1E3101

Roll No [redacted]

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1E3101

B. Tech. I - Sem. (Main / Back) Exam., - 2023

1FY2 – 01 Engineering Mathematics - I

Time: 3 Hours **Maximum Marks: 70**

Instructions to Candidates:

Attempt all ten questions from Part A, five questions out of seven questions from Part B and three questions out of five from Part C.

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used /calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL 2. NIL

PART – A [10×2=20]

(Answer should be given up to 25 words only)

All questions are compulsory

Q.1 Find the limit of the sequence $<x_n>$, where $x_n = \frac{5n-3}{7n+8}$.

Q.2 Write the power series expansion of logarithm function.

Q.3 Evaluate a_n in the Fourier series of the function $f(x) = x + x^2$, $-\pi < x < \pi$.

Q.4 Define Cauchy's $(E - \delta)$ definition of continuity.

Q.5 Write Euler's theorem on homogeneous function.

Q.6 Evaluate: $\int_0^\infty x^6 e^{-2x} dx$ by using beta – gamma function.

Q.7 Evaluate: $\iint xy\, dx\, dy$, where the region of integration is $x + y < 1$ in the positive quadrant.

Q.8 Change the order of integration of the following double integration:

$\int_0^4 \int_x^{2\sqrt{x}} f(x, y)dxdy$

Q.9 If $\vec{f} = x^2y\hat{i} - 2xy^2z\,\hat{j} + 3x^2z\hat{k}$, find div f at the point (3, –1, –2).

Q.10 State Stokes theorem. **[5×4=20]**

PART – B

(Analytical/Problem solving questions)

Attempt any five questions

Q.1 Prove that –

$$B(m,n) = \frac{\lceil m \lceil n}{\lceil (m+n)}$$

Q.2 Test the convergence of the following series –

$\frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \text{-------}$

Q.3 Find a Fourier series for the function $f(x) = x^2$ in the interval $-\pi < x < \pi$ and deduce the following: $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$

Q.4 Find the equations of the tangent plane and normal to the surface –

$x^3 + y^3 + 3xyz = 3$ at the point (1, 2, –1).

Q.5 Evaluate the point where the function –

$x^3y^2(1 - x - y)$

Will have maxima. Also find the maximum value.

Q.6 Evaluate the integral –

$\int_0^1 \int_0^x \frac{x^3 dxdy}{\sqrt{x^2+y^2}}$

by changing into polar coordinates.

Q.7 If $\vec{a}$ and $\vec{b}$ are differentiable vector point functions, then prove that –

$\text{div}\left(\vec{a} + \vec{b}\right) = \vec{b}.\text{curl}\,\vec{a} - \vec{a}.\text{curl}\,\vec{b}$

PART – C

[3×10=30]

(Descriptive/Analytical/Problem Solving/Design Questions)

Attempt any three questions

Q.1 Find the volume of spindle shaped solid generated by revolving the Astroid about the x – axis –

$$x^{2/3} + y^{2/3} = a^{2/3}$$

Q.2 If $u = \log x^3 + y^3 + z^3 - 3xyz$, then prove that –

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$$

Q.3 Find half range cosine series for the function –

$f(x) = 2x - 1, 0 < x < 1$

hence deduce that –

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \text{-------}$$

Q.4 Find the volume of the tetrahedron bounded by the co – ordinate planes and the plane –

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Q.5 State Gauss's divergence theorem. Verify Gauss's divergence theorem for $\vec{F} = xy\,\hat{i} + z^2\hat{j} + 2yz\hat{k}$ on the tetrahedron $x = y = z = 0$ and $x + y + z = 1$.

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B. Tech. I - Sem. (New Scheme) Main Exam., July – 2022

1FY1 – 01 Engineering Mathematics – I

Common to all Branches

Time: 2 Hours

Maximum Marks: 70

Min. Passing Marks:

Instructions to Candidates:

Part – A: Short answer questions (up to 25 words) 5×3 marks = 15 marks. Candidates have to answer 5 questions out of 10.

Part – B: Analytical/Problem Solving questions 3×5 marks = 15 marks. Candidates have to answer 3 questions out of 7.

Part – C: Descriptive/Analytical/Problem Solving questions 2×20 marks = 40 marks. Candidates have to answer 2 questions out of 5.

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL 2. NIL

PART– A

Q.1 What is the largest interval of x for which $f(x) = xe^{x^2}$ is concave upward?

Q.2 Find the points of inflexion of the curve $y = (x - 2)^2 (x - 3)^5$.

Q.3 Find the radius of curvature at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the Folium of Descartes

$x^3 + y^3 = 3axy, a > 0.$

Q.4 If $u = \sec^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u.$

Q.5 Solve the partial differential equation $p(1 + q) = 3q$.

Q.6 Solve the differential equation $ydx - xdy + x^2 \cos x\, dx = 0$

Q.7 If e^x is one of the linearly independent solution for the differential equation

$$x\frac{d^2y}{dx^2} - (2x - 1)\frac{dy}{dx} + (x - 1)\, y = 0,$$

Find the second linearly independent solution.

Q.8 Write a short note on double points.

Q.9 Find the values of p and q in the PDE $z^2(p^2 + q^2) = x^2 + y^2$ in term of x, y, z and arbitrary constant.

Q.10 Find the asymptotes of $y^2 (x - b) = x^3 + a^3,\ a, b > 0$.

PART– B

Q.1 Discuss the maxima and minima of the function $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$.

Q.2 Trace the Cartesian curve $y^2 (a + x) = x^2 (a - x),\ a > 0$.

Q.3 Show that the asymptotes of the following curve cut the curve again in eight points which lie on a circle of radius unity:

$$(x^2 - 4y^2)(x^2 - 9y^2) + 5x^2y - 5xy^2 - 30y^2 + xy + 7y^2 - 1 = 0$$

Q.4 Solve the differential equation -

$$\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2y = x^4$$

Q.5 The diameter and altitude of a right circular cylinder are measured as 4 cm and 6 cm respectively. If the possible error in each measurement is 0.1 cm, find approximately the maximum possible error in the value computed for the volume and lateral surface.

Q.6 Solve the ODE $y'' + 5y' + 4y = 0$ subject to the conditions $y(0) = 0$ and $y'(0) = 3$.

Q.7 Solve the PDE $yp = 2yx + \log q$.

PART–C

Q.1 Find the dimension of the rectangular box, open at the top, of maximum capacity whose surface is 432sq. cm.

Q.2 Solve by the method of variation of parameter -

$$(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (x+1)\,e^x$$

Q.3 Find the equation of circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$.

Q.4 Find a general solution of the PDE $p^2u^2 + q^2 = 1$ using Charpit's method.

Q.5 If z be a function of x and y and $u = lx + my$, $v = ly - mx$ be two other variables. Show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2)\frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial v^2}$

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B.Tech. I Sem. (Main) Examination, April/May - 2022

1FY2-01 Engineering Mathematics-I

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Attempt all ten questions From Part A, five Questions out of seven questions from Part B and three questions out of five questions from Part C.

Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination (As mentioned in form No. 205).

Part - A

(Answers should be given up to 25 words only)

All questions are compulsory. **(10×2=20)**

1. Define Beta function.

2. Write Euler's formula for a Fourier series.

3. Let $f = y^x$. What is $\frac{\partial^2 f}{\partial x \partial y}$ at $x = 2, y = 1$?

4. Consider a spatial curve in three-dimensional space given in parametric form by $x(t) = \cos t, y(t) = \sin t, z(t) = \frac{2}{\pi} t, 0 \le t \le \pi/2$. The length of the curve is......

5. In the Taylor series expansion of e^x about $x = 2$, the coefficient of $(x-2)^4$ is ....

6. Define the convergence of a power series.

7. The Directional derivative of the scalar function $f(x, y, z) = x^2 + 2y^2 + z$ at the point $P = (1,1,2)$ in the direction of the vector $\vec{a} = 3\hat{i} - 4\hat{j}$ is ......

8. Curl of vector $\vec{V}(x,y,z)=2x^2\hat{i}+3z^2\hat{j}+y^3\hat{k}$ at $x=y=z=1$ is ....

9. Velocity vector of a flow field is given as $\vec{V}(x,y,z)=2xy\hat{i}-3x^2z\hat{j}$. The vorticity vector at (1, 1, 1) is .....

10. The area enclosed between the curves $y^2=4x$ and $x^2=4y$ is ....

Part - B

(Analytical/Problem solving questions)

Attempt any five questions: **(5×4=20)**

1. Evaluate the following integrals:

   i) $\int_0^{\infty} x^4 e^{-x^4}\,dx$

   ii) $\int_0^{\pi/2} \sin^6\theta\cos^7\theta\,d\theta$.

2. The region in the first quadrant enclosed by the y-axis and the graphs of $y=\cos x$ and $y=\sin x$ is revolved about the x-axis to form a solid. Find its volume.

3. Test the convergence/divergence of the series.

$$\frac{1.2}{3^2.4^2}+\frac{3.4}{5^2.6^2}+\frac{5.6}{7^2.8^2}+....$$

4. Find the Fourier series expansion of the following periodic function with period 2π:

$$f(x)=\begin{cases}\pi+x, & if\ -\pi<x<0\\ 0, & if\ \ 0\le x<\pi\end{cases}$$

5. Consider the function:

$$f(x,y)=\sqrt{\frac{e^{\sin(x)}}{x^{2014}+\sqrt{x^{2012}+1}}}+\cos(xy)$$. Find the second partial derivative $\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$.

6. A scalar potential ϕ has the gradient $\nabla\phi=yz\hat{i}+xz\hat{j}+zy\hat{k}$. Evaluate the integral $\int_C \nabla\phi.d\vec{r}$ on the Curve $C:\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, if the curve C is parameterised as follows : $x=t, y=t^2, z=3t^2, 1\le t\le 3$.

7. Find the area of the region R in the xy-plane enclosed by the circle $x^2+y^2=4$, above the line y=1. and below the line $y=\sqrt{3x}$.

Part - C

(Descriptive/Analytical/Problem solving/Design Questions)

Attempt any three questions. **(3×10=30)**

1. If $f(x)=\begin{cases}\pi x, & 0<x<1\\ \pi(2-x), & 1<x<2\end{cases}$ using half range cosine series, show that

$$\frac{\pi^4}{96}=1+\frac{1}{3^4}+\frac{1}{5^4}+....$$

2. Show that $div(grad\ r^n)=n(n+1)r^{n-2}$, where $r=\sqrt{x^2+y^2+z^2}$. Hence , show that $\nabla^2\left(\frac{1}{r}\right)=0$.

3. The pressure P at any point (x,y,z) in space is $P=400xyz^2$. Find the highest pressure at the surface of a unit sphere $x^2+y^2+z^2=1$.

4. Find the work done by a force $\overline{F}=(y^2\cos x+z^3)\hat{i}+(2y\sin x-4)\hat{j}+(3xz^2+z)\hat{k}$ in moving a particle from $P(0,1,-1)$ to $Q\left(\frac{\pi}{2},-1,2\right)$.

5. Apply stoke's theorem to find the value of $\int_C(ydx+zdy+xdz)$. Where C is the curve of intersection of $x^2+y^2+z^2=a^2$ and $x+z=a$.