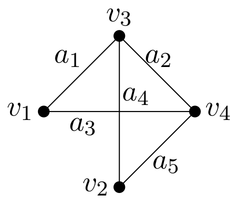
Walks, trails and paths

A **walk** in a graph G consists of an alternating (non empty) sequence of vertices and edges. $P = v_0 e_1 v_1 e_2 v_2 \ldots e_k v_k$, such that $v_0, v_1, \ldots, v_k \in V$, $e_1, e_2, \ldots, e_k \in E$ and vertices v_{i-1} and v_i are endpoints for edge e_i, with $i = 1, \ldots, k$. The vertex v_0 is called **inicial vertex**, the vertex v_k is called **end vertex** and the other $v_1, \ldots, v_{k-1}$ are intermediate vertices of the walk P.

The **lenght** of the walk P is the number of edges in the sequence, eventually repeated and it is denoted by **comp(**P**)**.

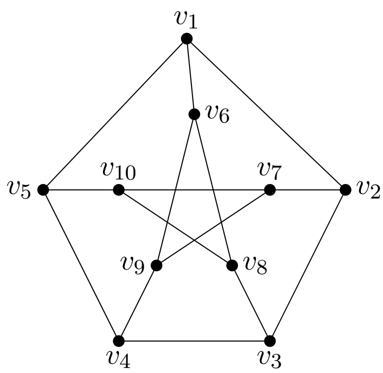
A **trail** is a walk in which all edges are distinct; a **path** is a walk in which all vertices are distinct.
A closed trail is called a **circuit**. A circuit in which the first vertex appears exactly twice (at the beginning and the end) and in which no other vertex appears more than once is a *cycle*.

Example 1. $v_1 v_3 v_4 v_2 v_3$ is a walk with lenght four in the following graph.



Because the graph is simple its enough to refer the vertices. The walk is also a trail because don't repeat edges but is not a path because the vertex v_3 appear twice.

Exercise 1. Considering the connected Petersen graph pictured, give an example of:



a. a walk that is not a path starting at vertex v_3;

b. a walk that is not a trail;

c. a cycle with lenght five.

Solution

a. *To be a walk that is not a pat,h starting at vertex 3 it must have repeated vertices, for example,* $v_3v_8v_6v_9v_7v_2v_3v_8v_{10}$.

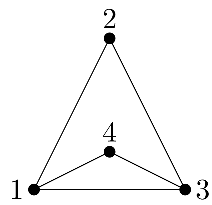
b. *To be a walk but not a trail, the previous example works, because the edge v3v8 is repeated.*

c. *An example of a cycle with lenght five, could be,* $v_6v_8v_{10}v_7v_9v_6$ *and also, the more evident,* $v_1v_2v_3v_4v_5v_1$.

Connectivity

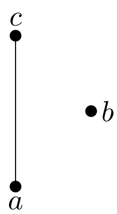
Let G be a graph. Two vertices v and w of G are connected if, and only if, there is a walk from v to w. The graph G is **connected** if, and only if, given any two vertices v and w in G, there is a walk from v to w.

Example 2. The graph G_1 is connected because there is a walk between all pairs of vertices.



G_1

Example 3. The graph pictured is not connected, because there is no walk, for example, between the vertex a and the vertex b.

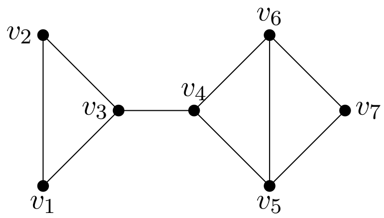


A graph H is a **connected component** of a graph G if, and only if,

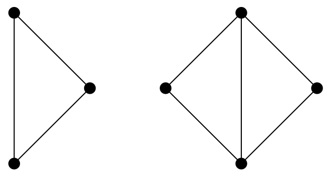
- H is subgraph of G;
- H is connected; and
- no connected subgraph of G has H as a subgraph and contains vertices or edges that are not in H.

A **bridge** is an edge that when deleted from the graph is obtained two connected component of the original graphs.

Example 4. The graph pictured is a connected graph with a bright, the edge v_3v_4.



If we delete the edge v_3v_4 we obtain the following subgraph with two connected components.



References

[1] Edgar Goodair and Michael Parmenter. *Discrete Mathematics with Graph Theory.* (3rd Ed.) Pearson, 2006.

[2] Kenneth H. Rosen. *Discrete Mathematics and its Application.* (7th Ed.) McGraw Hill, 2012.

Exercises in MathE platform