Project I - Error Control in Relay Networks

February 2023

1 Theoretic Analysis

1.1 AF relaying with end-to-end ARQ

Consider the queuing network for AF relaying with end-to-end ARQ shown in Fig. 2(a). Give the queuing model for the RS and MS nodes. Please derive:

- the end-to-end error probability of packet transmission ($p_{e,e2e}$), and the packet arrival rate at the first relay (λ_1),
- the average queuing delay and the average number of packets (waiting or under processing) at every RS and MS ($\overline{W}_j$ and $\overline{N}_j$),
- the average end-to-end delay, that is the average time from the generation of a packet until it is received successfully at the MS ($\overline{T}$).

Solution:

- $p_{e,e2e} = 1 - P(\text{no error}) = 1 - \Pi_{j=1}^{r+1}(1 - p_{e,j})$
  If the system is stable: $\lambda_1 = \lambda + p_{e,e2e}\lambda_1 = \dfrac{\lambda}{1 - p_{e,e2e}}$
- RS: $\overline{W}_j = \dfrac{\rho}{\mu_{AF} - \lambda_1} = \dfrac{\lambda_1}{\mu_{AF}^2 - \lambda_1 \mu_{AF}}$
  $\overline{N}_j = \dfrac{\rho_j}{1 - \rho_j} = \dfrac{\lambda_1}{\mu_{AF} - \lambda_1}$
  MS: $\overline{W}_j = \dfrac{\rho}{\mu_{MS} - \lambda_1} = \dfrac{\lambda_1}{\mu_{MS}^2 - \lambda_1 \mu_{MS}}$
  $\overline{N}_j = \dfrac{\rho_j}{1 - \rho_j} = \dfrac{\lambda_1}{\mu_{MS} - \lambda_1}$
- $\overline{T} = \dfrac{N}{\sum_{j=1}^{M} \gamma_j} = \dfrac{\sum_{j=1}^{r+1} \overline{N}_j}{\lambda}$

1.2 DF relaying with end-to-end ARQ

Consider the queuing network for DF relaying with end-to-end ARQ and with hop-by-hop ARQ shown in Fig. 2 (b) and (c), respectively. Please derive for both cases:

- the packet arrival rate at each RS and MS (λ_j),
- the average queuing delay and the average number of packets (waiting or under processing) at every RS and MS ($\overline{W}_j$ and $\overline{N}_j$),
- the average end-to-end delay, that is the average time from the generation of a packet until it is received successfully at the MS ($\overline{T}$).

Solution:

- $\lambda_1 = \lambda + \sum_{j=1}^{r+1} \lambda_j p_{e,j}$
  $\lambda_j = \lambda_{j-1}(1 - p_{e,j-1})$ for $2 \leq j \leq r+2$
  λ_j is a geometric serie, thus $\lambda_j = \lambda_1 \Pi_{i=1}^{j-1}(1 - p_{e,i})$
  Finally, we have:
  $\lambda_1 = \lambda + \sum_{j=1}^{r+1} \lambda_j p_{e,j} = \lambda + \sum_{j=1}^{r+1} \lambda_1 \Pi_{i=1}^{j-1}(1 - p_{e,i})p_{e,j}$
  $\lambda_1 = \dfrac{\lambda}{1 - \sum_{j=1}^{r+1} \Pi_{i=1}^{j-1}(1 - p_{e,i})p_{e,j}}$

- RS: $\overline{W}_j = \dfrac{\rho}{\mu_{DF} - \lambda_j} = \dfrac{\lambda_j}{\mu_{DF}^2 - \lambda_j \mu_{DF}}$
  $\overline{N}_j = \dfrac{\rho_j}{1 - \rho_j} = \dfrac{\lambda_j}{\mu_{DF} - \lambda_j}$
  MS: $\overline{W}_{r+1} = \dfrac{\rho}{\mu_{MS} - \lambda_{r+1}} = \dfrac{\lambda_{r+1}}{\mu_{MS}^2 - \lambda_{r+1}\mu_{MS}}$
  $\overline{N}_{r+1} = \dfrac{\rho_{r+1}}{1 - \rho_{r+1}} = \dfrac{\lambda_{r+1}}{\mu_{MS} - \lambda_{r+1}}$

- $\overline{T} = \dfrac{N}{\sum_{j=1}^{M} \gamma_j} = \dfrac{\sum_{j=1}^{r+1} \overline{N}_j}{\lambda}$

1.3 DF relaying with hop-by-hop ARQ

Consider the queuing network for DF relaying with end-to-end ARQ and with hop-by-hop ARQ shown in Fig. 2 (b) and (c), respectively. Please derive for both cases:

- the packet arrival rate at each RS and MS (λ_j),
- the average queuing delay and the average number of packets (waiting or under processing) at every RS and MS ($\overline{W}_j$ and $\overline{N}_j$),
- the average end-to-end delay, that is the average time from the generation of a packet until it is received successfully at the MS ($\overline{T}$).

Solution:

- $\lambda_1 = \lambda + \lambda_1 p_{e,1} = \dfrac{\lambda}{1 - p_{e,1}}$

  $\lambda_j = \lambda_{j-1}(1 - p_{e,j-1}) + \lambda_j p_{e,j} = \dfrac{\lambda_{j-1}(1 - p_{e,j-1})}{1 - p_{e,j}}$

  It is also a geometric serie, thus $\lambda_j = \lambda \dfrac{\Pi_{i=1}^{j-1}(1 - p_{e,i})}{\Pi_{i=1}^{j}(1 - p_{e,i})} = \dfrac{\lambda}{1 - p_{e,j}}$

- RS: $\overline{W}_j = \dfrac{\rho}{\mu_{DF} - \lambda_j} = \dfrac{\lambda_j}{\mu_{DF}^2 - \lambda_j \mu_{DF}}$

  $\overline{N}_j = \dfrac{\rho_j}{1 - \rho_j} = \dfrac{\lambda_j}{\mu_{DF} - \lambda_j}$

  MS: $\overline{W}_{r+1} = \dfrac{\rho}{\mu_{MS} - \lambda_{r+1}} = \dfrac{\lambda_{r+1}}{\mu_{MS}^2 - \lambda_{r+1}\mu_{MS}}$

  $\overline{N}_{r+1} = \dfrac{\rho_{r+1}}{1 - \rho_{r+1}} = \dfrac{\lambda_{r+1}}{\mu_{MS} - \lambda_{r+1}}$

- $\overline{T} = \dfrac{N}{\sum_{j=1}^{M} \gamma_j} = \dfrac{\sum_{j=1}^{r+1} \overline{N}_j}{\lambda}$

2 Numerical Analysis

Now you need to compare the performance of the three relaying systems shown in Fig. 2, based on the analytic results derived in Section 3.2, and for given system parameter values. You are strongly encouraged to use Matlab to draw the performance figures, but you are also allowed to use other programming languages and tools.

2.1 Stability region

Consider $p_{e,j} = 0,1$ with j = 1, 2, ..., r+1; $\mu_{DF} = 1$ and $\mu_{AF} = \mu_{MS} = 2\mu_{DF}$. Please draw the maximal allowed arrival rate of packets $\lambda_{max}(r)$ for the three relaying systems respectively as r increases from 1 to 8. The maximal allowed arrival rate $\lambda_{max}(r)$ is the rate where for all $\lambda < \lambda_{max}(r)$ the queuing network remains stable. (A queuing network is stable if all the queues are stable.) How does $\lambda_{max}(r)$ change as r goes to infinity? Give an intuitive explanation.

2.1.1 AF relaying with end-to-end ARQ

Stability condition:

$\rho = \dfrac{\lambda_j}{\mu_{AF}} = \dfrac{\lambda}{\mu_{AF}(1-p_{e,e2e})} = \dfrac{\lambda}{\mu_{AF}\Pi_{j=1}^{r+1}(1-p_{e,j})} < 1$

$\lambda_{max} = \mu_{AF}\Pi_{j=1}^{r+1}(0.9) = 2(0.9)^{r+1}$

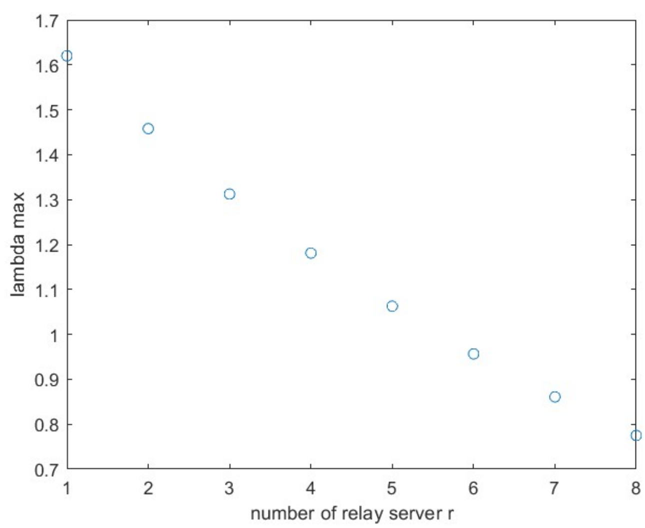

Figure 1: Relationship between lambda max and the number of relay server for AF relaying with end-to-end ARQ

As r increase, λ_{max} goes to 0. It can be explained because as the number of relay server increase the error will increase, leading to more retransmissions at the BS and allowing less new packets to be sent.

2.1.2 DF relaying with end-to-end ARQ

Stability condition:

$$\frac{\lambda_1}{\mu_{DF}} = \frac{\lambda}{\mu_{DF}(1 - \sum_{j=1}^{r+1} p_{e,j} \Pi_{i=1}^{j-1}(1 - p_{e,i}))} < 1$$

$$\frac{\lambda_j}{\mu_{DF}} = \frac{\lambda_1 \Pi_{i=1}^{j-1}(1 - p_{e,i})}{\mu_{DF}} < 1$$

We only need to verify the first condition since $1 - p_{e,i} < 1$.

Thus, $\lambda_{max} = \mu_{DF}(1 - \sum_{j=1}^{r+1} p_{e,j} \Pi_{i=1}^{j-1}(1 - p_{e,i})) = 1 - \sum_{j=1}^{r+1} 0.1(0.9)^{j-1}$

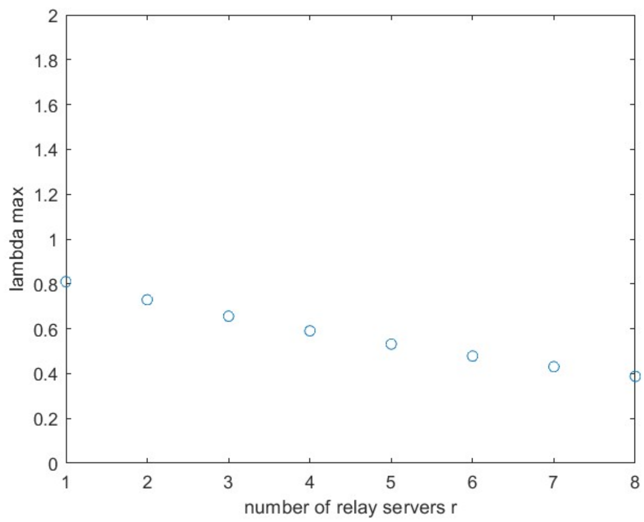

Figure 2: Relationship between lambda max and the number of relay server for DF relaying with end-to-end

As r increases, λ_{max} goes to 0. It can be explained because as the number of relay server increase the error will increase, leading to more retransmissions at the BS and allowing less new packets to be sent.

2.1.3 DF relaying with hop-by-hop ARQ

Stability condition:
$\lambda_j = \dfrac{\lambda}{1 - p_{e,1}} < 1$ for all j.
Thus, $\lambda_{max} = 1 - p_{e,1} = 0.9$.

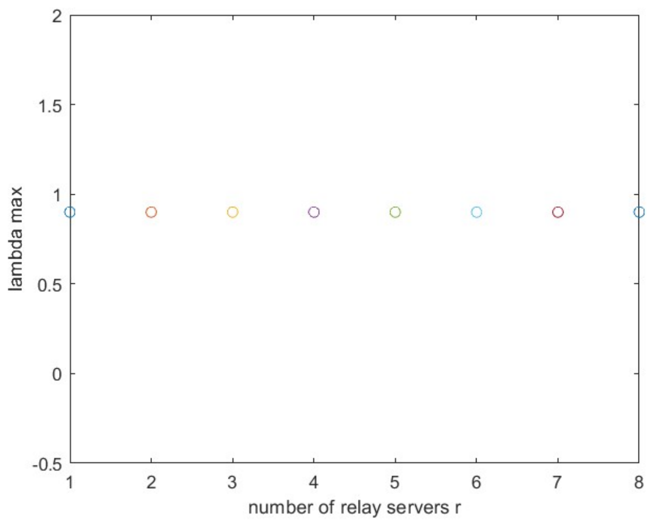

Figure 3: Relationship between lambda max and the number of relay server for DF relaying with hop-by-hop

As r increases, since λ_{max} is not a function of r, it stays constant to the same initial value. This can be explained because the retransmission is made at each DF, thus the number of relay server is not going to affect the first server.

2.2 Arrival rate-delay characteristics

With the same set of parameters considered in Section 3.3.1 and r = 4, plot the delay as a function of the arrival rate , for of the three relaying systems under their stability regions. The x axis is the arrival rate λ, whereas the y axis is the average delay from packet generation to correct reception. What happens with the arrival rate-delay curves when $p_{e,j}$ increases?

2.2.1 AF relaying with end-to-end ARQ

$\overline{T} = \frac{\sum_{j=1}^{r+1} \overline{N}_j}{\lambda} = \frac{1}{\lambda}(r\frac{\lambda_1}{\mu_{AF} - \lambda_1} + \frac{\lambda_1}{\mu_{MS} - \lambda_1})$ with
$\lambda_1 = \frac{\lambda}{1 - p_{e,e2e}}$, and $p_{e,e2e} = 1 - \Pi_{j=1}^{r+1}(1 - p_{e,j})$

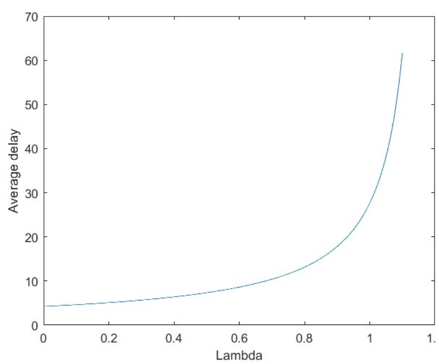

Figure 4: Relationship between lambda and the average delay for $p_{e,j} = 0.1$

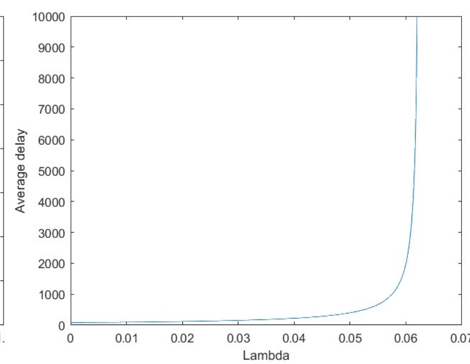

Figure 5: Relationship between lambda and the average delay for $p_{e,j} = 0.5$

We can see as we increase the $p_{e,j}$ that the delay increase exponentially for a λ much smaller. Thus, if the $p_{e,j}$ increases the arrival rate has to decrease a lot so that the average delay doesn't grow exponentially. Which makes sense since we have to spend time resending the packets that failed.

2.2.2 DF relaying with end-to-end ARQ

$$\lambda_1 = \frac{\lambda}{(1-\sum_{j=1}^{r+1} p_{e,j}\Pi_{i=1}^{j-1}(1-p_{e,i}))} = \frac{\lambda}{1-\sum_{j=1}^{r+1} 0.1(0.9)^{j-1}}$$

$$\lambda_j = \lambda_1 \Pi_{i=1}^{j-1}(1-p_{e,i}) = \lambda_1(1-p_{e,j})^{j-1}$$

$$\overline{T} = \frac{\sum_{j=1}^{r+1} \overline{N}_j}{\lambda} = \frac{1}{\lambda}(\sum_{j=1}^{r} \frac{\lambda_j}{\mu_{DF}-\lambda_j} + \frac{\lambda_{r+1}}{\mu_{MS}-\lambda_{r+1}})$$

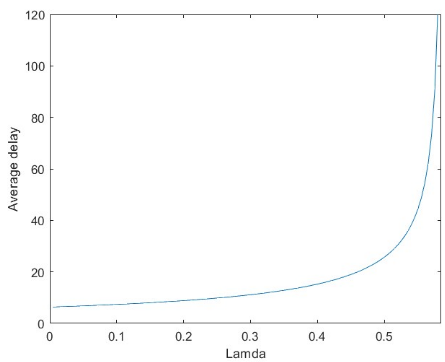

Figure 6: Relationship between lambda and the average delay for $p_{e,j} = 0.1$

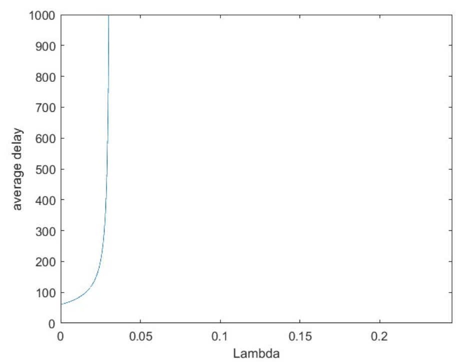

Figure 7: Relationship between lambda and the average delay for $p_{e,j} = 0.5$

We can see as we increase the $p_{e,j}$ that the delay increase exponentially for a λ much smaller. Thus, if the $p_{e,j}$ increases the arrival rate has to decrease a lot so that the average delay doesn't grow exponentially. Which makes sense since we have to spend time resending the packets that failed.

2.2.3 DF relaying with hop-by-hop ARQ

$\lambda_1 = \dfrac{\lambda}{1 - p_{e,j}}$

$\lambda_j = \dfrac{\lambda_1}{1 - p_{e,j}} = \dfrac{\lambda}{(1 - p_{e,j})^2}$

$\overline{T} = \dfrac{\sum_{j=1}^{r+1} \overline{N}_j}{\lambda} = \dfrac{1}{\lambda}(\sum_{j=1}^{r} \dfrac{\lambda_j}{\mu_{DF} - \lambda_j} + \dfrac{\lambda_{r+1}}{\mu_{MS} - \lambda_{r+1}})$

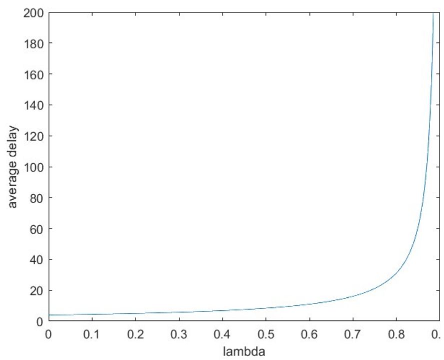

Figure 8: Relationship between lambda and the average delay for $p_{e,j} = 0.1$

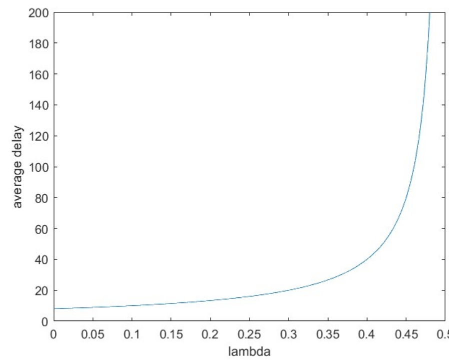

Figure 9: Relationship between lambda and the average delay for $p_{e,j} = 0.5$

We can see that as the $p_{e,j}$ increases, the average delay gets higher for a smaller lambda. Thus, if the error gets higher the lambda max is getting smaller. But the difference is much smaller than the other 2 systems. The average delay increases only when we get close to lambda max.

2.3 AF or DF

- To make a choice of the best system, we need to consider the system that will process the more packets and with the smallest delay. Thus, we consider λ and $\overline{T}$.

  First, let us compare the λ:

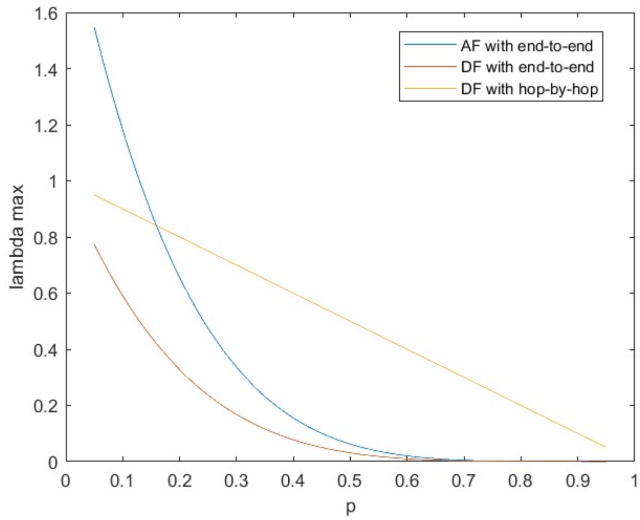

Figure 10: Relationship between p and lambda max

We can see that for low probability the AF with end-to-end system is the best. However, as the p increases, the DF with end-to-end becomes the best system since it decreases linearly and the other decreases exponentially.
Now let us compare the average delay:

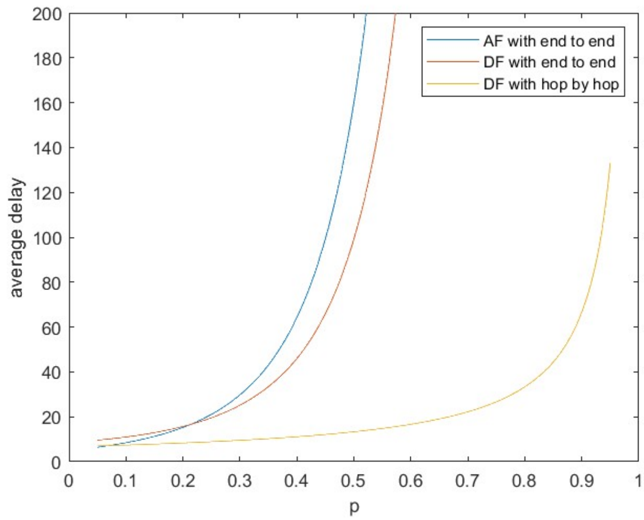

Figure 11: Relationship between p and average delay

We also see that the third system with DF hop-by-hop system is the best system.

- From the last two graphs, we see that for $p_{e,j} = 0.1$, the first system is the best one with the highest λ and the lowest average delay. If, we already have that and moreover $\mu_{AF} = k\mu_{DF}$, the service rate at each relay server will increase compared to the other systems which can only improve the system. Thus, the AF end-to-end ARQ is the best system.