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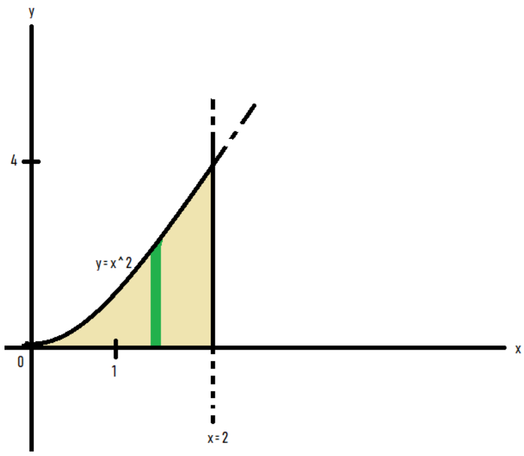
The objective of this question is to calculate the volume of solid generated by revolution of a planar region. Before proceeding into the solution, it is advised to check the theoretical part behind it.

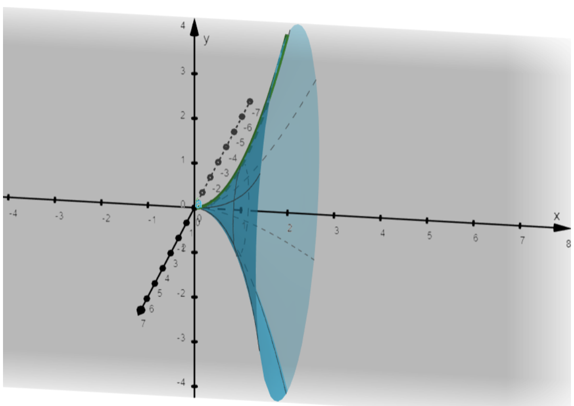


$y = x^2$ is a upward facing parabola with vertex (0,0).

$x = 2$ is a straight line.

The straight line $x = 2$ intersects the curve $y = x^2$ on $(2, 4)$

According to the question, we are supposed to revolve the region around the x-axis. On Revolving around the $x-$ axis, a solid of revolution is obtained.



Remember that, the volume of the solid of revolution formed by revolving region around the x-axis is given by,

$\mathbf{V} = \pi \displaystyle\int_a^b f^2(x) - g^2(x)\, dx$, where $f(x)$ **is the upper curve** and $g(x)$ **is the lower curve** and $x \in [a, b]$.

In this case, the upper function is $f(x) = x^2$ and lower function is $g(x) = 0$ and $x \in [0, 2]$.

$$\begin{aligned}
V &= \pi \int_a^b f^2(x) - g^2(x)\, dx \\
&= \pi \int_0^2 (x^2)^2\, dx \\
&= \pi \int_0^2 x^4\, dx \\
&= \pi \left[\frac{x^5}{5}\right]_0^2 \\
&= \pi \cdot \left(\frac{2^5}{5}\right) \\
&= \frac{32\pi}{5} \text{ cubic units}
\end{aligned}$$