

Portfolio (Part I)

1. Plot $f: [0, \infty) \rightarrow \mathbb{R}$, $f(x) = xe^x$.

Find and add to the plot the two Taylor polynomials of degree = 2 that approximate f locally at $x_0 = 0$ and x^* the maximum point of f .

2. Let $f: D \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be Lipschitz continuous
(i.e. $\exists L > 0$ s.t. $|f(x) - f(y)| \leq L|x - y| \quad \forall x, y \in D$)

Prove that

a) f Lip. cont. $\Rightarrow f$ cont. (on D)

b) $f: [0, \infty) \rightarrow \mathbb{R}$, $f(x) = \sqrt{x}$ is not Lip. cont. on $[0, \infty)$,
but f is Lip. cont. on $[a, \infty)$ for any $a > 0$.

3. Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}$, $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$

Study the continuity and partial differentiability of f at $(x, y) = (0, 0)$.

4. Find the box of fixed volume (=1) and minimal surface, i.e., solve

$$f(x, y, z) = 2xy + 2yz + 2zx \rightarrow \min!$$

subject to $g(x, y, z) = xyz - 1 = 0$.

What about the 2D case (rectangle of area 1 and minimal perimeter)?

Portfolio (Part II)

5. Find the antiderivative of $\frac{1}{(x^2+4)^2}$
(i.e. compute $\int \frac{dx}{(x^2+4)^2}$)
6. Compute $\iint_D \frac{xy}{x^2+1} dx dy$, where $D = [1, 2] \times [0, 1]$
7. Let D be delimited by $y=x^2$, $x=2$, $y=0$,
 $\iint_D (x+2y) = ?$
8. $D = \{(x, y, z) \in \mathbb{R}^3 : 1 \leq x^2 + y^2 + z^2 \leq 4, z \geq 0\}$
Compute $\iiint_D \frac{dx dy dz}{x^2 + y^2 + z^2}$
9. Using differentiation w.r.t. a parameter, compute
 $\int_0^1 \frac{x^5 - 1}{\ln x} dx$
10. a) Prove that $\Gamma(\frac{1}{2}) = 2 \int_0^\infty e^{-x^2} dx$
b) Compute $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$