Swami Keshvanand Institute of Technology, Management &Gramothan, Jaipur

I Mid Term Examination, Dec.-2022

Semester:	**B. Tech. I Sem. (Group-I)**	**Branch:**	**CSE, DS, CE & ME**
Subject:	**Engineering Mathematics-I**	**Subject Code:**	**1FY2-01**
Time:	**1.5 Hours**	**Maximum Marks:**	**20**
Session : I (First)			

PART A (short-answer type questions)

(All questions are compulsory) **(3*2=6)**

Q. I f $u = y^x$ **then find** $\dfrac{\partial^2 u}{\partial x \partial y}$ **at** $x = 2,\ y = 1.$

Q. 2 Change the order of integration in the integral

$$\int_0^4 \int_x^{2\sqrt{x}} f(x,y)dxdy$$

Q. 3 Find a**, if** $\vec{V} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$,

is a Solenoidal vector.

PART B (Analytical/Problem solving questions)

(Attempt any 2 Questions) **(2*4=8)**

Q. 4 If $x^x y^y z^z = c$**, then prove that at** $x = y = z$,

$$\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$$

Q. 5 Evaluate $\int_{0}^{1}\int_{x}^{\sqrt{2x-x^2}} \sqrt{(x^2+y^2)}dxdy$

by changing into polar coordinates.

Q.6 Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

PART C (Descriptive/Analytical/Problem solving/Design questions)

(Attempt any 1 Question) (1*6=6)

Q.7 The pressure P at point (*x, y, z*) in space is $P = 400xyz^2$.

Find the highest pressure at the surface of the unit sphere

$x^2 + y^2 + z^2 = 1$.

Q.8 If $\vec{A} = (x+2y+az)\hat{i} + (bx-3y-z)\hat{j} + (4x+cy+2z)\hat{k}$

find *a, b, c,* so that $\vec{A}$ is irrotational vector. Also find the scalar

potential of $\vec{A}$.

Swami Keshvanand Institute of Technology, Management & Gramothan,
Ramnagaria, Jagatpura, Jaipur-302017

Solution of Question Paper

I Mid-Term Examination, December -2022

Branch/Semester:CSE, DS, CE & ME/I	Subject: Engineering Mathematics-I	Subject Code:1FY2-01
Duration: 1.5 hours	Date: 22.12.2022 Session : FIRST	Max Marks: 20
Submitted By: Dr. Nawal Kishor Jangid		Group – I

Part A

Q-1 (Sol.) Given $u = y^x$ —— ①

Differentiating ① partially w.r.to 'y', we get

$$\frac{\partial u}{\partial y} = x y^{x-1} \quad \text{—— } (\tfrac{1}{2}M)$$

Again Differentiating w.r.to 'x', we have

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial x}\left(x y^{x-1}\right)$$

$$\Rightarrow \frac{\partial^2 u}{\partial x \partial y} = y^{x-1}.1 - x.y^{x-1}\log y \quad \text{—— } (1M)$$

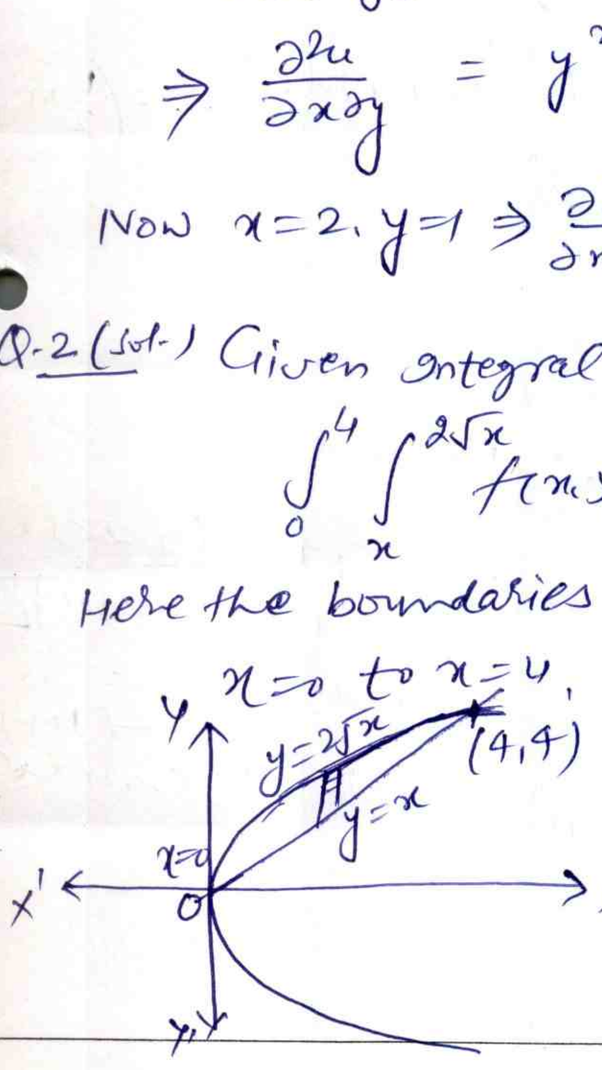
Now $x = 2, y = 1 \Rightarrow \frac{\partial^2 u}{\partial x \partial y} = 1$ Ans —— $(\tfrac{1}{2}M)$

Q-2 (Sol.) Given integral is

$$\int_0^4 \int_x^{2\sqrt{x}} f(x,y)\,dx\,dy$$

Here the boundaries are

$x = 0$ to $x = 4$, $y = x$ to $y = 2\sqrt{x} \Rightarrow y^2 = 4x$ (Parabola)



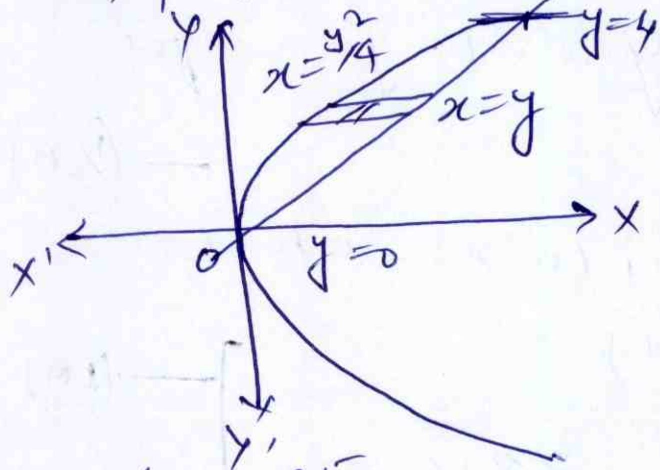
Now: $x^2 = 4x \Rightarrow x = 0, 4$

$y = 0, 4$

Point of intersections are $(0,0)$ and $(4,4)$ —— (1M)

Now to change the order, we make a horizontal strip which varies as:- $x = y^2/4$ to $x = y$

$y = 0$ to $y = 4$



Hence the integral becomes:- —— ($\frac{1}{2}$M)

$$\int_0^4 \int_x^{2\sqrt{x}} f(x,y)\,dx\,dy = \int_{y=0}^{4} \int_{x=y^2/4}^{x=y} f(x,y)\,dy\,dx \quad \text{—}(\tfrac{1}{2}\text{M})$$

<u>Q.3(Sol.)</u> Given vector

$$\vec{v} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$$

The vector field is solenoidal, so $\operatorname{div}(\vec{v}) = 0 \Rightarrow$

$$\vec{\nabla}\cdot\vec{v} = 0 \Rightarrow \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left((x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}\right) = 0 \quad \text{—(1M)}$$

$$\Rightarrow 1 + 1 + a = 0 \Rightarrow a = -2 \quad \text{——(1M)}$$

Hence the given vector $\vec{v}$ will be solenoidal if $a = -2$.

Solution of Question Paper
I Mid-Term Examination, December -2022

Branch/Semester: CSE, DS, CE & ME/I	Subject: Engineering Mathematics-I	Subject Code: 1FY2-01
Duration: 1.5 hours	Date: 22.12.2022 Session : FIRST	Max Marks: 20
Submitted By: Dr. Nawal Kishor Jangid		

Part-B

Q.4.(sol.) Given $x^x y^y z^z = c$

Taking log on both sides:-

$$x\log x + y\log y + z\log z = \log c \quad \text{--- (1)}$$

Since $z = f(x,y)$ so differentiating (1), partially w.r.to 'y' we get

$$0 + y\times\frac{1}{y} + 1.\log y + z\times\frac{1}{z}\frac{\partial z}{\partial y} + \log z.\frac{\partial z}{\partial y} = 0$$

$$\Rightarrow \frac{\partial z}{\partial y} = -\frac{(1+\log y)}{1+\log z} \quad \text{--- (2)}$$

Also differentiating (1) w.r.to 'x' we get

$$\frac{\partial z}{\partial x} = -\left(\frac{1+\log x}{1+\log z}\right) \quad \text{--- (3)} \qquad \text{--- (2M)}$$

Now differentiating (2) w.r.to 'x':

$$\frac{\partial^2 z}{\partial x\partial y} = -\frac{(1+\log y)}{(1+\log z)^2}.\frac{1}{z}\frac{\partial z}{\partial x} \qquad \text{--- (1M)}$$

$$\frac{\partial^2 u}{\partial x\partial y} = -\frac{(1+\log y)}{(1+\log z)^2}\times\frac{1}{z}\left(-\frac{(1+\log x)}{1+\log z}\right) \quad \left\{\begin{array}{l}\text{using}\\(3)\end{array}\right.$$

At $x=y=z \Rightarrow \frac{\partial^2 u}{\partial x\partial y} = -\frac{(1+\log x)^2}{(1+\log x)^3}.\frac{1}{x} = -(x\log ex)^{-1}$

— (1/2 + 1/2 M) Ans

Q-5 (Sol.) Given Integral is

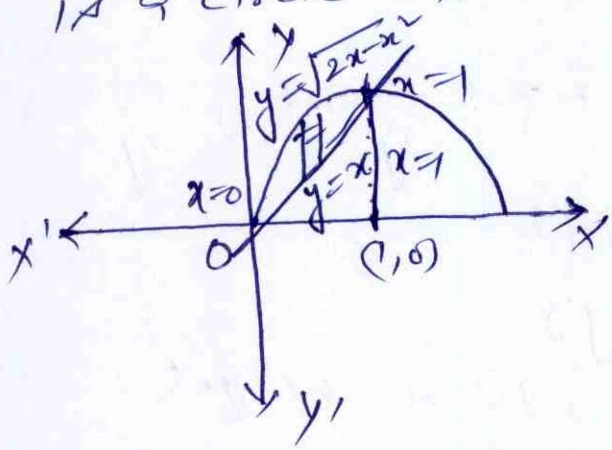
$$\int_0^1 \int_x^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dx\,dy$$

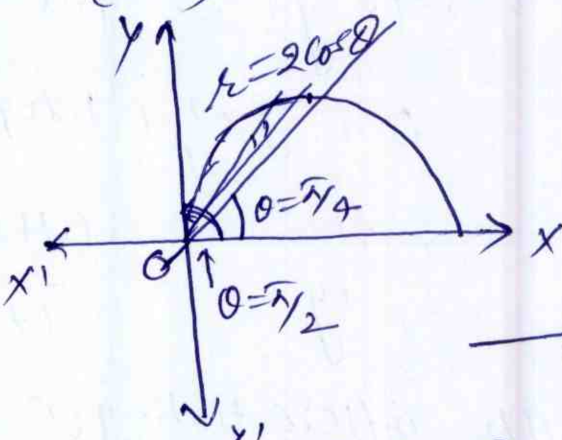
Here the boundaries are

$x=0$ to $x=1$, $y=x$ to $y=\sqrt{2x-x^2}$

$\Rightarrow x^2+y^2-2x=0$

is a circle with centre $(1,0)$ and radius 1





— (1m)

from circle $\Rightarrow x^2+y^2-2x=0 \Rightarrow r^2-2r\cos\theta=0$

$r(r-2\cos\theta)=0 \Rightarrow r=0,\ r=2\cos\theta$

From $y=x \Rightarrow r\sin\theta = r\cos\theta \Rightarrow \tan\theta=1 \Rightarrow \theta=\pi/4$

$x=0 \Rightarrow r\cos\theta=0 \Rightarrow \cos\theta=0 \Rightarrow \theta=\pi/2$

Hence on changing into Polar coordinates, the Radial strip varies as

$r=0$ to $2\cos\theta$.

$\theta=\pi/4$ to $\pi/2$,

Hence $\displaystyle\int_0^1 \int_x^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dx\,dy = \int_{\theta=\pi/4}^{\pi/2} \int_{r=0}^{2\cos\theta} r \cdot r\, d\theta\, dr$ — (1m)

$$= \int_{\theta=\pi/4}^{\pi/2} \left\{ \int_{r=0}^{2\cos\theta} r^2\,dr \right\} d\theta$$

$$= \int_{\pi/4}^{\pi/2} \left(\frac{r^3}{3}\right)_0^{2\cos\theta} d\theta = \frac{8}{3}\int_{\pi/4}^{\pi/2} \cos^3\theta\, d\theta$$

$$= \frac{8}{3}\int_{\pi/4}^{\pi/2} \frac{\cos 3\theta + 3\cos\theta}{4}\, d\theta \qquad \left\{ \because \cos 3\theta = 4\cos^3\theta - 3\cos\theta \right.$$

$$= \frac{2}{3}\left[\frac{\sin 3\theta}{3} + \frac{3\sin\theta}{1}\right]_{\pi/4}^{\pi/2}$$

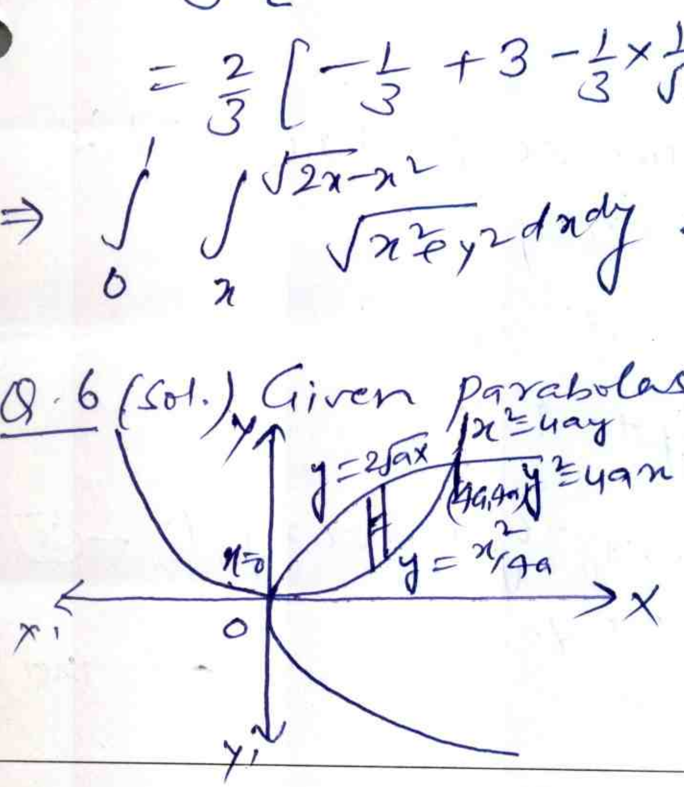
$$= \frac{2}{3}\left[\frac{1}{3}\sin 3\pi/2 + 3\sin\pi/2 - \frac{1}{3}\sin 3\pi/4 - 3\sin\pi/4\right]$$

$$= \frac{2}{3}\left[-\frac{1}{3} + 3 - \frac{1}{3}\times\frac{1}{\sqrt{2}} - 3\times\frac{1}{\sqrt{2}}\right] \quad \text{—}\left(1\tfrac{1}{2}M\right)$$

$$\Rightarrow \int_0^1\int_x^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\,dx\,dy = \frac{16}{9}\left(1-\frac{5}{4\sqrt{2}}\right) = \frac{2}{9}\left(8-5\sqrt{2}\right)$$

— $(\frac{1}{2}M)$ Ans

Q.6 (Sol.) Given parabolas $y^2=4ax$ and $x^2=4ay$



Also $\left(\frac{x^2}{4a}\right)^2 = 4ax$

$\Rightarrow x^4 - 64a^3x = 0$

$\Rightarrow x(x-64a^3) = 0$

$\Rightarrow x=0,\ x=4a$ — $(1\frac{1}{2}M)$

Solution of Question Paper
I Mid-Term Examination, December -2022

Branch/Semester:CSE, DS, CE & ME/I	Subject: Engineering Mathematics-I	Subject Code:1FY2-01
Duration: 1.5 hours	Date: 22.12.2022 Session : FIRST	Max Marks: 20
Submitted By: Dr. Nawal Kishor Jangid		

So point of intersection of two parabolas are $(0,0)$, $(4a, 4a)$.

Now Area $= \int_{x=0}^{4a}\int_{y=\frac{x^2}{4a}}^{2\sqrt{ax}} dx\,dy = \int_{x=0}^{4a}\left\{\int_{y=\frac{x^2}{4a}}^{2\sqrt{ax}} dy\right\}dx$

$$= \int_0^{4a}\left\{-\frac{x^2}{4a} + 2\sqrt{ax}\right\}dx$$

$$= \left[-\frac{x^3}{12a} + 2\sqrt{a}\frac{x^{3/2}}{3/2}\right]_0^{4a} = \left[\frac{4}{3}\sqrt{a}\,x^{3/2} - \frac{x^3}{12a}\right]_0^{4a}$$ —— (2M)

$\Rightarrow$ Area $= \frac{16}{3}a^2$ Ans. —— ($\frac{1}{2}$M)

Part - C

Q. 7 (Sol.) Given the pressure at point $P(x,y,z)$ is

$$P = 400xyz^2 \quad \text{—— (1)}$$

s.t. $\phi(x,y,z) = x^2+y^2+z^2-1=0$ —— (2)

Now $dP = \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy + \frac{\partial P}{\partial z}dz$

$$dP = 400yz^2dx + 400xz^2dy + 800xyz\,dz \quad \text{—— (3)}$$

$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz$$ —— (2M)

$\Rightarrow d\phi = 2x\,dx + 2y\,dy + 2z\,dz$ — (4)

Now $du + \lambda\, d\phi = 0$

$\Rightarrow 400yz^2 + \lambda 2x = 0$ — (5)

$400xz^2 + \lambda 2y = 0$ — (6)

$800xyz + \lambda \cdot 2z = 0$ — (7)

(5) $\times x +$ (6) $\times y +$ (7) $\times z \Rightarrow 1600xyz^2 + 2\lambda(x^2+y^2+z^2) = 0$

$\Rightarrow 4P + 2\lambda \cdot 1 = 0 \Rightarrow \boxed{\lambda = -2P}$ ——— (2m)

(5) $\Rightarrow 400xyz^2 + (-2P)2x^2 = 0$

$P - 4Px^2 = 0 \Rightarrow x = \pm \frac{1}{2}$

(6) $\Rightarrow y = \pm\frac{1}{2}$ ——— (1m)

(7) $\Rightarrow z = \pm\frac{1}{\sqrt{2}}$ ——— (1m)

The pressure is $P_{max} = 400 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 50$ Ans

Q: 8 (sol.) Given vector $\vec{A} = (x+2y+az)\hat{i} + (bx-3y-z)\hat{j} + (4x+cy+2z)\hat{k}$

Since $\vec{A}$ is irrotational so curl $\vec{A} = 0 \Rightarrow$

$$\vec{\nabla} \times \vec{A} = 0 \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x+2y+az & bx-3y-z & 4x+cy+2z \end{vmatrix} = 0$$ ——— (1m)

Solution of Question Paper
I Mid-Term Examination, December -2022

Branch/Semester:CSE, DS, CE & ME/I	Subject: Engineering Mathematics-I	Subject Code:1FY2-01
Duration: 1.5 hours	Date: 22.12.2022 Session : FIRST	Max Marks: 20
Submitted By: Dr. Nawal Kishor Jangid		

$\Rightarrow \hat{i}(c+1)+\hat{j}(a-4)+\hat{k}(b-2)=0$

$\Rightarrow a=4,\ b=2,\ c=-1$ —— (1 M)

$\Rightarrow \vec{A}=(x+2y+4z)\hat{i}+(2x-3y-z)\hat{j}+(4x-y+2z)\hat{k}$

since $\vec{\nabla}\times\vec{A}=0 \Rightarrow \vec{A}=\text{grad}\,\phi$ where ϕ is scaler potential of $\vec{A}$ so

$\vec{A}=\text{grad}\,\phi \Rightarrow$

$(x+2y+4z)\hat{i}+(2x-3y-z)\hat{j}+(4x-y+2z)\hat{k}$

$= \hat{i}\frac{\partial\phi}{\partial x}+\hat{j}\frac{\partial\phi}{\partial y}+\hat{k}\frac{\partial\phi}{\partial z}$ —— (1m)

$\Rightarrow \frac{\partial\phi}{\partial x}=x+2y+4z,\ \frac{\partial\phi}{\partial y}=2x-3y-z,\ \frac{\partial\phi}{\partial z}=4x-y+2z$

Now $d\phi=\frac{\partial\phi}{\partial x}dx+\frac{\partial\phi}{\partial y}dy+\frac{\partial\phi}{\partial z}dz$

$\Rightarrow d\phi=(x+2y+4z)dx+(2x-3y-z)dy+(4x-y+2z)dz$

$\Rightarrow \phi=\frac{x^2}{2}-\frac{3y^2}{2}+z^2+2xy+4xz-yz+C$ (2m)

Ans (1m)

—— x ——