

Note: All questions are compulsory. Each question carries equal marks.

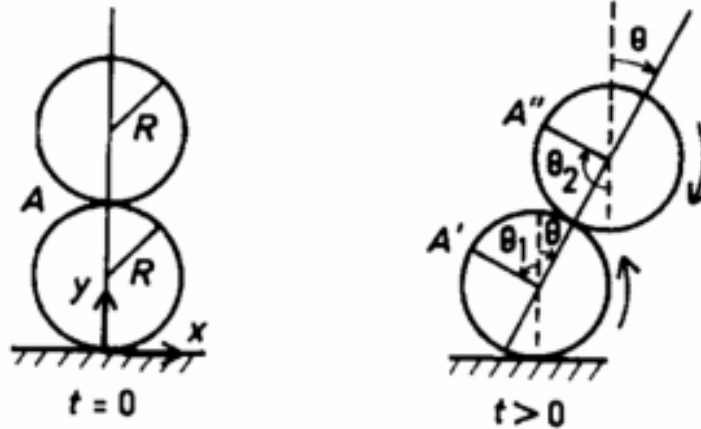
1) A particle moving in 1-D is in a stationary state described by wave function given by; (2+3+2+3=10)

$$\Psi(x) = \begin{cases} 0 & x < -L \\ A \left(1 + \cos\left(\frac{\pi x}{4L}\right) \right) & -L \leq x \leq L \\ 0 & x > L \end{cases}$$

where A and L is a constant.

- 1.1) Check the physical acceptability of the wave function? Explain with proper justification.
- 1.2) Find the value of A for which the wave function is normalized.
- 1.3) Find the corresponding energy eigenvalues.
- 1.4) Evaluate the value of Δx and Δp and verify the uncertainty relation.

2) A solid cylinder of radius R and mass M rests on a horizontal plane. A similar cylinder is resting on it, touching along the highest generator, as shown in Fig. 2. The upper cylinder is given an infinitesimal displacement so that both cylinders roll without slipping. (2.5*4=10)



- 2.1) What is the Lagrangian of the system?
- 2.2) What is the constant of the motion?

- 2.3) Find the Hamiltonian of the system.
- 2.4) Show that as long as the cylinder remains in contact

$$\dot{\theta}^2 = \frac{12g}{R} \left[\frac{(1 - \cos\theta)}{(17 + 4\cos\theta(1 - \cos\theta))} \right]$$

Where θ is the angle, the plane containing the axes makes with the vertical.

- 3) Energy expression for a crystal under tight binding is given by: (2.5*4=10)

$$E(k) = E_o - 2\gamma [\cos(k_x a) + \cos(k_y a)] \text{ eV}$$

Where $k^2 = k_x^2 + k_y^2$. Determine

- 3.1) The bandwidth of the lower band.
- 3.2) The density of states near the bottom of the band.
- 3.3) Wave velocity near the center of the Brillouin zone.
- 3.4) Effective mass at the top of the band.

4) The acceptor concentration in p-type Silicon is 2 atoms per 10^8 Silicon atoms. Assume that m_p is equal to $0.7m_e$. So determine

4.1) How far is the Fermi level from the valence band's edge at 290 K temperature?" Is E_F (Fermi energy) above or below E_V valence band energy? The concentration of silicon atoms is $5 \times 10^{24} \text{ atoms/cm}^3$.

4.2) Silicon is doped with acceptors to a concentration of 10^{17} 1/cm^3 . Plot the hole concentration as a function of temperature, indicating the intrinsic, extrinsic, and freeze-out regimes. At what temperature does the transition from intrinsic to extrinsic take place? Also, determine the minority carrier concentration at 300 K.

For Silicon at 300 K:

$E_g = 1.12 \text{ eV}$, $N_c = 2.78 \times 10^{19} \text{ 1/cm}^3$, $N_v = 9.84 \times 10^{18} \text{ 1/cm}^3$, and $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$, $\epsilon_r = 11.9$.

(5*2=10)