The Magnetoviscous-thermal Instability

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ABSTRACT

From very generic arguments, one can demonstrate that accretion flows onto black holes must be dilute; therefore, the accretion properties of such objects are modified by their large viscosities and thermal conductivities. In this paper, using an incompressible fluid treatment of an ionized gas, we expand on previous research by considering the stability properties of a magnetized rotating plasma wherein the thermal conductivity and viscosity are not negligible. We present the dispersion relation and relevant fluxes associated with this instability before discussing the implications for numerical simulations of mildly dilute astrophysical plasmas.

1. Introduction

The study of astrophysical rotating plasmas in general, and accreting plasmas in particular, has undergone a renaissance within the past fifteen years. Balbus & Hawley (1991); Hawley & Balbus (1991) reapplied the magnetorotational instability (MRI, Velikhov (1959); Chandrasekhar (1960)) into an appropriate astrophysical context. They demonstrated that in rotating astrophysical plasmas, even arbitrarily small magnetic forces can provide a channel by which free energy source of outwardly decreasing angular velocity, rather than angular momentum, can destabilize a Rayleigh-stable flow, generate magnetic fields with energies of order the thermal energy, and drive the right form of turbulence that allows for accretion to occur. Numerous numerical simulations (Hawley et al. 1996; Wardle & Ng 1999; Sano & Stone 2002; De Villiers & Hawley 2003; Fromang et al. 2004) have borne out the fact that magnetic fields can play an essential role in driving efficient accretion for a wide class of astrophysical objects.

Although the MRI is one of the more well-known free-energy gradient instabilities with astrophysical applications, recent research has shown that in dilute plasmas large viscosities (Balbus 2004; Islam & Balbus 2005) and thermal conductivities (Balbus 2001) can also destabilize plasmas that possess free energy gradients. This unusual behaviour arises from the fact that even a magnetic field with dynamically unimportant Lorentz forces can easily be strong enough that the ion Larmor radius is smaller than the collisional mean free path. Under these conditions, the viscosity and thermal conductivity are large and directed along magnetic field lines (Spitzer 1962; Braginskii 1965).

The explanation for these instabilities is conceptually simple. An equilibrium is perturbed so that there is a component of field line, hence of thermal conductivity (viscosity), along the thermal (angular velocity) gradient; heat (momentum) can then be transferred outwards, which further deforms magnetic field lines along the direction of the gradient – the process runs away. These are the essential natures of the magnetothermal (MTI) and magnetoviscous instability (MVI). Similar behaviour arises in the collisionless regime, in which large heat fluxes (momentum fluxes) along magnetic fields, limited by collisionless damping of long-wavelength modes parallel to the magnetic field, act to destabilize the plasma; this phenomenon is shown in collisionless and mildly collisional treatments of the MRI (Quataert et al. 2002; Sharma et al. 2003). Numerical simulations have also begun to demonstrate how large anisotropic transport coefficients can destabilize a plasma that contains free energy gradients: a recent simulation of the MTI into the nonlinear regime demonstrates a rough steady-state characterized by a vigorously convecting and largely isothermal core situated between and driven by a hot lower boundary and a cold upper boundary (Parrish & Stone 2005).

This paper expands on previous research by considering an ncompressible fluid treatment of instability in a rotating plasma with large anisotropic viscosities as well as electron thermal conductivities, hence the magnetoviscous-thermal instability (MVTI). The organization of this paper is as follows. In §2 we carefully demonstrate that the MVTI may play an important role in describing the dynamics of accretion flows about dim galactic nuclei, taking Sagittarius A as an example. In §3 we lay down the equations of the system. In §4 we derive the dispersion relation associated with the system. In §5 we justify and derive bulk fluxes for the MVTI, demonstrating that turbulence driven by this MHD instability can drive accretion onto radiatively inefficient flows. And in §6 we provide a short discussion of the conclusions, its main astrophysical implications, as well as directions for further research.

2. Preliminaries

In this section, we briefly review the conditions of validity for the approximations used in this calculation. First, we note that a fluid approach for a low density plasma requires that the ion and electon mean free paths be small compared to global length scales. In our calculation, the ions and electrons are assumed to have the same temperature, so the ion-ion and electron-electron mean free paths are identical. For convenience, however, we will always refer to the ion mean free path λ_i,

$$\lambda_i = 1.5 \times 10^{13} \frac{T_4^2}{n_1 \ln \Lambda} \text{ cm}, \tag{1}$$

Where T_4 is the temperature in units of 10^4 K, n_1 is the number density in cm^{-3}, and $\ln \Lambda$ is the Coulomb logarithm. The second condition is that this mean free path be large compared to the ion Larmor radius r_L,

$$r_L = 9.5 \times 10^7 B_{\mu G}^{-1} T_4^{1/2} \text{ cm}, \tag{2}$$

so that transport is directed along magnetic lines of force. Here $B_{\mu G}$ is the magnetic field strength in μG. The electron Larmor radius is of course a factor of 43 smaller than the ion

Larmor radius.

The formal regime of validity is $r_L \ll \lambda_i \ll R$, where R is a typical global scale of the flow. In practice the lower bound on λ_i is expected to be well-satisfied in the applications of interest (the protogalactic interstellar medium and under luminous black hole accretion). The upper bound on λ_i, on the other hand, breaks down in the innermost regions of black hole accretion models for the Galactic Center (Quataert 2004). Our fluid approximation is expected to be well-satisfied throughout the bulk of this flow, however.

3. Formulation of the Problem

Dim mass-starved accretion flows onto black holes are radiatively inefficient and develop comparable rotation and sound velocities. Under these circumstances, the turbulent energy flux should contain a substantial purely thermal contribution (from correlated fluctuations in the density and temperature) as well the usual contribution from the angular momentum transport (Balbus 2003). Because of its role in redistributing energy, this additional thermal flux could have a profound effect on the observational properties of the flow. It is the possibility that the MTVI could be the source of a profoundly enhanced thermal energy flux that motivates our study, beginning with an elucidation of its linear properties.

We examine the rotational and convective stability of a disk under the effects of a magnetic field, under the effects of a large Braginskii viscosity (Balbus 2004; Islam & Balbus 2005) as well as large thermal conductivity (Balbus 2001). Our coordinate system for the disk will be a standard cylindrical system: radius R, azimuth ϕ, and axial variable Z. As in Balbus (2001), we consider the stability of an equilibrium plasma at the midplane with only radial gradients in pressure and temperature, and nonradial equilibrium magnetic field $\mathbf{b} = B_0 \left(\cos\chi\hat{\boldsymbol{\phi}} + \sin\chi\hat{\boldsymbol{Z}}\right)$, where B_0 is the magnitude of the equilibrium magnetic field. We analyze unstable axisymmetric modes in the Boussinesq limit, which implies incompressible flow and isobaric perturbations. We calculate the linear stability of an idealized plasma with a single temperature T with physical radially outwardly decreasing temperature and pressure that remains convectively stable, and demonstrate that its dispersion relation reduces to that of the MRI, MVI, and MTI in specific limiting cases. We also demonstrate that the quadratic heat fluxes and Reynolds stresses associated with the MVTI are of the right sense to drive accretion in radiatively inefficient astrophysical plasmas.

3.1. Constituent Equations

The fluid equations consist of continuity, force balance, energy balance, and the induction equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \tag{3}$$

$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{v} \right) = -\nabla \left(P + \frac{\mathbf{B} \cdot \mathbf{B}}{4\pi} \right) + \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi} - \rho \nabla \Phi - \nabla \cdot \boldsymbol{\sigma} \tag{4}$$

$$\frac{3}{2} P \frac{d \ln P \rho^{-5/3}}{dt} = -\nabla \cdot (q \mathbf{b}) - \boldsymbol{\sigma} : \nabla \mathbf{V} \tag{5}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) \tag{6}$$

Where ρ is density; $\mathbf{V}$ is the velocity; Φ is the gravitational potential; $\mathbf{B}$ is the magnetic field; $\boldsymbol{\sigma}$ is the viscous stress tensor; q is the anisotropic heat flux; and P is the pressure. $\mathbf{b} = \mathbf{B}/B$ is the unit vector along the magnetic field line. The viscous stress tensor and heat flux are given in Braginskii (1965):

$$\boldsymbol{\sigma} = -3\rho_0 \nu \left(\mathbf{bb} - \frac{1}{3}\mathbb{I} \right) \left(\mathbf{b} \cdot \nabla \mathbf{V} \cdot \mathbf{b} - \frac{1}{3} \nabla \cdot \mathbf{V} \right), \tag{7}$$

$$q = -\kappa \rho k_B / m_i \mathbf{b} \cdot \nabla T. \tag{8}$$

Where ν and κ are the viscous and (electron) thermal diffusivity along the magnetic field line, respectively.

3.2. Dispersion Relation

Assume small perturbations δa about equilibrium fluid quantities a_0; thus $\mathbf{V} = R\Omega(R)\hat{\boldsymbol{\phi}} + \delta\mathbf{v}$, $\mathbf{B} = \mathbf{b} + \delta\mathbf{B}$, $T = T_0 + \delta T$, and $\rho = \rho_0 + \delta\rho$. We assume nonaxisymmetric perturbations of the form:

$$\delta a \propto \exp\left(ik_R R + ik_Z z + \gamma t\right) \tag{9}$$

In equilibrium the viscous stress and heat flux are zero. One can demonstrate that the perturbed viscous stress tensor and heat flux are given by the following, where $\delta\bar{\mathbf{B}} = \delta\mathbf{B}/B_0$:

$$\delta\boldsymbol{\sigma} = -3\rho_0\nu\left(\mathbf{bb} - \frac{1}{3}\mathbb{I}\right)\left(i\left(\mathbf{k}\cdot\mathbf{b}\right)\left(\delta\mathbf{v}\cdot\mathbf{b}\right) + \Omega' R\delta\bar{B}_R\cos\chi\right) \tag{10}$$

$$\delta q = -\kappa P_0\delta\bar{B}_R\frac{\partial\ln T_0}{\partial R} - \kappa P_0\left(i\mathbf{k}\cdot\mathbf{b}\right)\frac{\delta T}{T_0} \tag{11}$$

The perturbed induction equation, from Eq. (6) reduces to the following:

$$\begin{aligned}
\gamma\delta\bar{B}_R &= ik_Z\sin\chi\delta v_R \\
\gamma\delta\bar{B}_\phi &= ik_Z\sin\chi\delta v_\phi + \Omega' R\delta\bar{B}_R \\
\gamma\delta\bar{B}_Z &= ik_Z B_0\sin\chi\delta v_Z,
\end{aligned} \tag{12}$$

From Eq. (12) the perturbed viscous stress tensor has the following nonzero components:

$$\delta\sigma_{\phi\phi} = \delta\sigma_{ZZ} = \left(\cos^2\chi - \frac{1}{3}\right)\delta\sigma_{Z_bZ_b}, \tag{13}$$

$$\delta\sigma_{RR} = -\frac{1}{3}\delta\sigma_{Z_bZ_b}, \tag{14}$$

$$\delta\sigma_{Z\phi} = \delta\sigma_{\phi Z} = \sin\chi\cos\chi\delta\sigma_{Z_bZ_b}, \tag{15}$$

Where $\delta\sigma_{Z_bZ_b}$ is the anisotropic pressure, or equivalently $\delta\sigma_{Z_bZ_b}/2$ is the component of viscous stress along the magnetic field:

$$\delta\sigma_{Z_bZ_b} = -3\rho_0\nu\gamma\left(\cos\chi\delta\bar{B}_\phi - \frac{k_R}{k_Z}\delta\bar{B}_R\sin\chi\right), \tag{16}$$

The perturbed energy balance equation, from Eq. (5) is given by the following, where $\theta = P_0/\rho_0$ is the isothermal sound speed squared:

$$\gamma \left(\frac{\delta T}{T_0} - \frac{5}{3}\frac{\delta \rho}{\rho_0} \right) + \delta v_R \frac{\partial \ln P_0 \rho_0^{-5/3}}{\partial R} = \\ \frac{2}{3}\kappa \left(i k_Z \sin\chi \left(\delta \bar{B}_R \frac{\partial \ln T_0}{\partial R} \right) - k_Z^2 \sin^2\chi \frac{\delta T}{T_0} \right) - \frac{2}{3}\theta^{-1} \delta \boldsymbol{\sigma} : \nabla \left(R\Omega \hat{\boldsymbol{\phi}} \right) \tag{17}$$

First, the form of the perturbed stress tensor as given in Eqs. (13), (14), and (15) implies that $\delta\boldsymbol{\sigma} : \nabla \left(R\Omega\hat{\boldsymbol{\phi}} \right) = 0$. Second, Eq. (17), with isobaric perturbations, implies the following perturbed density:

$$\frac{\delta\rho}{\rho_0} = \frac{3}{5} \times \frac{\delta v_R \partial \ln P_0 \rho_0^{-5/3}/\partial R - \frac{2}{3} i\kappa k_Z \sin\chi \delta \bar{B}_R \partial \ln T_0/\partial R}{\gamma + \frac{2}{5}\kappa k_Z^2 \sin^2\chi} \tag{18}$$

Therefore, the perturbed form of the force balance equation, Eq. (4)

$$\frac{\partial \delta \mathbf{v}}{\partial t} + 2\Omega \hat{\boldsymbol{Z}} \times \delta \mathbf{v} + \Omega' R \hat{\boldsymbol{\phi}} = - \frac{1}{\rho_0} \nabla \left(\delta P + \frac{\mathbf{b} \cdot \delta \mathbf{B}}{4\pi} \right) + \frac{\mathbf{b} \cdot \nabla \delta \mathbf{B}}{4\pi \rho_0} - \rho_0^{-1} \nabla \cdot \delta \boldsymbol{\sigma} + \\ \frac{\delta\rho}{\rho_0} \theta \frac{\partial \ln P_0}{\partial R} \hat{\boldsymbol{R}} \tag{19}$$

In component form reduces to the following:

$$\gamma \delta v_R - 2\Omega \delta v_\phi = - i k_R \left(\frac{\delta P}{\rho_0} + \frac{B_0 \cos\chi \delta B_\phi + B_0 \sin\chi \delta B_Z}{4\pi\rho_0} \right) + \frac{i k_Z B_0 \sin\chi}{4\pi\rho_0} \delta B_R - \\ i k_R \rho_0^{-1} \delta\sigma_{RR} - i k_Z \rho_0^{-1} \delta\sigma_{zR} + \frac{\delta\rho}{\rho_0} \theta \frac{\partial \ln P_0}{\partial R} \tag{20}$$

$$\gamma \delta v_\phi + \left(2\Omega + \Omega' R \right) \delta v_R = \frac{i k_Z B_0 \sin\chi}{4\pi\rho_0} \delta B_\phi - i k_R \rho_0^{-1} \delta\sigma_{\phi R} - i k_Z \rho_0^{-1} \delta\sigma_{\phi z} \tag{21}$$

$$\gamma \delta v_Z = - i k_Z \left(\frac{\delta P}{\rho_0} + \frac{B_0 \cos\chi \delta B_\phi + B_0 \sin\chi \delta B_Z}{4\pi\rho_0} \right) - \frac{i k_Z B_0 \sin\chi}{4\pi\rho_0} \delta B_Z - \\ i k_R \rho_0^{-1} \delta\sigma_{zR} - i k_Z \rho_0^{-1} \delta\sigma_{ZZ} \tag{22}$$

Eq. (22) results in the following expression for the total pressure, where $v_A^2 = B_0^2/\left(4\pi\rho\right)$:

$$\frac{\delta P}{\rho_0} + v_A^2 \cos\chi \delta \bar{B}_\phi = \nu\gamma \left(3\sin^2\chi - 1 \right) \left(\delta \bar{B}_\phi \cos\chi - \frac{k_R}{k_Z} \delta \bar{B}_R \sin\chi \right) - \frac{i k_R}{k_Z^2} \gamma \delta v_R \tag{23}$$

From the induction equation, Eq. [12], the force balance equations, Eqs. (20, 21, 23) can be rewritten in terms of $\delta \bar{B}_R$ and $\delta \bar{B}_\phi$:

$$\left(1+\frac{k_R^2}{k_Z^2}\right)\gamma^2\delta\bar{B}_R - 2\Omega\left(\gamma\delta\bar{B}_\phi - \Omega' R\delta\bar{B}_R\right) = 3\nu\gamma k_R k_Z \sin^3\chi\left(\delta\bar{B}_\phi\cos\chi - \frac{k_R}{k_Z}\delta\bar{B}_R\sin\chi\right) - \left(k_R^2+k_Z^2\right)v_A^2\sin^2\chi\delta\bar{B}_R + \frac{3}{5}\theta\left(\frac{\partial\ln P_0}{\partial R}\right)\frac{\gamma\partial\ln P_0\rho_0^{-5/3}/\partial R + \frac{2}{3}\kappa k_Z^2\sin^2\chi\partial\ln T_0/\partial R}{\gamma+\frac{2}{5}\kappa k_Z^2\sin^2\chi}\delta\bar{B}_R \tag{24}$$

$$\gamma^2\delta\bar{B}_\phi + 2\Omega\gamma\delta\bar{B}_R = -3\nu\gamma k_Z^2\sin^2\chi\cos\chi\left(\delta\bar{B}_\phi\cos\chi - \frac{k_R}{k_Z}\delta\bar{B}_R\sin\chi\right) - k_Z^2 v_A^2\sin^2\chi\delta\bar{B}_\phi \tag{25}$$

Resulting in the following dispersion relation:

$$\left(\frac{k^2}{k_Z^2}\gamma^2 + \frac{d\Omega^2}{d\ln R} + 3\nu k_R^2\gamma\sin^4\chi - 3\nu k_R k_Z\Omega' R\sin^3\chi\cos\chi + k^2 v_A^2\sin^2\chi - \frac{3}{5}\theta\left(\frac{\partial\ln P_0}{\partial R}\right)\frac{\gamma\partial\ln P_0\rho_0^{-5/3}/\partial R + \frac{2}{3}\kappa k_Z^2\sin^2\chi\partial\ln T_0/\partial R}{\gamma+\frac{2}{5}\kappa k_Z^2\sin^2\chi}\right)\times \left(\gamma^2 + k_Z^2 v_A^2\sin^2\chi + 3\nu k_Z^2\gamma\sin^2\chi\cos^2\chi\right) + \gamma^2\left(4\Omega^2 - 9\nu^2 k_R^2 k_Z^2\sin^6\chi\cos^2\chi\right) = 0 \tag{26}$$

The Prandtl number for our regime of interest, where electrons dominate the thermal conductivity, is given in Braginskii (1965):

$$\mathrm{Pr} \equiv \nu/\kappa \approx \frac{0.96}{3.2}\left(\frac{2m_e}{m_i}\right)^{1/2} \approx 1/101 \tag{27}$$

After making the following normalizations, where α_P, α_S, and α_T are the normalized inverse scale heights of pressure, entropy, and temperature, respectively:

$$\begin{aligned}
\hat{\mathbf{k}} &= \mathbf{k}v_A/\Omega \\
\hat{\gamma} &= \gamma/\Omega \\
\hat{\nu} &= \nu\Omega/v_A^2 \\
\hat{\kappa} &= \kappa\Omega/v_A^2 \\
\alpha_P &= -H\frac{\partial\ln P_0}{\partial R} \\
\alpha_T &= -H\frac{\partial\ln T_0}{\partial R} \\
\alpha_S &= -H\frac{\partial\ln P_0\rho_0^{-5/3}}{\partial R} = \frac{5}{3}\alpha_T - \frac{2}{3}\alpha_P
\end{aligned} \tag{28}$$

One can show that Eq. (26) reduces to the following:

$$\left(\frac{\hat{k}^2}{\hat{k}_Z^2}\hat{\gamma}^2 + 2\frac{d\ln\Omega}{d\ln R} + 3\hat{\nu}\hat{k}_R^2\hat{\gamma}\sin^4\chi - 3\hat{\nu}\hat{k}_R\hat{k}_z\frac{d\ln\Omega}{d\ln R} + \hat{k}^2\sin^2\chi - \right.$$
$$\left.\frac{3}{5}\alpha_P\frac{\alpha_S\hat{\gamma} + \frac{2}{3}\alpha_T\mathrm{Pr}^{-1}\hat{\nu}\hat{k}_Z^2\sin^2\chi}{\hat{\gamma} + \frac{2}{5}\mathrm{Pr}^{-1}\hat{\nu}\hat{k}_Z^2\sin^2\chi}\right) \times \left(\hat{\gamma}^2 + \hat{k}_Z^2\sin^2\chi + 3\hat{\nu}\hat{k}_Z^2\hat{\gamma}\sin^2\chi\cos^2\chi\right) + \tag{29}$$
$$\hat{\gamma}^2\left(4 - 9\hat{\nu}^2\hat{k}_R^2\hat{k}_Z^2\sin^6\chi\cos^2\chi\right) = 0$$

Physically speaking, in this fluid analysis we are always in the regime in which $\kappa > \nu$, hence where $\mathrm{Pr} > 1$. The result is that for sufficiently large transport coefficient $\nu\Omega/v_A^2 > 1$ we then have a density perturbation given by:

$$\frac{\delta\rho}{\rho_0} \approx \frac{\delta v_R}{\gamma}\left(\frac{\partial\ln T}{\partial R}\right)$$

And the following substitution occurs in the above dispersion equation:

$$\frac{3}{5}\alpha_P\frac{\alpha_S\hat{\gamma} + \frac{2}{3}\alpha_T\mathrm{Pr}^{-1}\hat{\nu}\hat{k}_Z^2\sin^2\chi}{\hat{\gamma} + \frac{2}{5}\mathrm{Pr}^{-1}\hat{\nu}\hat{k}_Z^2\sin^2\chi} \rightarrow \alpha_P\alpha_T$$

Note the following limits of Eq. (29): 1) in the limit that the transport coefficients go to zero and there are no equilibrium gradients, we reproduce the MRI; 2) in the limit of no equilibrium gradients and finite viscosity, we reproduce the MVI; 3) in the limit of no viscosity but finite thermal conductivity, we reproduce the MTI dispersion relation (Balbus 2001); and 4) in the limit of no thermal conductivity or viscosity, but finite equilibrium gradients, we reproduce the dispersion relation for convectively unstable modes in a rotating magnetized plasma (Balbus 1995). To put some physical parameters in perspective, in physical terms the condition under which the magnetoviscous instability operates is one in which $\nu\Omega/v_A^2 > 1$. The viscosity and Alfvén velocity in dimensional units is given by the following:

$$\nu = 1.4 \times 10^{19}\frac{T_4^{5/2}}{n_1\ln\Lambda}\ \mathrm{cm^2\ s^{-1}}, \tag{30}$$

$$v_A = 2.2 \times 10^5 B_{\mu G} n_1^{-1/2}\ \mathrm{cm\ s^{-1}}, \tag{31}$$

One must thus have an angular velocity of the flow such that:

$$\Omega \gtrsim 3.5 \times 10^{-9} B_{\mu G}^2 \, (\ln \Lambda) \, T_4^{-5/2} \ \mathrm{s}^{-1} \tag{32}$$

For the viscosity to be dynamically important. Furthermore, the thermal diffusivity $\kappa = \mathrm{Pr}^{-1}\nu \gg \nu$, so that the range of applicability of thermal effects is significantly larger than that of viscous transport.

4. Results

In this section we calculate the quadratic azimuthal stress $W_{R\phi}$ and radial heat flux q_R associated with these instabilities. Since the MVTI operates in a regime of high thermal diffusivity and possibly high viscous diffusivity, we need to consider the local energetics and angular momentum transport modified by large viscous and thermal fluxes. A more careful analysis of the local energetics with large viscosity and thermal conductivity is described in Appendix A. Thus, for the accretion flows with significant diffusive transport, we have the following quadratic azimuthal stresses and radial heat flux in the Boussinesq limit:

$$T_{R\phi} = \left\langle \rho_0 \delta v_R \delta v_\phi - \frac{\delta B_R \delta B_\phi}{4\pi} - 3\rho_0 \nu \left(\delta \bar{B}_R \cos\chi \right) \left(\mathbf{b} \cdot \nabla \delta \mathbf{v} \cdot \mathbf{b} + \Omega' R \delta \bar{B}_R \cos\chi \right) \right\rangle \tag{33}$$

$$q_R = \left\langle \frac{5}{2} \rho_0 \delta v_R \delta\theta - \kappa \rho_0 \left(\mathbf{b} \cdot \nabla \delta\theta \right) \delta \bar{B}_R + \rho_0 \nu \delta v_R \left(\mathbf{b} \cdot \nabla \delta \mathbf{v} \cdot \mathbf{b} + \Omega' R \delta \bar{B}_R \cos\chi \right) \right\rangle \tag{34}$$

4.1. Growth Rate and Stability Characteristics

Here we consider two equilibrium configurations that illustrate the magnetoviscous-thermal dispersion relation. We only consider a physical pressure profile, hence $\alpha_P = 10$ (i.e., outwardly decreasing pressure). We also consider a flow that is convectively stable, hence with $H \frac{\partial}{\partial R} \ln P\rho^{-5/3} > 0$ or $\alpha_S < 0$. This implies the following:

$$0 < \alpha_T \leq \frac{2}{5} \alpha_P \tag{35}$$

Which implies that $\alpha_T \leq 4$. We focus our results on only vertical wavenumbers, $k_R = 0$. Then Eq. (29) reduces to the following quintic polynomial that we solve for, where we denote $x = (\mathbf{k} \cdot \mathbf{v}_A)/\Omega = \hat{k}_Z \sin\chi$:

$$\begin{aligned}&\left(\left[\hat{\gamma}^2 + 2\frac{d\ln\Omega}{d\ln R} + x^2\right]\left[\hat{\gamma} + \frac{2}{5}\mathrm{Pr}^{-1}\hat{\nu}x^2\right] - \frac{3}{5}\alpha_P\left[\alpha_S\hat{\gamma} + \frac{2}{3}\alpha_T\mathrm{Pr}^{-1}\hat{\nu}x^2\right]\right) \times \\ &\left(\hat{\gamma}^2 + x^2 + 3\hat{\nu}\hat{\gamma}x^2\cos^2\chi\right) + 4\hat{\gamma}^2\left(\hat{\gamma} + \frac{2}{5}\mathrm{Pr}^{-1}\hat{\nu}x^2\right) = 0\end{aligned} \tag{36}$$

In Fig. (1), we examine the instance where both magnetic tension and thermal conductivity are dynamically significant; dispersion relations match the salient characteristic of the magnetoviscous instability (Balbus 2004; Islam & Balbus 2005) – saturation of the growth rate at small wavenumbers, described by the condition $\nu k^2 \sim \Omega$. For the case $\alpha_T = 0$ (no equilibrium temperature gradient) we approach the magnetoviscous instability – the mode is reproduced only for $\alpha_P = 0$ and $\alpha_T = 0$. In Fig. (2), we examine the instance where only the thermal conductivity is dynamically important; the dispersion relations are similar to the magnetorotational instability. From an examination of the dispersion relation Eq. (26) or the normalized dispersion relation Eq. (36) that for physical $\alpha_T > 0$ the growth rate and wavenumber extent of the instability are increased – this is shown also in Fig. (1) and Fig. (2). One expects that equilibrium scale heights of temperature, pressure, and entropy are of order the radius R. Thus, in order to have signficant magnetoviscous and magnetothermal effects one requires that α_T, α_P, and α_S be at least of order unity. Hence, only thick disks $H \sim R$ are expected to be susceptible to these classes of instability.

4.2. Heat Flux and Azimuthal Stresses

The following are the calculated perturbed velocities, magnetic fields, density, and temperature in terms of the radial displacement $\xi_R = \gamma^{-1}\delta v_R$, that employing the induction

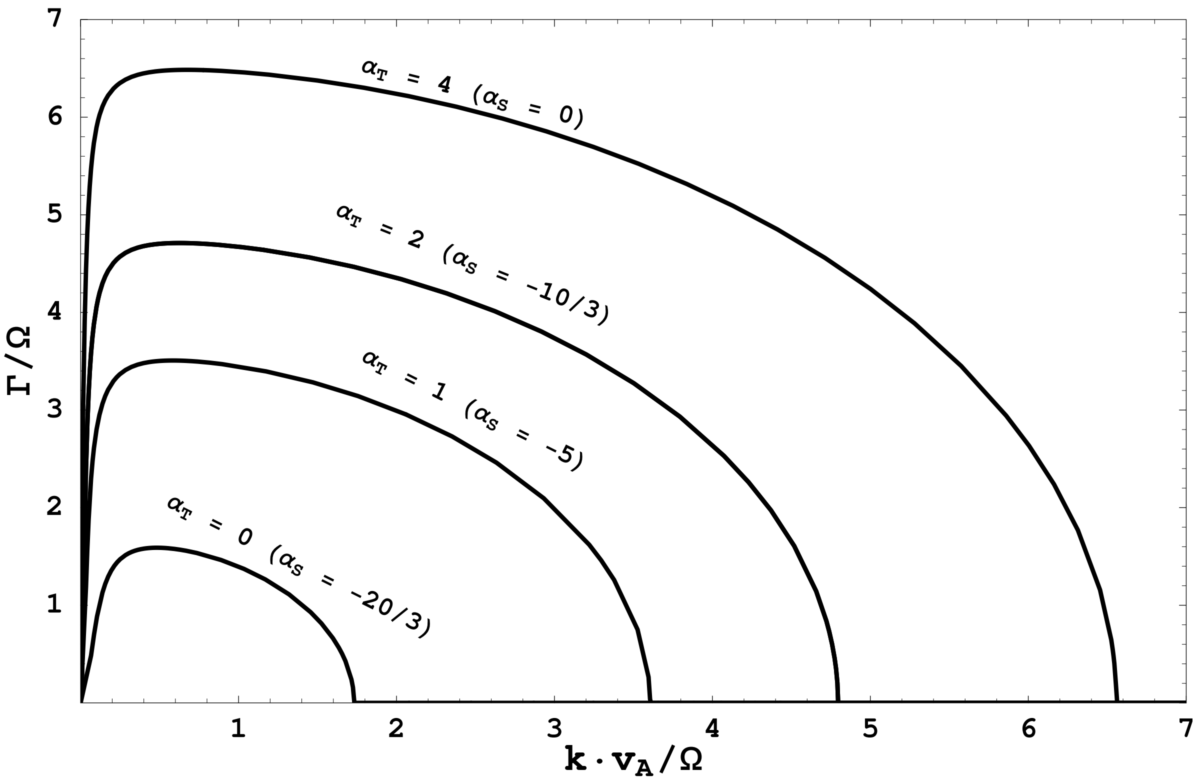

Fig. 1.— Plot of the real portion of the growth rate for various α_T for a Keplerian-like rotation profile, $\Omega \propto R^{-3/2}$, significant viscous diffusion coefficient $\nu\Omega/v_A^2 = 10^2$ and Prandtl number Pr $= 1/101$ (see Eq. [27]) – this corresponds to the case where both anisotropic viscous forces dominate over magnetic tension and where energy balance is determined by the magnetized thermal conductivity. The case $\alpha_T = 0$ approaches the magnetoviscous dispersion relation.

equations (Eq. [12]) and Eqs. (18) and (25). We examine nonradial modes, $k_R/k_Z = 0$:

$$\begin{aligned}
\delta v_R &= \hat{\gamma}\Omega\xi_R \\
\delta v_\phi &= -\left(\frac{2\hat{\gamma}^2}{\hat{\gamma}^2 + x^2 + 3\hat{\nu}x^2\hat{\gamma}\cos^2\chi} + \frac{d\ln\Omega}{d\ln R}\right)\Omega\xi_R \\
\delta B_R &= i\left(\mathbf{k}\cdot\mathbf{B}\right)\xi_R \\
\delta B_\phi &= -i\left(\mathbf{k}\cdot\mathbf{B}\right)\frac{2\hat{\gamma}}{\hat{\gamma}^2 + x^2 + 3\hat{\nu}x^2\hat{\gamma}\cos^2\chi}\xi_R \\
\delta\rho &= -\frac{3}{5}\times\frac{\hat{\gamma}\alpha_S + \frac{2}{3}\mathrm{Pr}^{-1}\hat{\nu}x^2\alpha_T}{\hat{\gamma} + \frac{2}{5}\mathrm{Pr}^{-1}\hat{\nu}x^2}\left(\rho H^{-1}\xi_R\right)
\end{aligned} \tag{37}$$

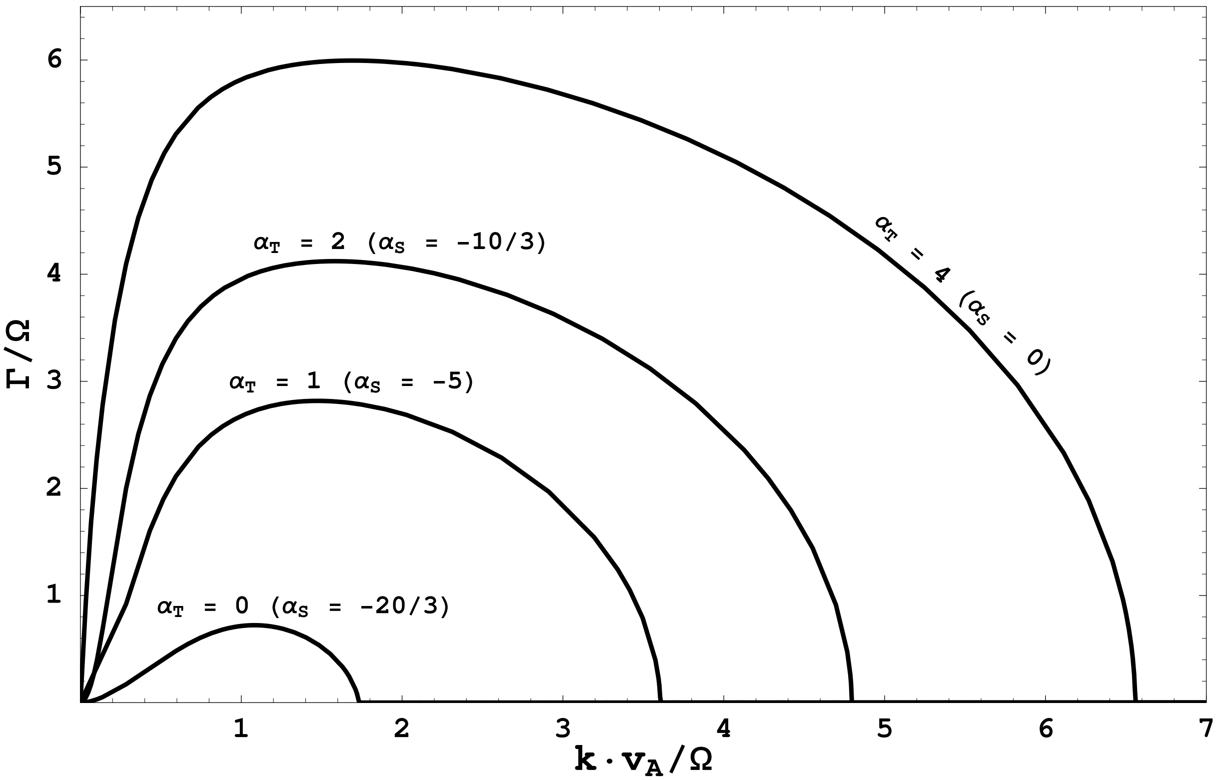

Fig. 2.— Plot of the dispersion relation for various α_T for a Keplerian-like rotation profile, $\Omega \propto R^{-3/2}$, a small viscous diffusion coefficient $\nu\Omega/v_A^2 = 1$ and Prandtl number $\mathrm{Pr} = 1/101$ (see Eq. [27]). The case $\alpha_T = 0$ approaches the magnetorotational dispersion relation.

with the form of the perturbed anisotropic pressure from Eq. (16), the turbulent heat flux from Eq. (34) yields the following:

$$\begin{aligned} q_R &= \rho_0 \mathrm{Re}\left(\frac{5}{2}\delta\theta\delta v_R^* + \rho_0\nu\gamma\cos\chi\delta v_R^*\delta\bar{B}_\phi\right) - \rho_0 \mathrm{Pr}^{-1}\nu k_Z \mathrm{Im}\left(\sin\chi\delta\theta\delta\bar{B}_R^*\right) \\ &= \left(\frac{3}{2}\hat{\gamma}\frac{\hat{\gamma}\alpha_S + \frac{2}{3}\hat{\nu}\mathrm{Pr}^{-1}x^2\alpha_T}{\hat{\gamma} + \frac{2}{5}\hat{\nu}\mathrm{Pr}^{-1}x^2}\right) P\Omega\left|\xi_R\right|^2 H^{-1}, \end{aligned} \tag{38}$$

And the azimuthal stress from Eq. (33) yields the following:

$$\begin{aligned} T_{R\phi} &= \mathrm{Re}\left(\rho_0\delta v_R\delta v_\phi^* - \frac{\delta B_R\delta B_\phi^*}{4\pi} - 3\rho_0\nu\cos^2\chi\gamma\delta\bar{B}_R^*\delta\bar{B}_\phi\right) \\ &= \hat{\gamma}\left(2 - \frac{d\ln\Omega}{d\ln R} - \frac{4\hat{\gamma}^2}{\hat{\gamma}^2 + x^2 + 3\hat{\nu}x^2\hat{\gamma}\cos^2\chi}\right)\rho_0\Omega^2\left|\xi_R\right|^2, \end{aligned} \tag{39}$$

In all the quasilinear plots we keep $\alpha_P = 10$, choose physical $\alpha_T > 0$ for which the flow is convectively stable, and fix the Prandtl number $\mathrm{Pr} = 1/101$. In Figs. (3) and (4) are plots of the normalized thermal flux q_R and azimuthal stress $T_{R\phi}$ for magnetoviscous-like dispersion modes ($\nu\Omega/v_A^2 = 10^2$, see Fig. [1]). In Figs. (5) and (6) are plots of normalized q_R and $T_{R\phi}$ for magnetorotational-like modes ($\nu\Omega/v_A^2 = 1$, see Fig. [2]). From Figs. (3) and (5),

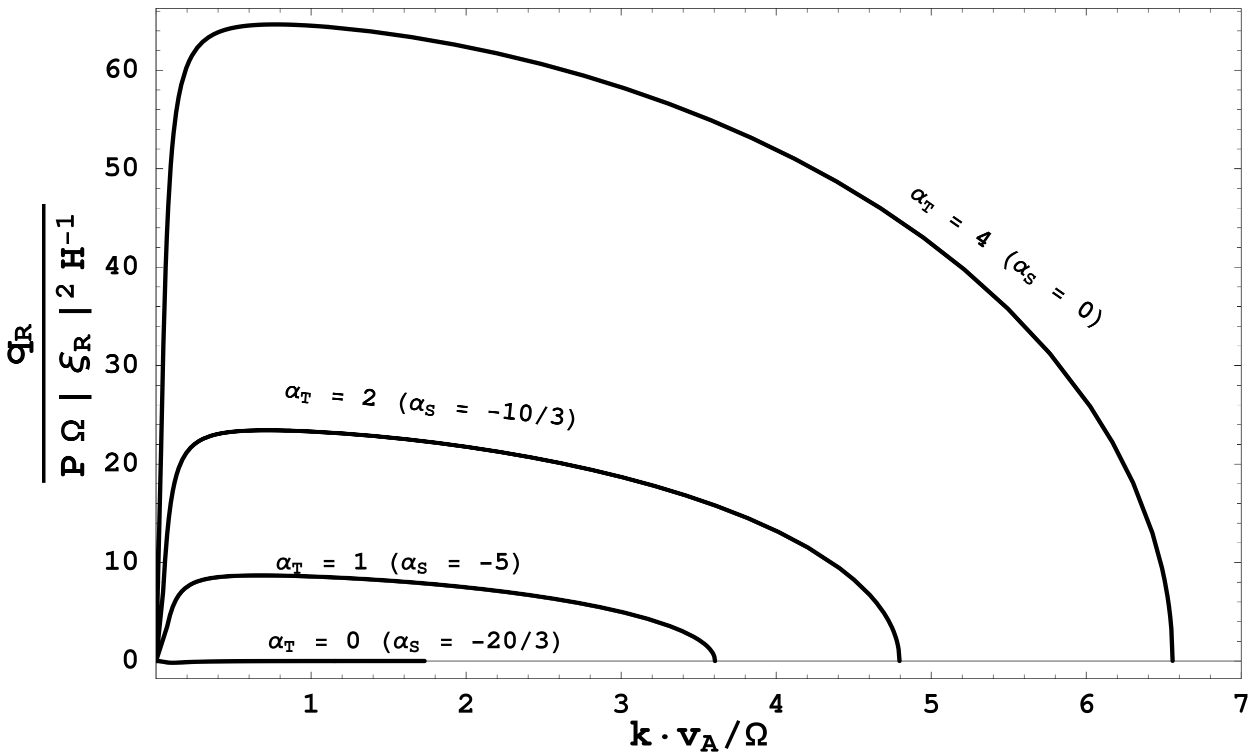

Fig. 3.— The normalized radial flux of thermal energy with parameters as described in Fig. (1)), where $\nu\Omega/v_A^2 = 10^2$.

one observes that the quadratic heat flux in the limit of large ($\nu\Omega/v_A^2 \gg 1$) and moderate ($\nu\Omega/v_A^2 \sim 1$) viscosities results in dynamically significant outwards heat fluxes, with $q_R \gtrsim T_{R\phi}\,(k_B\theta/m_i)^{1/2}$. This implies that the nonlinear MVTI can play an important role in transporting out the energy, generated via the azimuthal stress, in nonradiative accreting flows. Second, from Figs. (4) and (6), one notes that the flux of angular momentum for

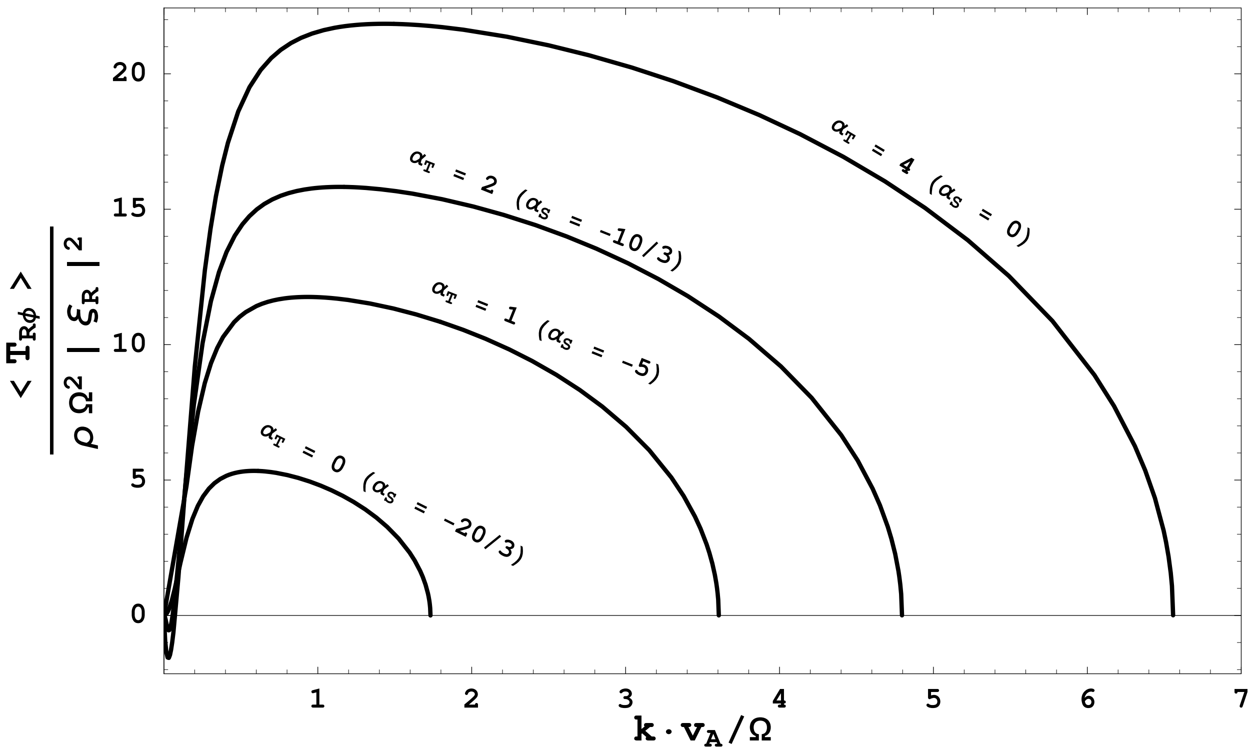

Fig. 4.— Normalized azimuthal momentum radial flux with parameters as described in Fig. (1), where $\nu\Omega/v_A^2 = 10^2$.

these modes can be either inwards or outwards. One also observes for a range of small wavenumbers that thermal gradients may cause a negative azimuthal stress, unlike the MRI.

An important point that must be made is that with a large thermal conductivity, angular momentum momentum may be transported inwards or outwards for large decreasing gradients of temperature. This effect can be seen even for the purely magnetothermal instability (MTI) in a plasma rotating without shear. We can rederive the dispersion relation and angular momentum flux for the MTI by setting $\mathrm{Pr}^{-1} = \kappa/\nu$, while letting

$\nu \rightarrow 0$. Then the normalized dispersion relation, Eq. (36), and azimuthal stress, Eq. (39):

$$\left(\left[\hat{\gamma}^2 + 2\frac{d\ln\Omega}{d\ln R} + x^2\right]\left[\hat{\gamma} + \frac{2}{5}\hat{\kappa}x^2\right] - \frac{3}{5}\alpha_P\left[\alpha_S\hat{\gamma} + \frac{2}{3}\alpha_T\hat{\kappa}x^2\right]\right)\left(\hat{\gamma}^2 + x^2\right) + 4\hat{\gamma}^2\left(\hat{\gamma} + \frac{2}{5}\hat{\kappa}x^2\right) = 0 \tag{40}$$

$$T_{R\phi} = \hat{\gamma}\frac{x^2\left(2 - d\ln\Omega/d\ln R\right) - \hat{\gamma}^2\left(2 + d\ln\Omega/d\ln R\right)}{\hat{\gamma}^2 + x^2}\rho\Omega^2\left|\xi_R\right|^2 \tag{41}$$

Fig. (7) demonstrates that in a rigidly rotating plasma $\Omega' R = 0$, the magnetothermal instability can drive a positive azimuthal stress depending on wavenumber. As the system approaches marginal convective stability $\alpha_S \rightarrow 0$ from isothermality ($\alpha_T = 0$), the range of wavenumbers for which the stress is outwards decreases. One must note, however, that in the absence of rotational shear no energy can be extracted from the flow.

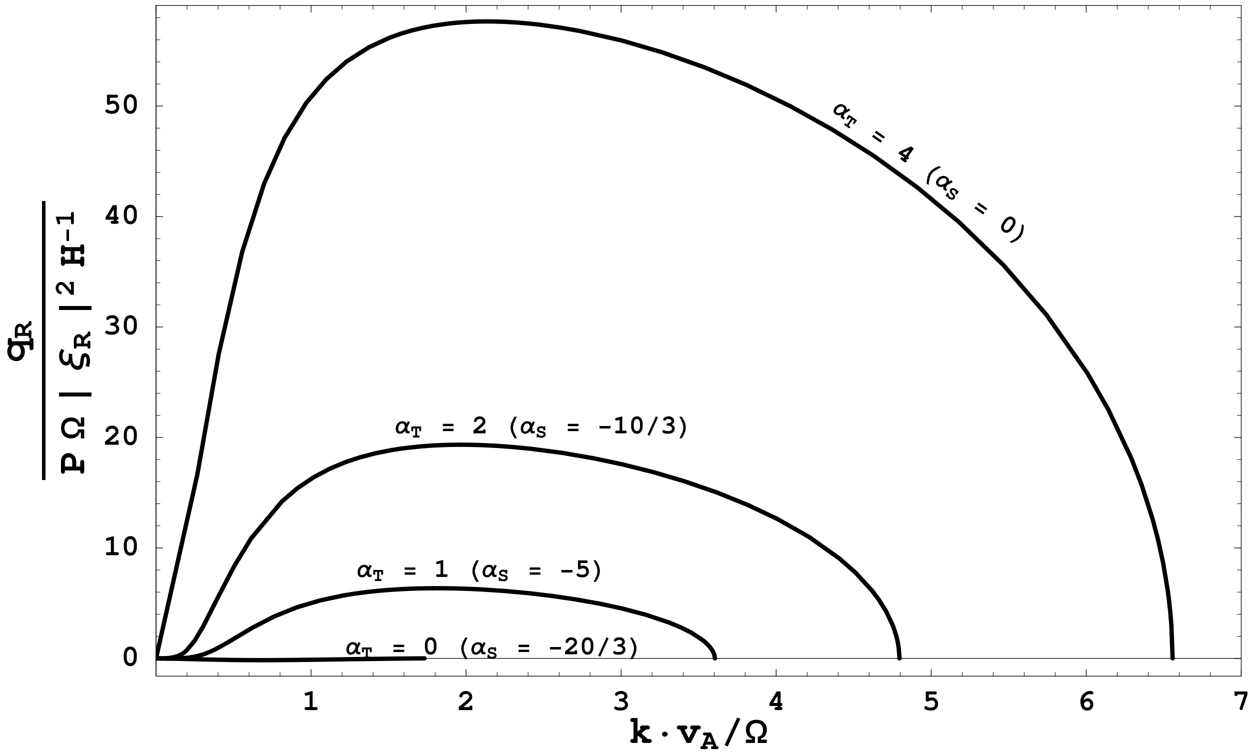

Fig. 5.— The normalized radial flux of thermal energy for a magnetorotational-like mode, with parameters as described in Fig. (2).

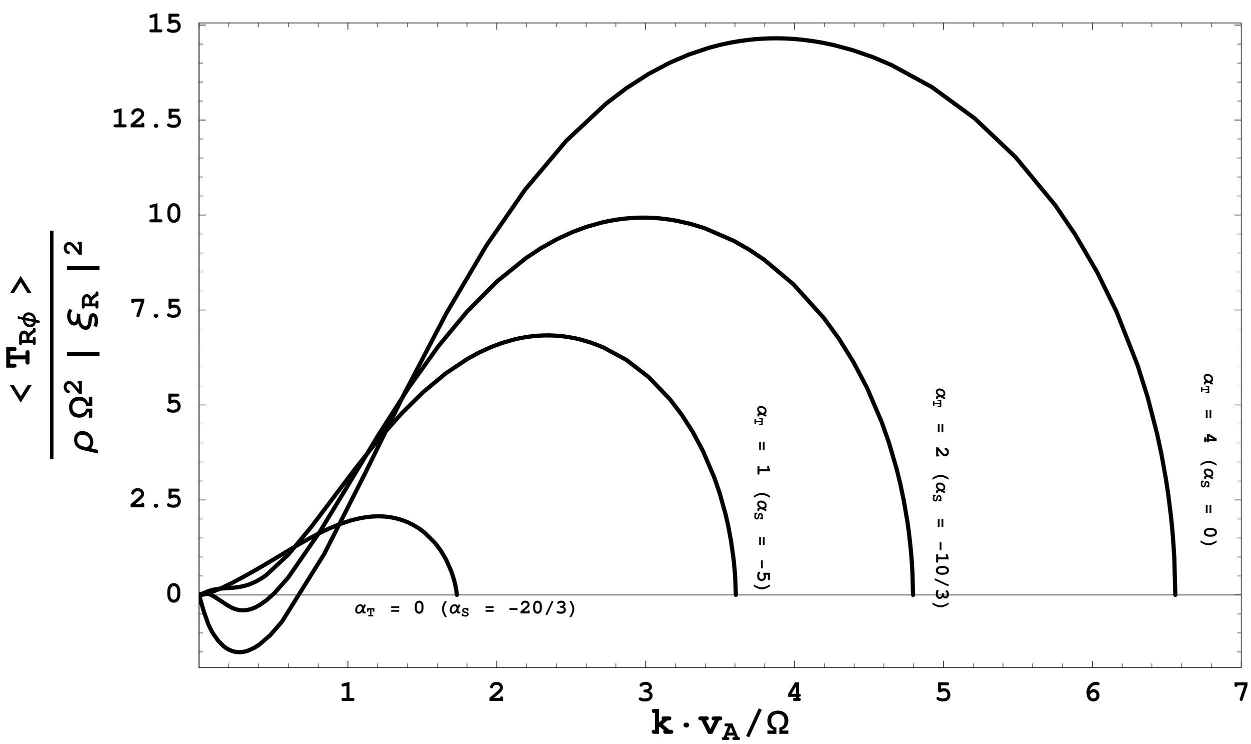

Fig. 6.— Normalized Reynolds stress $T_{R\phi}$ for a magnetorotational-like mode, with parameters as described in Figs. (2). For magnetoviscous-like modes, it is apparent that there is a range of low wavenumbers for which the Reynolds stress is inwards.

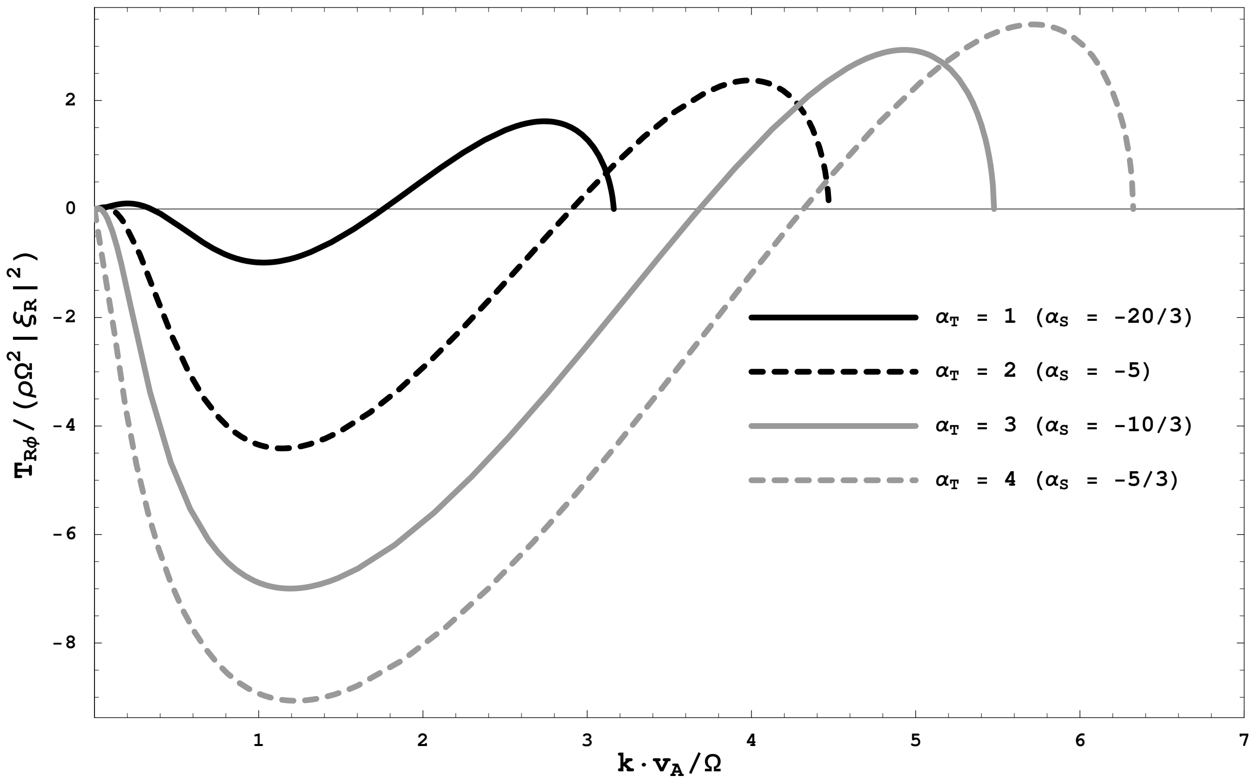

Fig. 7.— Normalized flux $T_{R\phi}/\left(\rho\Omega^2\,|\xi_R|^2\right)$ for a rigid rotation profile ($\Omega' R = 0$) for the magnetothermal instability (see Eq. [40]). There is a much larger range of negative Reynolds stress (inwards flux of angular momentum) than shown in Fig. (6).

5. Conclusions and Further Research

In this paper we attempt to characterize the stability of a mildly dilute rotating accretion flow to axisymmetric magnetoviscous-thermal modes, by incorporating both an anisotropic magnetized viscous stress tensor and anistropic electron thermal conductivities. We have demonstrated several important properties of the MVTI. First, we find that fat disks with sufficiently large thermal diffusion coefficients are susceptible to these classes of temperature gradient instabilities. Second, physical outwardly decreasing temperature gradients increase the range of wavenumbers for which the magnetorotational-like instability (where the viscous diffusion coefficient is small $\nu\Omega < v_A^2$) and magnetoviscous-like instability (where the viscous diffusion coefficient is large) are unstable, as well as increasing the growth rate of unstable wavenumbers. Third, that quadratic fluxes of angular momentum are substantially modified by both a large outward viscous transport in dilute plasmas and by the fact that temperature gradients can transport angular momentum either inwards or outwards.

The above basic characteristics are expected as well for a more carefully considered collisionless plasma. Previous treatments of the collisionless (Quataert et al. 2002) and mildly collisional (Sharma et al. 2003) MRI are limited by the fact that the thermal energy is overwhelmingly contained within the ions; this condition is expected to apply only within the innermost regions of a radiatively inefficient accretion flow (RIAF). In the outermost regions of a RIAF such as Sagittarius A, however, the collisional plasma's viscosity is dominated by the ions while thermal fluxes are dominated by electrons. Companion papers will consider a more general case of the collisionless and mildly collisional MVTI, in which both ion and electron thermal energies may be comparable. Our treatment of linear stability can then be applicable throughout a typical underluminous black hole accretion flow.

Although much work remains to be done in understanding the dynamics of hot dilute accretion flows, this paper and its companions (Balbus 2001, 2004; Islam & Balbus 2005) have demonstrated a class of instabilities that may roughly describe the type of turbulence expected to apply to these astrophysical objects. Global collisionless MHD simulations, employing codes that conserve total mechanical energy such as ATHENA (Gardiner & Stone 2005), show the most promise in beginning our understanding of the nonlinear stages of these newly explored free-energy channels.

6. Acknowledgements

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A. Local Energy Balance in Accretion Disk With Viscosity and Magnetized Thermal Conductivity

The force balance and internal energy balance equation with viscosity and thermal conductivity are given by the following:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \tag{A1}$$

$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = -\nabla \left(p + \frac{B^2}{8\pi} \right) - \rho \nabla \Phi + \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi} - \nabla \cdot \boldsymbol{\sigma} \tag{A2}$$

$$\frac{3}{2}\frac{\partial p}{\partial t} + \frac{3}{2}\mathbf{V} \cdot \nabla p + \frac{5}{2} p \left(\nabla \cdot \mathbf{V} \right) = -\nabla \cdot (q\mathbf{b}) - \boldsymbol{\sigma} : \nabla \mathbf{V} \tag{A3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\mathbf{V} \cdot \nabla \mathbf{B} + \mathbf{B} \cdot \nabla \mathbf{V} - \mathbf{B} \left(\nabla \cdot \mathbf{V} \right) \tag{A4}$$

Eqs. (A1) and (A2) can be used to derive the mechanical energy balance equation:

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 \right) + \nabla \cdot \left(\frac{1}{2}\rho v^2 \mathbf{V} \right) = -\nabla \cdot \left(\left[p + \frac{B^2}{8\pi} \right] \mathbf{V} \right) + \left[p + \frac{B^2}{8\pi} \right] \nabla \cdot \mathbf{V} + \frac{\mathbf{B} \cdot \nabla \mathbf{B} \cdot \mathbf{V}}{4\pi} - \nabla \cdot (\boldsymbol{\sigma} \cdot \mathbf{V}) + \boldsymbol{\sigma} : \nabla \mathbf{V} \tag{A5}$$

Furthermore, from the internal energy equation we get:

$$\frac{\partial}{\partial t}\left(\frac{3}{2} p \right) + \nabla \cdot \left(\frac{3}{2} p \mathbf{V} \right) = -p \nabla \cdot \mathbf{V} - \nabla \cdot (q\mathbf{b}) - \boldsymbol{\sigma} : \nabla \mathbf{v} \tag{A6}$$

The gravitational energy equation:

$$\frac{\partial}{\partial t}\left(\rho \Phi \right) + \nabla \cdot \left(\rho \mathbf{V} \Phi \right) = \rho \mathbf{V} \cdot \nabla \Phi \tag{A7}$$

The equation for the magnetic energy:

$$\frac{\partial}{\partial t}\left(\frac{B^2}{8\pi} \right) = -\nabla \cdot \left(\frac{B^2}{8\pi} \mathbf{V} \right) - \frac{B^2}{8\pi} \nabla \cdot \mathbf{V} + \frac{\mathbf{B} \cdot \nabla \mathbf{v} \cdot \mathbf{B}}{4\pi} \tag{A8}$$

Adding Eqs. (A5), (A6), (A7), and (A8) we then have the total energy balance equation:

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \frac{3}{2} p + \rho \Phi + \frac{B^2}{8\pi} \right) + \nabla \cdot \left(\rho \mathbf{v} \left[\frac{1}{2} v^2 + \frac{5}{2}\theta + \Phi \right] + q\mathbf{b} + \boldsymbol{\sigma} \cdot \mathbf{V} + \frac{1}{4\pi} \mathbf{B} \times (\mathbf{V} \times \mathbf{B}) \right) = 0 \tag{A9}$$

We attempt to derive the equations of local total energy balance, where the local energy $\mathcal{E} = \frac{1}{2}\rho v^2 + \frac{3}{2}p + \frac{B^2}{8\pi}$, where $\mathbf{v} = \mathbf{V} - R\Omega\hat{\boldsymbol{\phi}}$. Then the total energy balance equation is given by the following, where we average azimuthally:

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\frac{1}{2}\rho R^2\Omega^2 + \rho R\Omega v_\phi + \frac{1}{2}\rho v^2 + \rho\Phi + \frac{B^2}{8\pi}\right) + \nabla\cdot\left(\frac{1}{2}\rho v^2\mathbf{v}\right) + \\ &\nabla\cdot\left[\rho\mathbf{v}\left(\frac{1}{2}R^2\Omega^2 + \Phi + \frac{5}{2}\theta\right) + q\mathbf{b} + \boldsymbol{\sigma}\cdot R\Omega\hat{\boldsymbol{\phi}} + \right. \\ &\left. R\Omega\left(\rho v_\phi\mathbf{v} - \frac{B_\phi\mathbf{B}}{4\pi}\right) + \boldsymbol{\sigma}\cdot\mathbf{v}\right] \simeq 0\end{aligned} \tag{A10}$$

Where we have left out electromagnetic energy flux terms of the order $\frac{1}{4\pi}\mathbf{B}\times(\mathbf{v}\times\mathbf{B})$. Furthermore, we have that the equation describing conservation of local momentum $\rho\mathbf{v}$:

$$\begin{aligned}&\frac{\partial}{\partial t}(\rho\mathbf{v}) + \nabla\cdot(\rho\mathbf{u}\mathbf{v}) + \rho\left(2\Omega\hat{\boldsymbol{Z}}\times\mathbf{v} + \Omega' R v_R\hat{\boldsymbol{\phi}}\right) - \rho R\Omega^2\hat{\boldsymbol{R}} = -\nabla\cdot\boldsymbol{\sigma} + \\ &\nabla\cdot\left(\frac{\mathbf{B}\mathbf{B}}{4\pi} - \frac{B^2}{8\pi}\mathbb{I}\right) - \nabla p - \rho\nabla\Phi\end{aligned} \tag{A11}$$

Equilibrium is described by the following, where p_0 and ρ_0 are total equilibrium pressure and density.

$$R\Omega^2\hat{\boldsymbol{R}} = \nabla\Phi + \frac{1}{\rho_0}\nabla p_0 \tag{A12}$$

Now combining Eq. (A10) with Eqs. (A11) and (A12) yields the local energy balance equation:

$$\begin{aligned}&\frac{\partial\mathcal{E}}{\partial t} + \nabla\cdot\left(\rho\mathbf{v}\left[\frac{1}{2}v^2 + \frac{5}{2}\theta\right] + q\mathbf{b} + \boldsymbol{\sigma}\cdot\mathbf{v}\right) - \rho\mathbf{v}\cdot(\theta\nabla\ln p_0) \simeq \\ &-\Omega' R\left(\rho v_R v_\phi - \frac{B_R B_\phi}{4\pi} + \sigma_{R\phi}\right)\end{aligned} \tag{A13}$$

The steady state form of the local energy balance equation:

$$\begin{aligned}&\nabla\cdot\left(\rho\mathbf{v}\left[\frac{1}{2}v^2 + \frac{5}{2}\theta\right] + q\mathbf{b} + \boldsymbol{\sigma}\cdot\mathbf{v}\right) - \rho\mathbf{v}\theta\cdot\nabla\ln p_0 \simeq \\ &-\Omega' R\left(\rho v_R v_\phi - \frac{B_R B_\phi}{4\pi} + \sigma_{R\phi}\right)\end{aligned} \tag{A14}$$

We suppose a system with small turbulent fluctuations about the equilibrium.

$$\begin{aligned}
&\nabla \cdot \left(\frac{5}{2} \rho_0 \left\langle \delta \mathbf{u} \delta \theta \right\rangle + \left\langle q \mathbf{b} \right\rangle + \left\langle \boldsymbol{\sigma} \cdot \mathbf{v} \right\rangle \right) + \frac{3}{2} \theta \left\langle \rho \mathbf{u} \right\rangle \cdot \nabla \ln P_0 \rho_0^{-5/3} \simeq \\
&- \Omega' R \left\langle \rho_0 \delta v_R \delta v_\phi - \frac{\delta B_R \delta B_\phi}{4\pi} + \sigma_{R\phi} \right\rangle
\end{aligned} \tag{A15}$$

For a collisional fluid, we use following forms of the viscous stress tensor and heat flux. However, Eq. (A15) can be applied more generally to a magnetized plasma, in which the particle distribution is a function of velocities parallel and perpendicular to the magnetic field, in either the collisional or collisionless limits.