Confidence_Intervals_Differences_Population_Parameters

February 10, 2022

1 Confidence Intervals

This tutorial is going to demonstrate how to load data, clean/manipulate a dataset, and construct a confidence interval for the difference between two population proportions and means.

We will use the 2015-2016 wave of the NHANES data for our analysis.

*Note: We have provided a notebook that includes more analysis, with examples of confidence intervals for one population proportions and means, in addition to the analysis I will show you in this tutorial. I highly recommend checking it out!

For our population proportions, we will analyze the difference of proportion between female and male smokers. The column that specifies smoker and non-smoker is "SMQ020" in our dataset.

For our population means, we will analyze the difference of mean of body mass index within our female and male populations. The column that includes the body mass index value is "BMXBMI".

Additionally, the gender is specified in the column "RIAGENDR".

```
In [1]: import pandas as pd
    import numpy as np
    import matplotlib
    matplotlib.use('Agg')
    import seaborn as sns
    %matplotlib inline
    import matplotlib.pyplot as plt
    import statsmodels.api as sm
In [2]: url = "nhanes_2015_2016.csv"
    da = pd.read_csv(url)
```

1.0.1 Investigating and Cleaning Data

4	No
5	No
6	Yes
7	No
8	No
9	No
10	Yes
11	Yes
12	Yes
13	No
14	No
15	No
16	No
17	No
18	Yes
19	No
20	No
21	No
22	Yes
23	No
24	No
25	No
26	Yes
27	Yes
	Ma
28	No
28 29	No
29	No
29 5705	No ··· Yes
29 5705 5706	No ···· Yes Yes
29 5705 5706 5707	No Yes Yes
5705 5706 5707 5708	No Yes Yes No No
29 5705 5706 5707 5708 5709	No Yes Yes No No Yes
5705 5706 5707 5708 5709 5710	No Yes Yes No No Yes
5705 5706 5707 5708 5709 5710 5711	No Yes Yes No No Yes No
29 5705 5706 5707 5708 5709 5710 5711 5712	No Yes Yes No No Yes No Yes No
5705 5706 5707 5708 5709 5710 5711 5712 5713	No Yes Yes No No Yes No Yes No
5705 5706 5707 5708 5709 5710 5711 5712 5713 5714	No Yes Yes No No Yes No Yes No Yos No
5705 5706 5707 5708 5709 5710 5711 5712 5713 5714 5715	No Yes Yes No No Yes No Yes No Yoo No
5705 5706 5707 5708 5709 5710 5711 5712 5713 5714 5715 5716	No Yes Yes No No Yes No Yes No Yes No Yes No Yes
5705 5706 5707 5708 5709 5710 5711 5712 5713 5714 5715 5716 5717	No Yes Yes No No Yes No Yes No Yes No Yes Yes Yes
5705 5706 5707 5708 5709 5710 5711 5712 5713 5714 5715 5716 5717	No Yes Yes No No Yes No Yes No No No No No No No Yes Yes No
5705 5706 5707 5708 5709 5710 5711 5712 5713 5714 5715 5716 5717 5718 5719	No Yes Yes No Yes No Yes No Yes No No No Yes No Yes Yes Yes
5705 5706 5707 5708 5709 5710 5711 5712 5713 5714 5715 5716 5717 5718 5719 5720	No Yes Yes No
5705 5706 5707 5708 5709 5710 5711 5712 5713 5714 5715 5716 5717 5718 5719 5720 5721	No Yes Yes No No Yes No No Yes No No Yes No No Yes No Yes No No Yes No
5705 5706 5707 5708 5709 5710 5711 5712 5713 5714 5715 5716 5717 5718 5719 5720 5721 5722	No Yes Yes No Yes No Yes No No Yes No No No Yes No No Yes No No Yes No No No
5705 5706 5707 5708 5709 5710 5711 5712 5713 5714 5715 5716 5717 5718 5719 5720 5721 5722 5723	No Yes Yes No Yes No Yes No No Yes No No Yes No Yes No Yes No Yes No Yes
5705 5706 5707 5708 5709 5710 5711 5712 5713 5714 5715 5716 5717 5718 5719 5720 5721 5722	No Yes Yes No Yes No Yes No No Yes No No No Yes No No Yes No No Yes No No No

```
5726
                 Yes
        5727
                  No
        5728
                  No
        5729
                  No
        5730
                 Yes
        5731
                  No
        5732
                 Yes
        5733
                 Yes
        5734
                  No
        Name: SMQ020x, Length: 5735, dtype: object
In [4]: # Recode RIAGENDR from 1/2 to Male/Female into new variable RIAGENDRx
        da["RIAGENDRx"] = da.RIAGENDR.replace({1: "Male", 2: "Female"})
        da["RIAGENDRx"]
Out[4]: 0
                   Male
        1
                   Male
        2
                   Male
        3
                 Female
        4
                 Female
        5
                 Female
        6
                   Male
        7
                 Female
        8
                   Male
        9
                   Male
        10
                   Male
        11
                   Male
        12
                 Female
        13
                 Female
        14
                   Male
        15
                 Female
                 Female
        16
        17
                 Female
        18
                 Female
        19
                 Female
        20
                   Male
        21
                 Female
        22
                 Female
        23
                 Female
        24
                   Male
        25
                 Female
        26
                   Male
        27
                 Female
        28
                   Male
        29
                 Female
        5705
                   Male
```

5706

Male

```
5707
                 Female
        5708
                 Female
        5709
                   Male
        5710
                 Female
        5711
                   Male
        5712
                 Female
        5713
                   Male
        5714
                   Male
        5715
                 Female
        5716
                 Female
        5717
                   Male
        5718
                   Male
        5719
                 Female
        5720
                   Male
        5721
                 Female
        5722
                 Female
        5723
                 Female
        5724
                 Female
        5725
                   Male
        5726
                   Male
        5727
                 Female
        5728
                   Male
        5729
                   Male
        5730
                 Female
        5731
                   Male
        5732
                 Female
        5733
                   Male
        5734
                 Female
        Name: RIAGENDRx, Length: 5735, dtype: object
In [5]: dx = da[["SMQ020x", "RIAGENDRx"]].dropna()
        dx
              SMQ020x RIAGENDRx
Out[5]:
        0
                  Yes
                            Male
        1
                  Yes
                            Male
        2
                  Yes
                            Male
        3
                   No
                          Female
         4
                          Female
                   No
        5
                          Female
                   No
        6
                  Yes
                            Male
        7
                   No
                          Female
        8
                   No
                            Male
        9
                   No
                            Male
         10
                  Yes
                            Male
         11
                  Yes
                            Male
         12
                  Yes
                          Female
                          Female
         13
                   No
```

14	No	Male
15	No	Female
16	No	Female
17	No	Female
18	Yes	Female
19	No	Female
20	No	Male
21	No	Female
22	Yes	Female
23	No	Female
24	No	Male
25	No	Female
26	Yes	Male
27	Yes	Female
28	No	Male
29	No	Female
	· · ·	 М-Л-
5705	Yes	Male
5706	Yes	Male
5707	No	Female
5708	No	Female
5709	Yes	Male
5710	No	Female
5711	Yes	Male
5712	No	Female
5713	No	Male
5714	No	Male
5715	No	Female
5716	Yes	Female
5717	Yes	Male
5718	No	Male
5719	Yes	Female
5720	No	Male
5721	No	Female
5722	No	Female
5723	Yes	Female
5724	No	Female
5725	No	Male
5726	Yes	Male
5727	No	Female
5728	No	Male
5729	No	Male
5730	Yes	Female
5731	No	Male
5732	Yes	Female
5733	Yes	
		Male
5734	No	Female

```
[5725 rows x 2 columns]
In [6]: pd.crosstab(dx.SMQ020x, dx.RIAGENDRx)
Out[6]: RIAGENDRx Female Male
        SMQ020x
        No
                     2066 1340
                      906 1413
        Yes
In [9]: # Recode SMQ020x from Yes/No to 1/0 into existing variable SMQ020x
        dx["SMQ020x"] = dx.SMQ020x.replace({"Yes": 1, "No": 0})
        TypeError
                                                   Traceback (most recent call last)
        <ipython-input-9-0a796a728ba0> in <module>()
          1 # Recode SMQ020x from Yes/No to 1/0 into existing variable SMQ020x
    ----> 2 dx["SMQ020x"] = dx.SMQ020x.replace({"Yes": 1, "No": 0})
        /opt/conda/lib/python3.6/site-packages/pandas/core/series.py in replace(self, to_repla
       3839
                    return super(Series, self).replace(to_replace=to_replace, value=value,
       3840
                                                        inplace=inplace, limit=limit,
    -> 3841
                                                        regex=regex, method=method)
       3842
                @Appender(generic._shared_docs['shift'] % _shared_doc_kwargs)
       3843
        /opt/conda/lib/python3.6/site-packages/pandas/core/generic.py in replace(self, to_replace)
       6496
       6497
                        return self.replace(to_replace, value, inplace=inplace,
    -> 6498
                                            limit=limit, regex=regex)
       6499
                    else:
       6500
        /opt/conda/lib/python3.6/site-packages/pandas/core/series.py in replace(self, to_repla
                    return super(Series, self).replace(to_replace=to_replace, value=value,
       3839
       3840
                                                        inplace=inplace, limit=limit,
    -> 3841
                                                        regex=regex, method=method)
       3842
       3843
                @Appender(generic._shared_docs['shift'] % _shared_doc_kwargs)
        /opt/conda/lib/python3.6/site-packages/pandas/core/generic.py in replace(self, to_replace)
       6545
                                                                    dest_list=value,
       6546
                                                                    inplace=inplace,
```

```
-> 6547
                                                                    regex=regex)
       6548
                            else: # [NA, ''] -> 0
       6549
        /opt/conda/lib/python3.6/site-packages/pandas/core/internals/managers.py in replace_li
                        return _compare_or_regex_match(values, s, regex)
        557
        558
    --> 559
                    masks = [comp(s, regex) for i, s in enumerate(src_list)]
        560
        561
                    result_blocks = []
        /opt/conda/lib/python3.6/site-packages/pandas/core/internals/managers.py in <listcomp>
        557
                        return _compare_or_regex_match(values, s, regex)
        558
    --> 559
                    masks = [comp(s, regex) for i, s in enumerate(src_list)]
        560
        561
                    result_blocks = []
        /opt/conda/lib/python3.6/site-packages/pandas/core/internals/managers.py in comp(s, re
                            return _compare_or_regex_match(maybe_convert_objects(values),
        555
        556
                                                            getattr(s, 'asm8'), regex)
    --> 557
                        return _compare_or_regex_match(values, s, regex)
        558
        559
                    masks = [comp(s, regex) for i, s in enumerate(src_list)]
        /opt/conda/lib/python3.6/site-packages/pandas/core/internals/managers.py in _compare_or
       1949
                    raise TypeError(
       1950
                        "Cannot compare types {a!r} and {b!r}".format(a=type_names[0],
                                                                       b=type_names[1]))
    -> 1951
       1952
                return result
       1953
        TypeError: Cannot compare types 'ndarray(dtype=int64)' and 'str'
In [10]: dx
Out[10]:
               SMQ020x RIAGENDRx
         0
                     1
                            Male
         1
                     1
                            Male
         2
                     1
                            Male
                          Female
         3
                     0
                          Female
```

5	0	Female
6	1	Male
7	0	Female
8	0	Male
9	0	Male
10	1	Male
11	1	Male
12	1	Female
13	0	Female
14	0	Male
15	0	Female
16	0	Female
17	0	Female
18	1	Female
19	0	Female
20	0	Male
21	0	Female
	1	Female
22		
23	0	Female
24	0	Male
25	0	Female
26	1	Male
27	1	Female
28	0	Male
29	0	Female
	• • •	
5705	1	Male
5706	1	Male
5707	0	Female
5708	0	Female
5709	1	Male
5710	0	Female
5711	1	Male
5712	0	Female
5713	0	Male
5714	0	Male
5715	0	Female
5716	1	Female
5717	1	Male
5718	0	Male
5719	1	Female
5720	0	Male
5721	0	Female
5722	0	Female
5723	1	Female
5724	0	Female
5725	0	Male
5726	1	Male

```
5727
                     0
                          Female
         5728
                            Male
         5729
                     0
                            Male
                     1
         5730
                          Female
                     0
         5731
                            Male
         5732
                     1
                          Female
         5733
                            Male
         5734
                          Female
         [5725 rows x 2 columns]
In [11]: dz = dx.groupby("RIAGENDRx").agg({"SMQ020x": [np.mean, np.size]})
         dz
Out[11]:
                     SMQ020x
                        mean size
         RIAGENDRx
                   0.304845
         Female
                              2972
                    0.513258 2753
         Male
In [12]: dz.columns = ["Proportion", "Total n"]
         dz
Out[12]:
                    Proportion Total n
         RIAGENDRx
         Female
                      0.304845
                                   2972
         Male
                      0.513258
                                   2753
```

1.0.2 Constructing Confidence Intervals

Now that we have the population proportions of male and female smokers, we can begin to calculate confidence intervals. From lecture, we know that the equation is as follows:

Best Estimate
$$\pm$$
 Margin of Error

Where the *Best Estimate* is the **observed population proportion or mean** from the sample and the *Margin of Error* is the **t-multiplier**.

The equation to create a 95% confidence interval can also be shown as:

Population Proportion or Mean $\pm (t - multiplier * Standard Error)$

The Standard Error (SE) is calculated differenly for population proportion and mean:

$$Standard\ Error\ for\ Population\ Proportion = \sqrt{\frac{Population\ Proportion*(1-Population\ Proportion)}{Number\ Of\ Observations}}$$

$$Standard\ Error\ for\ Mean = \frac{Standard\ Deviation}{\sqrt{Number\ Of\ Observations}}$$

Lastly, the standard error for difference of population proportions and means is:

Standard Error for Difference of Two Population Proportions Or Means
$$=\sqrt{(SE_1)^2+(SE_2)^2}$$

Difference of Two Population Proportions

In [13]: p = .304845

```
n = 2972
         se_female = np.sqrt(p * (1 - p)/n)
         se_female
Out[13]: 0.00844415041930423
In [14]: p = .513258
        n = 2753
         se_male = np.sqrt(p * (1 - p)/ n)
         se_male
Out[14]: 0.009526078787008965
In [15]: se_diff = np.sqrt(se_female**2 + se_male**2)
         se_diff
Out[15]: 0.012729880335656654
In [16]: # no calculo do intervalo de confianca, usar o standard erro da diferenca (se_diff) e
         d = .304845 - .513258
         lcb = d - 1.96 * se_diff
         ucb = d + 1.96 * se_diff
         (1cb, ucb)
Out[16]: (-0.23336356545788706, -0.18346243454211297)
Difference of Two Population Means
In [18]: da["BMXBMI"].head()
Out[18]: 0
              27.8
         1
              30.8
         2
              28.8
         3
              42.4
              20.3
         Name: BMXBMI, dtype: float64
In [19]: da.groupby("RIAGENDRx").agg({"BMXBMI": [np.mean, np.std, np.size]})
Out[19]:
                       BMXBMI
                         mean
                                    std
                                           size
         RIAGENDRx
         Female
                    29.939946 7.753319 2976.0
                    28.778072 6.252568 2759.0
         Male
In [20]: sem_female = 7.753319 / np.sqrt(2976)
         sem_male = 6.252568 / np.sqrt(2759)
         (sem_female, sem_male)
```