Evaluate $\int_{-1}^{0} x\sqrt{1+x}\,dx$

* All the conditions required for fundamental theorem of calculus are fulfilled.

- $I(x) = \int \underbrace{x}_{g(x)} \underbrace{\sqrt{1+x}}_{f(x)}\,dx$

A.C.I,

In this case, 'x' should be $g(x)$ because it reduces the degree to '0', thus avoiding the repetition of integration by parts.

$$\int f(x)\cdot g(x)\,dx = F(x)\,g(x) - \int F(x)\,g'(x)\,dx$$

$$g'(x) = (x)' = 1$$

$$F(x) = \int f(x)\,dx = \int \sqrt{1+x}\,dx$$

$$= \frac{2(1+x)^{3/2}}{3} + C$$

- $I(x) = \int x\cdot\sqrt{1+x}\,dx$

$$\overset{A.C}{=} \frac{2x}{3}(1+x)^{3/2} - \frac{2}{3}\int (1+x)^{3/2}\,dx$$

$$= \frac{2x}{3}(1+x)^{3/2} - \frac{2}{3}\times\frac{(1+x)^{5/2}}{\frac{5}{2}} + C$$

$$= \frac{2x}{3}(1+x)^{3/2} - \frac{4}{15}(1+x)^{5/2} + C$$

Now,

$$\int_{-1}^{0} x\sqrt{1+x}\,dx = \int_{-1}^{0} I(x)\,dx$$

$$= \left[\frac{2x}{3}(1+x)^{3/2} - \frac{4}{15}(1+x)^{5/2}\right]_{-1}^{0}$$

$$= 0 - \frac{4}{15} - (0-0)$$

$$= -\frac{4}{15} \;//$$