Integration by Trigonometric Substitution

Example

$$\int \sqrt{1 - x^2}\, dx = ... = \int \sqrt{1 - \sin^2(t)}\cos(t)\, dt = ...$$

Do the following substitution:

$$\sin(t) = x$$
$$\Rightarrow \quad \cos(t)\, dt = dx$$

- This substitution leads to a simplification in the radicand.
- At the end it is necessary to apply an inverse substitution (to obtain the first variable).

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Table of Trigonometric Substitution

Integrand	Substitution	Trigonometric Identity
$\sqrt{1-f^2}$	$f = \sin(t)$ $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$	$1 - \sin^2(t) = cos^2(t)$
$\sqrt{1+f^2}$	$f = \text{tg}(t)$ $-\frac{\pi}{2} < t < \frac{\pi}{2}$	$1 + \text{tg}^2(t) = \sec^2(t)$
$\sqrt{f^2-1}$	$f = \sec(t)$ $0 \leq t < \frac{\pi}{2}$ ou $\pi \leq t < \frac{3\pi}{2}$	$\sec^2(t) - 1 = \text{tg}^2(t)$