NHANES Hypothesis Testing Walkthrough

February 26, 2022

1 Hypothesis Testing

In this notebook we demonstrate formal hypothesis testing using the NHANES data.

It is important to note that the NHANES data are a "complex survey". The data are not an independent and representative sample from the target population. Proper analysis of complex survey data should make use of additional information about how the data were collected. Since complex survey analysis is a somewhat specialized topic, we ignore this aspect of the data here, and analyze the NHANES data as if it were an independent and identically distributed sample from a population.

```
In [1]: import pandas as pd
    import numpy as np
    import matplotlib
    matplotlib.use('Agg') # workaround, there may be a better way
    import seaborn as sns
    %matplotlib inline
    import matplotlib.pyplot as plt
    import statsmodels.api as sm
    import scipy.stats.distributions as dist
```

Below we read the data, and convert some of the integer codes to text values.

```
Out[4]: 0 Male

1 Male
2 Male
3 Female
4 Female
Name: RIAGENDRx, dtype: object
```

1.0.1 Hypothesis Tests for One Proportion

The most basic hypothesis test may be the one-sample test for a proportion. This test is used if we have specified a particular value as the null value for the proportion, and we wish to assess if the data are compatible with the true parameter value being equal to this specified value. One-sample tests are not used very often in practice, because it is not very common that we have a specific fixed value to use for comparison. For illustration, imagine that the rate of lifetime smoking in another country was known to be 40%, and we wished to assess whether the rate of lifetime smoking in the US were different from 40%. In the following notebook cell, we carry out the (two-sided) one-sample test that the population proportion of smokers is 0.4, and obtain a p-value of 0.43. This indicates that the NHANES data are compatible with the proportion of (ever) smokers in the US being 40%.

```
In [5]: x = da.SMQ020x.dropna() == "Yes"
In [8]: x.head()
Out[8]: 0
              True
              True
        1
        2
              True
        3
             False
             False
        Name: SMQ020x, dtype: bool
In [6]: p = x.mean()
In [7]: p
Out[7]: 0.4050655021834061
In [10]: se = np.sqrt(.4 * (1 - .4) / len(x))
Out[10]: 0.00647467353462031
In [11]: test_stat = (p - 0.4) / se
         test_stat
Out[11]: 0.7823563854332805
In [12]: pvalue = 2 * dist.norm.cdf(-np.abs(test_stat))
         print(test_stat, pvalue)
```

The following cell carries out the same test as performed above using the Statsmodels library. The results in the first (default) case below are slightly different from the results obtained above because Statsmodels by default uses the sample proportion instead of the null proportion when computing the standard error. This distinction is rarely consequential, but we can specify that the null proportion should be used to calculate the standard error, and the results agree exactly with what we calculated above. The first two lines below carry out tests using the normal approximation to the sampling distribution of the test statistic, and the third line below carries uses the exact binomial sampling distribution. We can see here that the p-values are nearly identical in all three cases. This is expected when the sample size is large, and the proportion is not close to either 0 or 1.

```
In [13]: sm.stats.proportions_ztest(x.sum(), len(x), 0.4)
Out[13]: (0.7807518954896244, 0.43494843171868214)
In [14]: sm.stats.binom_test(x.sum(), len(x), 0.4)
Out[14]: 0.4340360854459431
```

1.0.2 Hypothesis Tests for Two Proportions

Comparative tests tend to be used much more frequently than tests comparing one population to a fixed value. A two-sample test of proportions is used to assess whether the proportion of individuals with some trait differs between two sub-populations. For example, we can compare the smoking rates between females and males. Since smoking rates vary strongly with age, we do this in the subpopulation of people between 20 and 25 years of age. In the cell below, we carry out this test without using any libraries, implementing all the test procedures covered elsewhere in the course using Python code. We find that the smoking rate for men is around 10 percentage points greater than the smoking rate for females, and this difference is statistically significant (the p-value is around 0.01).

```
In [15]: dx = da[["SMQ020x", "RIDAGEYR", "RIAGENDRx"]].dropna()
         dx.head()
Out [15]:
           SMQ020x
                     RIDAGEYR RIAGENDRx
         0
               Yes
                           62
                                   Male
         1
               Yes
                           53
                                   Male
         2
                           78
               Yes
                                   Male
                                 Female
         3
                Nο
                           56
         4
                Nο
                           42
                                 Female
In [16]: p = dx.groupby("RIAGENDRx")["SMQ020x"].agg([lambda z: np.mean(z == "Yes"), "size"])
         p.columns = ["Smoke", "N"]
         print(p)
```

```
        Smoke
        N

        RIAGENDRx
        8

        Female
        0.304845
        2972

        Male
        0.513258
        2753
```

Essentially the same test as above can be conducted by converting the "Yes"/"No" responses to numbers (Yes=1, No=0) and conducting a two-sample t-test, as below:

```
In [17]: p_{comb} = (dx.SMQ020x == "Yes").mean()
         va = p_comb * (1 - p_comb)
         se = np.sqrt(va * (1 / p.N.Female + 1 / p.N.Male))
In [18]: (p_comb, va, se)
Out [18]: (0.4050655021834061, 0.2409874411243111, 0.01298546309757376)
In [19]: test_stat = (p.Smoke.Female - p.Smoke.Male) / se
         p_value = 2 * dist.norm.cdf(-np.abs(test_stat))
         (test_stat, p_value)
Out[19]: (-16.049719603652488, 5.742288777302776e-58)
In [20]: dx_females = dx.loc[dx.RIAGENDRx == "Female", "SMQ020x"].replace({"Yes": 1, "No": 0})
         dx_females
Out[20]: 3
                 0
         4
                 0
                 0
         7
                 0
         12
                 1
         13
                 0
         15
                 0
         16
                 0
         17
                 0
         18
                 1
         19
                 0
         21
                 0
         22
                 1
         23
                 0
         25
                 0
         27
                 1
         29
                 0
         30
                 1
         33
                 0
         34
                 0
         35
                 1
         36
                 0
```

```
39
                  0
         43
                  0
         46
                  0
         47
                  0
                  0
         50
         52
                  0
         54
                  0
         5678
                  1
         5679
                  0
         5681
                  0
         5682
                  1
         5683
                  0
         5684
                  0
         5685
                  0
         5686
                  0
         5689
                  0
         5692
                  0
         5696
                  1
         5697
                  0
         5699
                  0
         5703
                  1
         5704
                  0
         5707
                  0
         5708
                  0
         5710
                  0
         5712
                  0
         5715
                  0
         5716
                  1
         5719
                  1
         5721
                  0
         5722
                  0
         5723
                  1
         5724
                  0
         5727
                  0
         5730
                  1
         5732
         5734
                  0
         Name: SMQ020x, Length: 2972, dtype: int64
In [21]: dx_males = dx.loc[dx.RIAGENDRx == "Male", "SMQ020x"].replace({"Yes": 1, "No": 0})
         dx_{males}
Out[21]: 0
                  1
                  1
         2
                  1
         6
                  1
```

8 9 10 11 14 20 24 26	0 0 1 1 0 0 0
28	0
31 32	0
37	0
40	1
41 42	0
44	1
45	1
48 49	0
51	0
53	1
56 57	1
59	0
60	1
64	1
5672	0
	1 1
5673	1
5677	
	0
5677 5680 5687 5688	0 1 0
5677 5680 5687 5688 5690	0 1 0 1
5677 5680 5687 5688	0 1 0
5677 5680 5687 5688 5690 5691 5693 5694	0 1 0 1 0 0
5677 5680 5687 5688 5690 5691 5693 5694 5695	0 1 0 1 0 0 0
5677 5680 5687 5688 5690 5691 5693 5694 5695 5698	0 1 0 1 0 0 0 0
5677 5680 5687 5688 5690 5691 5693 5694 5695	0 1 0 1 0 0 0
5677 5680 5687 5688 5690 5691 5693 5694 5695 5698 5700 5701 5702	0 1 0 1 0 0 0 0 1 1 0
5677 5680 5687 5688 5690 5691 5693 5694 5695 5698 5700 5701 5702 5705	0 1 0 1 0 0 0 0 1 1 0 0
5677 5680 5687 5688 5690 5691 5693 5694 5695 5698 5700 5701 5702	0 1 0 1 0 0 0 0 1 1 0
5677 5680 5687 5688 5690 5691 5693 5694 5695 5700 5701 5702 5705 5706 5709 5711	0 1 0 1 0 0 0 0 1 1 0 0 1 1 1 0
5677 5680 5687 5688 5690 5691 5693 5694 5695 5698 5700 5701 5702 5705 5706 5709	0 1 0 1 0 0 0 0 1 1 0 0

```
5717
                  1
         5718
                  0
         5720
                  0
         5725
                  0
         5726
                  1
         5728
                  0
         5729
                  0
         5731
                  0
         5733
                  1
         Name: SMQ020x, Length: 2753, dtype: int64
In [22]: sm.stats.ttest_ind(dx_females, dx_males)
Out [22]: (-16.42058555898443, 3.032088786691117e-59, 5723.0)
```

1.0.3 Hypothesis Tests Comparing Means

Tests of means are similar in many ways to tests of proportions. Just as with proportions, for comparing means there are one and two-sample tests, z-tests and t-tests, and one-sided and two-sided tests. As with tests of proportions, one-sample tests of means are not very common, but we illustrate a one sample test in the cell below. We compare systolic blood pressure to the fixed value 120 (which is the lower threshold for "pre-hypertension"), and find that the mean is significantly different from 120 (the point estimate of the mean is 126).

```
In [23]: dx = da[["BPXSY1", "RIDAGEYR", "RIAGENDRx"]].dropna()
          dx
                         RIDAGEYR RIAGENDRx
Out [23]:
                BPXSY1
          0
                 128.0
                                62
                                         Male
          1
                 146.0
                                53
                                         Male
          2
                 138.0
                                78
                                         Male
          3
                 132.0
                                56
                                      Female
          4
                 100.0
                                42
                                      Female
          5
                 116.0
                                72
                                      Female
          6
                 110.0
                                22
                                         Male
          7
                                32
                 120.0
                                      Female
          9
                 178.0
                                56
                                         Male
          10
                 144.0
                                46
                                         Male
                 116.0
                                45
          11
                                         Male
          12
                 104.0
                                30
                                      Female
          13
                 124.0
                                67
                                      Female
          14
                 132.0
                                67
                                         Male
          15
                                57
                 134.0
                                      Female
          16
                 102.0
                                19
                                      Female
          17
                                24
                                      Female
                 110.0
          18
                 138.0
                                27
                                      Female
          19
                 136.0
                                54
                                      Female
          20
                 110.0
                                49
                                         Male
          21
                 148.0
                                80
                                      Female
```

```
22
        140.0
                       69
                             Female
23
        116.0
                       58
                             Female
24
        136.0
                                Male
                      56
25
        108.0
                       27
                             Female
26
        122.0
                       22
                                Male
        142.0
27
                       60
                             Female
28
        132.0
                      51
                                Male
29
        122.0
                       68
                             Female
30
        146.0
                       69
                             Female
. . .
          . . .
5702
        116.0
                      38
                                Male
5703
        178.0
                       64
                             Female
5704
        134.0
                      75
                             Female
5705
        174.0
                       80
                                Male
5706
        124.0
                      72
                                Male
5707
        130.0
                       25
                             Female
5708
        102.0
                       29
                             Female
5709
        132.0
                       38
                                Male
5711
        144.0
                                Male
                       62
5712
        114.0
                       27
                             Female
5713
        116.0
                       43
                                Male
5714
        162.0
                       39
                                Male
5715
        124.0
                       34
                             Female
5717
        112.0
                       32
                                Male
5718
        128.0
                       45
                                Male
5720
        110.0
                                Male
                       38
5721
        118.0
                       35
                             Female
5722
        114.0
                       34
                             Female
5723
        142.0
                      72
                             Female
5724
        132.0
                       41
                             Female
5725
        110.0
                       34
                                Male
5726
        132.0
                       53
                                Male
5727
        164.0
                       69
                             Female
5728
        112.0
                       32
                                Male
5729
        112.0
                       25
                                Male
5730
        112.0
                      76
                             Female
5731
        118.0
                       26
                                Male
5732
        154.0
                      80
                             Female
5733
        104.0
                       35
                                Male
5734
        118.0
                       24
                             Female
```

[5401 rows x 3 columns]

```
In [24]: dx = dx.loc[(dx.RIDAGEYR >= 40) & (dx.RIDAGEYR <= 50) & (dx.RIAGENDRx == "Male"), :] dx
```

Out[24]: BPXSY1 RIDAGEYR RIAGENDRx 10 144.0 46 Male

11	116.0	45	Male
20	110.0	49	Male
42	128.0	42	Male
51	118.0	50	Male
66	124.0	41	Male
70	104.0	40	Male
72	140.0	48	Male
94	112.0	49	Male
101	104.0	43	Male
116	124.0	45	Male
119	132.0	43	Male
133	134.0	49	Male
135	120.0	40	Male
144	130.0	40	Male
152	154.0	43	Male
173	112.0	44	Male
176	102.0	46	Male
197	136.0	40	Male
204	120.0	45	Male
224	104.0	46	Male
246	192.0	45	Male
249	152.0	46	Male
251	156.0	43	Male
252	152.0	46	Male
269	106.0	45	Male
299	148.0	50	Male
323	116.0	41	Male
339	114.0	40	Male
358	98.0	42	Male
5309	144.0	44	Male
5317	124.0	46	Male
5330	118.0	40	Male
5358	114.0	49	Male
5369	114.0	41	Male
5370	136.0	46	Male
5376	142.0	49	Male
5378	110.0	43	Male
5379	138.0	42	Male
5388	128.0	50	Male
5421	116.0	46	Male
5448	162.0	48	Male
5486	116.0	40	Male
5501	132.0	47	Male
5555	124.0	44	Male
5593	126.0	48	Male
5596	146.0	50	Male
5601	114.0	50	Male

```
5610
        106.0
                      47
                               Male
5612
        124.0
                      46
                               Male
5625
        114.0
                      47
                               Male
5628
        104.0
                      41
                               Male
5644
        134.0
                      48
                               Male
5662
        146.0
                      47
                               Male
5666
        106.0
                      50
                               Male
5680
        134.0
                      50
                               Male
5690
        138.0
                      48
                               Male
5693
        96.0
                      41
                               Male
5713
        116.0
                      43
                               Male
5718
        128.0
                      45
                               Male
```

[421 rows x 3 columns]

```
In [25]: print(dx.BPXSY1.mean())
125.86698337292161
In [26]: sm.stats.ztest(dx.BPXSY1, value=120)
Out[26]: (7.469764137102597, 8.033869113167905e-14)
```

In the cell below, we carry out a formal test of the null hypothesis that the mean blood pressure for women between the ages of 50 and 60 is equal to the mean blood pressure of men between the ages of 50 and 60. The results indicate that while the mean systolic blood pressure for men is slightly greater than that for women (129 mm/Hg versus 128 mm/Hg), this difference is not statistically significant.

There are a number of different variants on the two-sample t-test. Two often-encountered variants are the t-test carried out using the t-distribution, and the t-test carried out using the normal approximation to the reference distribution of the test statistic, often called a z-test. Below we display results from both these testing approaches. When the sample size is large, the difference between the t-test and z-test is very small.

```
In [36]: dx = da[["BPXSY1", "RIDAGEYR", "RIAGENDRx"]].dropna()
         dx = dx.loc[(dx.RIDAGEYR >= 50) & (dx.RIDAGEYR <= 60), ["BPXSY1", "RIDAGEYR", "RIAGEN
         dx.head()
Out [36]:
                     RIDAGEYR RIAGENDRx
             BPXSY1
         1
              146.0
                            53
                                    Male
         3
              132.0
                                  Female
                            56
         9
              178.0
                            56
                                    Male
         15
              134.0
                            57
                                  Female
         19
              136.0
                            54
                                  Female
In [37]: bpx_female = dx.loc[dx.RIAGENDRx=="Female", "BPXSY1"]
         bpx_male = dx.loc[dx.RIAGENDRx=="Male", "BPXSY1"]
         print(bpx_female.mean(), bpx_male.mean())
```

```
In [38]: print(sm.stats.ztest(bpx_female, bpx_male))
(-1.105435895556249, 0.2689707570859362)
In [39]: print(sm.stats.ttest_ind(bpx_female, bpx_male))
(-1.105435895556249, 0.26925004137768577, 952.0)
```

Another important aspect of two-sample mean testing is "heteroscedasticity", meaning that the variances within the two groups being compared may be different. While the goal of the test is to compare the means, the variances play an important role in calibrating the statistics (deciding how big the mean difference needs to be to be declared statistically significant). In the NHANES data, we see that there are moderate differences between the amount of variation in BMI for females and for males, looking within 10-year age bands. In every age band, females having greater variation than males.

```
In [40]: dx = da[["BMXBMI", "RIDAGEYR", "RIAGENDRx"]].dropna()
         da["agegrp"] = pd.cut(da.RIDAGEYR, [18, 30, 40, 50, 60, 70, 80])
         da.groupby(["agegrp", "RIAGENDRx"])["BMXBMI"].agg(np.std).unstack()
Out[40]: RIAGENDRx
                      Female
                                  Male
         agegrp
         (18, 30]
                    7.745893
                              6.649440
         (30, 40]
                    8.315608
                              6.622412
         (40, 50]
                    8.076195 6.407076
         (50, 60]
                    7.575848 5.914373
         (60, 70]
                    7.604514 5.933307
                    6.284968 4.974855
         (70, 80]
In [41]: da.groupby(["agegrp", "RIAGENDRx"])["BMXBMI"].agg(np.std)
Out[41]: agegrp
                   RIAGENDRx
                   Female
         (18, 30]
                                7.745893
                   Male
                                6.649440
                   Female
         (30, 40]
                                8.315608
                   Male
                                6.622412
         (40, 50]
                                8.076195
                   Female
                   Male
                                6.407076
                   Female
         (50, 60]
                                7.575848
                   Male
                                5.914373
         (60, 70]
                   Female
                                7.604514
                   Male
                                5.933307
         (70, 80]
                  Female
                                6.284968
                   Male
                                4.974855
         Name: BMXBMI, dtype: float64
```

The standard error of the mean difference (e.g. mean female blood pressure minus mean male blood pressure) can be estimated in at least two different ways. In the statsmodels library, these approaches are referred to as the "pooled" and the "unequal" approach to estimating the variance. If the variances are equal (i.e. there is no heteroscedasticity), then there should be little difference between the two approaches. Even in the presence of moderate heteroscedasticity, as we have here, we can see that the results for the two differences are quite similar. Below we have a loop that considers each 10-year age band and assesses the evidence for a difference in mean BMI for women and for men. The results printed in each row of output are the test-statistic and p-value.

```
In [42]: for k, v in da.groupby("agegrp"):
             bmi_female = v.loc[v.RIAGENDRx=="Female", "BMXBMI"].dropna()
             bmi_female = sm.stats.DescrStatsW(bmi_female)
             bmi_male = v.loc[v.RIAGENDRx=="Male", "BMXBMI"].dropna()
             bmi_male = sm.stats.DescrStatsW(bmi_male)
             print("pooled: ", sm.stats.CompareMeans(bmi_female, bmi_male).ztest_ind(usevar='p
             print("unequal: ", sm.stats.CompareMeans(bmi_female, bmi_male).ztest_ind(usevar=')
             print()
(18, 30]
pooled:
         (1.7026932933643388, 0.08862548061449649)
unequal:
          (1.7174610823927268, 0.08589495934713022)
(30, 40]
pooled:
         (1.4378280405644916, 0.1504828511464818)
          (1.4437869620833494, 0.14879891057892475)
unequal:
(40, 50]
pooled:
         (2.8933761158070186, 0.003811246059501354)
          (2.9678691663536725, 0.0029987194174035366)
unequal:
(50, 60]
pooled:
         (3.362108779981367, 0.0007734964571391746)
unequal:
          (3.375494390173923, 0.0007368319423226574)
(60, 70]
         (3.6172401442432753, 0.000297761021031936)
pooled:
unequal:
          (3.62848309454456, 0.0002850914147149227)
(70, 80]
pooled: (2.926729252512258, 0.0034254694144858636)
unequal: (2.937779886769224, 0.003305716331519299)
```