Change of variables

Definition of Jacobian

Let $T : D^* \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ of class $\mathscr{C}^1$ defined by $x = x(u, v)$ e $y = y(u, v)$. The **Jacobian** of T, denoted by $J = \frac{\partial(x,y)}{\partial(u,v)}$ is the determinant of the matrix $DT(u, v)$:

$$J = \left|\frac{\partial(x, y)}{\partial(u, v)}\right| = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

Change of variables

Theorem

Let D and D^* be elementary regions (in the plane) and $T : D^* \longrightarrow D$ a function of $\mathscr{C}^1$ class and e tal que $D = T(D^*)$.
Then, for all integrable function $f : D \longrightarrow \mathbb{R}$:

$$\iint_D f(x, y)\,\mathrm{d}x\,\mathrm{d}y = \iint_{D^*} f(x(u, v), y(u, v)) \times |J|\,\mathrm{d}u\,\mathrm{d}v$$

Changing to polar coordinates

Polar Coordinates

A coordinate system represents a point in the plane by a pair of real numbers denominated coordinates.

Examples

- Cartesian coordinates: (x, y)
- Polar coordinates:

- We must choose a point in the plane for origin or **polo**; let it be $\mathscr{O}$.
- We draw a line (from left to right) with origin in $\mathscr{O}$ — the **polar axis**.
- P is a point in the plane with ρ the distance from $\mathscr{O}$ to P and θ the angle between the polar axis and the line $\overline{\mathscr{O}P}$.

Polar Coordinates

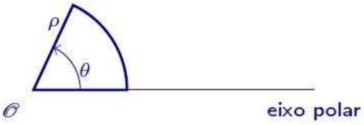
Changing to Polar Coordinates

We have:

- **polo** $= (0, 0)$.
- $x = \rho \cos(\theta)$
- $y = \rho \operatorname{sen}(\theta)$

or, generally,

- **polo** $= (x_0, y_0)$.
- $x - x_0 = \rho \cos(\theta)$
- $y - y_0 = \rho \operatorname{sen}(\theta)$



Elementary Polar Region

Theorem

If f é is continuous in the polar rectangle R_p defined by:
$\rho \in [a, b]$ and $\theta \in [\alpha, \beta]$, with $0 \leq \beta - \alpha \leq 2\pi$ then,

$$\iint_{R_p} f(x, y)\, dA = \int_{\alpha}^{\beta} \int_{a}^{b} f(\rho\cos(\theta), \rho\cos(\theta)) \cdot \rho\, d\rho\, d\theta$$

General polar region

Theorem

If f é is continuous in the polar rectangle D defined by:
$\theta \in [\alpha, \beta]$ and $h_1(\theta) \leq \rho \leq h_2(\theta)$ then,

$$\iint_D f(x,y)\,\mathrm{d}A = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(\rho\cos(\theta), \rho\cos(\theta)) \cdot \rho\,\mathrm{d}\rho\,\mathrm{d}\theta$$