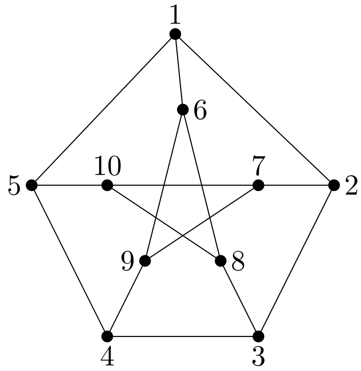
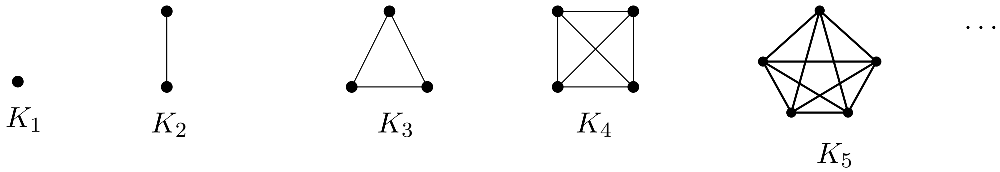
Particular Graphs

A simple graph is called **regular** if every vertex of this graph has the same degree. A regular graph is called n-**regular** if every vertex in this graph has degree n.

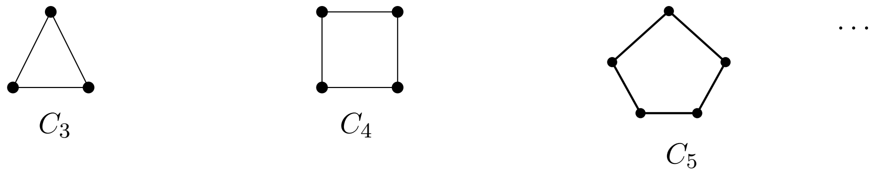
Example 1. The graph pictured is called Petersen graph and it is a 3-regular graph.



A simple graph is called **complete** on n vertices, denoted by K_n, if it contains exactly one edge between each pair of distinct vertices.

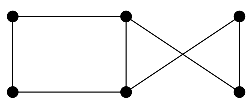


A **cycle** C_n, $n \geq 3$, consists of n vertices $v_1,\ v_2,\ \cdots,\ v_n$ and edges $\{v_1,\ v_2\}$, $\{v_2,\ v\ 3\}$, $\cdots$, $\{v_{n-1},\ v_n\}$ and $\{v_n,\ v_1\}$.



A simple graph G is called **bipartite** if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and a vertex in V_2 (so that no edge in G connects either two vertices in V_1 or two vertices in V_2). When this condition holds, we call the pair $(V_1,\ V_2)$ a bipartition of the vertex set V of G.

Example 2. The following graph is bipartite.



Exercise 1. The cycle C_6 is bipartite?

Solution:

Yes, the cycle C_6 is a bipartite graph because its vertex set can be partitioned into the two sets $V_1 = \{v_1, v_3, v_5\}$ and $V_2 = \{v_2, v_4, v_6\}$, and every edge of C_6 connects a vertex in V_1 and a vertex in V_2.

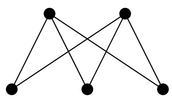
Exercise 2. Show that K_3 is not bipartite.

Solution:

To show that K_3 is not bipartite, note that if we divide the vertex set of K_3 into two disjoint sets, one of the two sets must contain two vertices. If the graph were bipartite, these two vertices could not be connected by an edge, but in K_3 each vertex is connected to every other vertex by an edge.

A **complete bipartite** graph $K_{m,n}$ is a graph that has its vertex set partitioned into two subsets of m and n vertices, respectively with an edge between two vertices if and only if one vertex is in the first subset and the other vertex is in the second subset.

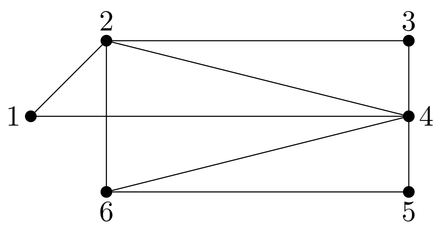
Example 3. The following graph displayed is the complete bipartite graph $K_{2,\,3}$



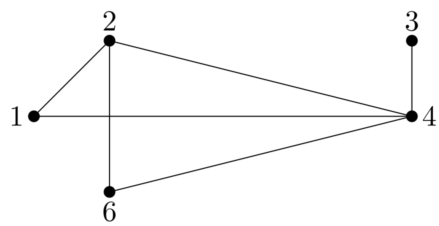
Subgraphs

A graph H is a **subgraph** of another graph G if, and only if, every vertex in H is also a vertex in G, every edge in H is also an edge in G, and every edge in H has the same endpoints as it has in G.

Example 4. The graph H is a subgraph of the graph G.



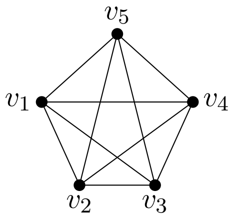
G



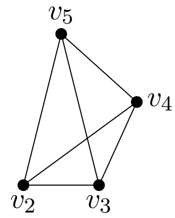
H

Let $G = (V\,, E)$ be a simple graph. The **subgraph induced** by a subset W of the vertex set V is the graph $H = (W\,, F)$, where the edge set F contains an edge in E if and only if both endpoints of this edge are in W.

Example 5. Considering the complete graph K_5 the subgraph induced by $W = \{v_2, v_3, v_4, v_5\}$ is the graph H pitcured



K_5



H

Exercise 3. In the Example 4 the subgraph H is not an induced subgraph of graph $G = (V, E)$, why?
Solution:
Considering the subset $W = \{1, \, , 2\, , 3\, , 4, \; 6\}$ *from the vertex set* V *is missing an edge, the edge 23.*

References

[1] Domingos Cardoso, Jerzy Szymanski, and Mohammad Rostami. *Matemática Discreta: Combinatória, Teoria dos Grafos, Algoritmos*. Escolar Editora, 2009.

[2] Susanna Epp. *Discrete Mathematics and Applications.* (4th Ed.) Brooks/Cole CENGAGE Learning, 2011.

Exercises in MathE platform