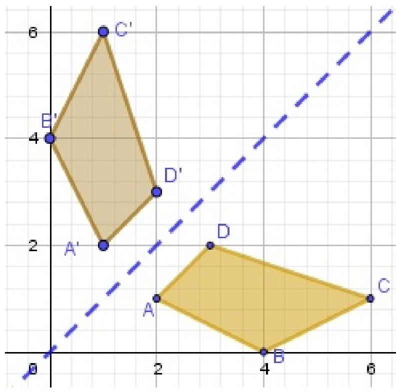
Reflections and translations

Computer graphics deals with the manipulation of images, through their positioning through linear transformations such as reflections, dilations and contractions, orthogonal projections and rotations.

Some reflections

The linear operator $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (y, x)$ translates a reflection around the line $y = x$, according to the figure beside. In fact:

- $T(A) = T(2, 1) = (1, 2) = A'$;
- $T(B) = T(4, 0) = (0, 4) = B'$;
- $T(C) = T(6, 1) = (1, 6) = C'$;
- $T(D) = T(3, 2) = (2, 3) = D'$.



Following are some of some of the most common reflections and their matrix representations when considering the canonical basis

Reflection in $\mathbb{R}^2$	Operator	Matrix
around of the OX axis	$T(x, y) = (x, -y)$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
around of the Oy axis	$T(x, y) = (-x, y)$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

Reflection in $\mathbb{R}^3$	Operator	Matrix
around the xy-plane	$T(x, y, z) = (x, y, -z)$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
around the yz-plane	$T(x, y, z) = (-x, y, z)$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
around the xz-plane	$T(x, y, z) = (x, -y, z)$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

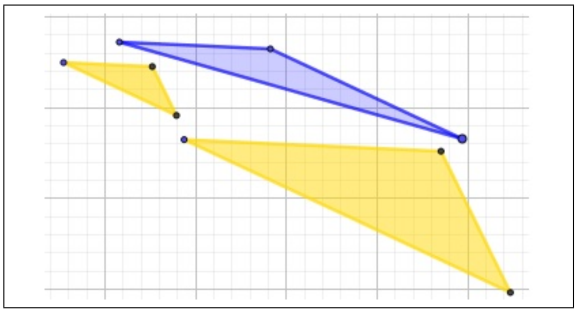
Dilation and contraction

Dilation or contraction is the operator stretching or shrinking a vector by a factor $k \in \mathbb{R}^+$, but keeping the direction unchanged. We call the operator a dilation if the transformed vector is at least as long as the original vector, and a contraction if the transformed vector is at most as long as the original vector.

The linear operator f such that

$$f(x,y) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- is a contraction with factor k on $\mathbb{R}^2$, if $0 < k < 1$.
- is a dilatation with factor k on $\mathbb{R}^2$, if $k > 1$.



The linear operator $f : \mathbb{R}^3 \to \mathbb{R}^3$ such that

$$f(x,y,z) = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- is a dilation on $\mathbb{R}^3$, if $k > 1$.
- is a contraction on $\mathbb{R}^3$, if $0 < k < 1$.

Example: The linear operator f such that

$$f(x,y) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

is a contraction with factor $1/2$.