Determine $|\mathcal{P}(\mathcal{P}(\{\phi, \tau\}))|$.

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Subset

Set A is a subset of set B iff each element of set A is also an element of set B. If set A is a subset of set B then we write as $A \subset B$.

1. If each element of set A is also an element of set B and B may be equal to A, then set A is an **improper subset** of set B.

   For example: $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2, 3, 4, 5\}$ then $A \subseteq B$ and $B \subseteq A$.

2. If each element of set A is also element of set B but set B is not equal to set A then Set A is **proper subset** of set B.

   For example: $A = \{2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5\}$ then $A \subset B$ but $A \not\subset B$

Properties of Subset

Properties

1. A set with n elements has 2^n subsets.
2. Every set is subset of itself.
3. Empty set ($\emptyset$) is subset of every set.
4. t $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
5. A is a subset of B if and only if their intersection is equal to A, that is,
   $A \subseteq B \iff (A \cap B) = A$
6. Set A is a subset of B if and only if their union is equal to B, that is,
   $A \subseteq B \iff (A \cup B) = B$

Example

What are the subsets of set $A = \{x, y, z\}$?

- $\emptyset$
- $\{x\}$
- $\{y\}$
- $\{z\}$
- $\{x, y\}$
- $\{x, z\}$
- $\{y, z\}$
- $\{x, y, z\}$

Notice, there are 8 subsets of set A which is also the result of $= 2^{|A|} = 2^3 = 8$

Superset

A set A is a superset of another set B if all elements of the set B are elements of the set A. The notation for superset is $A \supset B$.

Properties

- $A \supset \emptyset$.
- Since every set is a subset of itself, then every set is also a superset of itself.

Power Set

The set of all subsets of a set A is called the power set of A. The power set of A is denoted with the symbol $\mathcal{P}(A)$.

Example

If A is the set $\{1, 2, 3\}$, then what is $\mathcal{P}(A)$?

$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

Determine $|\mathcal{P}(\mathcal{P}(\{\phi, \tau\}))|$

As we know, for any set A, $|\mathcal{P}(A)| = 2^{|A|}$.

In this case,

$|\{\phi, \tau\}| = 2$ Therefore,

$$|\mathcal{P}(\{\phi, \tau\})| = 2^2 = 4$$
$$|\mathcal{P}(\mathcal{P}(\{\phi, \tau\}))| = 2^4 = 16$$

So, $|\mathcal{P}(\mathcal{P}(\{\phi, \tau\}))| = 16$.