Diagonals of a polygon

➢ How to calculate the number of diagonals of a polygon?

Remember:

A diagonal of a polygon is a line segment joining two non-consecutive vertices of that polygon.

Check the number of diagonals drawn on the polygons shown in figures 1, 2, 3, 4, and 5, respectively, a triangle, a quadrilateral, a pentagon, a hexagon, and a heptagon.

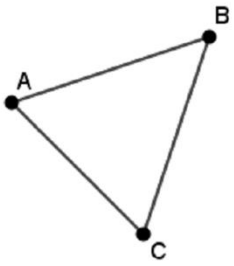

Figure 1

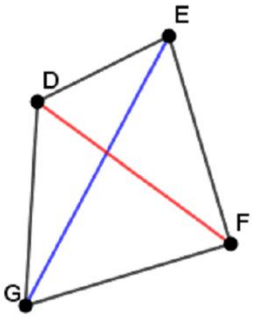

Figure 2

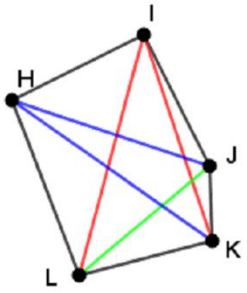

Figure 3

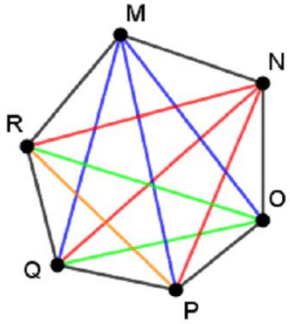

Figure 4

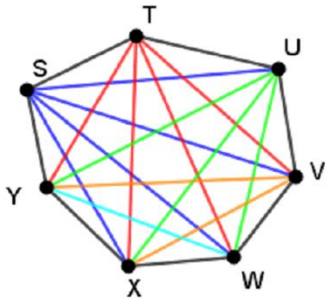

Figure 5

The following table shows the **number of diagonals** drawn as a function of the number of sides of each polygon.

Number of polygon sides	3	4	5	6	7
Number of diagonals	0	2	5	9	14
		+2	+3	+4	+5

Let's understand what is happening from a vertex.

Regarding the number of diagonals, for example in figure 3 (pentagon), we observe 2 diagonals from each vertex.

So, apparently, we would have 10 diagonals (5×2).

However, we only observe 5, because the line segments [IL] and [LI] represent the same diagonal, as well as [HJ] and [JH], and the same is true for the remaining diagonals.

Thus:

Number of polygon sides	3	4	5	6	7
Number of diagonals from a vertex	0	1	2	3	4

-3

+1 +1 +1 +1

And how do we determine the number of diagonals of a polygon with 12 sides without using the number of sides of the polygon with 11 sides and so on?

The number of diagonals from a vertex is equal 9, i.e., $12 - 3$. Apparently there seem to be 108 diagonals (12×9). But we find that there are 54, i.e., half of 108.

In other words:

$$12 \times (12 - 3) = 12 \times 9 = 108$$
$$108 : 2 = 54$$

Generalizing, to calculate the number of diagonals of a polygon we use the formula:

$$d = [n \times (n - 3)] : 2$$

where:
d – total number of diagonals
n – number of sides of the polygon
$(n - 3)$ – number of diagonals from a vertex