Swami Keshvanand Institute of Technology, Management &Gramothan, Jaipur

I Mid Term Examination, Dec.-2022

Semester:	I	Branch:	CSE, DS, ME, CE
Subject:	Engineering Physics	Subject Code:	1FY2-02
Time:	1.5 Hours	Maximum Marks:	20
Session (I/II/III):			

PART A (Short-answer type questions)
(All questions are compulsory) (3*2=6)

Q.1 Explain the reason of formation of concentric circular fringes in Newton's ring experiment.

Q.2 How is coherent source produced in Michelson Interferometer?

Q.3 What are matter waves? State de Broglie hypothesis.

PART B (Analytical/Problem solving questions)
(Attempt any 2 Questions) (2*4=8)

Q.4 (a) How will you determine the wavelength of monochromatic light with the help of Michelson's Interferometer? (2)

(b) Newton's rings are observed between a spherical surface of radius of curvature 120 cm and a plane plate. The diameters of 5th and 16th bright rings are 0.314 cm and 0.584 cm respectively. Calculate the diameter of 37th bright rings and also the wavelength used. (2)

Q.5 (a) Apply Rayleigh's criterion of resolution to derive the expression for resolving power of diffraction grating. (2)

(b) Light of wavelength 6000 Å is incident on a slit of width 0.30 mm. The screen is placed 2.0 m away from the slit. Calculate (i) the position of the first dark fringe and (ii) the width of the central bright fringe. (2)

Q.6 (a) What is a wave function in quantum mechanics? Explain the physical significance of a wave function. (2)

(b) Wave function corresponding to a system is given as $\psi(x) = Ne^{icx}$. Determine the value of normalization constant over the range $-a < x < a$. (2)

PART C (Descriptive/Analytical/Problem solving/Design questions)
(Attempt any 1 Question) (1*6=6)

Q.7 Show that the intensity of light diffracted from a plane transmission grating is given by- $I = I_0 (\sin^2\alpha/ \alpha^2) (\sin^2 N\beta/ \sin^2\beta)$, where symbols have their usual meanings.

Q.8 Derive the Schrödinger's time dependent and time independent wave equation.

Swami Keshvanand Institute of Technology, Management & Gramothan, Ramnagaria, Jagatpura, Jaipur-302017

Solution of Question Paper

I Mid-Term Examination, December -2022

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Submitted By: Dr. Manasvi Dixit		

PART A (Short-answer type questions)

Q.1 Explain the reason of formation of concentric circular fringes in Newton's ring experiment. (2)

Solu: Newton's rings are concentric circular fringes because the air film enclosed between plano-convex lens and plane glass plate has a circular symmetry i.e. locus of all points corresponding to a specific thickness of air film is a circle.

Q.2 How is coherent source produced in Michelson Interferometer? (2)

Solu: In Michelson Interferometer, coherent sources are produced by division of amplitude method. When a monochromatic light is incident on a beam splitter plate, it is divided into two parts of equal intensities. One part is a partially reflected wave and other part is a partially refracted/transmitted wave and both are coherent. These coherent waves proceed in perpendicular directions and are incident normally on two mirrors.

Q.3 What are matter waves? State de Broglie hypothesis. (2)

Solu: According to Louis de Broglie Hypothesis, "All material particles like electron, proton exhibits wave-like properties when they are in motion", i.e. all material particles having dual nature. Such waves associated with the material particles are known as "matter waves" or de Broglie waves".

The de- Broglie wave length of material particle is, $\lambda = \frac{h}{mv} = \frac{h}{p}$

where, h = 6.63 x 10^{-34} J-s is the Planck's constant, m is the mass of particle moving with the velocity v and $P = mv$ is the momentum of the particle.

PART B (Analytical/Problem solving questions)

Q.4 (a) How will you determine the wavelength of monochromatic light with the help of Michelson's Interferometer? (2)

Solu:

- Light from a monochromatic source whose wavelength is to be determined is allowed to fall on the beam splitter plate G_1.
- Now mirror M_1 and M_2 are adjusted so that circular fringes are visible in the field of view.
- Suppose the separation between real mirror M_1 and virtual mirror M_2' is such that an n^{th} order dark ring is formed at the centre in the field of view.
- Note down the position of mirror M_1 with the help of micrometer screw. Let it be x_1.
- Thus, the condition for the dark fringe at the centre

$$2d\cos\theta = n\lambda$$

- For central fringe θ=0°

$$2d = n\lambda$$

- On shifting mirror M_1 by a distance $\lambda/2$, one fringe shifts in the field of view of telescope and next order dark fringe appears at the centre.
- Now moving mirror M_1 through any arbitrary distance, N number of dark fringes passes from the centre in the field of view of telescope.
- Let this new position of mirror M_1 is x_2 (as measured by micrometer screw)
- So, the distance by which mirror M is shifted is $x= (x_2-x_1)$

$\Rightarrow$ $$2(x_2-x_1)=2x=N\lambda$$

$\Rightarrow$ $$\lambda = \frac{2X}{N}$$

Thus, if we know the value of N and x, we can determine the wavelength of given monochromatic light source.

Q.4(b) Newton's rings are observed between a spherical surface of radius of curvature 120 cm and a plane plate. The diameters of 5th and 16th bright rings are 0.314 cm and 0.584 cm respectively. Calculate the diameter of 37th bright rings and also the wavelength used. (2)

Solu: Given, Radius of curvature of plano convex lens= R=120 cm

Diameter of 5th bright ring = $(D_5)_{bright} = 0.314$ cm

Diameter of 16th bright ring = $(D_{16})_{bright} = 0.584$ cm

The wavelength of light used is given as

$$\lambda = \frac{D^2_{(n+m)} - D^2_n}{4mR}$$; Here m=16-5=11

$$\lambda = \frac{D^2_{16} - D^2_5}{4\times 11\times R} = \frac{(0.584)^2-(0.314)^2}{4\times 11\times 120} cm = \frac{0.243}{44\times 120} cm$$

$$\lambda = 4.592\times 10^{-5} cm = 4592 \overset{o}{\text{A}}$$

Diameter of 37th bright ring is

$$(D_{37})_{bright} = \sqrt{2(2\times 37-1)R\lambda}$$

$$\Rightarrow (D_{37})_{bright} = \sqrt{2(2\times 37-1)\times 120cm\times 4.592\times 10^{-5} cm}$$

$$\Rightarrow (D_{37})_{bright} = \sqrt{2\times 73\times 120\times 4.592\times 10^{-5} cm^2}$$

$$\Rightarrow (D_{37})_{bright} = \sqrt{8045.18\times 10^{-4} cm^2}$$

$$\Rightarrow (D_{37})_{bright} = 89.69\times 10^{-2} cm = 0.897cm$$

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Solution of Question Paper

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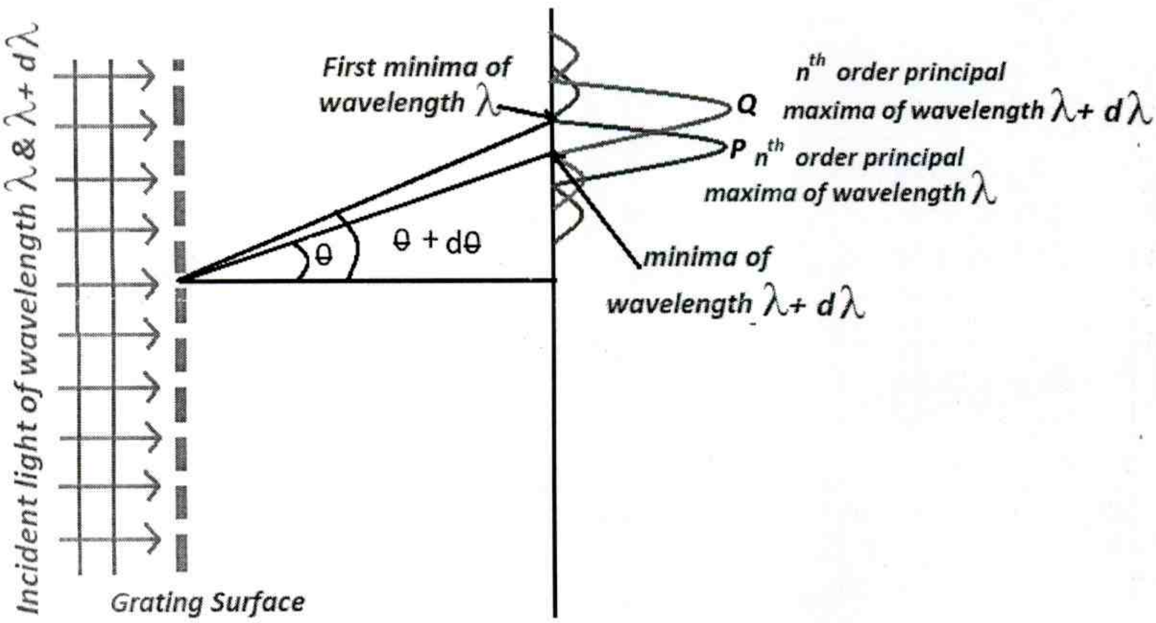
Q.5 (a) Apply Rayleigh's criterion of resolution to derive the expression for resolving power of diffraction grating. (2)

Solu: The resolving power of diffraction grating is the ability of grating to resolve two nearby spectral lines that is, to see these two spectral lines just as separate. It is measured in terms of the ratio $\frac{\lambda}{d\lambda}$. Here λ is the wavelength of any spectral line and $d\lambda$ is the difference in the two wavelengths.



Resolving Power of Diffraction Grating

Let a light beam of wavelength λ and $(\lambda+d\lambda)$ is incident normally on a grating surface. Let n^{th} order principal maxima due to wavelength λ is formed in the direction θ, then

$$(e+b)\sin\theta = n\lambda \qquad ...(1)$$

and n^{th} order principal maxima due to wavelength $(\lambda+d\lambda)$ is formed in the direction $(\theta+d\theta)$, then

$$(e+b)\sin(\theta+d\theta) = n(\lambda+d\lambda) \qquad ...(2)$$

According to Rayleigh criterion these spectral lines will appear just resolved if the principal maxima due to λ falls on the first minima due to $(\lambda+d\lambda)$ or vice-versa.

Let the position of first minima adjacent to n^{th} order principal maxima for wavelength λ in the direction $(\theta+d\theta)$ is given by

$$(e+b)\sin(\theta+d\theta) = \frac{p}{N}\lambda \qquad ...(3)$$

Where p has all integral value except p= 0,N,2N,3N,....

As the first minima adjacent n^{th} order principal maxima in the direction ($\theta + d\theta$) will be obtained for

$$p = nN+1 \qquad ...(4)$$

Using equations (3) and (4), we have

$$(e+b)\sin(\theta + d\theta) = \left(\frac{nN+1}{N}\right)\lambda \qquad ...(5)$$

On comparing equations (2) and (5), we have

$$n(\lambda + d\lambda) = \left(\frac{nN+1}{N}\right)\lambda$$

$$n\lambda + nd\lambda = n\lambda + \frac{\lambda}{N}$$

$$nd\lambda = \frac{\lambda}{N} \Longrightarrow \frac{\lambda}{d\lambda} = nN \qquad ...(6)$$

Thus, Resolving power R= nN

It is clear from equation (6) that

- Resolving power of grating is directly proportional to order of spectrum
- It is directly proportional to number of ruled lines
- Dependence on width of ruled surface

Q.5(b) Light of wavelength 6000 Å is incident on a slit of width 0.30 mm. The screen is placed 2.0 m away from the slit. Calculate (i) the position of the first dark fringe and (ii) the width of the central bright fringe. (2)

Solu: Given

Wavelength of incident light 'λ' = 6000 Å=6000 x 10^{-7} mm

Width of slit 'e' = 0.30 mm

The distance between screen and slit 'D = f'= 2000 mm

Condition of nth dark fringe (minima) in single slit is

$$e\sin\theta = n\lambda$$

For first dark, n = 1 then,

$$e\sin\theta = \lambda \quad \Rightarrow \quad \sin\theta = \frac{\lambda}{e}$$

As we know, $\sin\theta = \frac{x}{D}$

Then equating eq. (1) & (2),

$$\frac{\lambda}{e} = \frac{x}{D} \quad \Rightarrow \quad x = \frac{\lambda D}{e}$$

The position of first minima is, $x = \frac{2000\ mm \times 6000 \times 10^{-7} mm}{0.30\ mm}$

$$\Rightarrow x = 4\ mm = 0.4\ cm$$

And, the width of the central bright fringe is W = 2x = 8 mm = 0.8 cm

Q.6 (a) What is a wave function in quantum mechanics? Explain the physical significance of a wave function. (2)

Solu: The wave function $\psi(\vec{r},t)$ which is a solution of Schrödinger equation, describes the position of a particle with respect to time. It can be considered as 'probability amplitude since it is used to find the location of the particle. According to him ψ may be regarded as a measure of the probability of finding the particle at a particular position and a particular time.

The wave functions are usually complex. Therefore, the probability density for a complex wave function ψ is taken as the square of the absolute magnitude of ψ. Thus

The probability density $P(\vec{r},t) = |\psi(\vec{r},t)|^2 = \psi * (\vec{r},t)\psi(\vec{r},t)$

Here, $\psi * (\vec{r},t)$ is the complex conjugate of the wave function and for complex conjugate 'i' is replaced by '-i'.

Then, the probability of finding the particle in the volume element dV=dxdydz about any point $\vec{r}$ at time 't' is given by as: $P(\vec{r},t)dV = |\psi(\vec{r},t)|^2 dV = \psi * (\vec{r},t)\psi(\vec{r},t)\, dV$

Normalization condition: If the particle is somewhere in space then the total probability must be equal to unity for all space, i.e.

$$\int_{-\infty}^{\infty} |\psi(\vec{r},t)|^2 dV = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} \psi * (\vec{r},t)\psi(\vec{r},t)\, dV = 1$$

This is called Normalization condition. The wave function $\psi(\vec{r},t)$ which satisfies the above condition is said to be normalized wave function. For such normalization, the amplitude of ψ is adjusted to make the integral equal to unity.

Orthogonality condition: If ψ_i and ψ_j are two different wave functions satisfying the wave equation for a give system such that

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$$\int_{-\infty}^{\infty} \psi_i * (\vec{r}, t)\psi_j(\vec{r}, t)\, dV = \int_{-\infty}^{\infty} \psi_j * (\vec{r}, t)\psi_i(\vec{r}, t)\, dV = 0$$

Then this condition is known as orthogonal condition and the wave functions ψ_i and ψ_j are said to be mutually orthogonal.

Q.6 (b) Wave function corresponding to a system is given as $\psi(x) = Ne^{icx}$. Determine the value of normalization constant over the range $-a < x < a$. (2)

Solu:

Given

wave function $\psi(x) = Ne^{icx}$

from Normalization condition

$$\int_{-\infty}^{\infty} \psi^*(x)\, \psi(x)\, dx = 1$$

here $\psi^*(x) = Ne^{-icx}$

$$\int_{-a}^{a} N^2 e^{icx} e^{-icx}\, dx = 1$$

$$N^2 \int_{-a}^{a} dx = 1 \Rightarrow N^2 |x|_{-a}^{a} = N^2 2a = 1$$

$$\Rightarrow N = \frac{1}{\sqrt{2a}}$$

Hence normalized wave function is $\psi(x) = \frac{1}{\sqrt{2a}} e^{icx}$

PART C (Descriptive/Analytical/Problem solving/Design questions)

Q.7 Show that the intensity of light diffracted from a plane transmission grating is given by- $I = I_0 (\sin^2\alpha/\alpha^2)(\sin^2 N\beta/\sin^2\beta)$, where symbols have their usual meanings. (6)

Solu: Let us consider a plane transmission grating of N similar parallel slits and a parallel beam of light of wavelength 'λ' incident normally on the grating surface.

Let the width of each slit =e

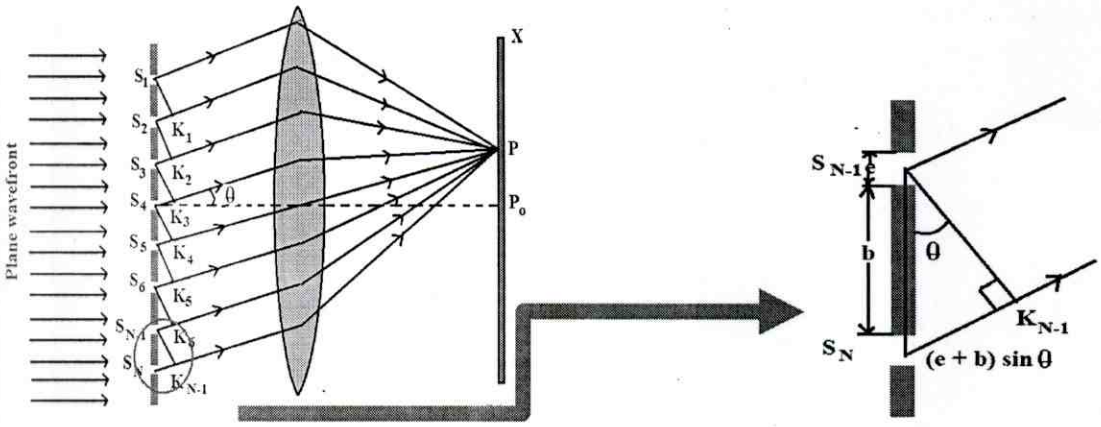
The width of each opaque space=b

Grating element = (e+b)= distance between mid points of successive slits.

According to Huygen's principle, each point of the incident wavefront acts like a secondary source and emits out secondary wavelets in all direction. Using the theory of single slit, the secondary wavelets emerging from all the points in a single slit in a direction 'θ', can be considered to be equivalent to a single wave of amplitude $R = pa\frac{\sin\alpha}{\alpha}$ [here $\alpha = \frac{\pi}{\lambda}e\sin\theta$]starting from the mid of the slit. Here we are considering only those secondary wavelets which are diffracted in a direction 'θ'.

As the total number of slits in grating is 'N', so the waves diffracted from all the slits in direction 'θ'are equivalent to N parallel waves each from the middle points of the slits S_1, S_2, S_3,...S_{N-1},S_N respectively.

These parallel waves interfere and gives interference pattern on the screen.



Let S_1K_1, S_2K_2, S_3K_3,........., $S_{N-1}K_{N-1}$ are the perpendicular drawn from the mid points of the slits S_1, S_2, S_3,...S_{N-1}.The path difference between the wavelets starting from the mid points of slits S_1 and S_2 and reaching at a point P in a direction 'θ' can be calculated as:

From right triangle $\Delta S_1K_1S_2$, we have

$$\frac{S_2K_1}{S_1S_2} = \sin\theta$$

$$S_2K_1 = S_1S_2\sin\theta$$

As $S_1S_2 = (e+b)$ $\Rightarrow S_2K_1 = (e+b)\sin\theta$ ...(1)

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Similarly the path difference between the wavelets mid points of slits S2 and S3 and reaching at the same point P in a direction 'θ' is

$$S_3K_2 = (e+b)\sin\theta \qquad ...(2)$$

Therefore equivalent phase difference will be

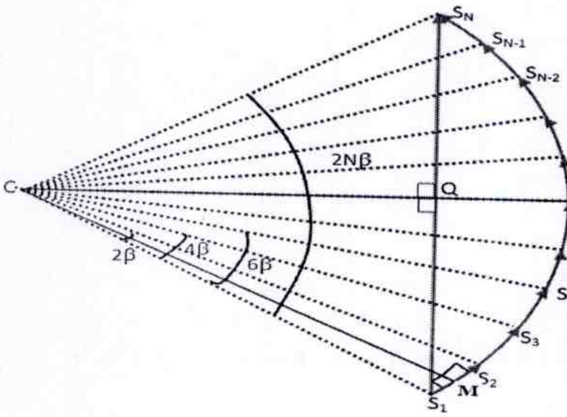
$$\delta = \frac{2\pi}{\lambda} \times path\ difference$$

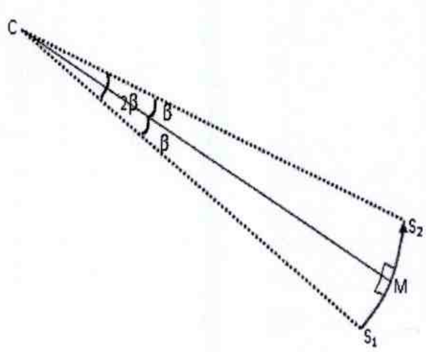
$$\delta = \frac{2\pi}{\lambda} \times (e+b)\sin\theta = 2\beta\ (let) \qquad ...(3)$$

So, it is clear that the phase difference between the successive wavelets is 2β. Now let us use vector polygon method to calculate the resultant amplitude of N waves in a direction 'θ' having common phase difference 2β and common amplitude ($R = pa\frac{\sin\alpha}{\alpha}$).

Let us draw a vector polygon of each side of equal length R and having phase difference of 2β among themselves.

i.e. $S_1S_2=S_2S_3=....=S_{N-1}S_N=R$





Here C is the centre of the polygon.

$$\angle S_1CS_N = 2N\beta$$
$$\angle S_1CO = \angle S_NCO = N\beta$$
$$\angle S_1CS_2 = \angle S_2CS_3 = 2\beta$$

From Figure, we get $S_1S_N = 2S_1Q$

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From ΔCQS_1 $\sin N\beta = \dfrac{S_1Q}{CS_1}$ ....(4)

$$S_1Q = (CS_1)\sin N\beta$$
$$S_1S_N = 2S_1Q = 2(CS_1)\sin N\beta \qquad ...(5)$$

Similarly $S_1S_2 = 2S_1M$ ...(6)

From ΔCMS_1 $\sin\beta = \dfrac{S_1M}{CS_1}$ (Since β is very small, so sinβ=tanβ)

$$S_1N = (CS_1)\sin\beta$$
$$S_1S_2 = 2S_1N = 2(CS_1)\sin\beta \qquad ....(7)$$

From equations (5) and (7), we have

$$\frac{S_1S_N}{S_1S_2} = \frac{2(CS_1)\sin N\beta}{2(CS_1)\sin\beta}$$
$$S_1S_N = S_1S_2 \times \frac{\sin N\beta}{\sin\beta} \qquad ...(8)$$

Since $S_1S_2 = R = pa\dfrac{\sin\alpha}{\alpha}$

$$S_1S_N = \left(pa\frac{\sin\alpha}{\alpha}\right)\left(\frac{\sin N\beta}{\sin\beta}\right)$$

Hence the resultant amplitude due to N-waves in a direction ‘θ, can be given as

$$R_N = S_1S_N = \left(pa\frac{\sin\alpha}{\alpha}\right)\left(\frac{\sin N\beta}{\sin\beta}\right) \qquad ...(9)$$

The resultant intensity at point of observation P is given as

As $I \propto R_N^2$

$$\Rightarrow I = Kp^2a^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2 \qquad ...(10)$$

Let $Kp^2a^2=I_o$ $\alpha = \frac{\pi}{\lambda} e\sin\theta$ and $\beta = \frac{\pi}{\lambda}(e+b)\sin\theta$

$$\Rightarrow I = I_o\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2 \quad ...(11)$$

Equation (11) gives the expression for intensity due to Fraunhofer diffraction at N slits.

Here the term $p^2a^2\left(\frac{\sin\alpha}{\alpha}\right)^2$ gives the intensity expression due to single slit diffraction pattern whereas the second term $\left(\frac{\sin N\beta}{\sin\beta}\right)^2$ gives the intensity expression due to interference of secondary wavelets from N-slits.

Q.8 Derive the Schrödinger's time dependent and time independent wave equation. (6)

Solu: Time Dependent Schrödinger's Equation: Let us consider a particle is moving in the +x direction. According to de Broglie, a wave is associated with this particle. Let us consider the wave function of this wave is:

$$\psi(x,t) = Ae^{i(kx-\omega t)} \quad ....(1)$$

This is a plane wave equation.

Here, the wave vector k is defined as; $k = \frac{2\pi}{\lambda} = \frac{p}{\hbar}$ ....(2) [where, $\lambda = \frac{h}{p}$ and $\hbar = \frac{h}{2\pi}$]

or $p = \hbar k$

and Angular frequency ω is defined as; $\omega = 2\pi\nu = \frac{2\pi h\nu}{h} = \frac{E}{\hbar}$ .... (3) [where, $E = h\nu$]

or $E = \hbar\omega$

Now put equation (2) & (3) in equation (1), then

$$\psi(x,t) = Ae^{\frac{i}{\hbar}(px-Et)} \quad ....(4)$$

where E is the total energy of the particle and p is its momentum.

Differentiating equation (4) with respect to 'x'

$$\frac{\partial\psi(x,t)}{\partial x} = \frac{i}{\hbar} pAe^{\frac{i}{\hbar}(px-Et)}$$

$or,\quad \frac{\partial\psi(x,t)}{\partial x} = \frac{i}{\hbar} p\,\psi(x,t)$ ....(5)

$\Rightarrow \quad p\,\psi(x,t) = \frac{\hbar}{i}\frac{\partial\psi(x,t)}{\partial x} = -i\hbar\frac{\partial\psi(x,t)}{\partial x}$ ....(6)

From equation (6) we get momentum operator in x-direction, i.e.

$\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$ ....(7)

Now, again differentiating equation (5) with respect to 'x'

$$\frac{\partial^2\psi(x,t)}{\partial x^2} = \frac{i^2}{\hbar^2} p\,\psi(x,t)$$

$$\Rightarrow \quad \frac{\partial^2\psi(x,t)}{\partial x^2} = -\frac{p^2}{\hbar^2}\psi(x,t)$$

Or, $p^2\psi(x,t) = -\hbar^2\frac{\partial^2\psi(x,t)}{\partial x^2}$ ....(8)

Now differentiating equation (4) with respect to 't'

$$\frac{\partial\psi(x,t)}{\partial t} = -\frac{i}{\hbar} EAe^{\frac{i}{\hbar}(px-Et)}$$

$or,\quad \frac{\partial\psi(x,t)}{\partial t} = -\frac{i}{\hbar} E\,\psi(x,t)$

$\Rightarrow \quad E\,\psi(x,t) = -\frac{\hbar}{i}\frac{\partial\psi(x,t)}{\partial t} = i\hbar\frac{\partial\psi(x,t)}{\partial t}$ ....(9)

From equation (9) we get Energy operator, i.e.

$\hat{E} = i\hbar\frac{\partial}{\partial t}$ ....(10)

For non-relativistic case where v << c, the total energy of the particle E is sum of kinetic energy and potential energy, i.e.

$E = \frac{p^2}{2m} + V$ ....(11)

here, potential energy 'V' is the function of x and t, i.e. V (x, t).

By acting this energy operator on wave function $\psi\ (x,t)$ or multiplying equation (11) by $\psi(x\ ,t)$ on both sides, then we obtain

$$E\psi(x,t) = \frac{p^2}{2m}\psi(x,t) + V\psi(x,t) \quad ....(12)$$

Substituting equations (8) and (9) in equation (12) we get,

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V\psi(x,t) \quad ....(13)$$

Equation (13) is known as one–dimension time dependent Schrödinger's equation.

Similarly, in three dimensions wave function $\psi(x,y,z,t) = \psi(\vec{r},t)$ it shall be

$$i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\psi(\vec{r},t) + V\psi(\vec{r},t) \quad ....(14)$$

$$or, \quad i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r},t) + V\psi(\vec{r},t) \quad ....(15)$$

here, $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right] = \nabla^2$ ....(16) ; known as Laplacian operator.

Equation (15) is known as three- dimensional time dependent Schrödinger's equation.

Time Independent Schrödinger's Equation: In most of the physical system the potential energy 'V' does not depend on the time but depends only on the position of the particle, *i.e.* $\mathrm{V}(x,t) = \mathrm{V}(x)$. For such a system, the wave function can be separated into time independent and time dependent part as:

$$\because \quad \psi\ (x,t) = Ae^{\frac{i}{\hbar}(px-Et)} \quad ....(1)$$

$$\therefore \quad \psi(x\ ,t) = Ae^{\frac{ipx}{\hbar}}e^{-\frac{iEt}{\hbar}} = \psi(x)e^{-\frac{iEt}{\hbar}} \quad ....(2)$$

Differentiating equation (2) with respect to 'x' twice, then

$$\frac{\partial^2\psi(x,t)}{\partial x^2} = \frac{d^2\psi(x)}{dx^2}e^{-\frac{iEt}{\hbar}} \quad ....(3)$$

Now, differentiating equation (2) with respect to 't'

$$\frac{\partial\psi(x,t)}{\partial t} = -\frac{i}{\hbar}E\,\psi(x)\,e^{-\frac{iEt}{\hbar}} \qquad \ldots(4)$$

Now put equations (3) & (4) in time dependent Schrödinger's equation. As time dependent Schrödinger's equation in 1-D case is

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

So, $$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}e^{-\frac{iEt}{\hbar}} + V\psi(x)e^{-\frac{iEt}{\hbar}} = i\hbar\left(\frac{-iE}{\hbar}\right)\psi(x)e^{-\frac{iEt}{\hbar}}$$

Or, $$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi(x) = 0 \qquad \ldots(5)$$

Equation (5) is known as time independent Schrödinger's equation in one–dimension.

For 3-D time independent Schrödinger's equation, let us consider $\psi(\vec{r}) = \psi(x,y,z)$, then

$$\frac{\partial^2\psi(\vec{r})}{\partial x^2} + \frac{\partial^2\psi(\vec{r})}{\partial y^2} + \frac{\partial^2\psi(\vec{r})}{\partial z^2} + \frac{2m}{\hbar^2}[E-V]\psi(\vec{r}) = 0$$

Or, $$\nabla^2\psi(\vec{r}) + \frac{2m}{\hbar^2}[E-V]\psi(\vec{r}) = 0 \qquad \ldots(6)$$

The above equation is called time independent Schrödinger equation in 3-D.