Linear Independence versus Linear Dependence

- **The vectors $(2,3)$ and $(1,-4)$ are linearly independents?**

Attend of the

Definition: Consider $u_1, u_2, u_3, \dots, u_n$ vectors of vectorial space V, and $c_1, c_2, c_3, \dots, c_n \in \mathbb{R}$. The vectors $u_1, u_2, u_3, \dots, u_n$ are **linearly independents** if

$$\boldsymbol{c_1 u_1 + c_2 u_2 + c_3 u_3 + \cdots + c_n u_n = 0_K \Rightarrow c_1 = c_2 = c_3 = \cdots = c_n = 0}.$$

Applying this definition to the example, and solving the system, we have:

$$c_1(2,3) + c_2(-1,4) = (0,0)$$

$$\begin{cases} 2c_1 - c_2 & = 0 \\ 3c_1 + 4c_2 & = 0 \end{cases} \Leftrightarrow \begin{cases} c_2 & = 2c_1 \\ 3c_1 + 4c_2 & = 0 \end{cases} \Leftrightarrow \begin{cases} c_2 & = 2c_1 \\ 3c_1 + 8c_1 & = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} c_2 & = 2c_1 \\ 11c_1 & = 0 \end{cases} \Leftrightarrow \begin{cases} c_2 & = 2c_1 \\ c_1 & = 0 \end{cases} \Leftrightarrow \begin{cases} c_2 & = 0 \\ c_1 & = 0 \end{cases}$$

Conclusion**: The vectors $(2,3)$ and $(-1,4)$ are linearly independents**.

- **The vectors $(-4,3)$ and $(12,-9)$ are linearly independents?**

Can we find c_1 and c_2 not simultaneously null, that

$$c_1(-4,3) + c_2(12,-9) = (0,0)?$$

Yes. If we consider $c_1 = 2$ and $c_2 = 1$, for example. Meet another values!

But, solving the system, how many solutions we meet?

$$c_1(-4,3) + c_2(12,-9) = (0,0)$$

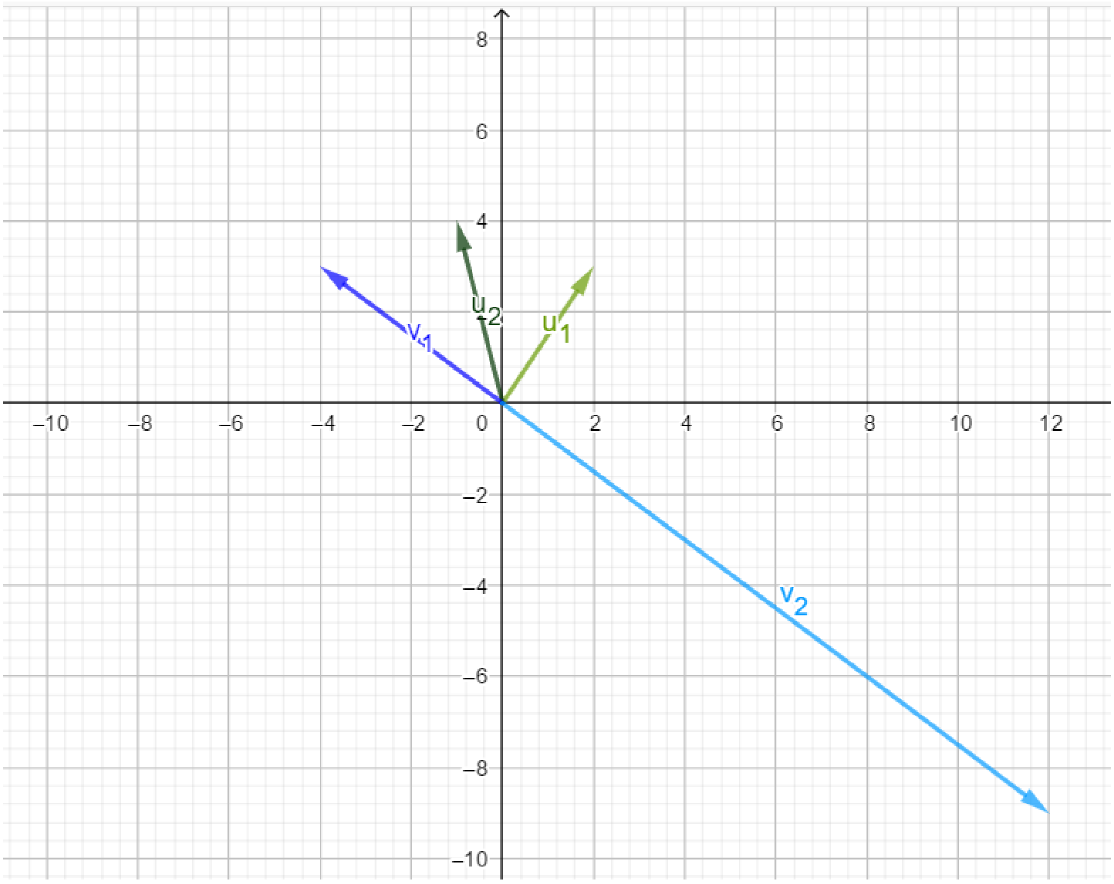
$$\begin{cases} -4c_1 + 12c_2 & = 0 \\ 3c_1 - 9c_2 & = 0 \end{cases} \Leftrightarrow \begin{cases} c_1 & = \dfrac{-12}{-4}c_2 \\ 3c_1 - 9c_2 & = 0 \end{cases} \Leftrightarrow \begin{cases} c_1 & = 3c_2 \\ 3c_1 - 9c_2 & = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} c_1 & = 3c_2 \\ 3 \times 3c_2 - 9c_2 & = 0 \end{cases} \Leftrightarrow \begin{cases} c_1 & = 3c_2 \\ 0 & = 0 \end{cases}$$

Conclusion**:** The system has an infinite number of solutions. So **the vectors $(-4,3)$ and $(12,-9)$ are linearly dependents.**
Note that $(12,-9) = -3(-4,3)$.

➢ What happens geometrically?

Consider $u_1 = (2,3)$, $u_2 = (-1,4)$, $v_1 = (-4,3)$ and $v_2 = (12,-9)$.



The vectors u_1 and u_2 aren't on the same line, they are linearly independents.

The vectors v_1 and v_2 are both in the same line: $y = -\frac{3}{4}x$, they are linearly dependents.

➢ Investigate what happen with three vectors of $\mathbb{R}^2$!

The vectors u_1, u_2 and v_1 are linearly dependents or independents?