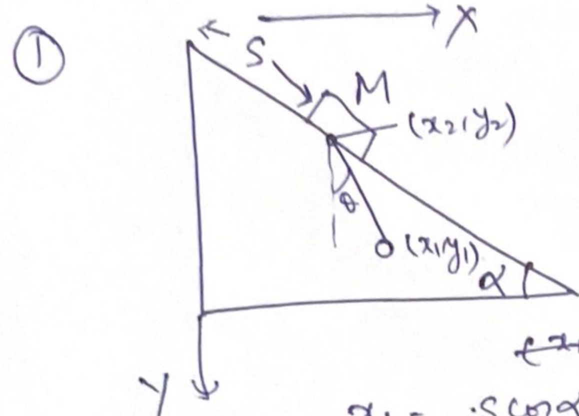
①



① DOF = 2

(S, θ) ①

② Transformation relation

$x_2 = S\cos\alpha \quad y_2 = S\sin\alpha$

$x_1 = S\cos\alpha + l\sin\theta \;;\; y_1 = S\sin\alpha + l\cos\theta$ ①

③ T & V in gernalized Coordinate

$$T = \frac{1}{2} m(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} M(\dot{x}_2^2 + \dot{y}_2^2)$$

$$V = -mgy_1 - Mgy_2$$

$$T = \frac{1}{2} m\left[\dot{s}^2 + l^2\dot{\theta}^2 + 2l\dot{s}\dot{\theta}\cos(\alpha+\theta)\right] + \frac{1}{2} M\dot{s}^2$$

$$V = -mg(S\sin\alpha + l\cos\theta) - Mgs\sin\alpha$$ ①

④ Lagrangian $L = T - V$

$$L = \frac{1}{2} m\left[\dot{s}^2 + l^2\dot{\theta}^2 + 2l\dot{s}\dot{\theta}\cos(\alpha+\theta)\right] + \frac{1}{2} M\dot{s}^2$$

$$+ Mg\,s\sin\alpha + mg(s\sin\alpha + l\cos\theta)$$ ①

⑤ Eq^n of motion

for S $\quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{s}}\right) - \frac{\partial L}{\partial s} = 0$

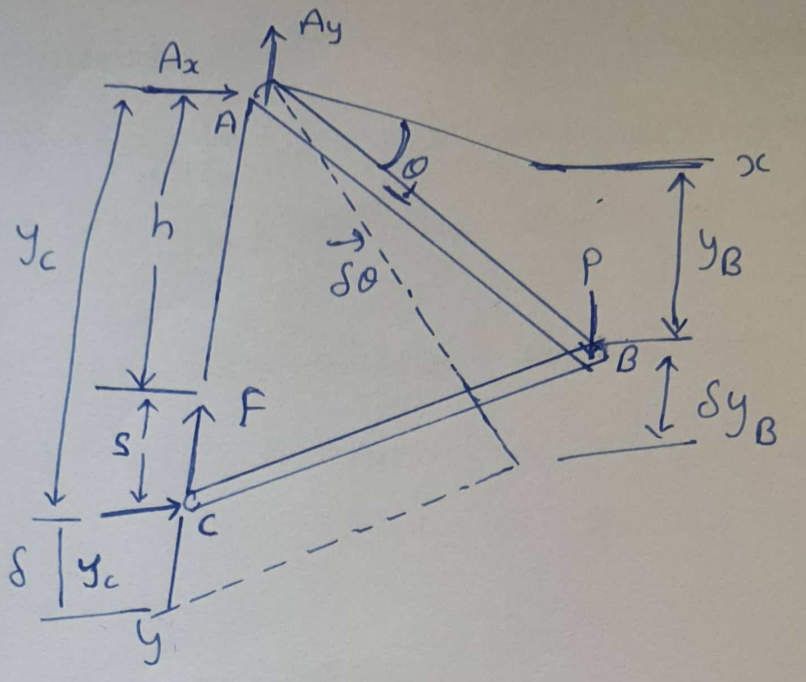
$$(m+M)\ddot{s} + ml\,\ddot{\theta}\cos(\alpha+\theta) + ml\dot{\theta}^2 . \sin(\alpha+\theta)$$
$$- (m+M)g\sin\alpha = 0$$ ①

for θ $\quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$

$$ml^2\ddot{\theta} + ml\ddot{s}\cos(\alpha+\theta) + mgl\sin\theta = 0$$

Problem:⇒ 2.

Solution.



— 1m

With the co-ordinate system show

$$y_B = l\sin\theta \qquad y_c = 2l\sin\theta$$

$$\delta y_B = l\cos\theta\,\delta\theta \qquad \delta y_c = 2l\cos\theta\,\delta\theta$$

1m

The elongation of the spring is

$$s = y_c - h = 2l\sin\theta - h$$

The magnitude of the force exerted at C by the spring is

$$F = ks = k(2l\sin\theta - h)$$

1m

Principle of virtual work. Since the reactions Ax, Ay, and C do not work, the total virtual work done by P and F must be Zero.

$$\delta U = 0: \quad P\,\delta y_B - F\,\delta y_c = 0$$

1m

②

$$P(l\cos\theta\,\delta\theta) - K(2l\sin\theta - h)(2l\cos\theta\,\delta\theta) = 0$$

$$\sin\theta = \frac{P + 2Kh}{4Kl}$$

1m

* Problem :⇒ 3 ①

Sol. Generalised Co-ordinate r and θ

Let 'r' be the distance from the swinging to the pulley. and let 'θ' be the angle of the swinging mass.

Hence, r, θ are generalised Co-ordinate

So, Degree of freedom $\rightarrow (y, \theta) / (r, \theta) \rightarrow$ 1M

Dof $\rightarrow 2$

The Lagrangian is

$$L = T - V$$

$$K.E. = T = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) \quad \text{(1)m}$$

$$P.E. = V = mgr - mgr\cos\theta \quad \text{(1m)}$$

2m

$$L = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - mgr + mgr\cos\theta$$

The last two terms are the (negatives of the) Potentials of each mass, relative to where they would be if the right mass were located at right Pulley. The equations of motion obtained varying 'r' and 'θ' are.

$$\frac{\partial L}{\partial \dot{r}} = m\dot{r} + m\dot{r} = 2m\dot{r}$$

$$\frac{\partial L}{\partial r} = mr\dot{\theta}^2 - mg + mg\cos\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = r^2\dot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -mgr\sin\theta$$

2m

Now, Equation of motion formula

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \left(\frac{\partial L}{\partial r}\right) = 0$$

$$2\ddot{r} = r\dot{\theta}^2 - g(1-\cos\theta) \quad \text{—①}$$

$$\frac{d}{dt}(r^2\dot{\theta}) = -gr\sin\theta \quad \text{—②}$$

The first equation deals with acceleration along the direction of string. The second equation equates the torque from gravity with changes in angular momentum.

If we do a small-angle approximation and keep only terms up to first order in 'θ', we find at $t=0$ (using the condition, $\dot{r}=0$)

$$\ddot{r} = 0 \quad \text{— (3)}$$

$$\ddot{\theta} + \frac{g}{r}\theta = 0 \quad \text{— (4)}$$

These say that the left mass stay still and right mass behaves just like a pendulum.

If we want to find leading term in the initial acceleration of the left mass (i.e. the leading term in $\ddot{r}$), we need to be a little less ~~waste~~ coarse in our approximation. So let keep terms in equation (4) up to second order in 'θ'. We then have at $t=0$ (using the initial condition $\dot{r}=0$)

$$2\ddot{r} = r\dot{\theta}^2 - \frac{1}{2}g\theta^2$$

$$\ddot{\theta} + \frac{g}{r}\theta = 0$$

The second equation says that right mass undergoes Harmonic motion.

where $\boxed{\omega = \sqrt{\frac{g}{r}}}$

(2)

3M

Hamiltonian

$$L = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2) - mgr + mgr\cos\theta$$

$$H = \sum p_i \dot{q}_i - L$$

Step 1. $p_i = \dfrac{\partial L}{\partial \dot{q}_i}$

Step 2. $H = \sum p_i \dot{q}_i - L$

Step 1.

$$\frac{\partial L}{\partial \dot{r}} = 2m\dot{r} = p_r \qquad \dot{r} = \frac{p_r}{2m}$$

$$\frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta} = p_\theta \qquad \dot{\theta} = \frac{p_\theta}{mr^2}$$

Step-2 $H = \sum p_i \dot{q}_i - L$

$$= \frac{p_r^2}{2m} + \frac{p_\theta^2}{r^2 m} - \frac{1}{2} m \frac{p_r^2}{4m^2} - \frac{1}{2} m \frac{p_r^2}{4m^2} - \frac{1}{2} m r^2 \frac{p_\theta^2}{r^4 m^2} + mgr - mgr\cos\theta$$

$$\boxed{H = \frac{p_r^2}{4m} + \frac{p_\theta^2}{2r^2 m} + mgr - mgr\cos\theta}$$

2M