Swami Keshvanand Institute of Technology, Management &Gramothan, Jaipur

I Mid Term Examination, December-2022

Semester:	I	Session (I/II/III):	
Subject:	Engineering Mathematics-I	Subject Code:	1FY2-01
Time:	1.5 Hours	Maximum Marks:	20
Branch :	AI,IT,IOT,EC,EE		

PART A (short-answer type questions)

(All questions are compulsory) (3×2=6)

Q.1.Use Euler's Theorem to prove that:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = tanu \;\; if \;\; u = sin^{-1}\left(\frac{x^2+y^2}{x+y}\right).$$

Q.2.Find the equation of the Tangent Plane to the surface

$$x^2 + 2y^2 + 3z^2 - 12 = 0 \qquad at\ (1,2,-1).$$

Q.3. $If\ \vec{F} = (3x^2 - 3yz)\hat{\imath} + (3y^2 - 3xz)\hat{\jmath} + (3z^2 - 3xy)\hat{k}$ then show that $\nabla \times \vec{F} = 0$

PART B (Analytical/Problem solving questions)

(Attempt any 2 Questions) (2×4=8)

Q.4.Use Lagrange's method to find the extreme value of $f = x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = p$.

Q.5. Solve $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}dxdy$ by changing into polar co-ordinates.

Q.6.Find the total work done in moving a particle in the force Field $\vec{F} = 3xy\hat{\imath} - 5z\hat{\jmath} + 10x\hat{k}$ along the curve $x = t^2 + 1$; $y = 2t^2$ & $z = t^3 from\ t = 1\ to\ t = 2$.

PART C (Descriptive/Analytical/Problem solving/Design questions)

(Attempt any 1 Question) **(1×6=6)**

Q.7.If $u = f(r) where\ x^2 + y^2 = r^2$, then prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r) \quad .$$

Q.8.Find by the double integration the area lying inside the circle $r = a sin\theta$ and outside the cardioid $r = a(1 - cos\theta)$.

Solution of Question Paper

I Mid-Term Examination, ~~Sept.~~ dec -2022

Branch/Semester: I, AI, IT, IoT EC, EE	Subject: Engg. Mathematics-I	Subject Code: 1FY2-01
Duration: 1.5 hours	Date: 21/12/22 Session (I/II/III): I	Max Marks: 20
Submitted By: C.P. Jain		

Part-A

Q.1. Here $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right) \Rightarrow \sin u = \frac{x^2+y^2}{x+y}$

or $\sin u = \frac{x^2\{1+y^2/x^2\}}{x\{1+y/x\}} = x^1\phi(y/x)$

So $\sin u$ is a Homogeneous function of degree '1' so by Euler's theorem $x\frac{\partial}{\partial x}\sin u + y\frac{\partial}{\partial y}\sin u = 1.\sin u$

or $x\cos u\frac{\partial u}{\partial x} + y\cos u\frac{\partial u}{\partial y} = \sin u$.

or $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{\sin u}{\cos u} = \tan u$ proved.

Q.2. here $f(x,y,z) = x^2+2y^2+3z^2-12=0$; $P(1,2,-1)$

so $\frac{\partial f}{\partial x} = 2x$, $\frac{\partial f}{\partial y} = 4y$, $\frac{\partial f}{\partial z} = 6z$.

& $\left(\frac{\partial f}{\partial x}\right)_{(1,2,-1)} = 2$, $\left(\frac{\partial f}{\partial y}\right)_P = 4\times 2 = 8$, $\left(\frac{\partial f}{\partial z}\right)_P = 6(-1) = -6$

We know that Eqn of tangent plane is

$\left(\frac{\partial f}{\partial x}\right)_P(x-x_0) + \left(\frac{\partial f}{\partial y}\right)_P(y-y_0) + \left(\frac{\partial f}{\partial z}\right)_P(z-z_0) = 0$

i.e. $2(x-1) + 8(y-2) - 6(z+1) = 0$.

or $2x+8y-6z-2-16-6=0$.

or $x+4y-3z=12$. Ans.

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Ramnagaria, Jagatpura, Jaipur-302017

Solution of Question Paper

I Mid-Term Examination, ~~Sept.~~ Dec -2022

Branch/Semester: I, AI, IOT, IT, EC, EE	**Subject:** Engg. Maths-I	**Subject Code:** 1FY2-01
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Q.3. here $\vec{F} = (3x^2-3yz)\hat{i} + (3y^2-3xz)\hat{j} + (3z^2-3xy)\hat{k}$

So

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (3x^2-3yz) & (3y^2-3xz) & (3z^2-3xy) \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}(3z^2-3xy) - \frac{\partial}{\partial z}(3y^2-3xz)\right] - \hat{j}\left[\frac{\partial}{\partial x}(3z^2-3xy) - \frac{\partial}{\partial z}(3x^2-3yz)\right]$$
$$+ \hat{k}\left[\frac{\partial}{\partial x}(3y^2-3xz) - \frac{\partial}{\partial y}(3x^2-3yz)\right]$$

$$= \hat{i}[-3x+3x] - \hat{j}[-3y+3y] + \hat{k}[-3z+3z]$$

$= 0$ Proved.

part-B

Q.4. Here Lagrangian function $V = f + \lambda\phi$

$$V = x^2+y^2+z^2 + \lambda(ax+by+cz-p)$$

Now for an extreme value of the given function we have

$$\frac{\partial V}{\partial x} = 0 \Rightarrow 2x + a\lambda = 0 \quad \text{——} (1)$$

$$\frac{\partial V}{\partial y} = 0 \Rightarrow 2y + b\lambda = 0 \quad \text{——} (2)$$

$$\frac{\partial V}{\partial z} = 0 \Rightarrow 2z + c\lambda = 0 \quad \text{——} (3)$$

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Now multiplying ① by a, ② by b & ③ by c and adding, we get

$$2(ax+by+cz)+\lambda(a^2+b^2+c^2)=0$$

but $ax+by+cz=p$ (given)

$$\text{so } \lambda = -\frac{2p}{a^2+b^2+c^2}$$

so from ①, ② & ③ respectively,

$$x=-\frac{a}{2}\lambda=\frac{ap}{a^2+b^2+c^2}$$

$$y=-\frac{b}{2}\lambda=\frac{bp}{a^2+b^2+c^2}$$

$$z=-\frac{c}{2}\lambda=\frac{cp}{a^2+b^2+c^2}\quad \text{Ans.}$$

Q.5. solve $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}\,dx\,dy$

changing to polar co-ordinates, we have

$x=r\cos\theta$, $y=r\sin\theta$, $x^2+y^2=r^2$ & $dx\,dy=r\,d\theta\,dr$

& here $y=0$ to $y=\sqrt{a^2-x^2}$ i.e. $x^2+y^2=a^2$

& $x=0$ to a

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Solution of Question Paper

I Mid-Term Examination, ~~Sept.~~ Dec -2022

Branch/Semester: I AI, IT, IOT EC, EE	Subject: Engg. maths - I	Subject Code: 1F42-01
Duration: 1.5 hours	Date: 21/12/22 Session (I/II/III): I	Max Marks: 20
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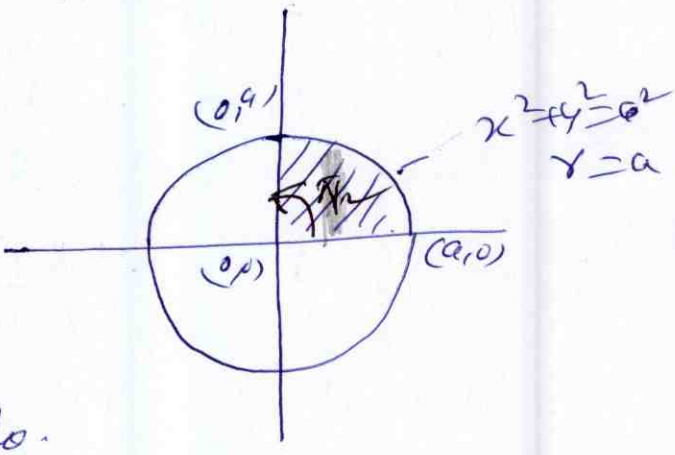
so given integration converts

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2\sqrt{(x^2+y^2)}\, dx\, dy$$



$$= \int_0^{\pi/2}\int_0^a r^2\sin^2\theta \cdot r \cdot r\, d\theta\, dr$$

$$\text{or} \int_0^{\pi/2} \sin^2\theta \left\{\int_0^a r^4\, dr\right\} d\theta$$

$$= \int_0^{\pi/2} \frac{a^5}{5}\sin^2\theta\, d\theta = \frac{a^2}{5}\int_0^{\pi/2}\left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{a^5}{5}\times\frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= \frac{a^5}{10}\left[\left(\frac{\pi}{2} - \frac{\sin\pi}{2}\right) - \left(0 - \frac{\sin 0}{2}\right)\right]$$

$$= \frac{\pi a^5}{20} \quad \text{Ans.}$$

Q.6. $\int_C \vec{F}\cdot d\vec{r}$ is the required work done

here

$$\therefore = \int_C (3xy\,\hat{i} - 5z\,\hat{j} + 10x\,\hat{k})\cdot(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

$$= \int_C (3xy\,dx - 5z\,dy + 10x\,dz) \quad \text{—— (1)}$$

But here curve C: is define by

$x = t^2+1$; $y = 2t^2$ & $z = t^3$ from $t=1$ to 2.

so $dx = 2t\,dt$ $\quad dy = 4t\,dt$ $\quad dz = 3t^2 dt$

So from Eqn (1) required Work done is

$$W = \int_1^2 3(t^2+1)2t^2 \cdot 2t\,dt - 5t^3 \cdot 4t\,dt + 10\cdot(t^2+1)\cdot 3t^2 dt$$

$$= \int_1^2 \{12t^5 + 12t^3 - 20t^4 + 30t^4 + 30t^2\} dt$$

$$= \int_1^2 (12t^5 + 10t^4 + 12t^3 + 30t^2)\,dt$$

or $$= \left[\frac{12}{6}t^6 + \frac{10}{5}t^5 + \frac{12}{4}t^4 + \frac{30}{3}t^3\right]_1^2$$

$$= \left[(2\cdot 2^6 + 2\cdot 2^5 + 3\cdot 2^4 + 10\cdot 2^3) - (2+2+3+10)\right]$$

$$= [128 + 64 + 48 + 80 - 17]$$

$$= [320 - 17] = 303 \text{ units} \quad \text{Ans.}$$

Part-C

Q.7. here $u = f(r)$ & $r^2 = x^2+y^2$ so $r = f(x,y)$

so $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial x} = \frac{\partial}{\partial r} f(r)\cdot\frac{x}{r} = f'(r)\frac{x}{r}$

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Similarly

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial y} = \frac{\partial}{\partial r} f(r)\, \frac{y}{r} = \frac{y}{r} f'(r)$$

&

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{x}{r} f'(r)\right)$$

$$= \frac{r \cdot \frac{\partial}{\partial x}(x f'(r)) - x f'(r) \frac{\partial}{\partial x} \cdot r}{r^2}$$

$$= \frac{r\left\{ x \cdot \frac{\partial}{\partial r} f'(r) \cdot \frac{\partial r}{\partial x} + f'(r) \frac{\partial x}{\partial x} \right\} - x f'(r) \cdot \frac{x}{r}}{r^2}$$

$$= \frac{r\left\{ x f''(r) \cdot \frac{x}{r} + f'(r) \right\} - x f'(r)\, x/r}{r^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{x^2 f''(r) + r f'(r)}{r^2} - \frac{x^2 f'(r)}{r^3} \quad \text{——} (1)$$

Similarly.

$$\frac{\partial^2 u}{\partial y^2} = \frac{y^2 f''(r) + r f'(r)}{r^2} - \frac{y^2 f'(r)}{r^3} \quad \text{——} (2)$$

Now by adding Eqn (1) & (2)

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{f''(r)\{x^2+y^2\}}{r^2}+\frac{2r f'(r)}{r^2}-\frac{(x^2+y^2)f'(r)}{r^3}$$

$$=f''(r)\frac{r^2}{r^2}+\frac{2f'(r)}{r}-\frac{r^2 f'(r)}{r^3}$$

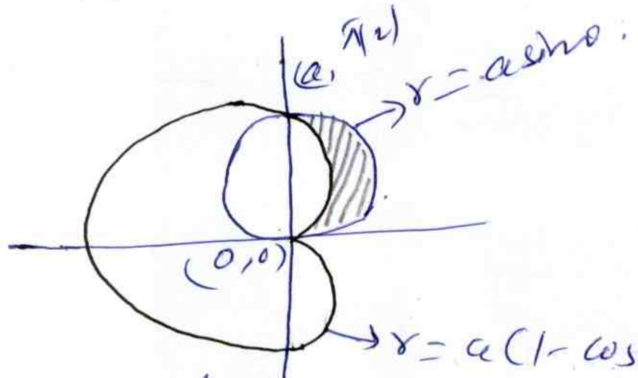
$$=f''(r)+\frac{1}{r}f'(r) \quad \text{proved.}$$

Q.B. We know that

Area by the double integration is

$$\iint dx\,dy=\int_0^{\pi/2}\int_{a(1-\cos\theta)}^{a\sin\theta} r\,d\theta\,dr$$

$$=\int_0^{\pi/2}\left[\frac{r^2}{2}\right]_{a(1-\cos\theta)}^{a\sin\theta}d\theta$$



$$=\frac{1}{2}\int_0^{\pi/2}\{a^2\sin^2\theta-a^2(1-\cos\theta)^2\}d\theta$$

$$=\frac{1}{2}a^2\int_0^{\pi/2}\left[\sin^2\theta-\{1-2\cos\theta+\cos^2\theta\}\right]d\theta$$

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$$= \frac{a^2}{2}\int_0^{\pi/2}\{\sin^2\theta - 1 + 2\cos\theta - \cos^2\theta\}d\theta.$$

$$= \frac{a^2}{2}\int_0^{\pi/2}\{2\cos\theta - 1 - \cos 2\theta\}d\theta.$$

$$= \frac{a^2}{2}\left[2\sin\theta - \theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= \frac{a^2}{2}\left[\left(2\sin\pi/2 - \pi/2 - \frac{\sin\pi}{2}\right) - 0\right]$$

$$= \frac{a^2}{2}\left[2 - \pi/2\right] = a^2\left(1 - \frac{\pi}{4}\right) \text{ Ans}$$