MathE - project

Differential Equation with Separable Variables

Definition *Let f_1, f_2 defined on $a \leq x \leq b$ and g_1, g_2 defined on $c \leq y \leq d$, continue, such that $f_2(x) \neq 0, \quad g_2(y) \neq 0$. A differential equation of the form*

$$f_1(x)g_1(y) + f_2(x)g_2(y)y' = 0 \Leftrightarrow f_1(x)g_1(y)dx + f_2(x)g_2(y)dy = 0. \quad (1)$$

called equation with separable variable.

For this equation we have the next result

Proposition *The general solution of the equation* (1) *is given by an implicit function in the following form*

$$\int \frac{f_1(x)}{f_2(x)}dx + \int \frac{g_1(y)}{g_2(y)}dy = C, \quad C \in R$$

Proof. It is easy to see that the equation (1) can be rewrite as follows

$$\frac{f_1(x)}{f_2(x)} = -\frac{g_1(y)}{g_2(y)},$$

and by integrating each side of the above equation we obtain the desired result.

Example. Integrate the next equation

$$(1 + x^2)dy + ydx = 0$$

Solution. The equation above is of the separable variable, and thus we have

$$(1+x^2)dy + ydx = 0, \quad \rightarrow \quad \frac{dx}{1+x^2} = -\frac{dy}{y} \quad \rightarrow \quad \int \frac{dx}{1+x^2} = -\int \frac{dy}{y} \quad \rightarrow$$

$$arctgx = -\ln y + C \quad \rightarrow \quad arctgx + \ln y = C$$

Exercises. Solve the next differential equations

$$1.\ \ yy' = -2xcosecy, \qquad 2.\ \ y' + \cos(x + 2y) = \cos(x - 2y),$$

$$3.\ \ 2x(2\cos y - 1)dx = (x^2 - 2x + 3)dy, \qquad 4.\ \ y' = \frac{\cos y - \sin y - 1}{\cos x - \sin x - 1}$$

5. $yy' + xe^y = 0, \quad y(1) = 0,$ 6. $y' = e^{x+y} + e^{x-y}, \quad y(0) = 0$

7. $\dfrac{dx}{x(y-1)} + \dfrac{dy}{y(x+2)}, \quad y(1) = 1,$ 8. $\dfrac{y^2+4}{\sqrt{x^2+4x+13}} = \dfrac{3y+2}{x+1} y', \ y(1) = 2$