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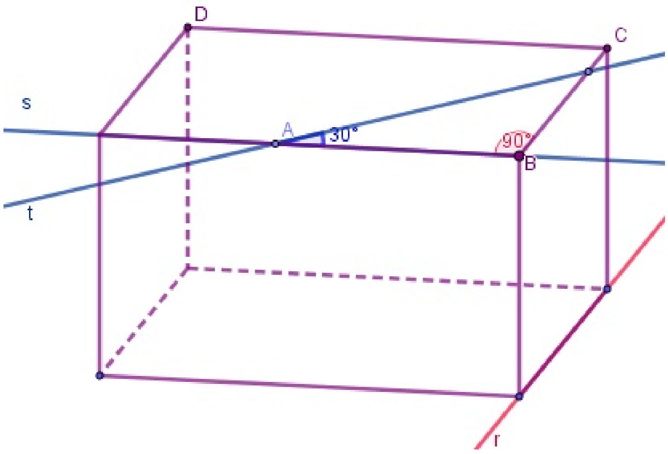
Angle between two lines

Definition: The angle between two lines is defined as as smallest angle between their directions.

In the figure to the side we can see that:

- The angle of the straight lines s and t belonging to the ABC plane measures $30^{\rm o}$.
- The angle of the reverse lines r and s is of $90^{\rm o}$ (equal to the angle between lines BC and s in the same plane).



Definition: The angle between two reverse lines (which do not intersect and are not parallel to each other) is the acute angle that one forms with a line parallel to the other.

Example: Let us consider the lines

$$r : (x, y, z) = (1, 2, 0) + k(2, 1, 3), k \in \mathbb{R} \quad \text{and} \quad s : (x, y, z) = (0, -1, -1) + t(3, 2, 1), t \in \mathbb{R}$$

of $\mathbb{R}^3$, whose directions are those of the non-collinear vectors $u = (2, 1, 3)$ and $v = (3, 2, 1)$, respectively. We can see that r and s do not intersect. In fact,

$$(1, 2, 0) + k(2, 1, 3) = (0, -1, -1) + t(3, 2, 1) \Leftrightarrow \left\{ \begin{array}{c} 2k - 3t = -1 \\ k - 2t = -3 \\ 3k - t = -1 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{c} k = -\dfrac{2}{7} \\ k = \dfrac{1}{5} \\ t = 3k + 1 \end{array} \right.$$

So r and s are reverse lines.

Besides that, $cos(\hat{rs}) = |cos(\hat{uv}| = \dfrac{|u \cdot v|}{|u||v|} = \dfrac{6+2+3}{\sqrt{14}\sqrt{14}} = \dfrac{11}{14}$, that is, $\hat{rs} = 23, 6^{\rm o}$.