Classical Mechanics

$$\frac{dr}{dt} = \dot{r} \qquad \frac{d^2r}{dt^2} = \ddot{r}$$

Conservation of Energy

→ $W_{12} = \int_1^2 F \cdot ds = \int_1^2 F \cdot dr$

$$= \int_1^2 \frac{d(m\dot{r})}{dt} \cdot dr = m\int_1^2 \frac{d^2r}{dt^2} dr$$

$$= m\int_1^2 \frac{d}{dt}(V) dr = m\int_1^2 V dV = \frac{1}{2}\left[mV^2\right]_1^2$$

$$= \frac{1}{2} m (V_2^2 - V_1^2) = \frac{1}{2} m \dot{r}_2^2 - \frac{1}{2} m \dot{r}_1^2$$

$$= T_2 - T_1$$

→ $F = -\nabla V$

$$W_{12} = \int_1^2 F \cdot dr = -\int_1^2 \nabla V \cdot ds = -\int_1^2 \frac{dV}{dr} \cdot dr$$

$$= V_1 - V_2$$

$\Rightarrow T_2 - T_1 = V_1 - V_2$

$$\boxed{T_2 + V_2 = T_1 + V_1}$$

$\Rightarrow$ Energy is conserved when particle moves from point 1 & 2.

Constraints of motion

- Holonomic (velocity is const.) → Bilateral
- Non-holonomic (velocity is not const.) → Unilateral
- Rheonomic (dependent on time)
- Scleronomic (Independent on time)

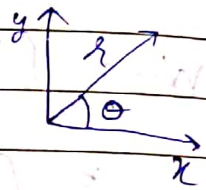
Generalized Coordinates

q_1, q_2 — 2-dimension

q_1, q_2, q_3 — 3-dimension

q_1, q_2 ------ q_N — n-dimension {Hypothetical}

$$q_1 = x \quad | = r \qquad\qquad x^2+y^2 = r^2$$
$$q_2 = y \quad | = \theta \qquad\qquad r = \sqrt{x^2+y^2}$$
$$\theta = \tan^{-1}(y/x)$$



Generalized Displacement

$$r_i = r_i \; (i=1,2,3 \text{ ------ } N)$$
$$= r_i(q_1, q_2, q_3 \text{ ------ } q_{3N}, t)$$

Euler's eq^n

$$X = X(x_1, x_2, x_3)$$

$$dX = \frac{\partial X}{\partial x_1} dx_1 + \frac{\partial X}{\partial x_2} dx_2 + \frac{\partial X}{\partial x_3} dx_3$$

$$\delta r_i = \sum_{j}^{3N} \frac{\partial r_i}{\partial \dot{q}_j} \delta q_j$$

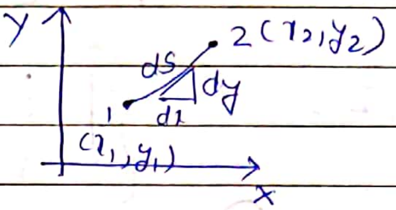
Virtual displacement or Generalized displacement

Generalized Velocity

Calculus of Variation

$y = mx + c$

$y = f(x)$



α = Variation

$I = \int_1^2 ds$

$= \int_1^2 \sqrt{(dx)^2 + (dy)^2} = \int_1^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_1^2 \sqrt{1 + \dot{y}^2}\, dx$

$= \int_1^2 f(\dot{y}) = \int_1^2 f(y, \dot{y}, x)\, dx = \int f(y(\alpha), \dot{y}(\alpha), x)\, dx$

$I(\alpha)$

$\boxed{f(\dot{y}) = \sqrt{1 + y^2}}$

$\frac{\partial x}{\partial \alpha} = 0$ —i) & $\left(\frac{\partial y}{\partial \alpha}\right)$ at end points $= 0$ —ii)

$\delta I = 0 \Rightarrow \delta \int f(y, \dot{y}, x)\, dx = 0$

$\rightarrow \delta I(\alpha) = \delta \int_1^2 f(y(\alpha), \dot{y}(\alpha), x)\, dx$

$\Rightarrow \frac{\partial I(\alpha)}{\partial \alpha} = 0$

$\frac{dI(\alpha)}{\partial \alpha} = \int_1^2 \frac{\partial y}{\partial \alpha} \cdot \frac{df}{dy}\, dx + \frac{\partial f}{\partial \dot{y}} \cdot \frac{d\dot{y}}{d\alpha}\, dx + \frac{df}{dx} \underbrace{\frac{\delta x}{\delta \alpha}}_{0\ \text{(by eq i)}}\, dx$

$\frac{\partial I(\alpha)}{\partial \alpha} = \int_{x_1}^{x_2} \left[\frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial \alpha} + \frac{\partial f}{\partial \dot{y}} \frac{d^2 y}{\partial x\, \partial \alpha}\right] dx$

$= \int_{x_1}^{x_2} \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial \alpha}\, dx + \frac{\partial f}{\partial \dot{y}} \frac{\partial y}{\partial \alpha} \Big|_{x_1}^{x_2}$

$- \int_{x_1}^{x_2} \frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) \frac{\partial y}{\partial \alpha}\, dx$

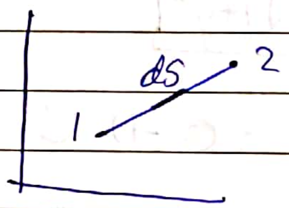
$$\left(\frac{\partial I}{\partial \alpha}\right) d\alpha = \int_{x_1}^{x_2} \left[\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right)\right] \frac{\partial y}{\partial \alpha}\, d\alpha\, dx$$

$$\delta I = \int_{x_1}^{x_2} \left\{\left[\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\delta f}{\delta \dot{y}}\right)\right] \delta y\right\} dx$$
$$= 0$$

$$\left[\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right)\right] = 0$$

Shortest Distance between 2 points

$$I = \int_1^2 dS$$



$$= \int_1^2 \sqrt{1+y^2}\, dx$$

$$= \int_1^2 f\, dx \qquad \text{where } f = \sqrt{1+\dot{y}^2}$$

$$\frac{\partial f}{\partial y} = 0 \qquad \frac{\partial f}{\partial \dot{y}} = \frac{\dot{y}}{\sqrt{1+(\dot{y})^2}}$$

$$\left[\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right)\right] = 0$$

$$0 - \frac{d}{dx}\left(\frac{\dot{y}}{\sqrt{1+\dot{y}^2}}\right) = 0$$

$$\frac{\dot{y}}{\sqrt{1+\dot{y}^2}} = c$$

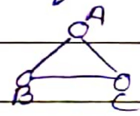
$$\dot{y}^2 = c^2(1+\dot{y}^2)$$
$$\frac{c^2}{1-c^2} = \dot{y}^2$$

$\dot{y} = a$

$\boxed{y = ax + c}$ $\Rightarrow$ S.D. b/w 2 points is a straight line.

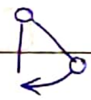
* Newton's law is a particular solⁿ of Langrangian eqⁿ.

* Total no. of d.o.f for rigid body $= 3\times3-3 = 6$

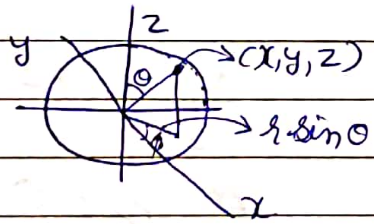


translation ← 3 ↙ ↘ 3 Rotational

* Total no. of d.o.f for simple pendulum $= 2-1 = 1$
(2-D System)

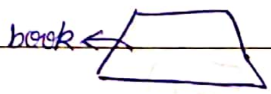


Spherical polar Co-ordinate



(x, y, z)
$\equiv (r\sin\theta\cos\phi, r\sin\theta\sin\phi, r\cos\theta)$

For motion of door, d.o.f $= 1$



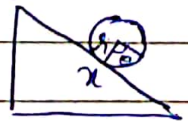
For a book placed on a table (2-D System), d.o.f $= 2\times4-4 = 4$

Constraints

i) Holonomic

$f(r_1, r_2, \text{----}, r_n, t) = 0$

Eg: Rolling of cylinder



$\dot{x} = r\dot{\theta}$

$\int \frac{dx}{dt} = r \int \frac{d\theta}{dt}$

$x = r\theta + c$ For $x=0$, $\theta=0$ $\Rightarrow c = 0$

$x = r\theta$; $\boxed{x - r\theta = 0}$

Scanned by CamScanner

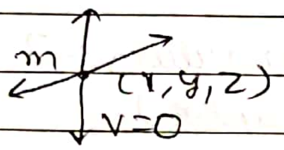
ii) Non-Holonomic
Non-Integrable eq.

Lagrange's Equation

$L = T - V$

Langrange's eq. = Difference of KE & PE

Eg(i)



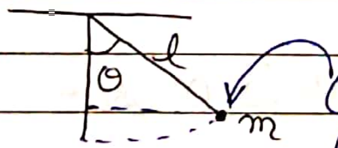
$L = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - 0$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

$\frac{\partial L}{\partial \dot{x}} = m\dot{x}$ $\quad$ $\frac{\partial L}{\partial \dot{y}} = m\dot{y}$ $\quad$ $\frac{\partial L}{\partial \dot{z}} = m\dot{z}$

$\frac{d}{dt}(p_x) = 0$ $\quad$ $\frac{d}{dt}(p_y) = 0$ $\quad$ $\frac{d}{dt}(p_z) = 0$

Eg(ii)



$(l\sin\theta, -l\cos\theta)$
(x, y)

$$L = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2) + mgl(1-\cos\theta)$$

$$= \frac{1}{2} m(l^2 \dot{\theta}^2) + mgl(1-\cos\theta)$$

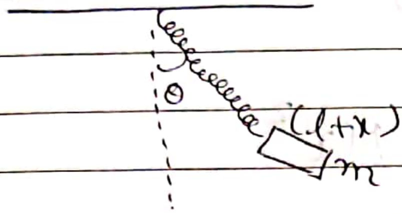
$\frac{\partial L}{\partial \dot{\theta}} = ml^2\dot{\theta}$ $\quad$ $\frac{\partial L}{\partial \theta} = -mgl\sin\theta$

$$ml^2\ddot{\theta} = -mgl\sin\theta$$

$$\boxed{\frac{d^2\theta}{dt^2} = \frac{-g}{l}\sin\theta}$$

*No. of constraint = No. of langrange's eq

Eg iii)
*



$[(l+x)\sin\theta, \; -(l+x)\cos\theta]$
$(X \qquad , \qquad Y)$

Ans $L = \frac{1}{2} m[\dot{X}^2 + \dot{Y}^2] + mg(l+x)\cos\theta - \frac{1}{2}Kx^2$

$= \frac{1}{2} m \left[\{\dot{x}\sin\theta + (x+L)\cos\theta\cdot\dot{\theta}\}^2 + \{\dot{x}\cos\theta - (x+L)\sin\theta\cdot\dot{\theta}\}^2\right]$

$+ mg(L+x)\cos\theta - \frac{1}{2}Kx^2$

$= \frac{1}{2} m[\dot{x}^2 + (x+L)^2(\dot{\theta})^2] - \frac{1}{2}Kx^2 + mg(L+x)\cos\theta$

$\frac{\partial L}{\partial \dot{x}} = m\dot{x}$

$\frac{\partial L}{\partial x} = mg\cos\theta - Kx + (x+L)(\dot{\theta})^2$

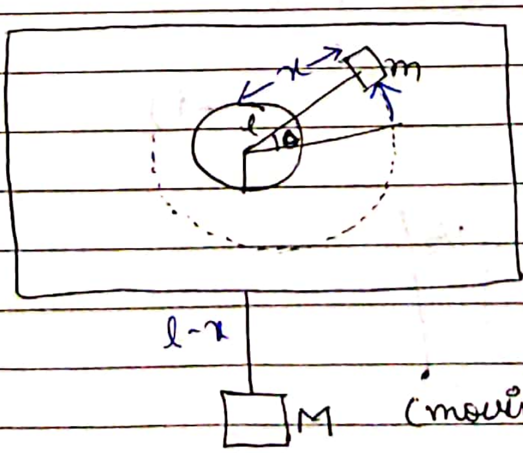
$\boxed{m\ddot{x} + mg\cos\theta + Kx - m(x+l)\dot{\theta}^2 = 0}$

$\frac{\partial L}{\partial \dot{\theta}} = m(l+x)^2\dot{\theta}$

$\frac{\partial L}{\partial \theta} = -mg(l+x)\sin\theta$

$\boxed{m(l+x)^2\ddot{\theta} + 2m(l+x)\dot{x}\dot{\theta} + mg(l+x)\sin\theta = 0}$

Que



Find lagrange's eq. for the system.

$m < M$

l = length of complete string.

(moving downward)

Ans $K_1 = \frac{1}{2} m r^2 \dot{\theta}^2 + \frac{1}{2} m \dot{r}^2$

$K_2 = \frac{1}{2} M (\dot{l} - \dot{r})^2 = \frac{1}{2} M \dot{r}^2$

$V = -Mg(l - r)$

$L = \frac{1}{2} m r^2 \dot{\theta}^2 + \frac{1}{2} m \dot{r}^2 + \frac{1}{2} M \dot{r}^2 + Mg(l - r)$

$\frac{\partial L}{\partial \dot{r}} = \dot{r}(M+m)$ $\qquad$ $\frac{\partial L}{\partial \dot{\theta}} = m r^2 \dot{\theta}$

$\frac{\partial L}{\partial r} = m r \theta^2 - Mg$ $\qquad$ $\frac{\partial L}{\partial \theta} = 0$

1st $\boxed{(M+m)\ddot{r} + Mg - m r \dot{\theta}^2 = 0}$

2nd $\boxed{\frac{d}{dt}(m r^2 \dot{\theta}) = 0}$ $\Rightarrow \frac{d}{dt}(L) = 0$

Angular Momentum remains conserved.

$(M+m)\ddot{r} = m r \dot{\theta}^2 - Mg$

$\ddot{r} = \frac{m r \dot{\theta}^2}{M+m} - \frac{Mg}{(M+m)}$ $\qquad$ $\ddot{r} = \frac{L^2}{m r^3 (M+m)} - \frac{Mg}{(M+m)}$ $\quad$ –i)

For circular motion, $\ddot{r} = 0$

$\frac{L^2}{m r_0^3} - Mg = 0$ $\quad$ –ii)

Equilibrium position $\boxed{r_0 = \left(\frac{L^2}{mMg}\right)^{1/3}}$

$$\frac{1}{x^3} = \frac{1}{(x_0+\delta x)^3} = \frac{1}{x_0^3}\frac{1}{\left(1+\frac{\delta x}{x_0}\right)^3}$$

$$= \frac{1}{x_0^3}\left(1+\frac{\delta x}{x_0}\right)^{-3} = \frac{1}{x_0^3}\left[1 - \frac{3\delta x}{x_0}\right]$$ (For $\delta x <<< x_0$)

$$= \frac{1}{x_0^3} - \frac{3\delta x}{x_0^4} \quad \text{--- iii)}$$

By eq i) & iii)

$$\frac{L^2}{m}\left[\frac{1}{x_0^3} - \frac{3\delta x}{x_0^4}\right] - Mg = (M+m)\ddot{x}$$

$$\cancel{\frac{L^2}{mx_0^3}} - \frac{3L^2\delta x}{mx_0^4} \cancel{-Mg} = (M+m)\ddot{x}$$

(By eq ii)

$\Rightarrow \ddot{x} \propto x$

$\Rightarrow$ SHM

$$\boxed{\omega^2 = \frac{3L^2}{m(M+m)x_0^4}}$$

$$\boxed{\omega = \sqrt{\frac{3Mg}{(M+m)x_0}}}$$

Conservative Energy: Independent of Path followed.

<u>Hamiltonian Equation</u>

$H(P, q, t)$

$P \rightarrow$ Momentum, $q \rightarrow$ Gen. coordinate

$$\boxed{H = p\dot{q} - L}$$

$$L = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

$$p = \frac{\partial L}{\partial \dot{q}} \qquad p_x = \frac{\partial L}{\partial \dot{x}} = m\dot{x}, \quad p_y = \frac{\partial L}{\partial \dot{y}} = m\dot{y}$$

$$p_z = \frac{\partial L}{\partial \dot{z}} = m\dot{z}$$

$$\dot{x} = \frac{p_x}{m} \qquad \dot{y} = \frac{p_y}{m} \qquad \dot{z} = \frac{p_z}{m}$$

$\dot{q}$ = Generalized Velocity $\qquad$ p = generalized momentum

$$H = \left[\frac{p_x \cdot p_x}{m} + \frac{p_y \cdot p_y}{m} + \frac{p_z p_z}{m}\right] - \frac{p_x^2}{2m} - \frac{p_y^2}{2m} - \frac{p_z^2}{2m}$$

$$= \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2)$$

Que $\quad \mathcal{L} = \frac{1}{2} m l^2 \dot{\theta}^2 + mgl\cos\theta \qquad H = ?$

Ans $\quad \frac{\partial L}{\partial \theta} = -mgL\sin\theta \qquad \frac{\partial L}{\partial \dot{\theta}} = ml^2\dot{\theta} = p_\theta$

Ans. $\quad H = \frac{P_\theta^2}{2ml^2} - mgl\cos\theta$

$$P_\theta = ml^2\dot{\theta} \qquad \dot{\theta} = \frac{P_\theta}{ml^2}$$

$$H = P\dot{q} - L = P_\theta\dot{\theta} - L = P_\theta\frac{P_\theta}{ml^2} - \left(\frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta\right)$$

$$= \frac{P_\theta^2}{2ml^2} - mgl\cos\theta$$

5414

$$\dot{p} = -\frac{\partial H}{\partial q} \qquad \dot{q} = \frac{\partial H}{\partial p}$$

$$\dot{p}_\theta = -mgl \sin\theta \qquad \text{here, } \dot{q} = \frac{p_\theta}{ml^2}$$

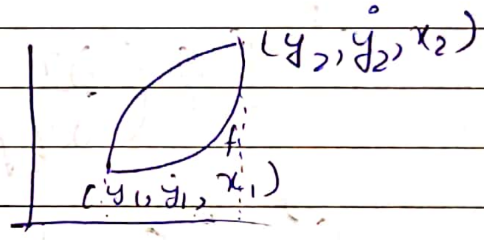
$$ml^2 \ddot{\theta} = -mgl \sin\theta$$

$$\boxed{\ddot{\theta} = -\frac{g}{l} \sin\theta}$$

Variational Principle

$f(y, \dot{y}, x)$

$\dot{y} = dy/dx$



We have a function of $f(y, \dot{y}, x)$ defined on a path $y = y(x)$ between two values x_1 & x_2, where $\dot{y} = dy/dx$. We wish to find a particular path $y(x)$ such that the line integral J of the function f between x_1 & x_2.

$$J = \int_{x_1}^{x_2} f(y, \dot{y}, x)\, dx$$

has a stationary value relative to paths differing infinitesimally from the correct function $y(x)$.

Hamiltonian Variational Principle

The motion of system between t_1 & t_2

$$I = \int_{t_1}^{t_2} L\, dt \qquad \boxed{L = T - V}$$

will give extremum part. This is known as Hamiltonian Variational Principle.

T is function of position & velocity both.

$T = T(q_j, \dot{q}_j)$

$V = V(q_j)$

$$\int_{t_1}^{t_2} L\,dt = \int_{t_1}^{t_2} (T-V)\,dt = \int_{t_1}^{t_2} \left[T(q_j, \dot{q}) - V(q_j)\right]$$

$$\delta \int_{t_1}^{t_2} \left[T(q_j \cdot \dot{q}_j) - V(q_j)\right] dt = 0$$

$$\int_{t_1}^{t_2} \left[\sum_j \left(\frac{\partial T}{\partial q_j}\,\delta q_j + \frac{\partial T}{\partial \dot{q}_j}\,\delta \dot{q}_j\right) - \frac{\partial L}{\partial q_j}\,\delta q_j\right] dt$$

$$\int_{t_1}^{t_2} \left[\sum_j \frac{\partial T}{\partial q_j}\,\delta q_j + \sum_j \frac{\partial T}{\partial \dot{q}_j}\,\delta \dot{q}_j\right] dt - \int_{t_1}^{t_2} \frac{\partial V}{\partial q_j}\,\delta q_j$$

$$\int_{t_1}^{t_2} \sum_j \left(\frac{\partial T}{\partial q_j} - \frac{\partial V}{\partial q_j}\right) \delta q_j\,dt + \sum_j \int_{t_1}^{t_2} \frac{\partial T}{\partial \dot{q}_j}\,\delta \dot{q}\,dt$$

$$\int_{t_1}^{t_2} \sum_j \left(\frac{\partial T}{\partial q_j} - \frac{\partial V}{\partial q_j}\right) \delta q_j\,dt + \int_{t_1}^{t_2} \frac{\partial T}{\partial \dot{q}_j} \frac{d}{dt}(\delta q)\,dt$$

$$\int_{t_1}^{t_2} \sum_j \left(\frac{\partial T}{\partial q_j} - \frac{\partial V}{\partial q_j}\right) \delta q_j\,dt + \cancel{\frac{\partial T}{\partial \dot{q}_j}\,\delta q_j \Big|_{t_1}^{t_2}} - \int_{t_1}^{t_2} \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) \delta q_j\,dt$$

$$\int_{t_1}^{t_2} \sum_j \left(\frac{\partial T}{\partial q_j} - \frac{\partial V}{\partial q_j}\right) \delta q_j\,dt - \int_{t_1}^{t_2} \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) \delta q_j$$

$$\int_{t_1}^{t_2} \left[\sum_j \left(\frac{\partial T}{\partial q_j} - \frac{\partial V}{\partial q_j}\right) - \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)\right] \delta q_j\,dt$$

$$\int_{t_1}^{t_2} \sum_j \left[\frac{\partial (T-V)}{\partial q_j} - \frac{d}{dt}\left(\frac{\partial (T-V)}{\partial \dot{q}}\right)\right] \delta q_j\,dt$$

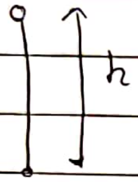
$$\int_{t_1}^{t_2} \sum_j \left(\frac{\partial L}{\partial q_j} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) \right) \delta q_j \, dt$$

$$\boxed{\underbrace{\frac{\partial L}{\partial q_j}}_{} - \underbrace{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right)}_{\text{generalized force}} = 0}$$

$\frac{\partial L}{\partial \dot{q}_j} \rightarrow$ Momentum

↳ generalized force

Eg



Ans

$L = T - V$

$L = \frac{1}{2} m \dot{x}^2 - mgx$

$\frac{\partial L}{\partial x} = -mg \qquad \frac{\partial L}{\partial \dot{x}} = m\dot{x}$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$

$\frac{d}{dt}(m\dot{x}) - (-mg) = 0$

$m\ddot{x} + mg = 0 \qquad \boxed{\ddot{x} = -g}$

$\boxed{\dot{x} = -gt + c}$

Hamiltonian Eq.

$H = H(q_j, P_j, t)$

$\delta q_j dt$

$H = T + V$

Hamilton's Eq. of Motion

$H(q_j, p_j, t) \Leftrightarrow L(q_j, \dot{q}_j, t)$

$L(q_i, \dot{q}_j) \Leftrightarrow H(q_j, p_j)$

Legendre transition (transformation)

$\frac{\partial L}{\partial \dot{q}_j} = P_j$

$L = T - V \qquad H = T + V$

Scanned by CamScanner

$$\Sigma p\dot{q}_j - L = \text{Const.}$$

$$H = \Sigma p\dot{q}_j - L$$

$$L = \Sigma p\dot{q}_j - H \qquad \frac{\partial L}{\partial \dot{q}_j} = p_j$$

$$dH = \Sigma dp_j\, \dot{q}_j + \Sigma p_j\, d\dot{q}_j - dL$$

$$H(q_j, p_j, t) = \Sigma p_j \dot{q}_j - L(q_j, \dot{q}_j, t)$$

$$\frac{\partial H}{\partial q_j}\boxed{dq_j} + \frac{\partial H}{\partial p_j}\boxed{dp_j} + \frac{\partial H}{\partial t}\boxed{dt} = \Sigma dp_j\, \dot{q}_j + \cancel{\Sigma p_j\, d\dot{q}_j}$$

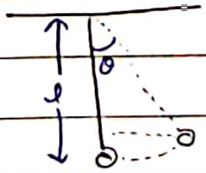
$$- \underbrace{\frac{\partial L}{\partial q_j}}_{\dot{p}_j} dq_j - \cancel{\underbrace{\frac{\partial L}{\partial \dot{q}_j}}_{p_j} d\dot{q}_j} - \frac{\partial L}{\partial t} dt$$

By comparing coefficients,

$$\frac{\partial H}{\partial q_j} = -\dot{p}_j \qquad \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$$

$$\frac{\partial H}{\partial p_j} = \dot{q}_j$$

Que



$mgl\cos\theta - mgl$

$(l\sin\theta, -l\cos\theta)$
(x, y)

Ans $H = T + V = \frac{1}{2} m l^2 \dot{\theta}^2 + mgl(1-\cos\theta)$

classmate
Date
Page

$\dot{P}_\theta = -\frac{\partial H}{\partial \theta} = -mgl \sin\theta$ $\quad P_\theta = \frac{\partial L}{\partial \dot{\theta}} = ml^2\dot{\theta}$

$$H = \Sigma P_\theta \dot{q}_\theta - L$$
$$= ml^2\dot{\theta}^2 - \left[\frac{1}{2}ml^2\dot{\theta}^2 - mgl(1-\cos\theta)\right]$$
$$= \frac{1}{2}ml^2\left(\frac{P_\theta}{ml^2}\right)^2 + mgl(1-\cos\theta)$$
$$= \frac{P_\theta^2}{2ml^2} + mgl(1-\cos\theta)$$

$$\frac{\partial H}{\partial P_\theta} = \frac{P_\theta}{ml^2}$$

Eg

Ans $T_M = \frac{1}{2}Li^2 = \frac{1}{2}L\dot{q}^2$ Electrical circuit

$$V_E = \frac{1}{2}\frac{q^2}{C}$$

$$L_E = T - V = T_M - V_E = \frac{1}{2}L\dot{q}^2 - \frac{1}{2}\frac{q^2}{C}$$

$$\frac{\partial L_E}{\partial q} - \frac{d}{dt}\left(\frac{\partial L_E}{\partial \dot{q}}\right) = 0$$

$$-\frac{q}{C} - \frac{d}{dt}(L\dot{q}) = 0$$

$$L\ddot{q} + \frac{q}{C} = 0$$

$$\ddot{q} + \frac{1}{LC}q = 0$$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$