The Magnetoviscous–Thermal Instability in Dilute Magnetized Astrophysical Plasmas

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Introduction

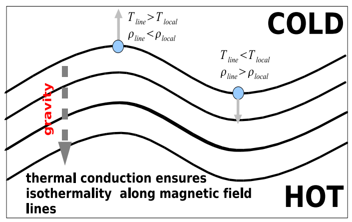
It has been shown that a differentially rotating plasma is linearly unstable in the presence of a magnetic field (Velikhov, 1959; Chandrasekhar, 1960). This has been shown to be important for accretion because this instability, the magnetorotatational instability (MRI), can efficiently transport angular momentum outwards (Balbus and Hawley, 1991). In hot, dilute accretion flows such as those around black holes, the particle collisional mean free path may be comparable or larger than the system scale. In such regions, if even a weak magnetic field is present, large anisotropic viscosities and thermal conductivities are directed along magnetic field lines. Previous research has demonstrated that viscous and thermal diffusive transport along magnetic field lines can also provide channels by which a plasma with differential rotation (Balbus, 2004a; Islam and Balbus, 2005) or temperature gradients (Balbus, 2001) can become unstable.

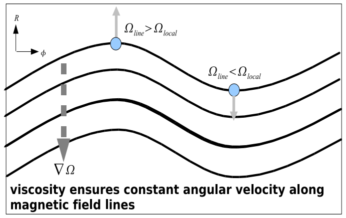
In addition to being dilute, certain types of accretion flows about black holes can also be radiatively inefficient: the cooling time from radiation is far longer than the timescale for matter to accrete into the central object. This property necessitates additional channels, other than radiation as in a classical accretion flow (Pringle, 1981; Frank et al., 2002), by which the energy generated through gravitational infall can be directed (Balbus, 2004b).

In this poster we examine the stability of a Keplerian plasma with equilibrium temperature and density gradients in the presence of a magnetic field and dynamically important anisotropic viscosities and thermal conductivities – we denote this as the magnetothermal-viscous instability (MVTI). We demonstrates that the growth rate and range of unstable wavenumbers is expanded over that of the MRI. We also demonstrate that the turbulence generated by the MVTI is of the right form to drive accretion in a dilute radiatively inefficient accretion flow.

Background

The physics behind the magnetothermal (Balbus, 2001) and magnetoviscous (Balbus, 2004a; Islam and Balbus, 2005) instabilities





In a steady state dilute radiatively inefficient rotating plasma, only certain types of fluctuation can drive accretion:

- **radial mass flux**: $\langle \delta\rho\,\delta v_R \rangle < 0$.
- **Azimuthal stress**: $T_{R\phi} = \left\langle \rho_0 \delta v_R \delta v_\phi - \frac{\delta B_R \delta B_\phi}{4\pi} + \mathbb{P}_{v,R\phi} \right\rangle > 0$
- **heat flux**: $q_R = \left\langle \frac{5\rho_0 k_B}{2m_i} \delta T \delta v_R + q_v \mathbf{b} + \mathbb{P}_v \cdot \delta\mathbf{v} \right\rangle > 0$.

Where $\langle \delta a \delta b \rangle$ is a spatially-averaged quadratic correlation of fluctuations. $\delta\rho$ is the fluctuation density, $\delta\mathbf{v}$ is the fluctuation velocity, $\delta\mathbf{B}$ is the fluctuation magnetic field, $\mathbb{P}_v$ is the anisotropic stress tensor, and q_v is the heat flux.

References

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Problem Formulation

We consider the stability of a rotating disk with a Keplerian profile ($\Omega \propto R^{-3/2}$) and weak ($B^2/(8\pi P) < 1$) nonradial magnetic field ($\mathbf{B}_0 = B_0 \left(\cos\chi\hat{\boldsymbol{\phi}} + \sin\chi\hat{\boldsymbol{z}}\right)$). The equilibrium state of the plasma is one of outwardly decreasing temperature and density that remains convectively stable. We consider axisymmetric modes of the form $\delta a \propto \exp(ik_Z z + \Gamma t)$, where k_Z is the vertical wavenumber and Γ is the growth rate.

We solve the following perturbed equations:

$$\nabla\cdot\delta\mathbf{v} = 0$$

$$\frac{\partial\delta\mathbf{v}}{\partial t} + 2\Omega\hat{\boldsymbol{z}}\times\delta\mathbf{v} + \Omega' R\hat{\boldsymbol{\phi}} = -\frac{1}{\rho_0}\nabla\left(\delta P + \frac{\mathbf{B}_0\cdot\delta\mathbf{B}}{4\pi}\right) + \frac{\mathbf{B}_0\cdot\nabla\delta\mathbf{B}}{4\pi\rho_0} - \frac{1}{\rho_0}\nabla\cdot\delta\mathbb{P}_v + \frac{\delta\rho}{\rho_0}\theta\frac{\partial\ln P_0}{\partial R}\hat{\boldsymbol{R}}$$

$$-\frac{5}{3}\Gamma\frac{\delta\rho}{\rho_0} + \delta v_R\frac{\partial\ln P_0\rho_0^{-5/3}}{\partial R} = -\frac{2}{3}P_0^{-1}\nabla\cdot(\delta q_v\mathbf{b})$$

$$\frac{\partial\delta\mathbf{B}}{\partial t} = \nabla\times\left[R\Omega(R)\hat{\boldsymbol{\phi}}\times\delta\mathbf{B} + \delta\mathbf{v}\times B_0\left(\cos\chi\hat{\boldsymbol{\phi}} + \sin\chi\hat{\boldsymbol{z}}\right)\right]$$

$$\delta\mathbb{P}_v = -\rho_0\nu\Gamma\left(3\mathbf{bb} - \mathbb{I}\right)\left(\cos\chi\frac{\delta B_\phi}{B_0} - \frac{k_R}{k_z}\sin\chi\frac{\delta B_R}{B_0}\right)$$

$$\delta q_v = -P_0\kappa\left(\frac{\delta B_R}{B_0}\times\frac{\partial\ln T_0}{\partial R} + i\left[\mathbf{k}\cdot\mathbf{b}\right]\frac{\delta T}{T_0}\right)$$

Where $\delta\mathbb{P}_v$ is the viscous stress tensor, δq_v is the thermal conductivity, $\theta = P_0/\rho_0$, ν is the viscous diffusivity, and κ is the thermal diffusivity.

Dispersion Relation

The dispersion relation for these unstable modes is given by the following:

$$\left(\Gamma^2 + \frac{d\Omega^2}{d\ln R} + k_Z^2 v_A^2\sin^2\chi - \frac{3}{5}\theta\left(\frac{\partial\ln P_0}{\partial R}\right)\frac{\Gamma\frac{\partial\ln P_0\rho_0^{-5/3}}{\partial R} + \frac{2}{3}\kappa k_Z^2\sin^2\chi\frac{\partial\ln T_0}{\partial R}}{\Gamma + \frac{2}{5}\kappa k_Z^2\sin^2\chi}\right)\times$$
$$\left(\Gamma^2 + k_z^2 v_A^2\sin^2\chi + 3\nu k_Z^2\Gamma\sin^2\chi\cos^2\chi\right) + 4\Omega^2\Gamma^2 = 0$$

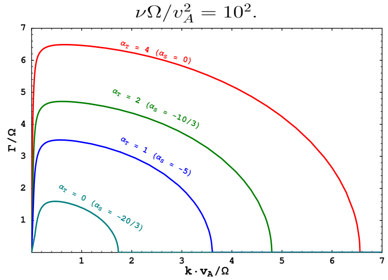
Where we find it convenient to use the following normalizations:

$$\alpha_T = -\left(\frac{\theta^{1/2}}{\Omega}\right)\frac{\partial\ln T_0}{\partial R} > 0, \alpha_P = -\left(\frac{\theta^{1/2}}{\Omega}\right)\frac{\partial\ln P_0}{\partial R} > 0$$

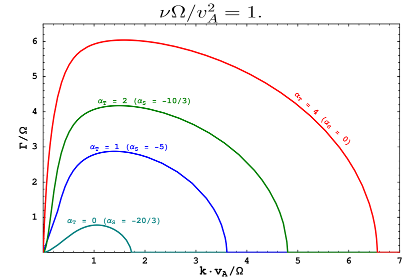
$$\alpha_S = -\left(\frac{\theta^{1/2}}{\Omega}\right)\frac{\partial\ln P_0\rho_0^{-5/3}}{\partial R} = \frac{5}{3}\alpha_T - \frac{2}{3}\alpha_P < 0$$

We take for all plots $\alpha_P = 10$, $\kappa \simeq 143\nu$, and weak magnetic field $v_A^2 = 10^{-2}\theta$.

Relative large viscous diffusivity, $\nu\Omega/v_A^2 = 10^2$.



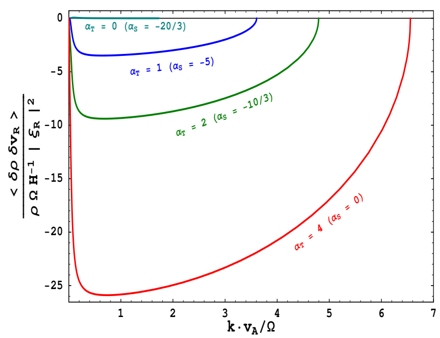
Relatively small viscous diffusivity, $\nu\Omega/v_A^2 = 1$.



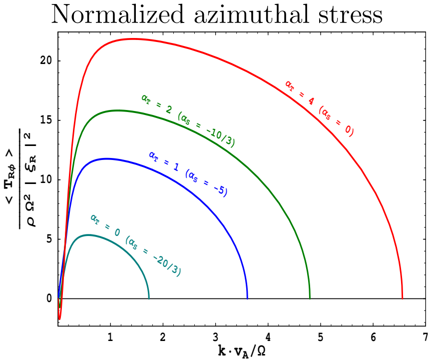
Quadratic Fluxes

Turbulent fluxes with large viscosity, $\nu\Omega/v_A^2 = 10^2$.

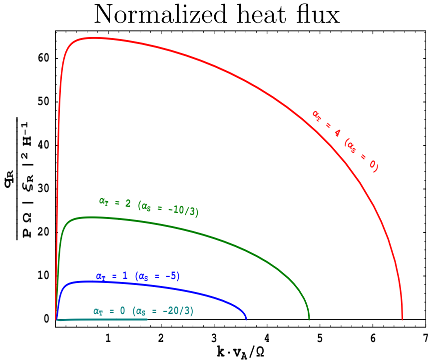
Normalized mass flux



Normalized azimuthal stress



Normalized heat flux



Conclusions

- The presence of thermal gradients enhances the growth rate and range of unstable wavenumbers beyond that of the MRI.
- Fat disks (disk height $H \sim R$) are unstable to the MVTI.
- Quadratic fluxes associated with the MVTI are of the right sign to drive accretion.
- Heat flux is also of the right magnitude to transport energy generated through accretion.
- Thermal gradients may also contribute to a Reynolds stress, whose sign may be positive or negative.