MATHEMATICS

5th hour

$A = \{$ people in this room $\}$

$B = \{$ chairs in this room $\}$

$f : A \to B$

$f(\text{person}) =$ which chair he/she sits on

f is INJECTIVE? YES, because if f was NOT injective, this would mean that there are two people sitting on the

same domain!

f is surjective? No, because there are "free" chairs

A = { professors in UNIGE }

B = { courses in UNIGE }

$f : A \to B$ $\quad f(a)$ = the course taught by a

Can I construct this function?

$g : B \to A$

$g(course)$ = the professor that teaches that course

g is NOT INJECTIVE because there are professors that teach more than one course

A = { deck officers }

B = { boats that are moving }

f(deck officer) = the boat at which he/she is assigned

f is for sure SURJECTIVE (every boat has at least a deck officer)

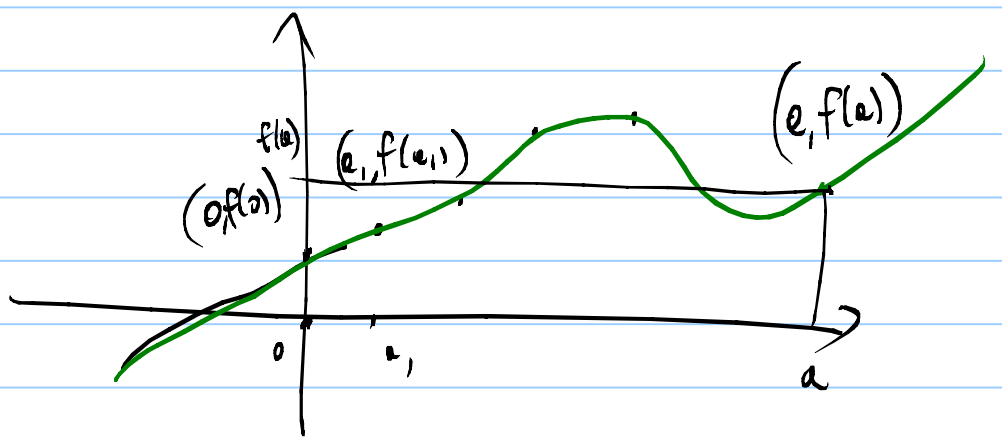
but it is not INJECTIVE (one boat could have more than one officer)

ELEMENTARY FUNCTIONS:

Graph of a funcin $f : A \to B$

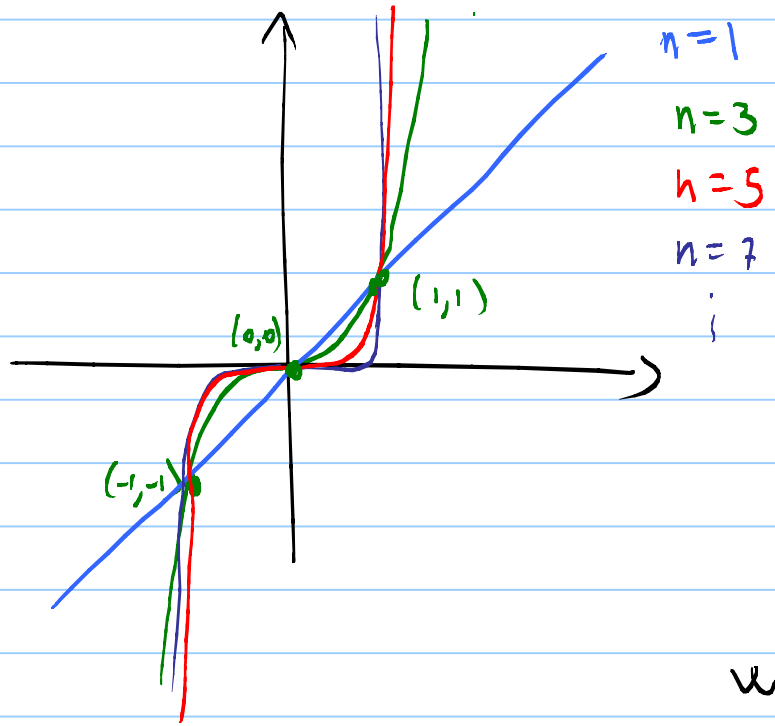
$$\text{graph}(f) = \{ (a,b) \in A \times B \; ; \quad b = f(a) \}$$

$$= \{ (a, f(a)) \; : \quad a \in A \}$$



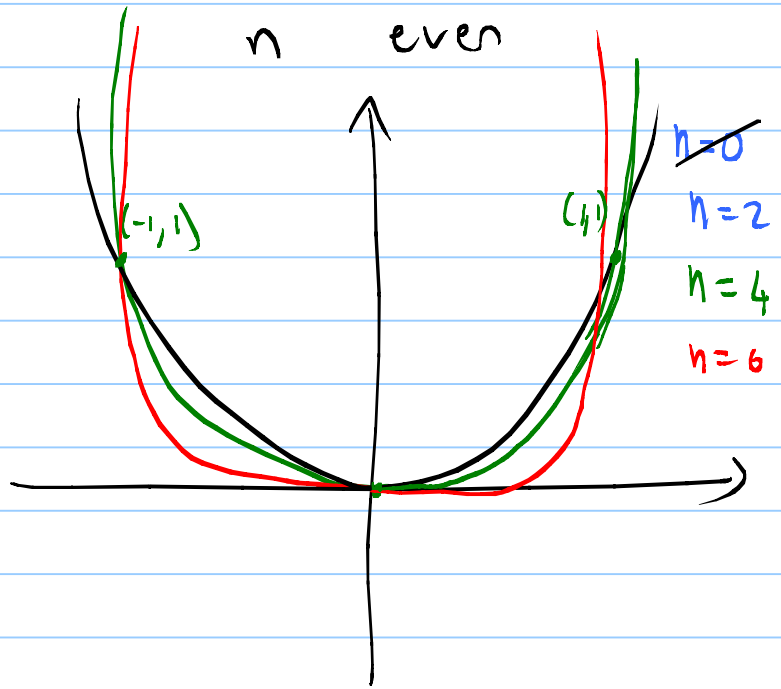
I) $f(x) = x^n$ $\quad n \in \mathbb{N} \quad n > 0$

I. a) n odd



n = 1
n = 3
n = 5
n = 7
⋮

n even



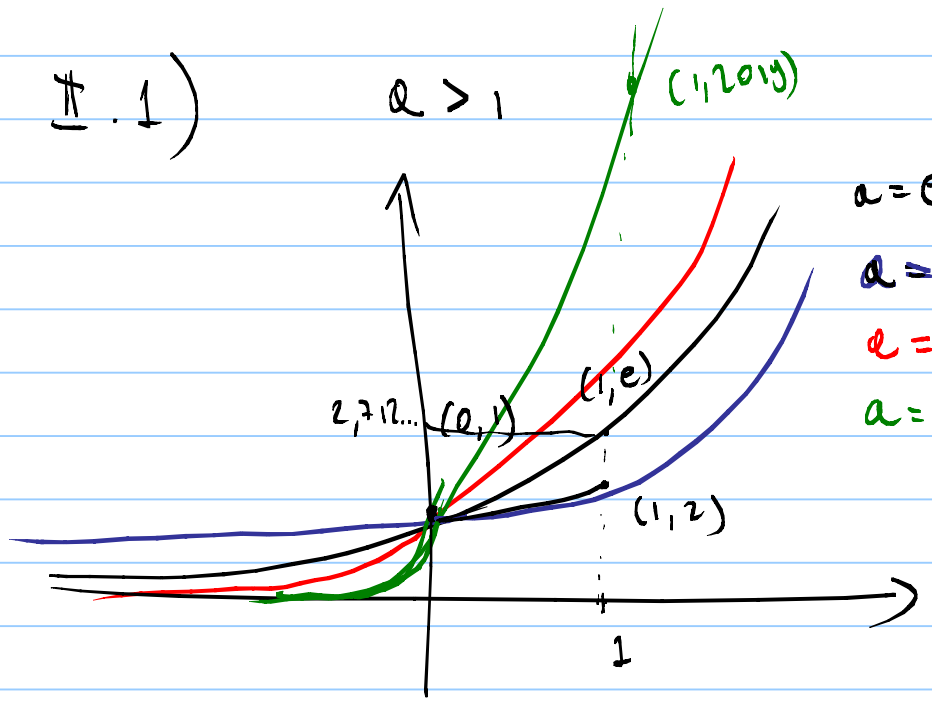
~~n = 0~~
n = 2
n = 4
n = 6

wolfram alpha

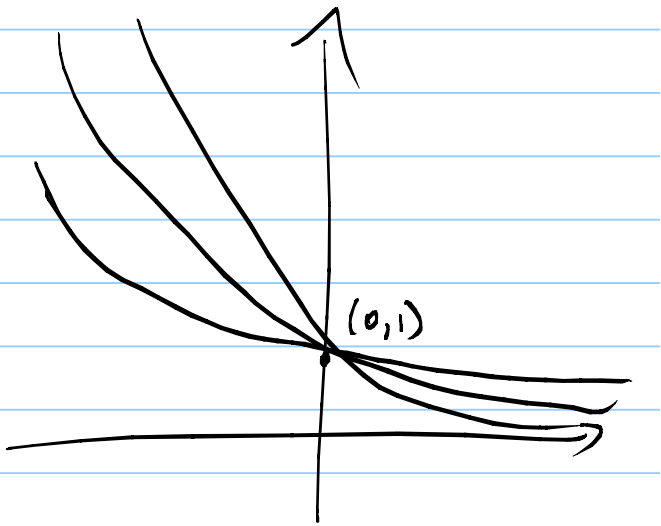
(important for derivatives and integrals) $\rightarrow \lim_{n \to \infty} \left(1+\frac{1}{n}\right)^n = e = 2,712\ldots$

II) EXPONENTIALS $f(x) = a^x$ $\quad a > 0 \quad a \in \mathbb{R}$

II.1) $a > 1$

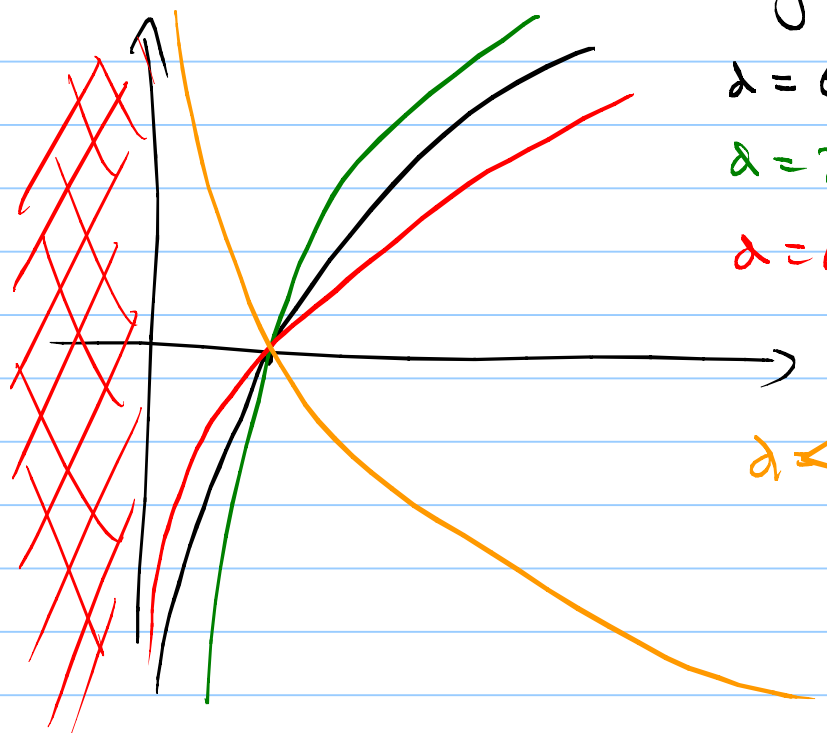


II.2) $a < 1$



III) Logarithms $f(x) = \log_a(x)$ $a > 0$ $a \in \mathbb{R}$

f is defined only if $x > 0$



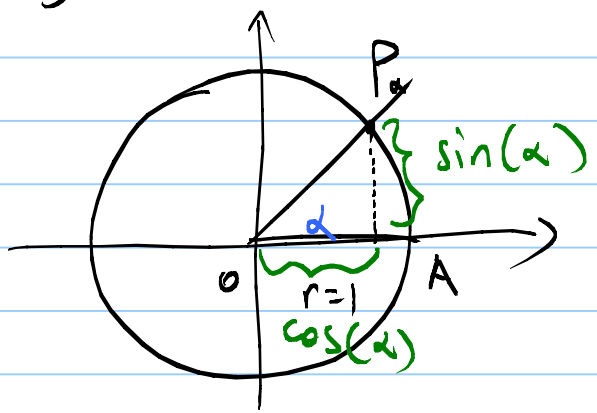
$a = e$ $\log(x)$ $\ln(x)$

$a = 2$

$a = 4$

$a < 1$

IV) TRIGONOMETRIC functions: sin, cos, tan, cotg



α is measured in radians

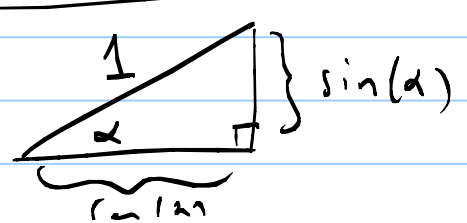
$$\alpha = \frac{\alpha \text{ degrees}}{360} \cdot 2\pi$$

right angle $\alpha = \frac{90}{360} \cdot 2\pi = \frac{\pi}{2}$

$\alpha = \frac{180}{360} \cdot 2\pi = \pi$

$\sin(\alpha)$ = altitude of the tringle $OP_\alpha A$

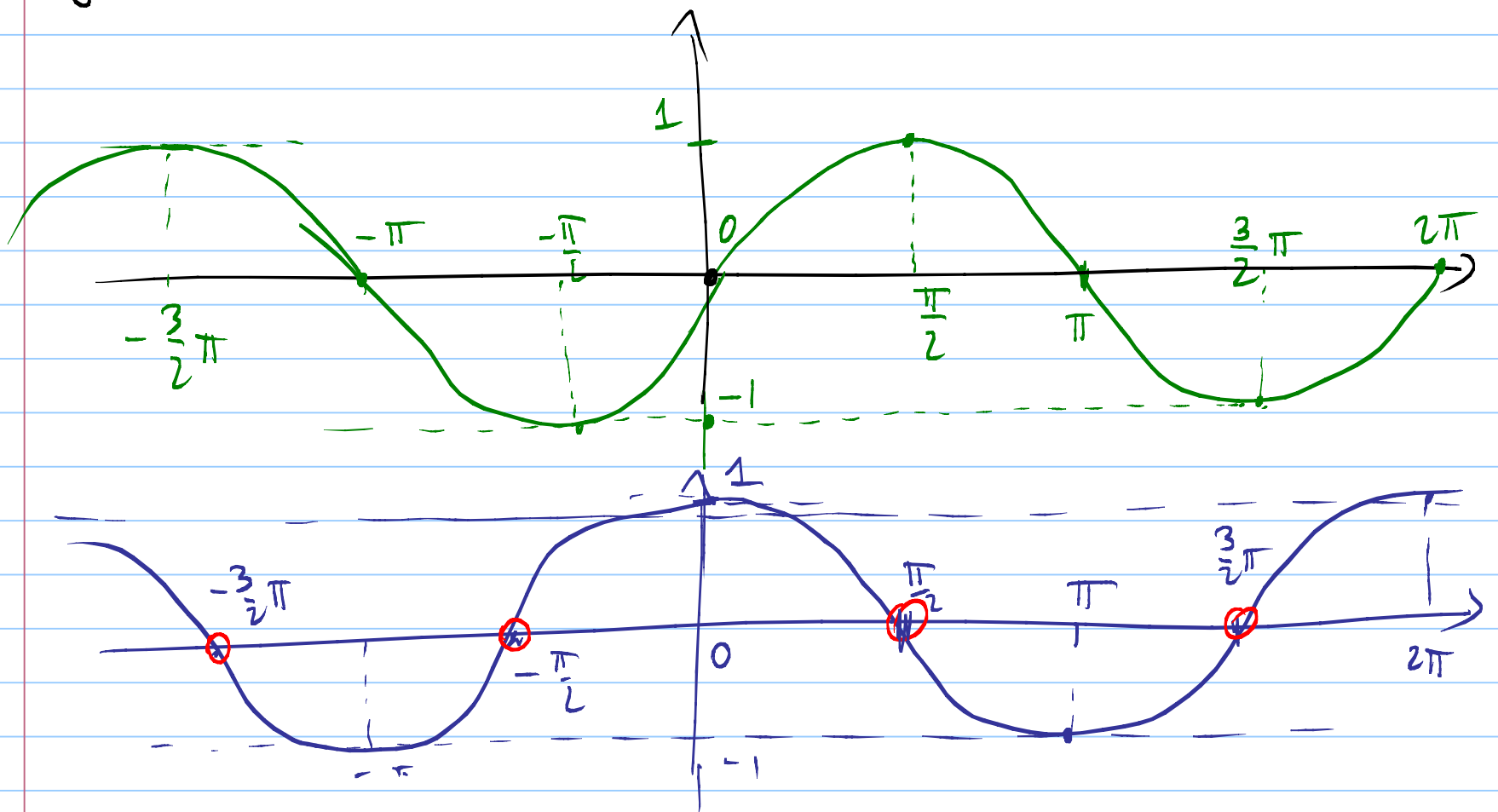
$\cos(\alpha)$ = projection of OP_α on the x axis



$\sin(\alpha)$ and $\cos(\alpha)$ are the length of the sides of a right triangle with hypoth=1

and an angle $= \alpha$

graph of $f(x) = \sin(x)$ and $f(x) = \cos(x)$



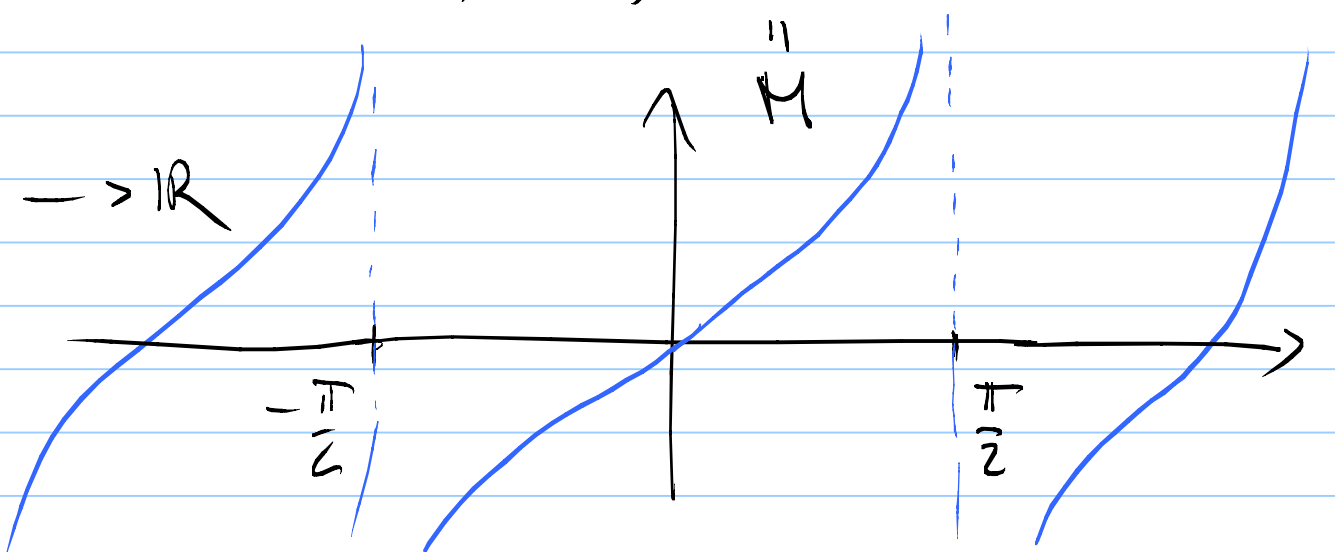
graph of $f(x) = \tan(x) = \dfrac{\sin(x)}{\cos(x)}$ $\quad f: \mathbb{R} \to \mathbb{R}$

in order to define $\tan$ we need to have $\cos(x) \neq 0$

$\cos(x) = 0 \qquad x = \frac{\pi}{2} + k\pi \quad \forall\, k \in \mathbb{Z}$

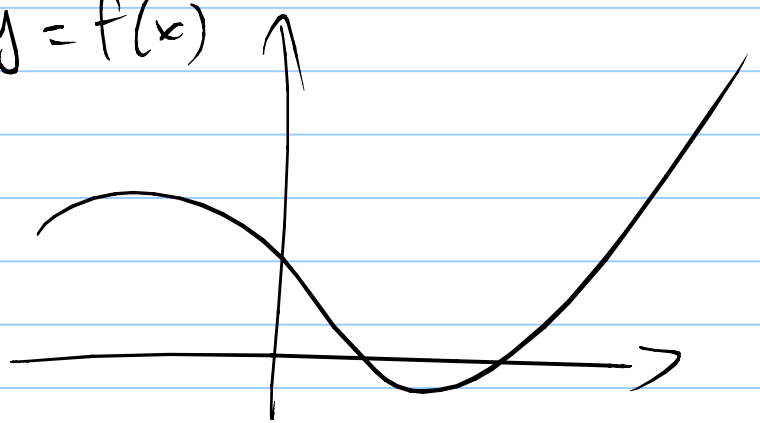
$x \in \{ \frac{\pi}{2}, -\frac{\pi}{2}, -\frac{3}{2}\pi, \frac{3}{2}\pi, \dots \}$

$\tan: \mathbb{R} \setminus M \to \mathbb{R}$

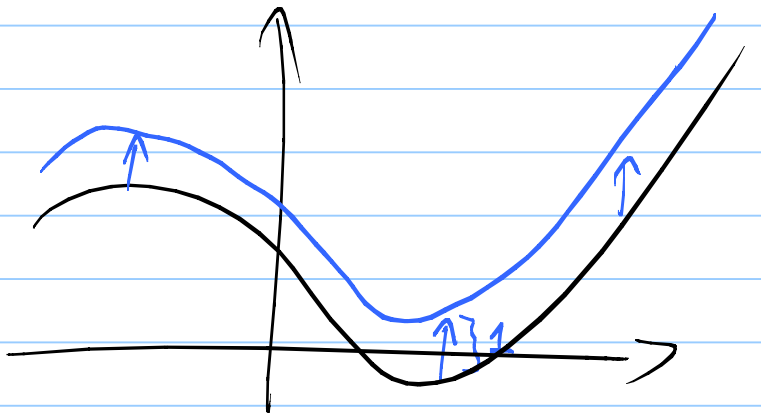


Operation on graphs

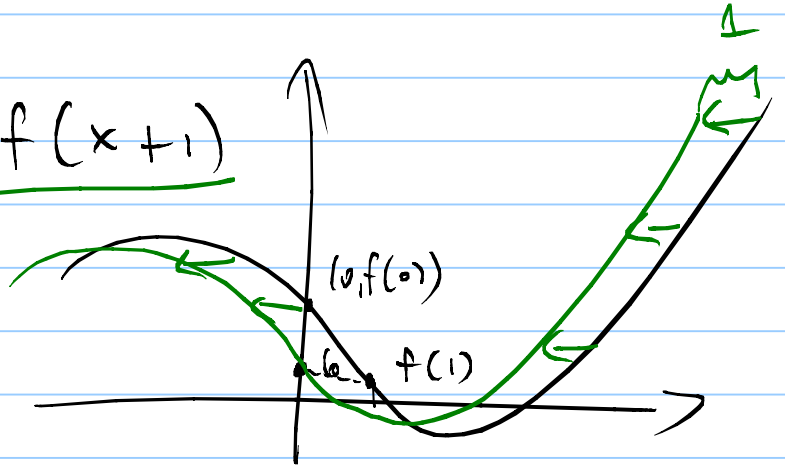
$y = f(x)$



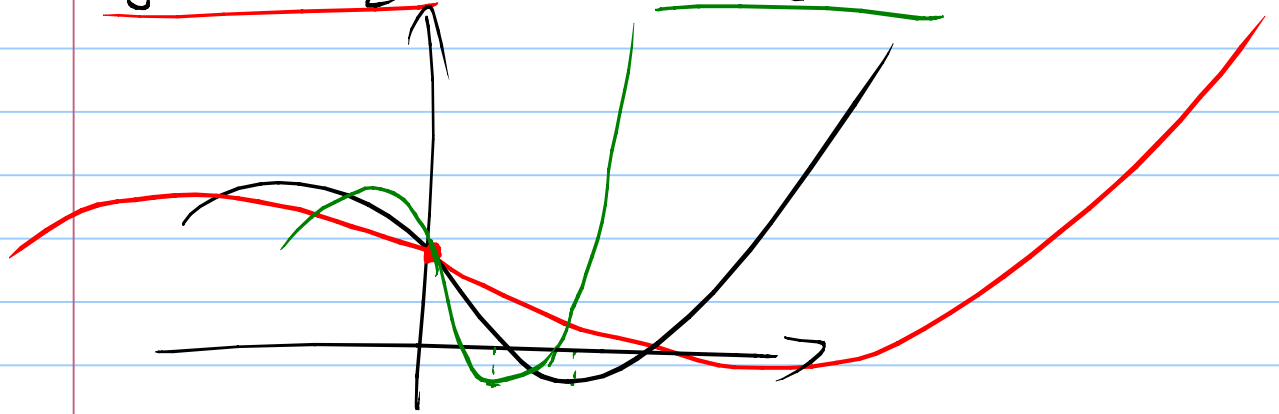
$y = f(x) + 1$



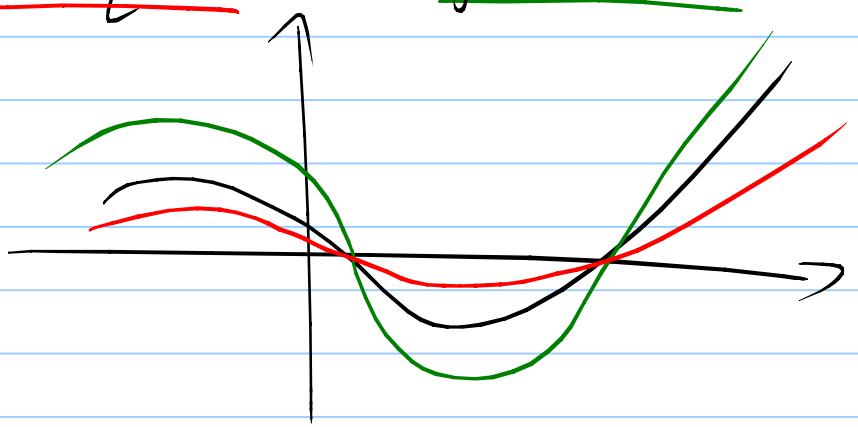
$y = f(x+1)$



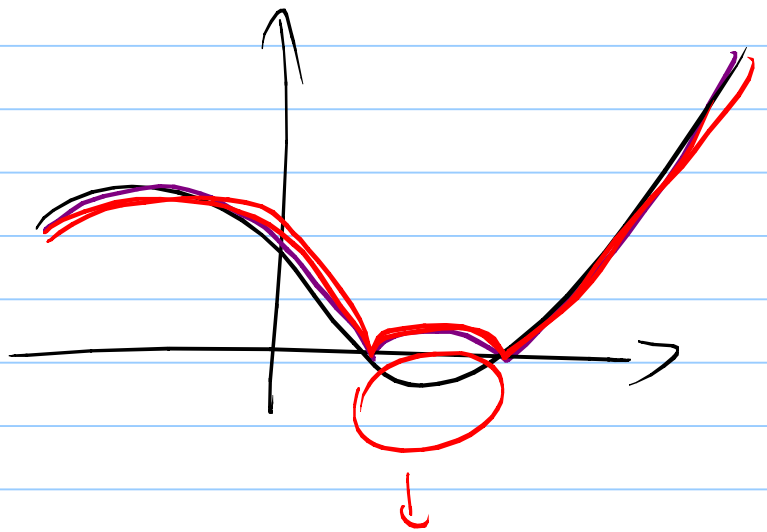
$y = f\left(\frac{x}{2}\right)$ $y = f(2x)$



$y = \frac{1}{2} f(x)$ $y = 2f(x)$



$y = |f(x)|$



I just flip this part that is below the x axis.