Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur

Assignment-I

B.Tech. / Semester: III/VI **Branch: CSE**
Subject: Information Theory & Coding **Subject Code: 5CS3-01**
Date of submission: 03.10.2024 **MM: 10**

CS SEC-D

22ESKCS180-22ESKCS184	**SET 1**
22ESKCS185-22ESKCS189	**SET 2**
22ESKCS190-22ESKCS194	**SET 3**
22ESKCS195-22ESKCS199	**SET 4**
22ESKCS800-22ESKCS804	**SET 5**
22ESKCS805-22ESKCS809	**SET 6**
22ESKCS810-22ESKCS814	**SET 7**
22ESKCS815-22ESKCS819	**SET 8**
22ESKCS820-22ESKCS824	**SET 9**
22ESKCS825-22ESKCS829	**SET 10**
22ESKCS830-22ESKCS834	**SET 11**
22ESKCS835-22ESKCS837	**SET 12**

Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur

Assignment-I

B.Tech. / Semester: III/VI **Branch: CSE**
Subject: Information Theory & Coding **Subject Code: 5CS3-01**
Date of submission: 03.10.2024 **MM: 10**

SET : 1

Q.1 If $I(x_1)$ is the information carried by symbols x_1 and $I(x_2)$ is the information carried by symbol x_2, then prove that the amount of information carried compositely due to x_1 and x_2 is $I(x_1, x_2) = I(x_1) + I(x_2)$. **[CO1, BL3]**

Q.2 If there are M equally likely and independent symbols, then prove that amount of information carried by each symbol will be $I(x_i) = N$ bits, where $M = 2^N$ and N is an integer. [**CO1, BL3]**

Q.3 Show that channel capacity of an ideal AWGN channel with infinite bandwidth is given by

$$C_\infty = \frac{1}{\ln 2}.\frac{S}{\eta} \cong 1.44\frac{S}{\eta}$$

Where S is the average signal power and $\frac{\eta}{2}$ is the power spectral density of white gaussian noise **[CO2, BL3]**

Q.4The second order extension of a DMS X, denoted by X^2, is formed by taking the source symbols two at a time. The coding of this extension has been shown in Table 1. Find the efficiency η and the redundancy **γ** of this extension code.

a_i	$P(a_i)$	Code
a_1=x_1x_1	0.81	0
a_2=x_1x_2	0.09	10
a_3=x_2x_1	0.09	110
$a_{4=x_2x_2}$	0.01	111

[CO2, BL3]

Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur

Assignment-I

B.Tech. / Semester: III/VI **Branch: CSE**
Subject: Information Theory & Coding **Subject Code: 5CS3-01**
Date of submission: 03.10.2024 **MM: 10**

<u>SET: 2</u>

Q.1 A DMS X has four symbols x_1, x_2, x_3 and x_4 with $P(x_1) = \frac{1}{2}, P(x_2) = \frac{1}{4}, P(x_3) = P(x_4) = \frac{1}{8}$.

Construct a Shannon- Fano code for X. **[CO2,BL3]**

Q.2 The second order extension of a DMS X, denoted by X^2, is formed by taking the source symbols two at a time. The coding of this extension has been shown in Table 1. Find the efficiency η and the redundancy **γ** of this extension code.

a_i	$P(a_i)$	Code
$a_1 = x_1x_1$	0.81	0
$a_2 = x_1x_2$	0.09	10
$a_3 = x_2x_1$	0.09	110
$a_4 = x_2x_2$	0.01	111

[CO2,BL3]

Q.3 Given a noiseless channel with m input symbols and m output symbols. Prove that $H(X) = H(Y)$ and $H\left(\frac{Y}{X}\right) = 0$.

[CO1,BL3]

Q.4 Verify the following expressions:

i) $H(X, Y) = H\left(\frac{X}{Y}\right) + H(Y)$

ii) $H(X, Y) = H\left(\frac{Y}{X}\right) + H(X)$

[CO1,BL3]

Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur

Assignment-I

B.Tech. / Semester: III/VI **Branch: CSE**
Subject: Information Theory & Coding **Subject Code: 5CS3-01**
Date of submission: 03.10.2024 **MM: 10**

SET: 3

Q.1 Show that the mutual information $I(X;Y)$ with the input probabilities $P(x_i), i = 1,2, \dots . m$ and the output probilities $P(y_j), j = 1,2, \dots . . n$ can be expressed as

$$I(X, Y) = \sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i, y_j) \log_2 \frac{P(x_i/y_j)}{P(x_i)}$$

[CO1,BL3]

Q.2 A channel has the following channel matrix

$$\left[P\left(\frac{Y}{X}\right)\right] = \begin{bmatrix} 1-p & p & 0 \\ 0 & p & 1-p \end{bmatrix}$$

(i) Draw the channel diagram

(ii) If the source has equally likely outputs, compute the possibilities associated with the channel outputs for p=0.2

[CO1, BL3]

Q.3 Determine the Huffmann code for the following messages with probabilities given

x_1	x_2	x_3	x_4	x_5	x_6	x_7
0.05	0.15	0.2	0.05	0.15	0.3	0.1

[CO2, BL3]

Q.4 Given a DMS X with two symbols x_1 and x_2 and $P(x_1) = 0.9$ and $P(x_2) = 0.1$

Symbols x_1 and Symbols x_2 are encoded as follows

x_i	$P(x_i)$	Code
x_1	0.9	0
x_1	0.1	1

Find the efficiency η and the redundancy $\boldsymbol{\gamma}$ of this code.

[CO2,BL3]

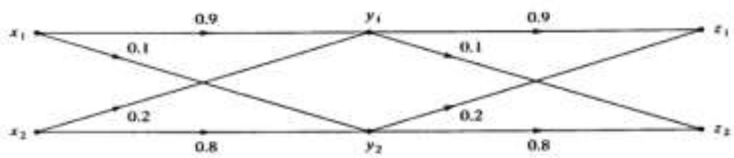
Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur

Assignment-I

B.Tech. / Semester: III/VI **Branch: CSE**
Subject: Information Theory & Coding **Subject Code: 5CS3-01**
Date of submission: 03.10.2024 **MM: 10**

SET: 4

1. Consider a telegraph source having two symbols; dot and dash. The dot duration is 0.2s.The dash duration is 3 times the dot duration. The probability of the dot's occurring is twice that of the dash, and the time between symbols is 0.2s.Calcukate the information rate of the telegraph source. **[CO1,BL3]**
2. Two binary channels are connected in cascade as shown in Fig 1.



a. Fig. 1

(a) Find the overall channel matrix and draw the resultant equivalent channel diagram.

(b) Find $P(z_1)$ and $P(z_2)$ when $P(x_1) = P(x_2) = 0.5$. **[CO1, BL3]**

3. A DMS X has five equally likely symbols

(a) Construct a Shannon Fano code for X, Calculate the efficiency

(b) Construct another Shannon Fano code and compare the results **[CO2, BL3]**

4. Consider a DMS X with symbols x_i, i=1,2, 3, 4. Table 1 shows four possible symbols.

Table 1

a.

x_i	Code *A*	Code *B*	Code *C*	Code *D*
x_1	00	0	0	0
x_2	01	10	11	100
x_3	10	11	100	110
x_4	11	110	110	111

(a) Show that all the codes except code B satisfy the Kraft's Inequality.

(b) Show that codes A and D are uniquely decodable but codes B and C are not uniquely decodable. **[CO2,BL3]**

Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur

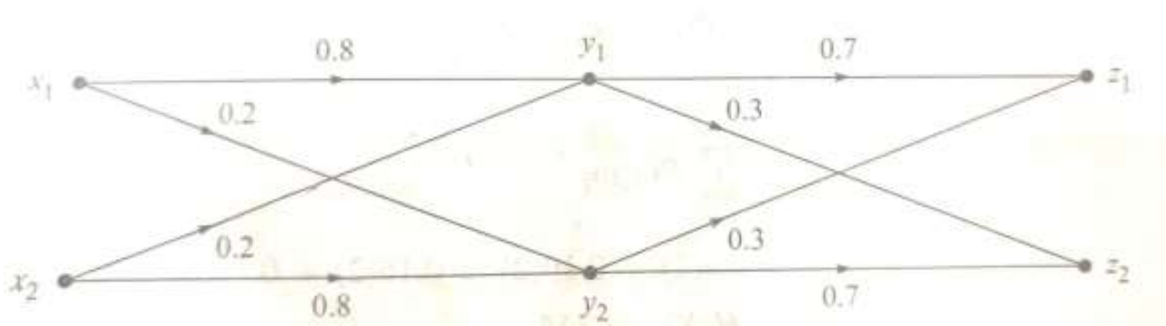
Assignment-I

B.Tech. / Semester: III/VI **Branch: CSE**
Subject: Information Theory & Coding **Subject Code: 5CS3-01**
Date of submission: 03.10.2024 **MM: 10**

SET: 5

Q.1 A high resolution black and white TV picture consists of about 2×10^6 picture element and 16 different brightness levels. Pictures are repeated at the rate of 32 per second. All picture elements are assumed to be independent, and all levels have equal likelihood of occurrence. Calculate the average rate of information conveyed by this TV picture source. **[CO1, BL3]**

Q.2 Two BSCs are connected in cascade as shown in figure



(a) Find the channel matrix of the resultant channel.

(b) Find $P(z_1)$ and $P(z_2)$ if $P(x_1)$=0.6 and $P(x_2)$=0.4 **[CO1, BL3]**

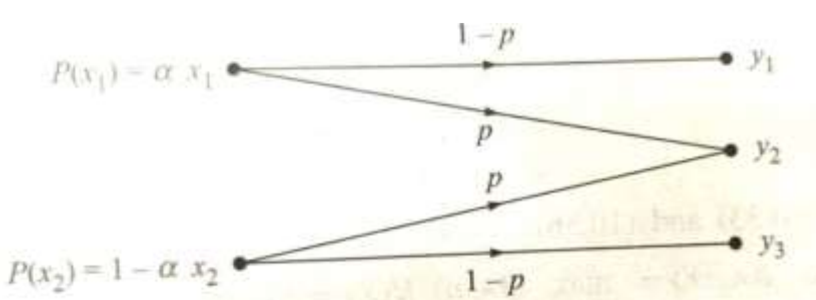
Q.3 A DMS X has five symbols x_1, x_2, x_3, x_4 and x_5 with $P(x_1)$=0.2, $P(x_2)$=0.15, $P(x_3)$=0.05, $P(x_4)$=0.10 and $P(x_5)$=0.50

(a) Construct a Shannon Fano code for X, and calculate the efficiency of the code.

(b) Repeat for the Huffman code and compare the results. **[CO2, BL3]**

Q.4 Find the channel capacity of the binary erasure channel of given figure **[CO2, BL3]**

$$P(x_1) = \alpha, \qquad P(x_2) = 1 - \alpha,$$



Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur

Assignment-I

B.Tech. / Semester: III/VI **Branch: CSE**
Subject: Information Theory & Coding **Subject Code: 5CS3-01**
Date of submission: 03.10.2024 **MM: 10**

SET 6:

Q.1 In a message conveyed through long sequence of dots and dashes, the probability of occurrence of a dash is one third of that of a dot. The duration of a dash is three times that of dot. If the dot lasts for 10 ms and the same time is allowed between symbols, determent the following :

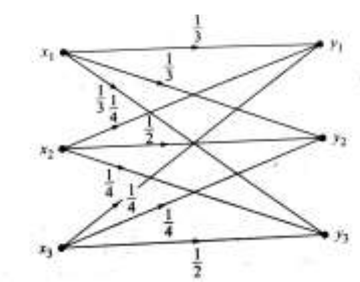
(a) The information in dot and dash b) Average information in the dot dash code

(c) Average information rate **[CO1, BL3]**

Q.2 Consider the DMC shown in figure

(a) Find the output probabilities if $P(x_1)=1/2$ and $P(x2)=P(x3)=1/4$

(b) Find the output entropy H(y).



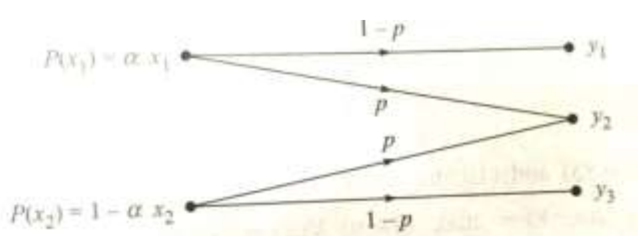
[CO1, BL3]

Q.3Consider a DMS X with symbols x_i, i=1,2,3,4. Table 1 shows four possible symbols.

x_i	Code *A*	Code *B*	Code *C*	Code *D*
x_1	00	0	0	0
x_2	01	10	11	100
x_3	10	11	100	110
x_4	11	110	110	111

(a) Show that all the codes except code B satisfy the Kraft's Inequality.
(b) Show that codes A and D are uniquely decodable but codes B and C are not uniquely decodable. **[CO2, BL3]**
Q.4Find the channel capacity of the binary erasure channel of given figure **[CO2, BL3]**

$$P(x_1) = \alpha, \qquad P(x_2) = 1 - \alpha,$$



Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur

Assignment-I

B.Tech. / Semester: III/VI **Branch: CSE**

Subject: Information Theory & Coding **Subject Code: 5CS3-01**

Date of submission: 03.10.2024 **MM: 10**

SET: 7

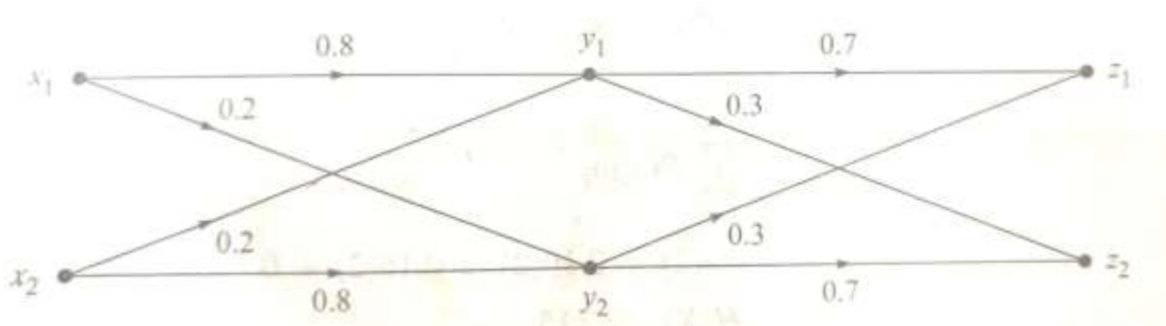
Q.1

Show that for a discrete binding channel **[CO1, BL3]**

$$H(x,y) = H\left(\frac{x}{y}\right) + H(y)$$

$$H(x,y) = H(x) + H(y)$$

Q.2Two BSCs are connected in cascade as shown in figure **[CO1, BL3]**



(a) Find the channel matrix of the resultant channel.

(b) Find $P(z_1)$ and $P(z_2)$ if $P(x_1)$=0.6 and $P(x_2)$=0.4

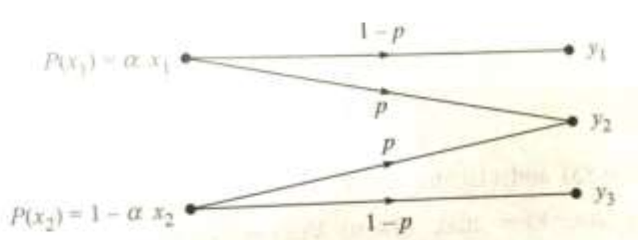
Q.3 Determine the Huffman code and Shannon code for the following message with their:

X1	X2	X3	X4	X5	X6	X7
0.05	0.15	0.20	0.05	0.15	0.30	0.10

[CO2, BL3]

Q.4Find the channel capacity of the binary erasure channel of given figure **[CO2, BL3]**

$$P(x_1) = \alpha, \qquad P(x_2) = 1 - \alpha,$$



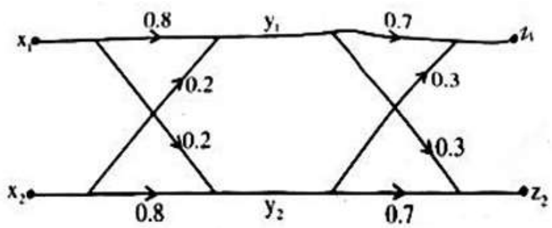
Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur

Assignment-I

B.Tech. / Semester: III/VI **Branch: CSE**
Subject: Information Theory & Coding **Subject Code: 5CS3-01**
Date of submission: 03.10.2024 **MM: 10**

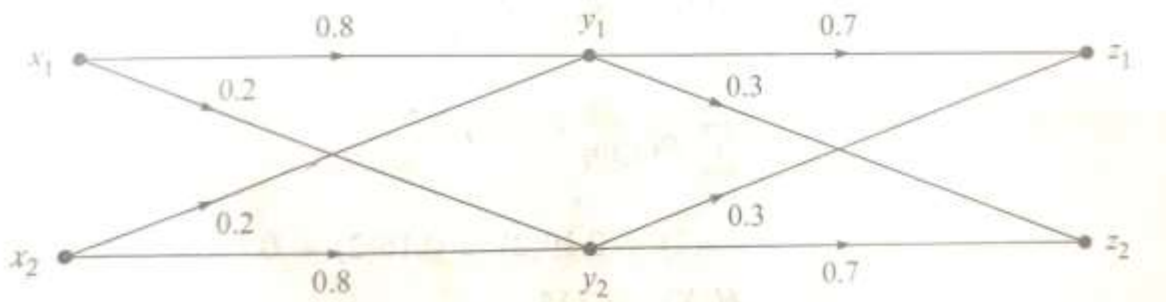
SET: 8

Q.1 The BSC are connected in cascade as shown in fig



a. Determine the transition matrix for discrete memory less channel
b. Determine $P(z_1)$ and $P(z_2)$ if $P(x_1)=0.6$ and $P(x_2)=0.4$ **[CO1, BL3]**

Q.2 Two BSCs are connected in cascade as shown in figure **[CO1, BL3]**



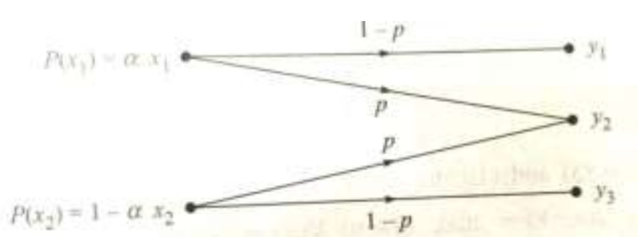
Q.3 Determine the Huffman code for the following message with their

X1	X2	X3	X4	X5	X6	X7
0.15	0.15	0.10	0.25	0.05	0.20	0.10

And also find the average code word length, entropy, code efficiency, compare the result with entropy. **[CO2, BL3]**

Q.4 Find the channel capacity of the binary erasure channel of given figure **[CO2, BL3]**

$$P(x_1) = \alpha, \qquad P(x_2) = 1 - \alpha,$$



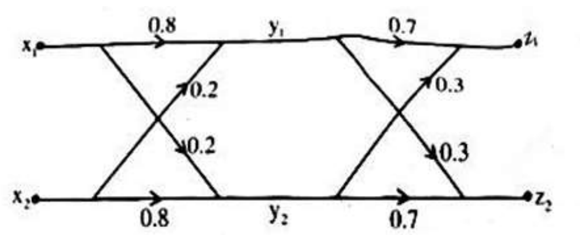
Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur

Assignment-I

B.Tech. / Semester: III/VI **Branch: CSE**
Subject: Information Theory & Coding **Subject Code: 5CS3-01**
Date of submission: 03.10.2024 **MM: 10**

SET-9

Q1. The BSC are connected in cascade as shown in fig



c. Determine the transition matrix for discrete memory less channel

d. Determine $P(z_1)$ and $P(z_2)$ if P(x1)=0.6 and P(x2)=0.4 **[CO1, BL3]**

Q2. Prove that the upper bound on Entropy is given as $H_{max} \le \log_2 m$. Here m is the number of messages emitted by the source.

[CO1, BL3]

Q3. Consider a BSC with P(x1) = α.

Show that the mutual information I(X;Y) is given by:

$$I(X;Y) = H(Y) + p\log_2 p + (1-p)\log_2(1-p)$$

Also calculate I(X;Y) for α=0.5 and p = 0.1

[CO2, BL3]

Q4. Consider an AWGN channel with 4 KHz bandwidth and noise power spectral density $\frac{\eta}{2} = 10^{-12} W/Hz$. The signal power required at the receiver is 0.1 mW. Calculate the capacity of this channel.

[CO2, BL3]

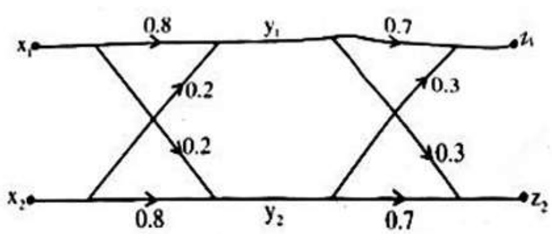
Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur

Assignment-I

B.Tech. / Semester: III/VI **Branch: CSE**
Subject: Information Theory & Coding **Subject Code: 5CS3-01**
Date of submission: 03.10.2024 **MM: 10**

SET-10

Q1. The BSC are connected in cascade as shown in fig



a. Determine the transition matrix for discrete memory less channel
b. Determine $P(z_1)$ and $P(z_2)$ if P(x1)=0.6 and P(x2)=0.4 **[CO1, BL3]**

Q2. Consider a telegraph source having two symbols; dot and dash. The dot duration is 0.2s.The dash duration is 3 times the dot duration. The probability of the dot's occurring is twice that of the dash, and the time between symbols is 0.2s.Calcukate the information rate of the telegraph source.

[CO1, BL3]

Q3. Consider a DMS X with symbols xi and corresponding probabilities $P(x_i) = P_i, i = 1,2, \dots .m$. Let ni be the length of the code word for xi such that

$$log_2 \frac{1}{P_i} \leq n_i \leq log_2 \frac{1}{P_i} + 1$$

Show that this relationship satisfies the Kraft inequality, and find the bound on K.

[CO2, BL3]

Q4. A DMS X has five symbols x_1, x_2, x_3, x_4 and x_5 with $P(x_1)$=0.2, $P(x_2)$=0.15, $P(x_3)$=0.05, $P(x_4)$=0.10 and $P(x_5)$=0.50.

(a) Construct a Shannon Fano code for X, and calculate the efficiency of the code.

(a) Repeat for the Huffman code and compare the results. **[CO2, BL3]**

Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur

Assignment-I

B.Tech. / Semester: III/VI **Branch: CSE**
Subject: Information Theory & Coding **Subject Code: 5CS3-01**
Date of submission: 03.10.2024 **MM: 10**

SET-11

Q1.Verfify the mutual information is:

$$I(X;Y) \geq 0$$

[CO1, BL3]

Q2. Consider a telegraph source having two symbols; dot and dash. The dot duration is 0.2s.The dash duration is 3 times the dot duration. The probability of the dot's occurring is twice that of the dash, and the time between symbols is 0.2s.Calcukate the information rate of the telegraph source.

[CO1, BL3]

Q3.Consider a BSC with P(x1) = α.Show that the mutual information I(X;Y) is given by:

$$\mathrm{I(X;Y) = H(Y) + p\log_2 p + (1-p)\log_2(1-p)}$$

Also calculate I(X;Y) for α=0.5 and p = 0.1

[CO2, BL3]

Q4.A DMS X has five symbols x_1, x_2, x_3, x_4 and x_5 with $P(x_1)=0.3$, $P(x_2)=0.05$, $P(x_3)=0.10$, $P(x_4)=0.10$ and $P(x_5)=0.45$

(b) Construct a Shannon Fano code for X, and calculate the efficiency of the code.

(c) Repeat for the Huffman code and compare the results. **[CO2, BL3]**

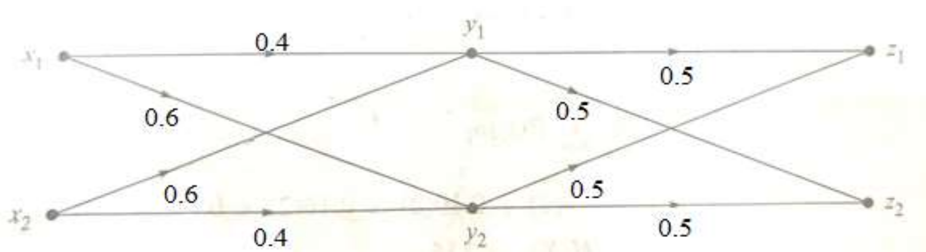
Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur

Assignment-I

B.Tech. / Semester: III/VI **Branch: CSE**
Subject: Information Theory & Coding **Subject Code: 5CS3-01**
Date of submission: 03.10.2024 **MM: 10**

SET-12

Q1. Two BSCs are connected in cascade as shown in figure



(c) Find the channel matrix of the resultant channel.

(d) Find $P(z_1)$ and $P(z_2)$ if $P(x_1)=0.6$ and $P(x_2)=0.4$ **[CO1, BL3]**

Q2. Consider a telegraph source having two symbols; dot and dash. The dot duration is 0.2s.The dash duration is 3 times the dot duration. The probability of the dot's occurring is twice that of the dash, and the time between symbols is 0.2s.Calcukate the information rate of the telegraph source. **[CO1, BL3]**

Q3.Consider a BSC with $P(x1) = \alpha$.Show that the mutual information I(X;Y) is given by:

$$I(X;Y) = H(Y) + p\log_2 p + (1-p)\log_2(1-p)$$

Also calculate I(X;Y) for $\alpha=0.5$ and $p = 0.1$

[CO2, BL3]

Q4. A DMS X has five symbols x_1, x_2, x_3, x_4 and x_5 with $P(x_1)=0.10$, $P(x_2)=0.05$, $P(x_3)=0.25$, $P(x_4)=0.30$ and $P(x_5)=0.30$

(a)Construct a Shannon Fano code for X, and calculate the efficiency of the code.

(b)Repeat for the Huffman code and compare the results. **[CO2, BL3]**