

Facultad de Ciencias, UNAM
Lenguajes de Programación
Tarea 2

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1. Define las siguientes funciones sobre expresiones del lenguaje WAE:

- (a) La función `libres`: $WAE \rightarrow (\text{listof symbol})$ que dada una expresión de tipo WAE devuelve una lista con los identificadores libres (sin repeticiones) contenidos en ésta.

SOLUCIÓN:

```
(define (libres expr)
  (type-case WAE expr
    [id (i) (list i)]
    [num (n) '()]
    [add (lhs rhs) (union (libres lhs) (libres rhs))]
    [sub (lhs rhs) (union (libres lhs) (libres rhs))]
    [with (id value body)
      (union (libres-aux value (list id))
              (libres-aux body (list id)))]))

(define (libres-aux expr lst)
  (type-case WAE expr
    [id (i) (if (not (member? i lst))
                 (list i)
                 '())]
    [num (n) '()]
    [add (lhs rhs) (union (libres-aux lhs lst) (libres-aux rhs lst))]
    [sub (lhs rhs) (union (libres-aux lhs lst) (libres-aux rhs lst))]
    [with (id value body)
      (union (libres-aux value (union lst (list id)))
              (libres-aux body (union lst (list id)))]))
```

- (b) La función `ligadas: WAE \rightarrow (listof symbol)` que dada una expresión de tipo WAE devuelve una lista con identificadores ligados (sin repeticiones) contenidos en ésta.

SOLUCIÓN:

```
(define (ligadas expr)
  (type-case WAE expr
    [id (i) '()]
    [num (n) '()]
    [add (lhs rhs) (union (ligadas lhs) (ligadas rhs))]
    [sub (lhs rhs) (union (ligadas lhs) (ligadas rhs))]
    [with (id value body)
      (union (ligadas-aux value (list id))
              (ligadas-aux body (list id))))])

(define (ligadas-aux expr lst)
  (type-case WAE expr
    [id (i) (if (member? i lst)
                 (list i)
                 '())]
    [num (n) '()]
    [add (lhs rhs) (union (ligadas-aux lhs lst) (ligadas-aux rhs lst))]
    [sub (lhs rhs) (union (ligadas-aux lhs lst) (ligadas-aux rhs lst))]
    [with (id value body)
      (union (ligadas-aux value (union lst (list id)))
              (ligadas-aux body (union lst (list id))))])
```

- (c) La función `de-ligado: WAE \rightarrow (listof symbol)` que dada una expresión de tipo WAE devuelve una lista con identificadores de ligado (sin repeticiones) contenidos en ésta.

SOLUCIÓN:

```
(define (de-ligado expr)
  (type-case WAE expr
    [id (i) '()]
    [num (n) '()]
    [add (lhs rhs) (union (de-ligado lhs) (de-ligado rhs))]
    [sub (lhs rhs) (union (de-ligado lhs) (de-ligado rhs))]
    [with (id value body)
      (union (list id) (de-ligado value) (de-ligado body))])
```

2. Sea e una expresión del lenguaje WAE. Suponiendo que $(\text{libres } e) = '()$, demostrar o dar un contraejemplo de la siguiente desigualdad.

$$(\text{length } (\text{ligada } e)) \leq (\text{length } (\text{de-ligado } e))$$

Proof. Sea e la siguiente expresión del lenguaje WAE

`{with {a 17} {+ a {+ a {+ a {+ a a}}}}}`

donde el símbolo de color azul es una variable de **de-ligado** y los símbolos de color rojo son variables **ligadas**. Notemos, además, que no tenemos variables **libres**.

Así,

$$(\text{length}(\text{ligada } e)) = 5 \not\leq 1 (\text{length}(\text{de-ligado } e))$$

Por lo tanto, la desigualdad $(\text{length}(\text{ligada } e)) \leq (\text{length}(\text{de-ligado } e))$ es falsa.

□

3. Realiza las siguientes sustituciones cuidando el alcance de las variables correspondientes. Indica para cada expresión los identificadores libres, de ligado y ligados.

De color **azul** tenemos los identificadores **de-ligado**, de color **rojo** los identificadores **ligados** y de **verde** los identificadores **libres**

(a) $\{\text{with } \{\text{w } \{- \text{u } 8\}\} \{\text{with } \{\text{v } 5\} \{+ \text{w } \{+ \text{y } \text{x}\}\}\} \} \text{[x := } \{+ \text{u } \text{v}\}]$

SOLUCIÓN:

$$\begin{aligned} & \{\text{with } \{\text{w } \{- \text{u } 8\}\} \{\text{with } \{\text{v } 5\} \{+ \text{w } \{+ \text{y } \text{x}\}\}\} \} \text{[x := } \{+ \text{u } \text{v}\}] \\ &= \{\text{with } \{\text{w } \{- \text{u } 8\} \text{[x := } \{+ \text{u } \text{v}\}] \} \{\text{with } \{\text{v } 5\} \{+ \text{w } \{+ \text{y } \text{x}\}\} \} \text{[x := } \{+ \text{u } \text{v}\}] \} \\ &= \{\text{with } \{\text{w } \{- \text{u } \text{[x := } \{+ \text{u } \text{v}\}] 8 \text{[x := } \{+ \text{u } \text{v}\}] \} \} \{\text{with } \{\text{v } 5 \text{[x := } \{+ \text{u } \text{v}\}] \} \{+ \text{w } \{+ \text{y } \text{x}\} \} \} \\ &= \{\text{with } \{\text{w } \{- \text{u } 8\}\} \{\text{with } \{\text{v } 5\} \{+ \text{w} \text{[x := } \{+ \text{u } \text{v}\}] \{+ \text{y } \text{x}\} \text{[x := } \{+ \text{u } \text{v}\}] \} \} \} \\ &= \{\text{with } \{\text{w } \{- \text{u } 8\}\} \{\text{with } \{\text{v } 5\} \{+ \text{w } \{+ \text{y} \text{[x := } \{+ \text{u } \text{v}\}] \text{x} \text{[x := } \{+ \text{u } \text{v}\}] \} \} \} \} \\ &= \{\text{with } \{\text{w } \{- \text{u } 8\}\} \{\text{with } \{\text{v } 5\} \{+ \text{w } \{+ \text{y } \{+ \text{u } \text{v}\}\} \} \} \} \} \end{aligned}$$

$$\{\text{with } \{\text{w } \{- \text{u } 8\}\} \{\text{with } \{\text{v } 5\} \{+ \text{w } \{+ \text{y } \{+ \text{u } \text{v}\}\} \} \} \}$$

(b) $\{\text{with } \{\text{y } \{+ \text{x } \text{v}\}\} \{\text{with } \{\text{z } \text{x}\} \{- \text{x } \{- \text{y } \text{z}\}\} \} \} \text{[x := } \{- \text{y } \text{z}\}]$

SOLUCIÓN:

$$\begin{aligned} & \{\text{with } \{\text{y } \{+ \text{x } \text{v}\}\} \{\text{with } \{\text{z } \text{x}\} \{- \text{x } \{- \text{y } \text{z}\}\} \} \} \text{[x := } \{- \text{y } \text{z}\}] \\ &= \{\text{with } \{\text{y } \{+ \text{x } \text{v}\} \text{[x := } \{- \text{y } \text{z}\}] \} \{\text{with } \{\text{z } \text{x}\} \{- \text{x } \{- \text{y } \text{z}\}\} \} \text{[x := } \{- \text{y } \text{z}\}] \} \\ &= \{\text{with } \{\text{y } \{+ \text{x} \text{[x := } \{- \text{y } \text{z}\}] \text{v} \text{[x := } \{- \text{y } \text{z}\}] \} \} \{\text{with } \{\text{z } \text{x} \text{[x := } \{- \text{y } \text{z}\}] \} \{- \text{x } \{- \text{y } \text{z}\} \} \} \\ &= \{\text{with } \{\text{y } \{+ \{- \text{y } \text{z}\} \text{v}\}\} \{\text{with } \{\text{z } \{- \text{y } \text{z}\} \} \{- \text{x} \text{[x := } \{- \text{y } \text{z}\}] \{- \text{y } \text{z}\} \text{[x := } \{- \text{y } \text{z}\}] \} \} \} \\ &= \{\text{with } \{\text{y } \{+ \{- \text{y } \text{z}\} \text{v}\}\} \{\text{with } \{\text{z } \{- \text{y } \text{z}\} \} \{- \{- \text{y } \text{z}\} \{- \text{y} \text{[x := } \{- \text{y } \text{z}\}] \text{z} \text{[x := } \{- \text{y } \text{z}\}] \} \} \} \\ &= \{\text{with } \{\text{y } \{+ \{- \text{y } \text{z}\} \text{v}\}\} \{\text{with } \{\text{z } \{- \text{y } \text{z}\} \} \{- \{- \text{y } \text{z}\} \{- \text{y } \text{z}\} \} \} \} \end{aligned}$$

$$\{\text{with } \{\text{y } \{+ \{- \text{y } \text{z}\} \text{v}\}\} \{\text{with } \{\text{z } \{- \text{y } \text{z}\} \} \{- \{- \text{y } \text{z}\} \{- \text{y } \text{z}\} \} \} \}$$

(c) $\{\text{with } \{\text{y } \{- \text{z } 3\}\} \{+ \text{x } \{+ \text{y } 11\}\} \} \text{[x := } \{- \text{y } \{z 23\}\}]$

SOLUCIÓN:

$$\begin{aligned} & \{\text{with } \{\text{y } \{- \text{z } 3\}\} \{+ \text{x } \{+ \text{y } 11\}\} \} \text{[x := } \{- \text{y } \{- \text{z } 23\}\}] \\ &= \{\text{with } \{\text{y } \{- \text{z } 3\} \text{[x := } \{- \text{y } \{- \text{z } 23\}\}] \} \{+ \text{x } \{+ \text{y } 11\} \text{[x := } \{- \text{y } \{- \text{z } 23\}\}] \} \} \\ &= \{\text{with } \{\text{y } \{- \text{z} \text{[x := } \{- \text{y } \{- \text{z } 23\}\}] 3 \text{[x := } \{- \text{y } \{- \text{z } 23\}\}] \} \} \{+ \text{x} \text{[x := } \{- \text{y } \{- \text{z } 23\}\}] \} \\ & \quad \{+ \text{y } 11 \text{[x := } \{- \text{y } \{- \text{z } 23\}\}] \} \} \\ &= \{\text{with } \{\text{y } \{- \text{z } 3\}\} \{+ \{- \text{y } \{- \text{z } 23\}\} \{+ \text{y} \text{[x := } \{- \text{y } \{- \text{z } 23\}\}] 11 \text{[x := } \{- \text{y } \{- \text{z } 23\}\}] \} \} \} \\ &= \{\text{with } \{\text{y } \{- \text{z } 3\}\} \{+ \{- \text{y } \{- \text{z } 23\}\} \{+ \text{y } 11\}\} \} \end{aligned}$$

$$\{\text{with } \{\text{y } \{- \text{z } 3\}\} \{+ \{- \text{y } \{z 23\}\} \{+ \text{y } 11\}\} \}$$

4. Convierte las siguientes expresiones a su respectiva versión usando índices de *De Bruijn*.

(a) $\{with\{a\ 2\}$
 $\{with\{b\ 3\}$
 $\{with\{c\ 4\}$
 $\{with\{d\{+ a\{- b\ c\}\}\}$
 $\{with\{f\{with\{a\{+ b\ c\}\} a\}\}$
 $\{+ d\{with\{b\{- d\ f\}\}\{- b\ c\}\}\}\}\}\}$

SOLUCIÓN:

$\{with\ 2$
 $\{with\ 3$
 $\{with\ 4$
 $\{with\{+\ <:2>\{- \<:1>\<:0>\}\}$
 $\{with\{with\{+\ <:2>\<:1>\}\<:0>\}$
 $\{+\ <:1>\{with\{- \<:1>\<:0>\}\{- \<:0>\<:2>\}\}\}\}\}\}$

(b) $\{with\{\{a\ 2\}\ \{b\ 3\}\ \{c\ \{with\{\{a\ 2\}\}\{+\ 2\ 3\}\}\}\}$
 $\{with\{\{d\ 8\}\}$
 $\{with\{\{a\ c\}\ \{b\{- 8\ d\}\}\ \{c\{+\ b\ b\}\}\}\{$
 $\{with\{\{g\ \{with\{\{z\ a\}\ \{y\ b\}\ \{z\ d\}\}\ 1\}\}\}$
 $\{+\ g\{- d\ c\}\}\}\}\}\}$

SOLUCIÓN:

$\{with\{2\ 3\ \{with\{2\}\{+\ 2\ 3\}\}\}$
 $\{with\{8\}$
 $\{with\{\{\<:1,\ 2>\}\{- 8\<:0,\ 0>\}\}\{\{+\ <:1,\ 1>\<:1,\ 1>\}\}\}$
 $\{with\{\{with\{\{\<:0,\ 0>\}\<:0,\ 1>\}\{\<:1,\ 0>\}\}\ 1\}\}$
 $\{+\ <:0,\ 0>\{- \<:2,\ 0>\<:1,\ 2>\}\}\}\}\}\}$

5. Dadas las siguientes expresiones representadas mediante índices de *De Bruijn*, obtén su respectiva versión usando identificadores de variables.

(a) $\{with\{+\ 2\ 3\}$
 $\{with\ 17$
 $\{with\{+\ <:0>\<:0>\}$
 $\{with\{- \<:0>\{+\ <:1>\<:2>\}\}$
 $\{with\{with\ 2\{+\ <:0>\ 3\}\}$
 $\{- \<:3>\{+\ <:2>\{+\ <:0>\<:1>\}\}\}\}\}\}$

SOLUCIÓN:

$\{with\{x\{+\ 2\ 3\}\}$
 $\{with\{y\ 17\}$
 $\{with\{z\{+\ y\ y\}\}$
 $\{with\{w\{- z\{+\ y\ x\}\}\}$
 $\{with\{v\{with\{a\ 2\}\{+\ a\ 3\}\}\}$
 $\{- y\{+\ z\{+\ v\ w\}\}\}\}\}\}\}$

(b) $\{with\{1\ 2\ 3\}$
 $\{with\{4\ 5\ 6\}$
 $\{with\{\{with\{\{+\ <:0\ 1>\<:1\ 2>\}\{- \<:1\ 1>\<:0\ 0>\}\}\ 3\}\}$

```
{with {<:0 0>}
  {+ <:3 2> {+ <:2 1> {+ <:1 0> <:0 0>}}}}}}
```

SOLUCIÓN:

```
{with {{a 1} {b 2} {c 3}}
  {with {{d 4} {e 5} {f 6}}
    {with {{g {with {{h {+ e c}} {i {- b d}}} 3}}}
      {with {j g}
        {+ c {+ e {+ g j}}}}}}}}
```

6. Determina el valor de la siguiente expresión y responde las siguientes preguntas: ¿puede haber otro resultado correcto? ¿por qué? ¿cuál es el correcto?

```
{with {a 2}
  {with {b 3}
    {with {c {+ a b}}
      {with {a -2}
        {with {b -3}
          {+ c c}}}}}}
```

SOLUCIÓN:

```
{with {a 2} {with {b 3} {with {c {+ a b}} {with {a -2} {with {b -3} {+ c c}}}}}
={with {c {+ 2 3}} {with {a -2} {with {b -3} {+ c c}}}
={with {c 5} {with {a -2} {with {b -3} {+ c c}}}
={+ 5 5}
=10
```

La expresión nos puede dar dos diferentes valores dependiendo si se ocupa alcance dinámico o estático y los dos valores son correctos dependiendo de cómo esté implementado el lenguaje en el que estemos trabajando.

Usando alcance estático nos da: 10

Usando alcance dinámico nos da: -10