disliked); the context and the methodology to be followed were explained and feed-back about thoughts and questions was asked. After a short period of discussion, students were introduced to several available collaborative online platforms to develop their weekly tasks, Google Docs being the chosen one, together with virtual.ipb.pt, the Polytechnic Institute of Bragança's platform for communication between teachers and students and for the management of all course related items (see Figure 4).

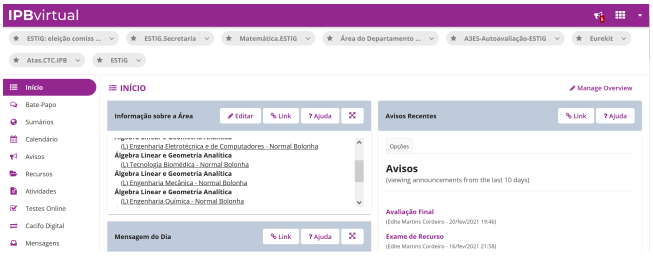


Fig. 4: Front page of virtual.ipb.pt.

Groups were formed according to the preferences of the students and, after a moment of reflection within each group, the assessment procedure was defined by both teachers and groups. It was decided that the workload would be divided into to 'big' written reports that would cover all the topics in the course program (the usual contents of any classical Linear Algebra and Analytic course, complex numbers, matrices and determinants, linear systems, vector spaces, linear transformations, eigenvalues and eigenvectors and analytic geometry), subject to weekly short presentations and in-class discussion as well as final exam for the validation of the process. Each of the two reports arose from the need to answer a particular question related the scientific area of Civil Engineering (it could either be physics, mechanics or another related subject) chosen by each group in the mentioned initial meeting (class). The clarification of the rules to be followed throughout the semester was a major concern of the teachers and in all classes attention was devoted to make sure everyone kept up with those rules. The evaluation of the tasks weekly developed by the groups included peer review of reports and group presentations, self-assessment and interviews. Every week, 24 hours before the class, the groups uploaded the written work they had carried out under IPB's platform forum in such a way that everyone had the chance to read and comment everyone else's work, as seen in Figure 5.

Feed-back from the teachers was provided before the class and adaptations were made in time for the weekly presentations and discussions (see Figure 6). The promptness of the feed-back from the teachers was also a big concern since it is a decisive step for the students' awareness of details and for the improvement and efficacy of student-centered and active approaches like this one.

The final results obtained were 19 approvals out of 25 evaluated students. The impressions collected in the final process evaluation questionnaire that the students answered in the last class of the semester echo the motivation and commitment that most students expressed, class by class. The students considered that the only point that needed improvement throughout the process was their own dedication to work. Almost all students (there was 1 exception in a universe of 25 students) considered the followed methodology (much) more engaging than the conventional one.

Fig. 5: Forum where weekly tasks were uploaded.

Fig. 6: Exchange of messages about the developed work.

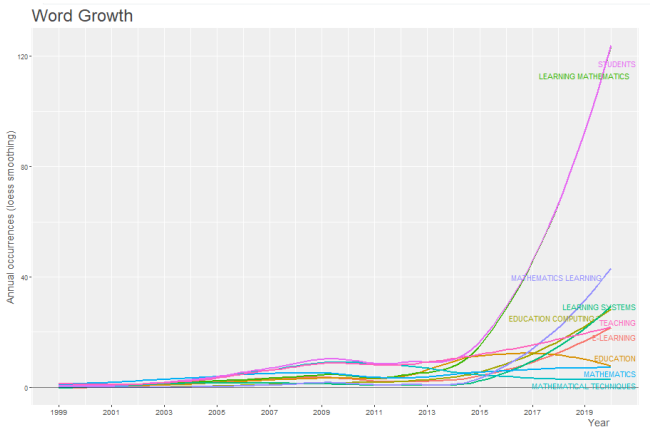
3 Learning Mathematics

The learning mathematics is associated with the topics learning strategies and learning outcomes. The main learning strategies in mathematics are indicated on the literature as Metacognitive Strategies, Cognitive Strategies and Social/Affective Strategies [34].

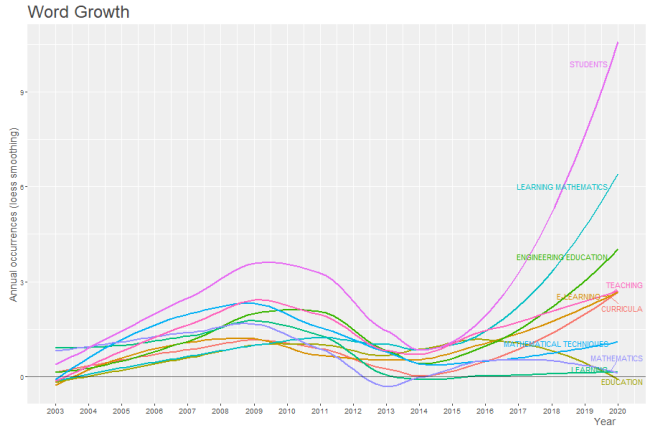
The Metacognitive Strategies consists on plan, monitorize and evaluate their learning mathematics concepts and skills. Cognitive Strategies are associated to the students interaction with the learn information, changing or organization mentally or physically. Social/Affective Strategies are based on the interactions between students to assist on learning, or attitudes in their learnings [34].

It was identified two databases. The first, named Database I, is based on 2133 documents from SCOPUS database, between 1999 and 2020, using the keyword learning mathematics. The second (Database II) is based the previous one refined

with keywords "higher education" or "university". This second database has 248 documents.



(a) Database I

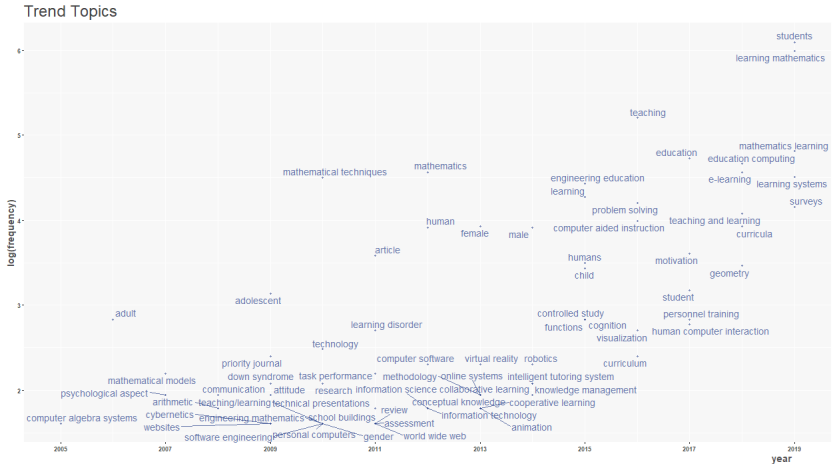


(b) Database II

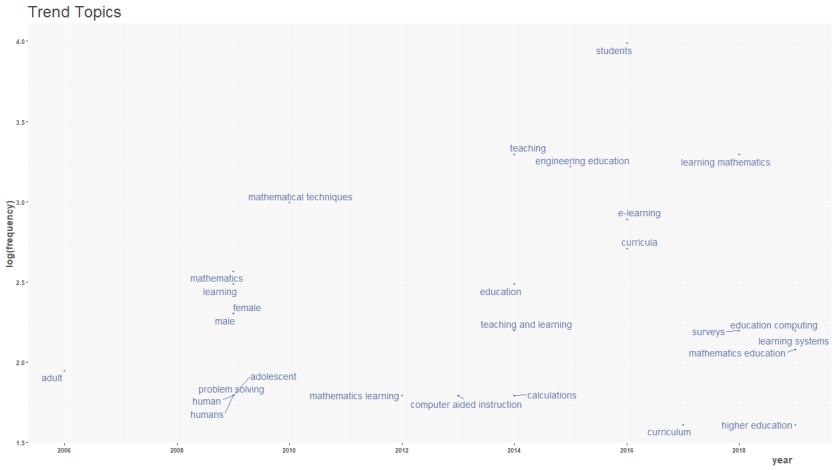
Fig. 7: Comparison of 2 databases

It is possible to observe in Figure 7a there is a trend of the topics "students", "learning mathematics", "mathematics learning" since 2015 until now. Associated to the university field, this trend is not so strong and "engineering education" is one of the research topics that had attract research interest as can be seen on Figure 7b.

In Figure 8 it is presented a keywords distribution considering the years and frequency. It is possible to confirm that in the recent years the topics associated to "students", "learning mathematics" have a higher increase on the research.



(a) Database I



(b) Database II

Fig. 8: Comparison of 2 databases

When we compare with the situation of higher education it is possible to observe that "learning mathematics" and "engineering education" had attract recent research.

It was selected some research documents based on citation on this field to be analysed and discuss.

In [21] investigate the influence of family on adolescent students' mathematical habits through a quantitative methodology on data from 563 Cypriot students on 'core' and 'advanced' mathematics pre-university courses. The aim of this study is to explore the association between students' perceptions of parental influence and their dispositions towards mathematics. Statistical analysis showed

that perceived parental influence is mediated by the mathematics course students are studying and their mathematical inclination. This study suggests that family influences on students' dispositions are significantly considered for by students' prior choice of mathematics course and students' mathematical inclination; influencing university choices.

In [11] focuses on who is teaching mathematics in one university. This author argues that mathematics teaching is overtaken by individuals who would not describe them as mathematics specialists. In fact, many teachers learned their mathematics autonomously in order to solve specific problems such as in the research while others are constantly learned from their teaching. For most individuals, mathematics learning is positioned in the contexts in which it is applied, being the teaching one such context. According to this author, it is necessary to explore the experience of learning mathematics in order to teach as it can provide specific understanding into the successful learning mathematics process.

In [28], the following three aspects of learning mathematics are investigated: intention, approach and outcome. The starting point for the study consisted of interviews with 22 students, second to fourth-year students majoring in an area of the mathematical sciences (statistics, mathematical finance, operations research), of the University of Technology, Sydney. The were asked about their experience of learning mathematics, their understanding of mathematics as a discipline field, and their perception of work as a mathematician. The interviewer was experienced in collecting oral histories and carrying out interviews in the social sciences, but was not a mathematician, Hence, students were encouraged to provide explanations of any mathematical aspects of their responses. The authors focus on the complex nature of the students' intentions for learning, approaches to learning and outcomes of learning, presenting a theoretical model based on their research findings, and performing a deep analysis and very interesting conclusions.

In [9] the question is considered, if it is better to use abstract or concrete examples in learning Mathematics. Starting point is a paper of Kaminski, Sloutsky, and Heckler (2008a) published in Science, where the authors claim that it is a bigger advantage for the students, to consider a single abstract example than many concrete examples. In this paper some criticisms to this idea are summarized, made by different authors, and this is starting point for an experimental study, that involved one hundred thirty undergraduate students in educational sciences. the results confirm the basic finding by Kaminski et al. (2008a): transfer to a new abstract domain is better enhanced by an abstract learning domain than by a concrete learning domain. However, the authors show that this is only one side of the coin: transfer in a new concrete domain is also enhanced more by concrete examples than by abstract examples.

In [34] the authors explore the ways that students learn in flipped classrooms, presenting the concept of self - regulated learning. In particularly it is present the Winne and Hadwin's self-regulated learning model consists in four stages: task definition, goal setting and planning, enactment, and adaptation applied to

flipped classrooms concept. The work also present some metrics to evaluated the strategies. The paper concludes that it is important to promote the self-regulated learning process (pre-class and in-class) and collaborative learning environments presenting some suggestion for that.

In [15] it is present the effectiveness of bridging courses in higher education. This courses bridges students' difficulties with mathematical concepts, helping develop strategies for learning mathematics and extends skills in thinking and reasoning. Concepts, methodologies and metrics are presented.

The works [8, 16] discuss two different types of learning mathematics at the university: surface learning and deep learning. Surface learning reduces mathematics to applying simple rules, which need to be memorized to perform a certain task, while deep learning is centred on meaning and understanding, with a particular focus on creating connections between different parts. The seminal paper [8] discusses the idea that different conceptions of mathematics, often coming from the primary-to-high school experience, lead to different types of learning. More precisely, students with a *fragmented* conception (i.e., mathematics as numbers and rules) often reach only a surface learning, with implications to the performance in the assessment and in the perception of the workload. On the other hand, students with a *cohesive* conception (i.e., mathematics as a way of thinking, and of understanding the world) often have a deep learning, leading to better results in the assessment and a more satisfactory learning experience. The paper [16] discusses a blended approach to calculus for first year university students, in which traditional lessons are combined with computer-assisted activities, in which the students can have a more direct experience with the mathematics of calculus (functions, graphs, areas, integrals, etc.). Compared to a control group, which followed a traditional approach, the experimental group had a higher number of students achieving a deep learning.

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