

1.



2 particles are maintaining fixed distance mutually, and they are confined to move only in a 2-dimensional plane. Number of degrees of freedom for the system is

a. 4 b. 2 c. 3 d. 5

☐ (a)

☐ (b)

☐ (c)

☐ (d)

2.

A particle is moving on the surface of a sphere and satisfies a constraint $r \geq a$, where a is the radius of the sphere. The constraint is-

- a. Holonomic and scleronomous
- b. Holonomic and rheonomous
- c. Non-holonomic and rheonomous
- d. Non-holonomic and scleronomous

☐ (a)

☐ (b)

☐ (c)

☐ (d)

☐ (c)

☐ (d)

3.

A simple pendulum of mass m is suspended from a point that is moving in a simple harmonic motion horizontally following $x = a \cos \omega t$. Its Lagrangian is (after removing a total derivative)

a. $L = \frac{1}{2}ml^2\dot{\phi}^2 + mla\omega^2 \cos \omega t \sin \phi + \frac{1}{2}ma^2\omega^2 \sin^2 \omega t + mgl \cos \phi$

b. $L = \frac{1}{2}ml^2\dot{\phi}^2 + mla\omega \cos \omega t \sin \phi + \frac{1}{2}ma^2\omega^2 \sin^2 t + mgl \cos \phi$

c. $L = \frac{1}{2}ml^2\dot{\phi}^2 + mgl \cos \phi$

d. $L = \frac{1}{2}ml^2\dot{\phi}^2 + mla\omega^2 \cos \omega t + mgl \cos \phi$

where ϕ is the angle of the string of the pendulum with the vertical and l is the length of the string.

☐ (a)

☐ (b)

☐ (c)

☐ (d)

4.

The full Lagrangian of a RL circuit contains a dissipative term of the form:

a. RI b. R^2I c. $\frac{1}{2}\dot{Q}^2R$ d. QV

☐ (a)

☐ (b)

☐ (c)

☐ (d)

5.

The Lagrangian of a particle of mass m moving in one dimension is

$$L = \exp(\alpha t) \left[\frac{m\dot{x}^2}{2} - \frac{kx^2}{2} \right]$$

where α and k are positive constants. The equation of motion is

- (a) $\ddot{x} + \alpha\dot{x} = 0$
- (b) $\ddot{x} + \frac{k}{m}x = 0$
- (c) $\ddot{x} - \alpha\dot{x} + \frac{k}{m}x = 0$
- (d) $\ddot{x} + \alpha\dot{x} + \frac{k}{m}x = 0$

- ☐ (a)
- ☐ (b)
- ☐ (c)

6.

The Lagrangian of a particle of unit mass moving in a plane, in Cartesian coordinates, is given by

$$L = \dot{x}\dot{y} - x^2 - y^2.$$

In plane polar (r, θ) coordinates, the expression for the conjugate momentum p_r (conjugate to the radial coordinate r) is:

- (a) $\dot{r} \sin \theta + r \dot{\theta} \cos \theta$
- (b) $\dot{r} \cos \theta + r \dot{\theta} \sin \theta$
- (c) $\dot{r} \cos 2\theta + r \dot{\theta} \sin 2\theta$
- (d) $\dot{r} \sin 2\theta + r \dot{\theta} \cos 2\theta$

☐ (a)

☐ (b)

☐ (c)

☐ (d)

7.

The Hamiltonian of a one dimensional system is

$$H = \frac{xp^2}{2m} + \frac{1}{2}kx$$

where m and k are positive constants. The corresponding equation of motion for the system is

(a) $m\ddot{x} + kx = 0$

(b) $m\ddot{x} + 2\dot{x} + kx^2 = 0$

(c) $2m\dot{x}\ddot{x} - m\dot{x}^2 + kx^2 = 0$

(d) $m\dot{x}\ddot{x} + 2m\dot{x}^2 + kx^2 = 0$

☐ (a)

☐ (b)

☐ (c)

☐ (d)

8.

The Lagrangian of a system is

$$L = \frac{13}{2}m\dot{x}^2 + 4m\dot{x}\dot{y} + 3m\dot{y}^2 - mg(x + 2y)$$

Which one of the following is conserved?

A. $11\dot{x} + \dot{y}$

B. $11\dot{x} - \dot{y}$

C. $\dot{x} + 11\dot{y}$

D. $\dot{x} - 11\dot{y}$

9.

A bead of mass m is constrained to move under gravity along a planar rigid wire that has a equation $y = ax^2/2l^3$, where x and y are, respectively, the horizontal and the vertical coordinates. Lagrangian for the system is

(a) $\frac{1}{2}m\dot{x}^2 \left(1 + \frac{a^2 x^2}{l^3}\right) - mgax^2/l^3$

(b) $\frac{1}{2}mx^2 \left(1 + \frac{a^2 \dot{x}^2}{l^3}\right) - mgax^2/l^3$

(c) $\frac{1}{2}m\dot{x}^2 \left(1 - \frac{a^2 x^2}{l^3}\right) - mgax^2/l^3$

(d) $\frac{1}{2}mx^2 \left(1 - \frac{a^2 \dot{x}^2}{l^3}\right) - mgax^2/l^3$

☐ (a)

☐ (b)

☐ (c)

☐ (d)

☐ (c)

☐ (d)

10.

Hamiltonian of simple pendulum consisting of a mass m attached to a mass less string of length l is $H = \frac{p_\theta^2}{2ml^2} + mgl(1 - \cos\theta)$. If L denotes the Lagrangian, The value of $\frac{dL}{dt}$ is

(a) $\frac{g}{l} p_\theta \cos\theta$

(b) $p_\theta^2 \cos\theta$

(c) $-\frac{2g}{l} p_\theta \sin\theta$

(d) $-\frac{g}{l} p_\theta \sin 2\theta$

☐ (a)

☐ (b)

☐ (c)

☐ (d)

☐ (b)

☐ (c)

☐ (d)

11.

The Hamiltonian for the Lagrangian $L = -m_0c^2\sqrt{1 - \frac{\dot{x}^2}{c^2}}$ is

(a) $-m_0c^2\sqrt{1 - \frac{\dot{x}^2}{c^2}}$ (b) $m_0c^2\sqrt{1 - \frac{\dot{x}^2}{c^2}}$ (c) $-\frac{m_0c^2}{\sqrt{1 - \frac{\dot{x}^2}{c^2}}}$ (d) $\frac{m_0c^2}{\sqrt{1 - \frac{\dot{x}^2}{c^2}}}$

☐ (a)

☐ (b)

☐ (c)

☐ (d)

12.

Which of the following is an example of non-holonomic constraint?

- (a) A bead moving on a circular wire (b) A particle sliding down a plane
(c) Molecules moving inside a container (d) A simple pendulum

☐ (a)

☐ (b)

☐ (c)

☐ (d)

☐ (b)

☐ (c)

☐ (d)

13.

Lagrangian for a charged particle with charge e and mass m moving in an electromagnetic field with scalar potential $\phi(x, y, z)$ and vector potential $\mathbf{A}(x, y, z)$ is given by

$$L = \frac{1}{2}mv^2 - e\phi + \frac{e}{c}\mathbf{A} \cdot \mathbf{v}$$

Expression for generalized momentum is

(a) zero

(b) $m\mathbf{v}$

(c) $m\mathbf{v} + \frac{e}{c}\mathbf{A}$

(d) $m\mathbf{v} - e\phi + \frac{e}{c}\mathbf{A}$

☐ (a)

☐ (b)

☐ (c)

☐ (d)

☐ (b)

☐ (c)

☐ (d)

14.

The geodesic on a surface enclosed by a surface of characteristic equations: $x^2 + y^2 = a^2$, $z = z$; where a is constant, is:

(a) A great circle

(b) a helix

(c) a straight line

(d) a catenary curve

☐ (a)

☐ (b)

☐ (c)

☐ (d)

15.

The functional $f = y'^2 - y^2 + 2xy$ will have its extremal only if:

(a) $y'' + y = x$

(b) $y'' - y = x$

(c) $y'' + x = y$

(d) $y'' + y' = x$

☐ (a)

☐ (b)

☐ (c)

☐ (d)