# Quantum Mechanics Tutorial I

Engineering Physics

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## Let us consider a problem which involve concepts of de Broglie wavelength

- **Find the wavelength of (a) a 46-g golf ball with a velocity 30 m/s and, (b) an electron with a velocity of $10^7$ m/s.**

**(a)** Momentum of golf ball p = mv

Since we know that p = h/λ therefore λ = h/mv

So now, $$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\ J.s}{(0.046\ kg)(30\ m/s)} = 4.8\times 10^{-34}\ \text{m}$$

- The wavelength of the golf ball is so small compared with its dimensions that we would not expect to find any wave aspects in its behavior.

**(b)** Mass of electron m = $9.1\times 10^{-31}$ kg

Now $$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\ J.s}{(9.1\times10^{-31}\ kg)(10^7\ m/s)} = 7.2\times 10^{-11}\ \text{m}$$

- The dimension of atoms are comparable with above finding (e. g. the radius of hydrogen atom is $\sim 5.3\times 10^{-11}\ m$). It is not surprising that the wave character of moving electron is the key to understand atomic structure and behaviour.

## Let's look at another problem

- **A proton has de Broglie wavelength 1 fm. Calculate the kinetic energy of the proton.**

Rest energy of proton $E_o = m_o c^2$ = 0.938 GeV

Where $m_0$ is rest mass of proton having value $1.672\times 10^{-27}$ kg

$$pc = \frac{hc}{\lambda} = \frac{[(6.63\times10^{-34})/1.6\times10^{-19})\ eV.s](3\times10^{8}\ m/s)}{1\times10^{-15}\ m} = 1.241\times 10^{9} \text{ eV} = 1.241 \text{ GeV}$$

Since $pc > E_o$ a relativistic calculation is required.

The expression for total energy ( including both kinetic energy and rest energy) of proton is given as-

$$E = \sqrt{E_o^2 + p^2c^2} = \sqrt{(0.938\ GeV)^2+(1.241\ GeV)^2} = 1.555\text{GeV}$$

Hence,

Kinetic Energy = $E - E_o$ = (1.555-0.938) GeV = 617 MeV

- **An electron has a de Broglie wavelength of 2$\times$ $10^{12}$ m. Find its (a) kinetic energy (b) phase and group velocity of its de Broglie wave.**

**(a)** Rest energy of electron $E_o = m_o c^2 = 511\ keV$ (Rest mass of electron $m_o$ = 9.1 $\times$ $10^{-31}$ kg)

Now to calculate pc: $pc = \frac{hc}{\lambda} = \frac{(4.136\times10^{-15}\ eV.s)(3\times10^{8}\ m/s)}{2\times10^{-12}\ m} = 620\ keV$

Since $pc > E_{\text{o}}$, so we will use the relativistic approach.

And kinetic energy of electron using relativistic approach is given by:

$$KE = E - E_o = \sqrt{E_o^2 + (pc)^2} - E_{\text{o}} = \sqrt{(511\ keV)^2 + (620\ keV)^2} - 511\ keV = 292\ keV$$

**(b)** The energy of electron with velocity *v* is of the fallowing form

$$E = \gamma E_{\text{o}} \qquad \text{where } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$E = \frac{E_o}{\sqrt{1 - v^2/c^2}}$$

Gives $$v = c\sqrt{1 - \frac{E_0^2}{E^2}} = \text{c}\sqrt{1 - \left(\frac{511\ keV}{803\ keV}\right)^2} = 0.771\ c$$

Hence the phase and group velocities are respectively

$$\text{v}_{\text{p}} = \frac{c^2}{v} = \frac{c^2}{0.771c} = 1.30\ \text{c} \qquad \text{And} \qquad \text{v}_{\text{g}} = 0.771\text{c}$$

## ➢ The Davisson – Germer experiment: An experiment that confirms the existence of de Broglie waves

Measured by XRD

n = 1, θ = $65^0$ (highest intensity observed with a 54 V) and d = 0.091 nm (spacing of crystalline planes of nickel)

The Bragg equation for maxima in the diffraction pattern

$$n\lambda = 2d\sin\theta = 2(0.091\text{ nm})(\sin 65^0) = 0.165\text{ nm}$$

Now we use de Broglie's formula to find expected wavelength of the electrons i.e. $\lambda = \frac{h}{\gamma m v}$

Kinetic energy of electron KE = eV = 54 eV

since KE < 0.51 MeV (rest energy of electron). So we can let $\gamma = 1$

We also know that $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ Gives $p = \sqrt{2mKE}$

$$\lambda = \frac{h}{\sqrt{2mKE}} = \frac{6.63\times10^{-34}\ J.s}{\sqrt{2(9.1\times10^{-31}\ kg)(54eV)\left(1.6\times10^{-19}\frac{J}{eV}\right)}} = 0.166\text{ nm}$$

Which agrees well with the observed wavelength of 0.165 nm. The Davisson - Germer experiments thus directly verifies de Broglie hypothesis of the wave nature of moving bodies.