Black-Scholes (from foundation to code)

- Financial options: "Book it at current price", may or may not act on it later.
  - ↳ call option: right to BUY
  - ↳ put option: right to SELL

European options: can only exercise on expiration date → we deal with this in Black-Scholes.

American options: can exercise anytime till expiration date

terms:-

- Strike Price (K):- booking price
- Expiration date
- Premium:- cost of booking
- Underlying Asset
- Exercise: actually executing the BUY or SELL action.

- Key assumptions:-
  i. Stock prices follow a random geometric Brownian Motion - ofc
  ii. Constant volatility - true in short period or stable markets
  iii. No transaction costs - scale
  iv. Continuous Trading - A bit of a stretch, but it's okay
  v. No dividents are paid by the stock during the option's life - fine
  vi. Constant risk free rate, i.e. Interest rates don't change - no worries
  vii. Markets are frictionless & liquid:- larger market ⇒ better liquidity
  viii. No Arbitrage opportunities (doubt)

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- let's define
  - → S_0 = current stock price
  - → K = Strike price (price at which I book the option)
  - → T = time to expiration (in years)
  - → r = risk-free interest rate (known value)
  - → σ = volatility (standard deviation of returns)

- $$d_1 = \frac{\left[\ln(S_0/K) + (r + \sigma^2/2) \times T\right]}{\sigma \times \sqrt{T}}$$

- $d_2 = d_1 - \sigma \times \sqrt{T}$

- $N(x)$ = cumulative standard normal dist f^n

- European Call Option & Put Option:-

* $C = S_0 \times N(d_1) - K \times e^{-r \times T} \times N(d_2)$

* $P = K \times e^{-rT} \times N(-d_2) - S_0 \times N(-d_1)$

definition:

Volatility (σ)

statistical measure of how much price of stock fluctuates over a period of time

1. Collect price data (T)
2. Compute log returns
   $r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$
3. Find mean of $\{r_i\}$
4. get std-dev of $\{r_i\}$
5. σ = std dev $\times \sqrt{T}$

Here C & P are basically the premium which should be charged for the resp option.

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- The "Greeks"

  - they help us understand how option price changes when market conditions change.

① Delta (Δ) - Price sensitivity

$\Delta =$ change in option's price for every 1\$ change in underlying stock price

Call options: $0 \leq \Delta \leq 1$

Put options: $-1 \leq \Delta \leq 0$

② Gamma (Γ) - Rate of change of delta

$\Gamma =$ change in delta value for every 1\$ change in underlying stock price

- always positive
- highest at the money i.e. when $S_0 = K$
- Approaches zero for deep in money / out of money options

→ High $\Gamma \Rightarrow \Delta$ & risk change rapidly. // imp for risk management

③ Theta (Θ) - Time Decay

$\Theta =$ how much an option loses value each day, due to passage of time

- usually -ve
- accelerates as expiration approaches
- Highest for at-the-money options

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④ Vega (ν) - Volatility Sensitivity

ν = how much an option's price changes when volatility changes by 1%

- always +ve
- higher for longer term options
- highest for at the money options.

issue: Volatility is an input which cannot be observed directly, it must be estimated or implied from market prices.

⑤ Rho (ρ) - Interest Rate Sensitivity

ρ = how much an option's price changes when interest rate changes by 1%

- +ve for CALLs & -ve for PUTs
- more significant for longer term options.