Indian Institute of Information Technology, Allahabad

End Sem Question Paper

B. Tech. 1st Semester

Course Name: Engineering Physics **Course Coordinator: Dr. Akhilesh Tiwari**

Course code: BS-AS-EGP102 **Exam Date: 11/12/23** **Max. marks: 40**

Note: Use of a non-programmable scientific calculator is allowed.

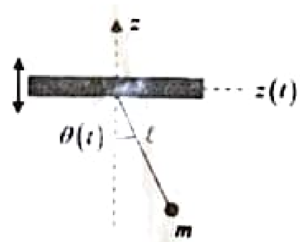
1. In terms of paraboloidal coordinates (ξ, η, ϕ), the Lagrangian of a free particle is given as

$$L = \frac{1}{2}m(\xi^2 + \eta^2)(\dot{\xi}^2 + \dot{\eta}^2) + \frac{1}{2}m\xi^2\eta^2\dot{\phi}^2$$

   a. Find momenta conjugate to (ξ, η, ϕ). Identify the conserved momenta.

   b. Determine Hamiltonian and Hamilton's equations of motion. [5]

2. The fulcrum of a simple pendulum oscillates vertically as $\sin z(t) = a \sin \omega t$ along the z axis, where ω is a constant (see the figure below). Find out the Lagrangian of the system. Find the Lagrange's equation of motion. [5]



3. A particle trapped in a box of length a, has as its initial wave function:

$$\Psi(x,0) = Ax(x - a).$$

   (a) Normalize $\Psi(x,0)$. Find the location where the probability of finding the particle is maximum at $t = 0$.
   (b) Find $<\hat{x}>$, $<\hat{p}>$, $<\hat{H}>$, at $t = 0$. [6]

4. Assume a material has an E-K diagram given by
   $E_{conduction}(K) = E_C + E_1 \sin^2(Ka)$
   $E_{valence}(K) = E_V - E_2 \sin^2(Ka)$. Let $a = 0.5nm$, $E_1 = 5eV$, and $E_2 = 4eV$.

   a) Sketch the E-K diagram for the first Brillouin zone. Label the axes completely.
   b) What is the effective mass for an electron near the bottom of the conduction band?
   c) What is the effective mass for holes near the top of the valence band? [5]

5. (a) Suppose you were to dope some silicon with $N_D = N_A = 10^{16}\ cm^{-3}$. Where do you expect the Fermi level to be?

(b) Band gap of Si depends on the temperature as

$$E_g = 1.17eV - 4.73 \times 10^{-4} \frac{T^2}{T + 636}$$

Find a concentration of electrons in the conduction band of intrinsic (undoped) Si at $T = 77$ K if at 300 K, $n_i = 1.05 \times 10^{10} cm^{-3}$.

[5]

6. A material is doped such that the electron concentration varies linearly across the sample. The sample is $0.5 \mu m$ thick. The donor concentration varies from $N_D = 0$ at $x = 0$ to $N_D = 10^{16}\ cm^{-3}$ at $x = 0.5\,\mu m$. Given $D_n = 30\, cm^2 V^{-1} s^{-1}$, and $D_p = 12\, cm^2 V^{-1} s^{-1}$

$250 \to N_c$

$\left(\frac{kT}{q}\right)$

(a) Write expressions for $n(x)$ and $p(x)$ in terms of x.

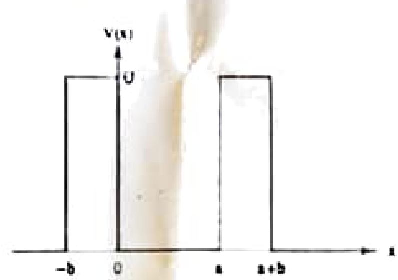
(b) Find the electron diffusion current density.

(c) Find the hole diffusion current density at $x = 0.5 \mu m$.

(d) Find an expression for $E_c(x) - E_f$ as a function of x. Locate the E_f at the edges of the material.

[6]

7. Consider an electron in a periodic potential (Kronig-Penney model) as shown below.



(a) Write down the general solutions for the Schrödinger equation(s).

(b) Use the appropriate boundary conditions to obtain a set of equations that lead to the appearance of energy bands.

(c) Find the limit for which the energy spectrum reduces to the spectrum of particle in a box. When the forbidden bands will disapear?

[8]

$N_c = 2\left(\frac{2\pi m k T}{h^2}\right)^{3/2}$