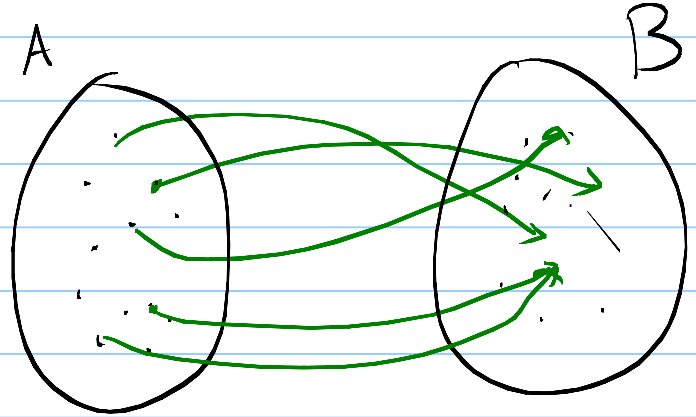
FUNCTIONS

A function is 3 things:

- A starting set
- An arrival set
- a rule that assigns only to each element of the starting set a unique element on the arrival set

DOMAIN — CO-DOMAIN

$$f : A \rightarrow B$$



Properties of fncts

f is INJECTIVE (ONE-TO-ONE) - if distinct elements in the domain go to distinct element in the co-domain

- if $\forall\ a_1 \neq a_2 \quad a_1, a_2 \in A$

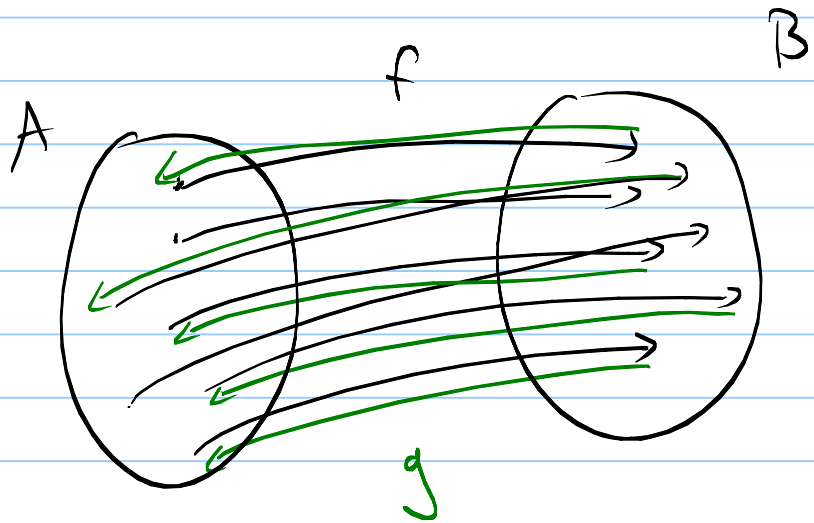
$\Rightarrow f(a_1) \neq f(a_2)$

f is SURJECTIVE (ONTO) - if every element in the codomain is reached by some element in the domain

- $\forall\ b \in B, \ \exists\ a \in A$ s.t. $f(a) = b$

f is BIJECTIVE if is INJECTIVE & surjective.

INVERTIBLE



$g : B \to A$ is a function the "inverse" of f

$g(f(a)) = a$

$g \circ f = id_A$

$f \circ g = id_B$

Beware! injectivity, surjectivity and bijectivity DEPENDS on the domain so, in particular

$\cos : \mathbb{R} \to \mathbb{R}$ is NON INJ.
NOT SURJ.

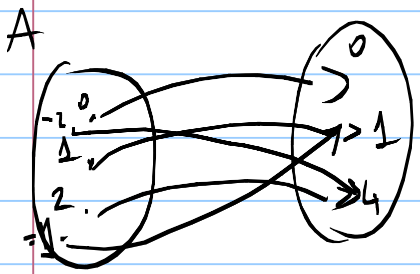
but $\cos : [0, \pi] \to [-1, 1]$ it is surj. and inj. and thus invert.

$\arccos : [-1, 1] \to [0, \pi]$ is its inverse

$f(x) = x^2$ $\qquad$ $f : A \to B$

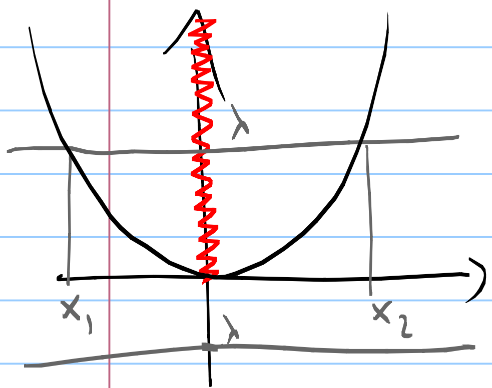
$\forall\, a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$

$(a_1, a_2 \in A)$



A	B	INJ.?	SURJ.?	BIJ.?
$\mathbb{R}$	$\mathbb{R}$	NO!	NO!	NO
$\mathbb{R}_{\geq 0}$	$\mathbb{R}$	YES	NO!	NO
$\mathbb{R}$	$\mathbb{R}_{\geq 0}$	NO	YES	NO
$\mathbb{R}_{\geq 0}$	$\mathbb{R}_{\geq 0}$	YES	YES	BIJ
$\mathbb{N}$	$\mathbb{N}$	YES	NO	NO
$\mathbb{Q}_{\geq 0}$	$\mathbb{Q}_{\geq 0}$	YES	NO	NO

$\to$ inv. $\sqrt{\ }$



Def. $\mathrm{Range}(f) = \{ f(a) : a \in A \}$

f surjective iff $\mathrm{Range}(f) = B$

$f(x) = x^2$ $\quad f : \mathbb{N} \to \mathbb{N}$ $\quad \mathrm{Range}(f) = \{ a^2 : a \in \mathbb{N} \} =$

$= \{ 0, 1, 4, 9, 16, \ldots \}$

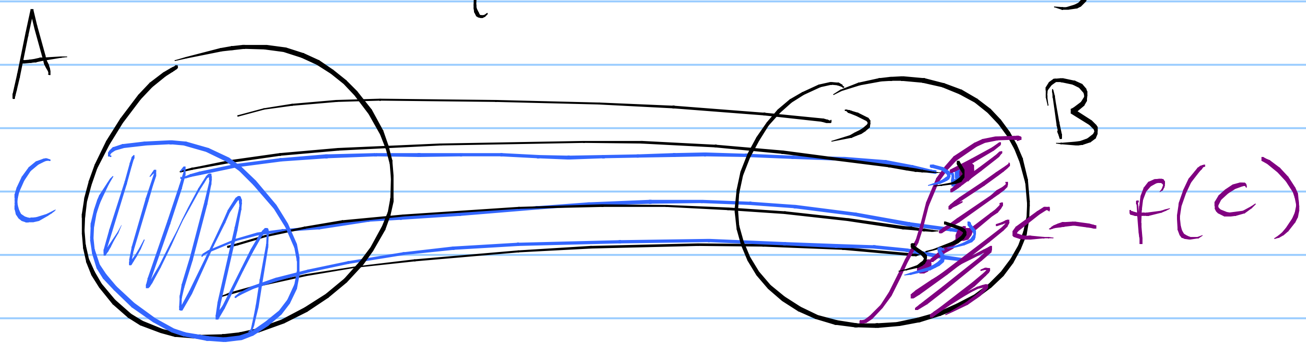
$= \{ \text{perfect squares} \} \neq \mathbb{N}$

Image and counter-image of a set

$f: A \to B$

for every $C \subseteq A$

$$f(C) = \{ b \in B \ ; \ \exists c \in C \text{ s.t. } b = f(c) \}$$
$$= \{ f(c) : c \in C \}$$



$\forall \quad D \subseteq B$

$$f^{-1}(D) = \{ a \in A \text{ s.t. } f(a) \in D \}$$