Geometric transformations
Similarity | Isometry

Geometric Transformation

A geometric transformation T is a correspondence that associates with each point P of the plane one and a single point P' of the plan, under the following conditions:

a) if P and Q are two distinct points, then the corresponding points P' and Q' are also distinct,

b) if R is any point of the plane, then there is a point S in the plane such that its correspondent by geometric transformation T is R.

Similarity

A geometric transformation S is a similarity if it preserves the ratios between lengths of segments, that is, given any three points A, B, and C, the equality $\overline{AB}/\overline{BC} = \overline{A'B'}/\overline{B'C'}$ is verified in which A'=S(A), B'=S(B), C'=S(C).

In a similarity, the distances between each two points are multiplied by a constant ($r = \overline{A'B'}/\overline{AB} = \overline{B'C'}/\overline{BC}$), called scale factor, usually represented by r.

Dilation of centre O and scale factor k

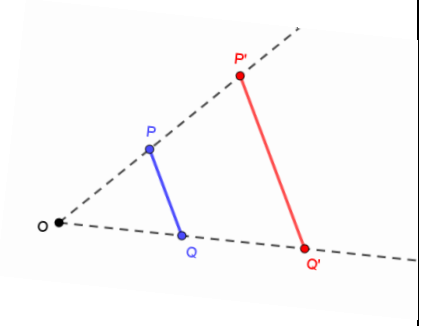
Dilation D of centre O and scale factor k does correspond to each point P of the plane the point P'=D(P) of the plane, under the following conditions:

a) D(O)=O, that is, the centre O is a fixed point for dilation,

b) If P≠O, P' is on the OP line and
- $\overline{OP'}/\overline{OP} = |k|$,
- P and P' are on the same side or on opposite sides, relative to O, as k is positive or negative.

[P'Q'] is the image of [PQ]

O: centre ; k=2



|k|>1, dilation image is larger than original figure – enlargement
|k|=1, dilation image is the same size than original figure – (isometry)
0<|k|<1, dilation image is smaller than original figure – reduction

Dilation involves "resizing" the figure, resulting in an enlargement or a reduction.

Fundamental Mathematics
Elementary Geometry

Isometry (Isometric Transformation)

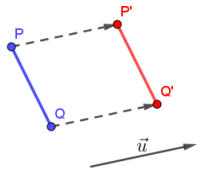
A geometric transformation I is an isometry (isometric transformation) if, for any two points P and Q, it has dist(P',Q')=dist(P,Q), where P'= I(P) and Q'=I(Q).

An isometry preserves the distances, and the figures are transformed into congruent figures.

Translation associated to vector $\vec{u}$

Translation associated to vector $\vec{u}$ is a geometric transformation in which each P point of the plane is transformed into another point P' (image of P), with P'=P+ $\vec{u}$.

[P'Q'] is the image of [PQ]



Translation involves "sliding" the figure from one position to another.

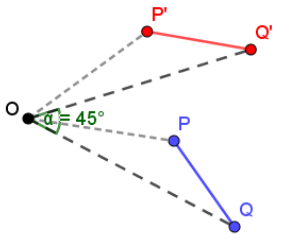
Rotation of centre O and amplitude α

Rotation of centre O and amplitude α is a geometric transformation such that:

a) whatever the point P of the plane, the distance from O to P is equal to the distance from O to the image of P (P'),

b) the amplitude of the oriented angle defined by P, O and P' is equal to α.

[P'Q'] is the image of [PQ]



Rotation involves "turning" the figure around a point (centre of rotation).

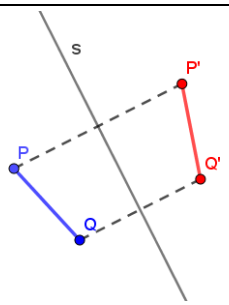
Reflection associated to line s

Reflection associated to line s is the geometric transformation that does correspond to each point P of the plane the point P' (image of P), in such a way that:

a) the line s is perpendicular to [PP'] and passes through the midpoint of [PP'] (or s is the mediator of [PP']),

b) if P belongs to s, its image coincides with P.

[P'Q'] is the image of [PQ]

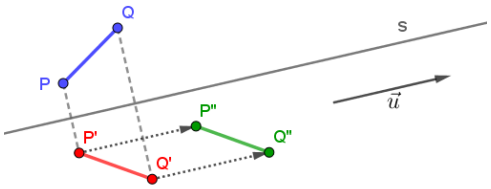


Reflection involves "flipping" the figure over a line (line of reflection).

Glide Reflection associated to line s and vector $\vec{u}$

Glide reflection is the geometric transformation that results from the sequence of a reflection associated to line s with a translation whose vector $\vec{u}$ has the same direction as s.

[P"Q"] is the image of [PQ]



Glide reflection involves "flipping" the figure over a line and "sliding" it, maintaining the direction of the line.