HINT:

Find $\int e^x \sin x \, dx$

Solution Take e^x as the first function and $\sin x$ as second function. Then, integrating by parts, we have

$$I = \int e^x \sin x \, dx = e^x(-\cos x) + \int e^x \cos x \, dx$$

$$= -e^x \cos x + I_1 \text{ (say)} \quad \ldots (1)$$

Taking e^x and $\cos x$ as the first and second functions, respectively, in I_1, we get

$$I_1 = e^x \sin x - \int e^x \sin x \, dx$$

Substituting the value of I_1 in (1), we get

$$I = -e^x \cos x + e^x \sin x - I \text{ or } 2I = e^x(\sin x - \cos x)$$

Hence, $$I = \int e^x \sin x \, dx = \frac{e^x}{2}(\sin x - \cos x) + C$$

Alternatively, above integral can also be determined by taking $\sin x$ as the first function and e^x the second function.