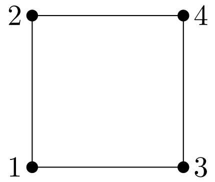
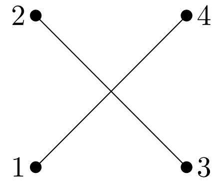
Complementary graphs

The **complementary graph** $\bar{G}$ of a simple graph G has the same vertices as G and two vertices are adjacent in $\bar{G}$ if and only if they are not adjacent in G. Describe each of these graphs.

Example 1. The following graphs are complementary.



G



$\bar{G}$

Isomorphic graphs

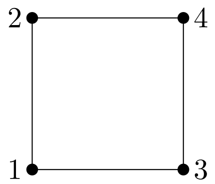
There is a distinction between a graph and its picture and it is importance to know when two graphs are the same or different, meaning that they differ only in the way they are labeled or drawn.

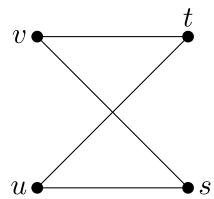
Let G and G' be graphs with vertex sets $V(G)$ and $V(G')$ and edge sets $E(G)$ and $E(G')$, respectively.

G is **isomorphic** to G' if, and only if, there exist one-to-one correspondences $f : V(G) \to V(G')$ and $g : E(G) \to E(G')$ that preserve the edge endpoint functions of G and G' in the sense that for all $v \in V(G)$ and $e \in E(G)$, v is an endpoint of $e \Leftrightarrow f(v)$ is an endpoint of $g(e)$.

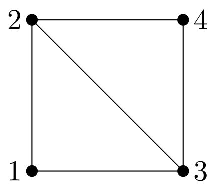
Two graphs isomorphic are equal, meaning the same graph.

Example 2. The following graphs are isomorphic.

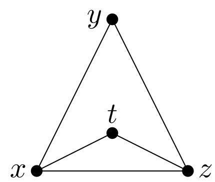




Exercise 1. The graphs pictured are isomorphic? Justify.



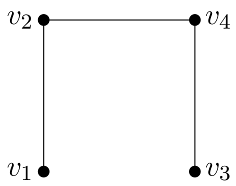
G



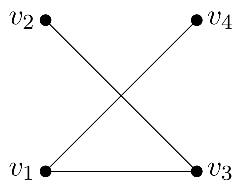
G'

Solution:
To solve this problem, you must find functions $f : V(G) \to V(G')$ *and* $g : E(G) \to E(G')$ *such that for all* $v \in V(G)$ *and* $e \in E(G)$, v *is an endpoint of* e *if, and only if,* $f(v)$ *is an endpoint of* $g(e)$. *Setting up such functions is partly a matter of trial and error and partly a matter of deduction.*
The function f *with* $f(1) = t$, $f(2) = x$, $f(3) = z$, *and* $f(4) = y$ *is a one-to-one correspondence between* V *and* V'. *To see that this correspondence preserves adjacency, note that adjacent vertices in* G *are 1 and 2, 1 and 3, 2 and 3, 2 and 4, and 3 and 4, and each of the pairs* $f(1) = t$ *and* $f(2) = x$, $f(1) = t$ *and* $f(3) = z$, $f(2) = x$ *and*$f(3) = z$, $f(2) = x$ *and* $f(4) = y$ *and* $f(3) = z$ *and* $f(4) = y$ *consists of two adjacent vertices in* G', *thus, or all* $v \in V(G)$ *and* $e \in E(G)$, v *is an endpoint of* $e \Leftrightarrow f(v)$ *is an endpoint of* $g(e)$.

Example 3. The following graph are complementary and isomorphic graphs.



G



$\bar{G}$

References

[1] Domingos Cardoso, Jerzy Szymanski, and Mohammad Rostami. *Matemática Discreta: Combinatória, Teoria dos Grafos, Algoritmos.* Escolar Editora, 2009.

[2] Edgar Goodair and Michael Parmenter. *Discrete Mathematics with Graph Theory.* (3rd Ed.) Pearson, 2006.

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