

## Chapter 1

### ***Visualization of the Silicon Crystal***

**1.1** (a) Please refer to Figure 1-2. The 8 corner atoms are shared by 8 unit cells and therefore contribute 1 atom. Similarly, the 6 face atoms are each shared by 2 unit cells and contribute 3 atoms. And, 4 atoms are located inside the unit cell. Hence, there are total 8 silicon atoms in each unit cell.

(b) The volume of the unit cell is

$$V_{unit\ cell} = (5.43\text{ \AA})^3 = (5.43 \times 10^{-8}\text{ cm})^3 = 1.60 \times 10^{-22}\text{ cm}^3,$$

and one unit cell contains 8 silicon atoms. The atomic density of silicon is

$$N_{Si} = \frac{8\text{ silicon atoms}}{V_{unit\ cell}} = 5.00 \times 10^{22}\text{ (silicon atoms) cm}^{-3}.$$

Hence, there are  $5.00 \times 10^{22}$  silicon atoms in one cubic centimeter.

(c) In order to find the density of silicon, we need to calculate how heavy an individual silicon atom is

$$Mass_{1\ Si\ atom} = \frac{28.1\text{ (g/mole)}}{6.02 \times 10^{23}\text{ (atoms/mole)}} = 4.67 \times 10^{-23}\text{ (g/atom)}.$$

Therefore, the density of silicon ( $\rho_{Si}$ ) in  $\text{g/cm}^3$  is

$$\rho_{Si} = N_{Si} \times Mass_{1\ Si\ atom} = 2.33\text{ g/cm}^3.$$

### ***Fermi Function***

**1.2** (a) Assume  $E = E_f$  in Equation (1.7.1),  $f(E)$  becomes  $\frac{1}{2}$ . Hence, the probability is  $\frac{1}{2}$ .

(b) Set  $E = E_c + kT$  and  $E_f = E_c$  in Equation (1.7.1):

$$f(E) = \frac{1}{1 + e^{[(E_c + kT) - E_c]/kT}} = \frac{1}{1 + e^1} = 0.27.$$

The probability of finding electrons in states at  $E_c + kT$  is 0.27.

\* For Problem 1.2 Part (b), we cannot use approximations such as Equations (1.7.2) or (1.7.3) since  $E - E_f$  is neither much larger than  $kT$  nor much smaller than  $-kT$ .

(c)  $f(E)$  is the probability of a state being filled at  $E$ , and  $1 - f(E)$  is the probability of a state being empty at  $E$ . Using Equation (1.7.1), we can rewrite the problem as

$$f(E = E_c + kT) = 1 - f(E = E_c + 3kT)$$

$$\frac{1}{1 + e^{(E_c + kT - E_f)/kT}} = 1 - \frac{1}{1 + e^{(E_c + 3kT - E_f)/kT}}$$

where

$$1 - \frac{1}{1 + e^{(E_c + 3kT - E_f)/kT}} = \frac{1 + e^{(E_c + 3kT - E_f)/kT} - 1}{1 + e^{(E_c + 3kT - E_f)/kT}} = \frac{e^{(E_c + 3kT - E_f)/kT}}{1 + e^{(E_c + 3kT - E_f)/kT}}$$

$$= \frac{1}{1 + e^{-(E_c + 3kT - E_f)/kT}}.$$

Now, the equation becomes

$$\frac{1}{1 + e^{(E_c + kT - E_f)/kT}} = \frac{1}{1 + e^{-(E_c + 3kT - E_f)/kT}}.$$

This is true if and only if

$$E_c + kT - E_f = -(E_c + 3kT - E_f).$$

Solving the equation above, we find

$$E_f = E_c + 2kT.$$

**1.3** (a) Assume  $E = E_f$  and  $T > 0K$  in Equation (1.7.1).  $f(E)$  becomes  $\frac{1}{2}$ . Hence, the probability is  $\frac{1}{2}$ .

(b)  $f(E)$  is the probability of a state being filled at  $E$ , and  $1 - f(E)$  is the probability of a state being empty at  $E$ . Using Equation (1.7.1), we can rewrite the problem as

$$f(E = E_c) = 1 - f(E = E_v)$$

$$\frac{1}{1 + e^{(E_c - E_f)/kT}} = 1 - \frac{1}{1 + e^{(E_v - E_f)/kT}}$$

where

$$1 - \frac{1}{1 + e^{(E_c - E_f)/kT}} = \frac{1 + e^{(E_v - E_f)/kT} - 1}{1 + e^{(E_v - E_f)/kT}} = \frac{e^{(E_v - E_f)/kT}}{1 + e^{(E_v - E_f)/kT}} = \frac{1}{1 + e^{-(E_v - E_f)/kT}}.$$

Now, the equation becomes

$$\frac{1}{1 + e^{(E_c - E_f)/kT}} = \frac{1}{1 + e^{-(E_v - E_f)/kT}}.$$

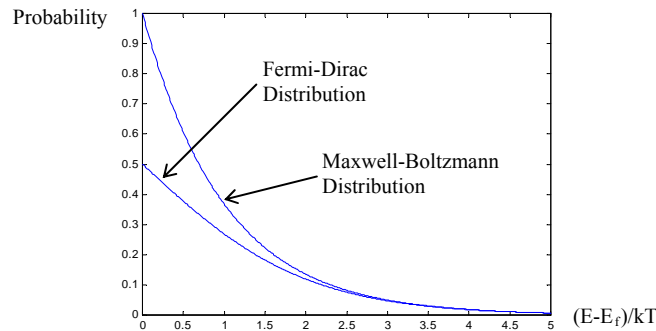
This is true if and only if

$$E_c - E_f = -(E_v - E_f).$$

Solving the equation above, we find

$$E_f = \frac{E_c + E_v}{2}.$$

- (c) The plot of the Fermi-Dirac distribution and the Maxwell-Boltzmann distribution is shown below.



The Boltzmann distribution considerably overestimates the Fermi distribution for small  $(E-E_f)/kT$ . If we set  $(E-E_f)/kT = A$  in Equations (1.7.1) and (1.7.2), we have

$$e^{-A} \leq 1.10 \left[ \frac{1}{1 + e^A} \right].$$

Solving for A, we find

$$e^A \geq \frac{1 + e^A}{1.10} \rightarrow e^A \geq 10.11 \rightarrow A \geq \ln(10.11) = 2.31.$$

Therefore, the Boltzmann approximation is accurate to within 10% for  $(E-E_f)/kT \geq 2.31$ .

**1.4 (a)** Please refer to the example in Sec. 1.7.2. The ratio of the nitrogen concentration at 10 km above sea level to the nitrogen concentration at sea level is given by

$$\frac{N(N_2)_{10 \text{ km}}}{N(N_2)_{\text{Sea Level}}} = \frac{e^{-E_{10 \text{ km}}/kT}}{e^{-E_{\text{Sea Level}}/kT}} = e^{-(E_{10 \text{ km}} - E_{\text{Sea Level}})/kT}$$

where

$$\begin{aligned} E_{10 \text{ km}} - E_{\text{Sea Level}} &= \text{altitude} \times \text{mass of } N_2 \text{ molecule} \times \text{acceleration of gravity} \\ &= 10^6 \text{ cm} \times 28 \times 1.66 \times 10^{-24} \text{ g} \times 980 \text{ cm} \cdot \text{s}^{-2} = 4.56 \times 10^{-14} \text{ erg}. \end{aligned}$$

The ratio is

$$\frac{N(N_2)_{10 \text{ km}}}{N(N_2)_{\text{Sea Level}}} = e^{-(4.56 \times 10^{-14} \text{ erg}) / (1.38 \times 10^{-16} \text{ erg} \cdot \text{K}^{-1} \times 273 \text{ K})} = e^{-1.21} = 0.30.$$

Since nitrogen is lighter than oxygen, the potential energy difference for nitrogen is smaller, and consequently the exponential term for nitrogen is larger than 0.25 for oxygen. Therefore, the nitrogen concentration at 10 km is more than 25% of the sea level  $N_2$  concentration.

(b) We know that

$$\frac{N(O_2)_{10 \text{ km}}}{N(O_2)_{\text{Sea Level}}} = 0.25, \quad \frac{N(N_2)_{10 \text{ km}}}{N(N_2)_{\text{Sea Level}}} = 0.30, \quad \text{and} \quad \frac{N(N_2)_{\text{Sea Level}}}{N(O_2)_{\text{Sea Level}}} = 4.$$

Then,

$$\begin{aligned} \frac{N(N_2)_{10 \text{ km}}}{N(O_2)_{10 \text{ km}}} &= \frac{N(N_2)_{10 \text{ km}}}{N(N_2)_{\text{Sea Level}}} \times \frac{N(N_2)_{\text{Sea Level}}}{N(O_2)_{\text{Sea Level}}} \times \frac{N(O_2)_{\text{Sea Level}}}{N(O_2)_{10 \text{ km}}} \\ &= 0.30 \times 4 \times \frac{1}{0.25} = 4.8. \end{aligned}$$

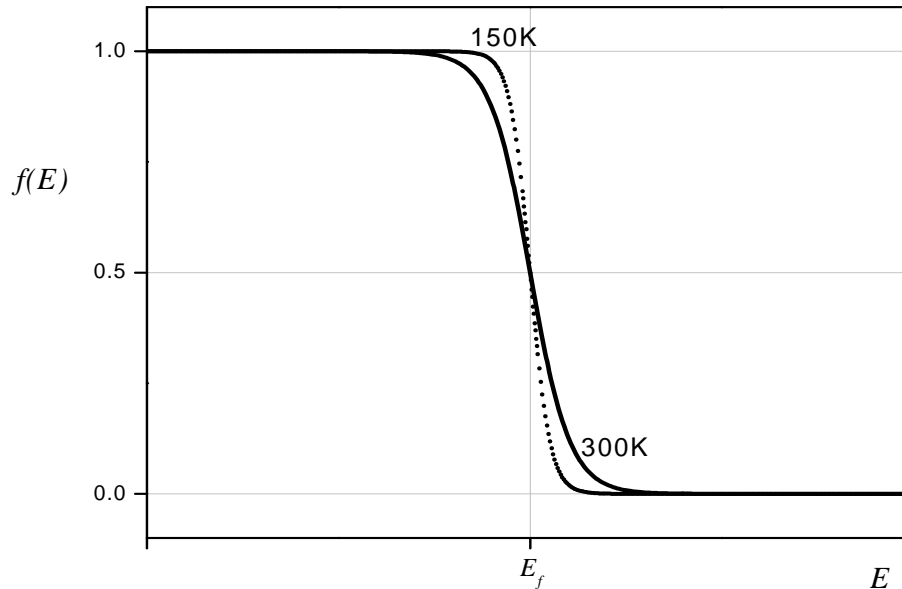
It is more  $N_2$ -rich than at sea level.

## 1.5

$$\begin{aligned} 1 - f(E_f - \Delta E) &= 1 - \frac{1}{1 + e^{(E_f - \Delta E - E_f)/kT}} \\ &= \frac{e^{(E_f - \Delta E - E_f)/kT}}{1 + e^{(E_f - \Delta E - E_f)/kT}} \\ &= \frac{1}{1 + e^{-(E_f - \Delta E - E_f)/kT}} \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{1 + e^{-(E_f - \Delta E - E_f)/kT}} \\
 &= \frac{1}{1 + e^{(E_f + \Delta E - E_f)/kT}} \\
 &= f(E_f + \Delta E)
 \end{aligned}$$

**1.6 (a)**



- (b) At 0K, the probability of a state below the Fermi level being filled is 1 and a state above the Fermi level being filled is 0. So a total of 7 states are filled which means there are 14 electrons (since 2 electrons can occupy each state) in the system.

## Density of States

**1.7** Since the semiconductor is assumed to be, We are asked to use Equations (1.7.2) and (1.7.4) to approximate the Fermi distribution. (This means that the doping concentration is low and  $E_f$  is not within a few  $kT$ s from  $E_c$  or  $E_v$ . A lightly doped semiconductor is known as a non-degenerate semiconductor.) The carrier distribution as a function of energy in the conduction band is proportional to

$$\text{Distribution}(E) \propto (E - E_c)^{1/2} e^{-(E - E_f)/kT},$$

where  $e^{-(E - E_f)/kT}$  is from Equation (1.7.2). Taking the derivative with respect to  $E$  and setting it to zero, we obtain

$$\frac{d}{dE} \left[ (E - E_c)^{1/2} e^{-(E - E_f)/kT} \right] = \frac{1}{2} \frac{1}{\sqrt{E - E_c}} e^{-(E - E_f)/kT} + \sqrt{E - E_c} \left( -\frac{1}{kT} \right) e^{-(E - E_f)/kT} = 0$$

The exponential terms cancel out. Solving the remaining equation yields

$$\frac{1}{2} (E - E_c)^{-1/2} = \frac{1}{kT} (E - E_c)^{1/2} \rightarrow (E - E_c) = \frac{kT}{2} \rightarrow E = E_c + \frac{kT}{2}.$$

So, the number of carriers in the conduction band peaks at  $E_c + kT/2$ .

Similarly, in the valence band, the carrier distribution as a function of energy is proportional to

$$\text{Distribution}(E) \propto (E_v - E)^{1/2} e^{-(E_f - E)/kT},$$

where  $e^{-(E_f - E)/kT}$  is Equation (1.7.2). Taking the derivative and setting it to zero, we obtain

$$\frac{d}{dE} \left[ (E_v - E)^{1/2} e^{-(E_f - E)/kT} \right] = \frac{1}{2} (E_v - E)^{-1/2} e^{-(E_f - E)/kT} + (E_v - E)^{1/2} \left( \frac{1}{kT} \right) e^{-(E_f - E)/kT} = 0.$$

Again, the exponential terms cancel out, and solving the remaining equation yields

$$\frac{1}{2} (E_v - E)^{-1/2} = \frac{1}{kT} (E_v - E)^{1/2} \rightarrow (E_v - E) = \frac{kT}{2} \rightarrow E = E_v - \frac{kT}{2}.$$

Therefore, the number of carriers in the valence band peaks at  $E_v - kT/2$ .

**1.8** Since it is given that the semiconductor is non-degenerate (not heavily doped),  $E_f$  is not within a few  $kT$ s from  $E_c$  or  $E_v$ . We can use Equations (1.7.2) and (1.7.4) to approximate the Fermi-Dirac distribution.

(a) The electron concentration in the conduction band is given by

$$n = \int_{C.B.} D_c(E) f(E) dE \approx \int_{E_c}^{\infty} A \sqrt{E - E_c} e^{-(E - E_f)/kT} dE.$$

In order to simplify the integration, we make the following substitutions:

$$\frac{E - E_c}{kT} = x \rightarrow E = kT x + E_c, \quad \frac{1}{kT} dE = dx \rightarrow dE = kT dx, \quad \text{and } x: \text{ from } 0 \text{ to } \infty.$$

Now the equation becomes

$$n = \int_0^{\infty} A \sqrt{kTx} e^{-(kTx + E_c - E_f)/kT} kT dx = A (kT)^{3/2} e^{-(E_c - E_f)/kT} \int_0^{\infty} \sqrt{x} e^{-x} dx$$

where

$$\int_0^{\infty} \sqrt{x} e^{-x} dx = \int_0^{\infty} (x)^{\frac{3}{2}-1} e^{-x} dx = \Gamma(3/2) = \frac{\sqrt{\pi}}{2}. \text{ (Gamma function)}$$

Hence, the electron concentration in the conduction band is

$$n = \frac{\sqrt{\pi}}{2} A (kT)^{3/2} e^{-(E_c - E_f)/kT}.$$

Similarly, the hole concentration is given by

$$p = \int_{V.B.} D_v(E) [1 - f(E)] dE \approx \int_{-\infty}^{E_v} B \sqrt{E_v - E} e^{-(E_f - E)/kT} dE.$$

Again, we make the following substitutions to simplify the integration:

$$\frac{E_v - E}{kT} = x \rightarrow E = -kTx + E_v, -\frac{1}{kT} dE = dx \rightarrow dE = -kT dx, \text{ and } x: \text{ from } \infty \text{ to } 0.$$

Now the equation becomes

$$p = \int_{\infty}^0 -B \sqrt{kTx} e^{-(E_f + kTx - E_v)/kT} kT dx = B (kT)^{3/2} e^{-(E_f - E_v)/kT} \int_0^{\infty} \sqrt{x} e^{-x} dx$$

where

$$\int_0^{\infty} \sqrt{x} e^{-x} dx = \int_0^{\infty} (x)^{\frac{3}{2}-1} e^{-x} dx = \Gamma(3/2) = \frac{\sqrt{\pi}}{2}. \text{ (Gamma function)}$$

Therefore, the hole concentration in the conduction band is

$$p = \frac{\sqrt{\pi}}{2} B (kT)^{3/2} e^{-(E_f - E_v)/kT}.$$

- (b) The word “Intrinsic” implies that the electron concentration and the hole concentration are equal. Therefore,

$$n = p \rightarrow \frac{\sqrt{\pi}}{2} A (kT)^{3/2} e^{-(E_c - E_i)/kT} = \frac{\sqrt{\pi}}{2} B (kT)^{3/2} e^{-(E_i - E_v)/kT}.$$

This simplifies to

$$A e^{-(E_c - E_i)/kT} = B e^{-(E_i - E_v)/kT}.$$

Solving for  $E_i$  yields

$$E_i = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln\left(\frac{1}{2}\right) = \frac{E_c + E_v}{2} - 0.009 \text{ eV}; k = 8.62 \times 10^{-5} \text{ eV K}^{-1}, T = 300 \text{ K}.$$

Hence, the intrinsic Fermi level ( $E_i$ ) is located at 0.009 eV below the mid-bandgap of the semiconductor.

**1.9** The unit step functions set the integration limits.  $D_c(E)$  is zero for  $E < E_c$ , and  $D_v(E)$  is zero for  $E > E_v$ . Since it is given that the semiconductor is non-degenerate (not heavily doped),  $E_f$  is not within a few  $kT$ s from  $E_c$  or  $E_v$ . We can use Equations (1.7.2) and (1.7.4) to approximate the Fermi-Dirac distribution.

(a) The electron concentration in the conduction band is given by

$$n = \int_{C.B.} D_c(E) f(E) dE \approx \int_{E_c}^{\infty} A (E - E_c) e^{-(E - E_f)/kT} dE.$$

In order to simplify the integration, we make the following substitutions:

$$\frac{E - E_c}{kT} = x \rightarrow E = kT x + E_c, \quad \frac{1}{kT} dE = dx \rightarrow dE = kT dx, \quad \text{and } x: \text{ from } 0 \text{ to } \infty.$$

Now the equation becomes

$$n = \int_0^{\infty} A kT x e^{-(kT x + E_c - E_f)/kT} kT dx = A (kT)^2 e^{-(E_c - E_f)/kT} \int_0^{\infty} x e^{-x} dx$$

where

$$\int_0^{\infty} x e^{-x} dx = 1.$$

Hence, the electron concentration in the conduction band is

$$n = A (kT)^2 e^{-(E_c - E_f)/kT}.$$

Similarly, the hole concentration is given by

$$p = \int_{V.B.} D_v(E) [1 - f(E)] dE \approx \int_{-\infty}^{E_v} B (E_v - E) e^{-(E_f - E)/kT} dE.$$

Again, we make the following substitutions to simplify the integration:



$$\frac{E_v - E}{kT} = x \rightarrow E = -kTx + E_v, -\frac{1}{kT}dE = dx \rightarrow dE = -kTdx, \text{ and } x: \text{ from } \infty \text{ to } 0.$$

Now the equation becomes

$$p = \int_{\infty}^0 -BkTx e^{-(E_f + kTx - E_v)/kT} kT dx = B(kT)^2 e^{-(E_f - E_v)/kT} \int_0^{\infty} x e^{-x} dx$$

where

$$\int_0^{\infty} x e^{-x} dx = 1.$$

Therefore, the hole concentration in the conduction band is

$$p = B(kT)^2 e^{-(E_f - E_v)/kT}.$$

- (b) The word “Intrinsic” implies that the electron concentration and the hole concentration are equal. Therefore,

$$n = p \rightarrow A(kT)^2 e^{-(E_c - E_i)/kT} = B(kT)^2 e^{-(E_i - E_v)/kT}.$$

This simplifies to

$$A e^{-(E_c - E_i)/kT} = B e^{-(E_i - E_v)/kT}.$$

If we solve for  $E_i$ , we obtain

$$E_i = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln\left(\frac{1}{2}\right) = \frac{E_c + E_v}{2} - 0.009 \text{ eV}; k = 8.62 \times 10^{-5} \text{ eV K}^{-1}, T = 300 \text{ K}.$$

Hence, the intrinsic Fermi level ( $E_i$ ) is located at 0.009 eV below the mid-bandgap of the semiconductor.

- 1.10** (a) The carrier distribution as a function of energy in the conduction band is proportional to

$$\text{Distribution}(E) \propto (E - E_c)^{1/2} e^{-(E - E_f)/kT},$$

where  $e^{-(E - E_f)/kT}$  is from Equation (1.7.2). Taking the derivative and setting it to zero, we obtain

$$\frac{d}{dE} \left[ (E - E_c)^{1/2} e^{-(E - E_f)/kT} \right] = \frac{1}{2} \frac{1}{\sqrt{E - E_c}} e^{-(E - E_f)/kT} + \sqrt{E - E_c} \left( -\frac{1}{kT} \right) e^{-(E - E_f)/kT} = 0.$$

The exponential terms cancel out. Solving the remaining equation yields

$$\frac{1}{2}(E - E_c)^{-1/2} = \frac{1}{kT}(E - E_c)^{1/2} \rightarrow (E - E_c) = \frac{kT}{2} \rightarrow E = E_c + \frac{kT}{2}.$$

Hence, the number of carriers in the conduction band peaks at  $E_c + kT/2$ .

(b) The electron concentration in the conduction band is given by

$$n = \int_{C.B.} D_c(E) f(E) dE = \int_{E_c}^{\text{Top of the Conduction Band}} D_c(E) f(E) dE.$$

We assume that the function  $f(E)$  falls off rapidly such that

$$\frac{\int_{\text{Top of the Conduction Band}}^{\infty} D_c(E) f(E) dE}{\int_{E_c}^{\text{Top of the Conduction Band}} D_c(E) f(E) dE} \approx 0.$$

Now we may change the upper limit of integration from the Top of the Conduction Band to  $\infty$ :

$$n = \int_{E_c}^{\infty} A \sqrt{E - E_c} e^{-(E - E_f)/kT} dE.$$

Also, in order to simplify the integration, we make the following substitutions:

$$\frac{E - E_c}{kT} = x \rightarrow E = kT x + E_c, \quad \frac{1}{kT} dE = dx \rightarrow dE = kT dx, \quad \text{and } x: \text{ from } 0 \text{ to } \infty.$$

The equation becomes

$$n = \int_0^{\infty} A \sqrt{kT x} e^{-(kT x + E_c - E_f)/kT} kT dx = A (kT)^{3/2} e^{-(E_c - E_f)/kT} \int_0^{\infty} \sqrt{x} e^{-x} dx$$

where

$$\int_0^{\infty} \sqrt{x} e^{-x} dx = \int_0^{\infty} (x)^{\frac{3}{2}-1} e^{-x} dx = \Gamma(3/2) = \frac{\sqrt{\pi}}{2}. \quad (\text{Gamma function})$$

Therefore, the electron concentration in the conduction band is

$$n = \frac{\sqrt{\pi}}{2} A (kT)^{3/2} e^{-(E_c - E_f)/kT}.$$

(b) The ratio of the peak electron concentration at  $E = E_c + (1/2)kT$  to the electron concentration at  $E = E_c + 40kT$  is

$$\begin{aligned}\frac{n(E_c + 40kT)}{n(E_c + \frac{1}{2}kT)} &= \frac{A(E_c + 40kT - E_c)^{1/2} e^{-(E_c + 40kT - E_f)/kT}}{A(E_c + 0.5kT - E_c)^{1/2} e^{-(E_c + 0.5kT - E_f)/kT}} = \\ &= (40kT / 0.5kT)^{1/2} e^{[-(E_c + 40kT - E_f) + (E_c + 0.5kT - E_f)]/kT} = (40/0.5)e^{-39.5} = 5.60 \times 10^{-16}.\end{aligned}$$

The ratio is very small, and this result justifies our assumption in Part (b).

- (c) The kinetic energy of an electron at E is equal to E-E<sub>C</sub>. The average kinetic energy of electrons is

$$\begin{aligned}\langle K.E. \rangle &= \frac{\text{sum of the kinetic energy of all electrons}}{\text{total number of electrons}} = \\ &= \frac{\int_{C.B.} (E - E_c) D_c(E) f(E) dE}{\int_{C.B.} D_c(E) f(E) dE} \\ &\approx \frac{\int_{E_c}^{\infty} (E - E_c) A \sqrt{E - E_c} e^{-(E - E_f)/kT} dE}{\int_{E_c}^{\infty} A \sqrt{E - E_c} e^{-(E - E_f)/kT} dE}.\end{aligned}$$

In order to simplify the integration, we make the following substitutions:

$$\frac{E - E_c}{kT} = x \rightarrow E = kTx + E_c, \quad \frac{1}{kT} dE = dx \rightarrow dE = kT dx, \quad \text{and } x: \text{ from } 0 \text{ to } \infty.$$

Now the equation becomes

$$\frac{\int_0^{\infty} A(kTx)^{3/2} e^{-(kTx + E_c - E_f)/kT} kT dx}{\int_0^{\infty} A(kTx)^{1/2} e^{-(kTx + E_c - E_f)/kT} kT dx} = \frac{A(kT)^{5/2} e^{-(E_c - E_f)/kT} \int_0^{\infty} (x)^{3/2} e^{-x} dx}{A(kT)^{3/2} e^{-(E_c - E_f)/kT} \int_0^{\infty} (x)^{1/2} e^{-x} dx}$$

where

$$\int_0^{\infty} (x)^{3/2} e^{-x} dx = \int_0^{\infty} (x)^{\frac{5}{2}-1} e^{-x} dx = \Gamma(5/2) = \frac{3\sqrt{\pi}}{4} \quad (\text{Gamma functions})$$

and

$$\int_0^{\infty} (x)^{1/2} e^{-x} dx = \int_0^{\infty} (x)^{\frac{3}{2}-1} e^{-x} dx = \Gamma(3/2) = \frac{\sqrt{\pi}}{2}. \quad (\text{Gamma functions})$$

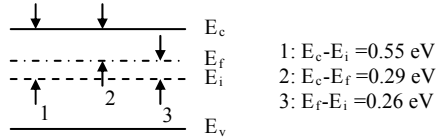
Hence, the average kinetic energy is (3/2)kT.

### ***Electron and Hole Concentrations***

**1.11** (a) We use Equation (1.8.11) to calculate the hole concentration:

$$n \times p = n_i^2 \rightarrow p = n_i^2 / n = (10^{10})^2 / 10^5 \text{ cm}^{-3} = 10^{15} \text{ cm}^{-3}.$$

- (b) Please refer to Equations (1.9.3a) and (1.9.3b). Since  $N_d - N_a \gg n_i$  and all the impurities are ionized,  $n = N_d - N_a$ , and  $p = (n_i)^2 / (N_d - N_a)$ .
- (c) Since the Fermi level is located 0.26 eV above  $E_i$  and closer to  $E_c$ , the sample is n-type. If we assume that  $E_i$  is located at the mid-bandgap ( $\sim 0.55$  eV), then  $E_c - E_f = 0.29$  eV.



Using Equations (1.8.5) and (1.8.11), we find

$$n = N_c e^{-(E_c - E_f)/kT} = 4.01 \times 10^{14} \text{ cm}^{-3} \text{ and } p = n_i^2 / n = 2.49 \times 10^5 \text{ cm}^{-3}.$$

Therefore, the electron concentration is  $4.01 \times 10^{14} \text{ cm}^{-3}$ , and the hole concentration is  $2.49 \times 10^5 \text{ cm}^{-3}$ .

\* There is another way to solve this problem:

$$n = n_i e^{(E_f - E_i)/kT} = 2.20 \times 10^{14} \text{ cm}^{-3} \text{ and } p = n_i^2 / n = 4.55 \times 10^5 \text{ cm}^{-3}.$$

- (d) If  $T = 800$  K, there is enough thermal energy to free more electrons from silicon-silicon bonds. Hence, using Equation (1.8.12), we first calculate the intrinsic carrier density  $n_i$  at 800 K:

$$n_i = \sqrt{N_c(800 \text{ K}) N_v(800 \text{ K})} e^{-(E_g)/(2kT)} = 2.56 \times 10^{16} \text{ cm}^{-3}.$$

where

$$N_c(T = 800 \text{ K}) = 2 \left[ \frac{2\pi m_{dn} kT}{h^2} \right]^{3/2} = 2.8 \times 10^{19} \times \left( \frac{T}{300 \text{ K}} \right)^{3/2} \text{ cm}^{-3} = 1.22 \times 10^{20} \text{ cm}^{-3}$$

and

$$N_v(T = 800 \text{ K}) = 2 \left[ \frac{2\pi m_{dp} kT}{h^2} \right]^{3/2} = 1.04 \times 10^{19} \times \left( \frac{T}{300 \text{ K}} \right)^{3/2} \text{ cm}^{-3} = 4.53 \times 10^{19} \text{ cm}^{-3}.$$

Clearly,  $n_i$  at 800K is much larger than  $N_d - N_a$  (which is equal to  $n$  from the previous part). Hence the electron concentration is  $n \approx n_i$ , and the hole concentration is  $p = (n_i)^2 / n \approx n_i$ . The semiconductor is intrinsic at 800K, and  $E_f$  is located very close to the mid-bandgap.

### Nearly Intrinsic Semiconductor

**1.12** Applying Equation (1.8.11) to this problem yields

$$n_i^2 / p = n = 2p \rightarrow 2p^2 = n_i^2 \rightarrow p = \frac{1}{\sqrt{2}} n_i = 7.07 \times 10^{12} \text{ cm}^{-3} \text{ and } n = 1.41 \times 10^{13} \text{ cm}^{-3}.$$

**1.13** (a) B is a group III element. When added to Si (which belongs to Group IV), it acts as an acceptor producing a large number of holes. Hence, this becomes a P-type Si film.

(b) At  $T = 300 \text{ K}$ , since the intrinsic carrier density is negligible compared to the dopant concentration,  $p = N_a = 4 \times 10^{16} \text{ cm}^{-3}$ , and  $n = (n_i = 10^{10} \text{ cm}^{-3})^2 / p = 2500 \text{ cm}^{-3}$ .  
At  $T = 600 \text{ K}$ ,

$$n_i = \sqrt{N_c(600 \text{ K}) N_v(600 \text{ K})} e^{-(E_g)/(2kT)} = 1.16 \times 10^{15} \text{ cm}^{-3}$$

where

$$N_c(T = 600 \text{ K}) = 2 \left[ \frac{2\pi m_{dn} kT}{h^2} \right]^{3/2} = 2.8 \times 10^{19} \times \left( \frac{T}{300 \text{ K}} \right)^{3/2} \text{ cm}^{-3} = 7.92 \times 10^{19} \text{ cm}^{-3}$$

and

$$N_v(T = 600 \text{ K}) = 2 \left[ \frac{2\pi m_{dp} kT}{h^2} \right]^{3/2} = 1.04 \times 10^{19} \times \left( \frac{T}{300 \text{ K}} \right)^{3/2} \text{ cm}^{-3} = 2.94 \times 10^{19} \text{ cm}^{-3}.$$

The intrinsic carrier concentration is no more negligible compared to the dopant concentration. Thus, we have

$$p = N_a + n_i = (4 \times 10^{16} + 1.16 \times 10^{15}) \text{ cm}^{-3} = 4.12 \times 10^{16} \text{ cm}^{-3}, \text{ and}$$

$$n = n_i^2 / p = (1.16 \times 10^{15} \text{ cm}^{-3})^2 / 4.12 \times 10^{16} \text{ cm}^{-3} = 3.27 \times 10^{13} \text{ cm}^{-3}.$$

The electron concentration has increased by many orders of magnitude.

- (c) At high temperatures, there is enough thermal energy to free more electrons from silicon-silicon bonds, and consequently, the number of intrinsic carriers increases.  
(d) Using Equation 1.8.8, we calculate the position of the Fermi level with respect to  $E_v$ .

$$E_f - E_v = kT \ln[N_v(T) / p(T)] = 0.34 \text{ eV}, \quad T = 600 \text{ K}.$$

At 600 K, the Fermi level is located 0.34 eV above the valence band.

### ***Incomplete Ionization of Dopants and Freeze-out***

**1.14** From Equation (1.9.1), we know that  $n + N_a^- = p + N_d^+$ . Since  $N_d^+$  is much larger than  $N_a^-$ , all the samples are n-type, and  $n \approx N_d^+ - N_a^- = 3 \times 10^{15} / \text{cm}^3$ . This value is assumed to be constant. Using the Equations (1.8.10) and (1.9.3b),

$$p = n_i^2 / (N_d^+ - N_a^-) = N_c N_v \exp(-E_g / kT) = CT^3 \exp(-E_g / kT),$$

where C is a temperature independent constant. Using the sensitivity of p defined by  $\partial p / \partial T$ ,

$$\partial p / \partial T = (3 + E_g / kT) \times CT^2 \exp(-E_g / kT)$$

Therefore, the larger the energy gap is the less sensitive to temperature the minority carrier is. For the definition of the sensitivity of p,

$$(\partial p / \partial T) / p = (3 + E_g / kT) / T$$

The temperature sensitivity of the minority carrier is greater for larger  $E_g$ .

- 1.15 (a)** Let us first consider the case of n-type doping. The dopant atoms are located at energy  $E_d$  inside the bandgap, near the conduction band edge. The problem states that we are considering the situation in which half the impurity atoms are ionized, i.e.  $n = N_d / 2$ . In other words, the probability of dopant atoms being ionized is  $1/2$ , or conversely, the probability that a state at the donor energy  $E_D$  is filled is  $1/2$ .

From Problem 1.2 part (a), we know that if  $f(E_D) = 1/2$ , then  $E_D = E_F$ . From Equation 1.8.5,

$$n = N_c e^{-(E_c - E_F) / kT}.$$

We also know that  $E_F = E_D$  and  $E_c - E_D = 0.05 \text{ eV}$ .

$$N_c(T) = 2 \left[ \frac{2\pi m_{dn} kT}{h^2} \right]^{3/2} = 2.8 \times 10^{19} \times \left( \frac{T}{300 \text{ K}} \right)^{3/2} \text{ cm}^{-3}.$$

$$N_c(T) e^{-(E_c - E_F) / kT} = N_c(T) e^{-(E_c - E_D) / kT} = \frac{N_D}{2} \rightarrow \frac{2N_c(T)}{N_d} = e^{(E_c - E_D) / kT}.$$

This equation can be solved iteratively. Starting with an arbitrary guess of 100K for T, we find T converges to 84.4 K.

Similarly, for boron

$$N_v(T) = 2 \left[ \frac{2\pi m_{dp} kT}{h^2} \right]^{3/2} = 1.04 \times 10^{19} \times \left( \frac{T}{300 \text{ K}} \right)^{3/2} \text{ cm}^{-3}.$$

$$N_v(T) e^{-(E_F - E_v) / kT} = N_v(T) e^{-(E_d - E_v) / kT} = \frac{N_a}{2} \rightarrow \frac{2N_v(T)}{N_a} = e^{(E_d - E_v) / kT}.$$

Starting from T = 100K, we find T converges to 67.7K.

(b) We want to find T where  $n_i$  is  $10N_d$ . This can be written as

$$n_i = \sqrt{N_c(T)N_v(T)} e^{-(E_g)/(2kT)} = 1.71 \times 10^{19} \times \left( \frac{T}{300K} \right)^{3/2} e^{-(E_g)/(2kT)} = 10N_d$$

where

$$N_c(T) = 2 \left[ \frac{2\pi m_{dn} kT}{h^2} \right]^{3/2} = 2.8 \times 10^{19} \times \left( \frac{T}{300K} \right)^{3/2} \text{ cm}^{-3} \text{ and}$$

$$N_v(T) = 2 \left[ \frac{2\pi m_{dp} kT}{h^2} \right]^{3/2} = 1.04 \times 10^{19} \times \left( \frac{T}{300K} \right)^{3/2} \text{ cm}^{-3}.$$

We need to solve the equation iteratively, as in part (a) for  $n_i = 10N_d = 10^{17} \text{ cm}^{-3}$ . Starting from  $T = 300K$ , we get  $T = 777 K$  for  $n_i = 10N_d$ .

For  $n_i = 10N_a$ , we simply replace  $N_d$  in the equation above with  $N_a$ . Starting from  $T = 300K$ , we find  $T = 635 K$ .

(c) If we assume full ionization of impurities at  $T = 300 K$ ,

$$\text{For arsenic: } n \cong N_d = 10^{16} \text{ cm}^{-3} \gg n_i, \quad p = \frac{n_i^2}{N_d} = 2.1 \times 10^4 \text{ cm}^{-3}$$

$$\text{For boron: } p \cong N_a = 10^{15} \text{ cm}^{-3} \gg n_i, \quad n = \frac{n_i^2}{N_a} = 2.1 \times 10^5 \text{ cm}^{-3}$$

(d) Please refer to the example in Section 2.8. For arsenic,

$$E_f - E_v = kT \ln \frac{N_v}{p} = \frac{1.04 \times 10^{19} \text{ cm}^{-3}}{2.1 \times 10^4 \text{ cm}^{-3}} = 0.88 \text{ eV}.$$

For boron,

$$E_f - E_v = kT \ln \frac{N_v}{p} = \frac{1.04 \times 10^{19} \text{ cm}^{-3}}{10^{15} \text{ cm}^{-3}} = 0.24 \text{ eV}.$$

(e) In case of arsenic + boron,

$$n \cong N_d - N_a = 9 \times 10^{15} \text{ cm}^{-3}, \text{ and } p = \frac{n_i^2}{n} = \frac{(10^{10} \text{ cm}^{-3})^2}{9 \times 10^{15} \text{ cm}^{-3}} = 1.11 \times 10^4 \text{ cm}^{-3}, \text{ and}$$

$$E_f - E_v = kT \ln \left( \frac{N_v}{p} \right) = 0.026(\text{eV}) \ln \left( \frac{1.04 \times 10^{19} \text{ cm}^{-3}}{1.11 \times 10^4 \text{ cm}^{-3}} \right) = 0.90 \text{ eV}.$$

- 1.16** (a) If we assume full ionization of impurities, the electron concentration is  $n \approx N_d = 10^{17} \text{ cm}^{-3}$ . The hole concentration is  $p = (n_i)^2 / n = (10^{10} \text{ cm}^{-3})^2 / 10^{17} \text{ cm}^{-3} = 10^3 \text{ cm}^{-3}$ . The Fermi level position, with respect to  $E_c$ , is

$$E_c - E_f = kT \ln[N_c / n] = 0.026 \ln[2.8 \times 10^{19} \text{ cm}^{-3} / 10^{17} \text{ cm}^{-3}] = 0.15 \text{ eV}.$$

$E_f$  is located 0.15 eV below  $E_c$ .

- (b) In order to check the full ionization assumption with the calculated Fermi level, we need to find the percentage of donors occupied by electrons.

$$E_D - E_f = (E_c - E_f) - (E_c - E_D) = 0.1 \text{ eV}, \text{ and}$$

$$n_D = N_d \frac{1}{1 + e^{(E_D - E_f)/kT}} = \frac{10^{17} (\text{cm}^{-3})}{1 + e^{(0.1 \text{ eV} / 0.026 \text{ eV})}} = 2.09 \times 10^{15} \text{ cm}^{-3} \equiv 2\% \text{ of } N_d.$$

Since only 2% of dopants are not ionized, it is fine to assume that the impurities are fully ionized.

- (c) We assume full ionization of impurities, the electron concentration is  $n \approx N_d = 10^{19} \text{ cm}^{-3}$ . The hole concentration is  $p = (n_i)^2 / n = (10^{10} \text{ cm}^{-3})^2 / 10^{19} \text{ cm}^{-3} = 10 \text{ cm}^{-3}$ . The Fermi level position, with respect to  $E_c$ , is

$$E_c - E_f = kT \ln[N_c / n] = 0.026 \times \ln[2.8 \times 10^{19} \text{ cm}^{-3} / 10^{19} \text{ cm}^{-3}] = 0.027 \text{ eV}.$$

It is located 0.027 eV below  $E_c$ .

Again, we need to find the percentage of donors occupied by electrons in order to check the full ionization assumption with the calculated Fermi level.

$$E_D - E_f = (E_c - E_f) - (E_c - E_D) = -0.023 \text{ eV}, \text{ and}$$

$$n_D = N_d \frac{1}{1 + e^{(E_D - E_f)/kT}} = \frac{10^{19} (\text{cm}^{-3})}{1 + e^{(-0.023 \text{ eV} / 0.026 \text{ eV})}} = 7.08 \times 10^{18} \text{ cm}^{-3} \equiv 71\% \text{ of } N_d.$$

Since 71% of dopants are not ionized, the full ionization assumption is not correct.

- (d) For  $T=30 \text{ K}$ , we need to use Equation (1.10.2) to find the electron concentration since the temperature is extremely low. First, we calculate  $N_c$  and  $N_v$  at  $T=30 \text{ K}$ :

$$N_c(T = 30 \text{ K}) = 2 \left[ \frac{2\pi m_{dn} kT}{h^2} \right]^{3/2} = 2.8 \times 10^{19} \times \left( \frac{T}{300 \text{ K}} \right)^{3/2} \text{ cm}^{-3} = 8.85 \times 10^{17} \text{ cm}^{-3}$$

and

$$N_v(T = 30 \text{ K}) = 2 \left[ \frac{2\pi m_{dp} kT}{h^2} \right]^{3/2} = 1.04 \times 10^{19} \times \left( \frac{T}{300 \text{ K}} \right)^{3/2} \text{ cm}^{-3} = 3.29 \times 10^{17} \text{ cm}^{-3}.$$

The electron concentration is



$$n = \sqrt{\frac{N_c(30\text{ K})N_d}{2}} e^{-(E_c-E_D)/(2kT)} = 8.43 \times 10^8 \text{ cm}^{-3}.$$

And, the hole concentration is

$$p = n_i^2 / n \approx 0$$

where

$$n_i = \sqrt{\frac{N_c(30\text{ K})N_v(30\text{ K})}{2}} e^{-(E_g)/(2kT)} = 2.32 \times 10^{-75} \text{ cm}^{-3}.$$

Since  $n_i$  is extremely small, we can assume that all the electrons are contributed by ionized dopants. Hence,

$$n = N_d \left( 1 - \frac{1}{1 + e^{(E_D-E_f)/kT}} \right) = 8.43 \times 10^8 \text{ cm}^{-3} \rightarrow \frac{8.43 \times 10^8 \text{ cm}^{-3}}{10^{17} \text{ cm}^{-3}} = 8.43 \times 10^{-9}.$$

The full ionization assumption is not correct since only  $8.43 \times 10^{-70}\%$  of  $N_d$  is ionized. To locate the Fermi level,

$$E_D - E_f = kT \ln \left[ \left( 1 - \frac{n}{N_d} \right)^{-1} - 1 \right] = -0.048 \text{ eV}.$$

$E_c - E_f = 0.05 - 0.048 = 0.002 \text{ eV}$ . Therefore, the Fermi level is positioned  $0.002 \text{ eV}$  below  $E_c$ , between  $E_c$  and  $E_D$ .

- 1.17** (a) We assume full ionization of impurities, the electron concentration is  $n \approx N_d = 10^{16} \text{ cm}^{-3}$ . The hole concentration is  $p = (n_i)^2 / n = (10^{10} \text{ cm}^{-3})^2 / 10^{16} \text{ cm}^{-3} = 10^4 \text{ cm}^{-3}$ . The Fermi level position, with respect to  $E_c$ , is

$$E_c - E_f = kT \ln[N_c / n] = 0.026 \ln[2.8 \times 10^{19} \text{ cm}^{-3} / 10^{16} \text{ cm}^{-3}] = 0.21 \text{ eV}.$$

It is located  $0.21 \text{ eV}$  below  $E_c$ .

We need to find the percentage of donors occupied by electrons in order to check the full ionization assumption with the calculated Fermi level.

$$E_D - E_f = (E_c - E_f) - (E_c - E_D) = 0.16 \text{ eV}, \text{ and}$$

$$n_D = N_d \frac{1}{1 + e^{(E_D-E_f)/kT}} = \frac{10^{16} (\text{cm}^{-3})}{1 + e^{(0.16 \text{ eV} / 0.026 \text{ eV})}} = 2.12 \times 10^{13} \text{ cm}^{-3} \equiv 2.12 \times 10^{-1}\% \text{ of } N_d.$$

Since only  $0.21\%$  of dopants are not ionized, the full ionization assumption is correct.

- (b) We assume full ionization of impurities, the electron concentration is  $n \approx N_d = 10^{18} \text{ cm}^{-3}$ . The hole concentration is  $p = (n_i)^2 / n = (10^{10} \text{ cm}^{-3})^2 / 10^{18} \text{ cm}^{-3} = 10^2 \text{ cm}^{-3}$ . The Fermi level position with respect to  $E_c$  is

$$E_c - E_f = kT \ln[N_c / n] = 0.026 \ln[2.8 \times 10^{19} \text{ cm}^{-3} / 10^{18} \text{ cm}^{-3}] = 0.087 \text{ eV}.$$

It is located 0.087 eV below  $E_c$ .

We need to find the percentage of donors occupied by electrons in order to check the full ionization assumption with the calculated Fermi level.

$$E_D - E_f = (E_c - E_f) - (E_c - E_D) = 0.037 \text{ eV}, \text{ and}$$

$$n_D = N_d \frac{1}{1 + e^{(E_D - E_f) / kT}} = \frac{10^{18} (\text{cm}^{-3})}{1 + e^{(0.037 \text{ eV} / 0.026 \text{ eV})}} = 1.94 \times 10^{17} \text{ cm}^{-3} \equiv 19\% \text{ of } N_d.$$

Since 19% of dopants are not ionized, the full ionization assumption is not accurate but acceptable.

- (c) We assume full ionization of impurities, the electron concentration is  $n \approx N_d = 10^{19} \text{ cm}^{-3}$ . The hole concentration is  $p = (n_i)^2 / n = (10^{10} \text{ cm}^{-3})^2 / 10^{19} \text{ cm}^{-3} = 10 \text{ cm}^{-3}$ . The Fermi level position, with respect to  $E_c$ , is

$$E_c - E_f = kT \ln[N_c / n] = 0.026 \ln[2.8 \times 10^{19} \text{ cm}^{-3} / 10^{19} \text{ cm}^{-3}] = 0.027 \text{ eV}.$$

It is located 0.027 eV below  $E_c$ .

Again, we need to find the percentage of donors occupied by electrons in order to check the full ionization assumption with the calculated Fermi level.

$$E_D - E_f = (E_c - E_f) - (E_c - E_D) = -0.023 \text{ eV}, \text{ and}$$

$$n_D = N_d \frac{1}{1 + e^{(E_D - E_f) / kT}} = \frac{10^{19} (\text{cm}^{-3})}{1 + e^{(-0.023 \text{ eV} / 0.026 \text{ eV})}} = 7.08 \times 10^{18} \text{ cm}^{-3} \equiv 71\% \text{ of } N_d.$$

Since 71% of dopants are not ionized, the full ionization assumption is not correct.

Since  $N_d$  is not fully ionized and  $N_d(\text{ionized}) \ll N_d(\text{not-ionized})$ ,

$$n = N_d [1 - f(E_D)] \approx N_d e^{(E_D - E_f) / kT} = N_c e^{-(E_c - E_f) / kT}.$$

Solving the equation above for  $E_f$  yields

$$E_f = \frac{(E_D + E_c)}{2} + \frac{kT}{2} \ln \left( \frac{N_d}{N_c} \right).$$