Manipulation of Algebraic expressions

Solving Logarithmic Equations

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Remember!

- When solving log equations, always check that each term has the same base. If this is not the case, the change of base rule must first be used to change to a common base.

- If no base is given, the equation holds true for all bases.

- If $log_a b = log_a c$, then $b = c$.

- If $log_a b = k$, then $b = a^k$

- Check all solutions to make sure they do not produce logs of negative numbers as these are not defined!

The Laws of Logarithms

1. $\log_a xy = \log_a x + \log_a y$

2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

3. $\log_a x^n = n\log_a x$

4. $\log_a a = 1$

5. $\log_a 1 = 0$

6. $\log_a x = \dfrac{\log_b x}{\log_b a}$

Worked Example 1

Solving Logarithmic Equations

$$\mathbf{\log_5 x - \log_5 (10 - x) = 1}$$

- $\underbrace{\log_5 x - \log_5}(10 - x) = 1$
- $\log_5 \frac{x}{10-x} = 1$
- $\frac{x}{10-x} = 5^1$
- $\frac{x}{10-x} = 5$

This law was applied here:

$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

Covert to indices

$\log_a b = k$, then $b = a^k$

Solving Logarithmic Equations

- $5(10 - x) = x$
- $50 - 5x = x$
- $-x - 5x + 50 = 0$
- $-6x = -50$
- $6x = 50$
- $x = \frac{50}{6}$
- $x = 8.33$

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Worked Example 2

Solving Logarithmic Equations

Example 2

- $\log_3 x + 3\log_x 3 = 4$

Note both terms have different bases! we need to change base 3 to base x or visa versa.

- $\log_3 x = \left(\frac{\log_x x}{\log_x 3}\right) = \left(\frac{1}{\log_x 3}\right)$

$\log_x x$=1

- $\left(\frac{1}{\log_x 3}\right) + 3\log_x 3 = 4$
- Using the substitution $\log_x 3 = y$
- $\frac{1}{y} + 3y = 4$

Solving Logarithmic Equations

Example 2 cont'd

- $1 + 3y^2 \; = 4y$
- $3y^2 - 4y + 1 = 0$
- $(3y - 1)(y - 1) \; = 0$
- Therefore,
- $y = 1 \; or \; y = \frac{1}{3}$
- Remember we substituted $\log_x 3 = y$

Solving Logarithmic Equations

Example 2: cont'd

- $\log_x 3 = y, where\ y\ is\ equal\ to\ 1\ and\ \frac{1}{3}$

$\log_x 3 = 1$	$\log_x 3 = \frac{1}{3}$
$x^1 = 3$	$x^{\frac{1}{3}} = 3$
$x = 3$	$x = 27$

Both solutions give positive logs and thus are acceptable
Note this example could also be repeated using the base 3 instead of the base *x*. The same results are achieved.

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