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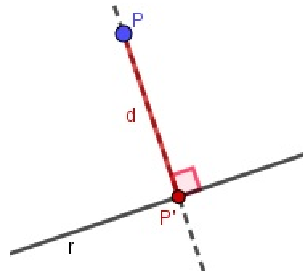
Distances

Distance from a point to a line

The distance from a point A to a line r is equal to the distance from A to its orthogonal projection A' on the line r, according to the figure beside.



We calculate the distance d by doing:

1. Determine the line PP' that is perpendicular to r containing P;
2. Determine $P' = PP' \cap r$;
3. Determine $d = P\bar{P}'$.

Example:

Consider in $\mathbb{R}^3$, $P = (2,1,1)$ and $r : (x,y,z) = (0,0,-1) + k(1,-1,1), \quad k \in \mathbb{R}$. Let us determine the distance from P to r.

For example, $u = (1,2,1)$ is orthogonal to $v = (1,-1,1)$ because $u \cdot v = 0$.

Then, $PP' : (x,y,z) = (2,1,1) + t(1,2,1), t \in \mathbb{R}$.

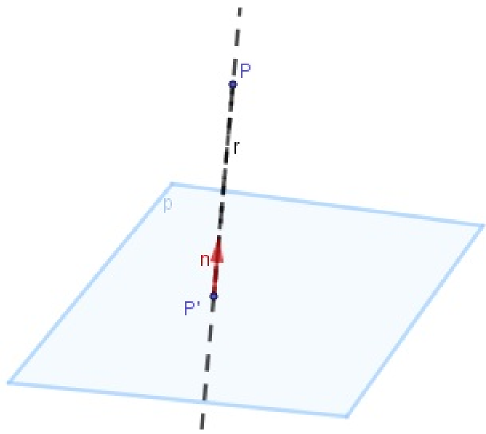
Besides that $P' = (x,y,z) = PP' \cap r$ is such that $\begin{cases} x = \dfrac{y}{-1} = z+1 \\ x-2 = \dfrac{y-1}{2} = z-1 \end{cases} \Leftrightarrow \begin{cases} x = 1 \\ y = -1 \\ z = 0 \end{cases}$.

That is $P' = (1,-1,0)$.

Finally $d = P\bar{P}' = \sqrt{(2-1)^2 + (1+1)^2 + (1-0)^2} = \sqrt{6}$.

Distance from a point to a plane:

We can determine the distance from point P to plane p, by performing:



- Calculate the line r that contains the point P and is normal to the plane p;
- Calculate $P' = r \cap p$;
- Determine the distance from P to P'.

Example: To calculate the distance from $P = (1,2,-1)$ to the plane $p : x - y + z = 0$, we can take

$n = (2, -2, 1) \perp p$ and the line r that contains P and is normal to the plane p is $r : \dfrac{x-1}{2} = \dfrac{y-2}{-2} = z+1$.

$$P' = r \cap p = \left\{ \begin{array}{c} x - 1 = -y + 2 \\ -y + 2 = z + 1 \\ x - y + z = 1 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} x = 2 \\ y = 0 \\ z = 1 \end{array} \right.$$

Then, $d(P, p) = d(P, P') = \sqrt{(1-2)^2 + (2-0)^2 + (-1-1)^2} = 3$

Distance from a straight line to a parallel plane:

Given a line r parallel to a plane p, the distance d from the line r is the distance from any point p on the line to the plane, that is,

$$d(r, p) = d(P, p)$$

Example: To calculate the distance from $r : \dfrac{x-1}{2} = -y = \dfrac{z+1}{-3}$ to the plane $p : x - y + z = 0$ is the distance from $P = (1, 0, -1) \in r$ to the plane p.

Distance between two parallel planes:

To calculate the distance between two planes α and β parallel to each other, we can perform:

- Calculate the line r that is normal to the planes α and β;
- Calculate $A = r \cap \alpha$;
- Calculate $B = r \cap \beta$;
- Determine the distance from A to B.