

Evaluate $\int_1^9 \frac{y-1}{\sqrt{y}} dy$

- $I = [1, 9]$ is a closed interval.
- $f(x) = \frac{y-1}{\sqrt{y}}$ is continuous on I .
- $$F(x) = \int \frac{y-1}{\sqrt{y}} dy = \int \frac{y}{\sqrt{y}} dy - \int \frac{1}{\sqrt{y}} dy$$
$$= \int \sqrt{y} dy - \int y^{-\frac{1}{2}} dy$$
$$= \frac{2}{3} \cdot y^{\frac{3}{2}} - 2 \cdot \sqrt{y} + C$$

Then, by using Fundamental theorem of Calculus

$$\int_1^9 \frac{y-1}{\sqrt{y}} dy = \left[\frac{2}{3} \cdot y^{\frac{3}{2}} - 2 \cdot \sqrt{y} \right]_1^9 dx$$

$$= \frac{2}{3} \cdot \left[y^{\frac{3}{2}} \right]_1^9 - 2 \cdot \left[\sqrt{y} \right]_1^9 = \frac{40}{3}$$