What is $A \cup \emptyset$ and $A \cap \emptyset$?

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Union and Intersection

Union

1. The union of two sets is the set containing all of the elements from both of those sets. It is represented by the symbol $\cup$.
2. $A \cup B = \{x \mid x \in A \text{ and } x \in B\}$

Intersection

1. The intersection of two sets is the set containing just the elements that are in both of those sets. It is represented by the symbol $\cap$.
2. $A \cap B = \{x \mid x \in A \text{ or } x \in B\}$

Union and Intersection - Venn Diagram

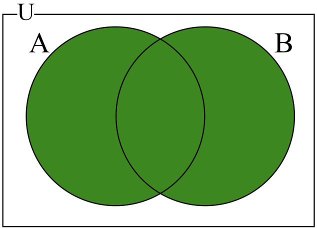

Figure 1: $A \cup B$

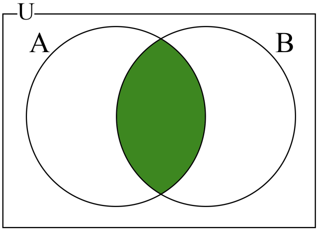

Figure 2: $A \cap B$

Union and Intersection - Example

If $A = \{1, 3, 5, 7, 9, 11, 13, 15\}$ **and**
$B = \{2, 3, 5, 7, 11, 13\}$**,**
then:

- $A \cup B = \{1, 2, 3, 5, 7, 9, 11, 13, 15\}$
- $A \cap B = \{3, 5, 7, 11, 13\}$

What is $A \cup \emptyset$ and $A \cap \emptyset$?

Finding $A \cup \emptyset$.

The empty set is the set with no elements so, the union of any set A and the $\emptyset$ is always going to be A.

$$A \cup \emptyset = A$$

Finding $A \cap \emptyset$.

An empty set is a set with no elements so, the intersection of any set A and $\emptyset$ is always going to be $\emptyset$ as there is no element simultaneously belonging to both the sets.

$$A \cap \emptyset = \emptyset$$

Other Set Operations

Difference

1. The difference of any two sets A and B written as $A - B$ which is the set containing the elements that are in A but not in B.
2. $A - B = \{x \mid x \in A \text{ and } x \notin B\}$
3. For two disjoint sets A and B, $A - B = A$ and $B - A = B$.

Compliment

1. For a set A in a universe U, the compliment of A or $\overline{A}$ is set of all the elements that are in the universe but not in A.
2. $\overline{A} = \{x \in U \mid x \notin A\}$

Difference and Compliment - Venn Diagram

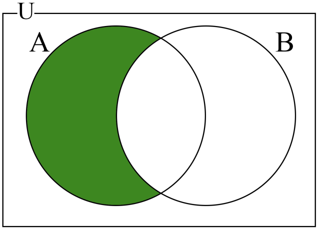

Figure 3: $A - B$

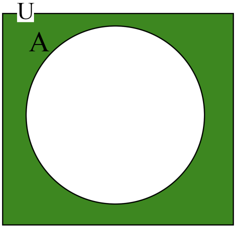

Figure 4: $\overline{A}$

Difference - Example

Consider $A = \{a, e, i, o, u\}$ **and** $B = \{a, b, c, d, e\}$**,**
and find $A - B$ **and** $B - A$**.**

- $A - B = \{i, o, u\}$
- $B - A = \{b, c, d\}$

Notice that $A - B \neq B - A$

Compliment - Example

If the universe U is the set of letters in the English alphabet and A is the set of the consonant letters of the same alphabet, what is $\overline{A}$?

- $\overline{A} = \{a, e, i, o, u\}$
- Also, $\overline{U} = \emptyset$ and $\overline{\emptyset} = U$

Properties of Set Operations

Commutative

1. $A \cup B = B \cup A$
2. $A \cap B = B \cap A$

Associative

1. $A \cup (B \cup C) = (A \cup B) \cup C$
2. $A \cap (B \cap C) = (A \cap B) \cap C$

Idempotent

1. $A \cup A = A$
2. $A \cap A = A$

De Morgan's Law

1. $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$
2. $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$

Click here to check the proof here