Indian Institute of Information Technology, Allahabad

C2 Review Test Answers: Engineering Physics

Total Marks: 40 **Total Time:** 60 minutes

✓ Objective Questions. Choose one of the options. ✓

Note: All questions in this section carry equal marks.

Que. 1 When we impose boundary conditions on an electron confined in a quantum well, only discrete values of k are permitted. When we impose periodic boundary conditions on a string of N atoms, the same thing happens. How many discrete values of k are there and what is their spacing?
(a) $2N$ discrete values spaced $2\pi/Na$ (b) $2N$ discrete values spaced π/Na
(c) N discrete values spaced $2\pi/Na$ (d) N discrete values spaced π/Na

Que. 2 What is a Brillouin zone?
(a) A region of energy-space that encompasses all of the unique values of energy
(b) A region of k-space that contains all of the unique solutions of the wave equation
(c) A region of position-space that the electron is allowed to reside within
(d) A region of k-space where the group velocity is positive

Que. 3 Exactly what is a "hole" in semiconductor terminology?
(a) Another name for a positron
(b) A fictitious particle that is really just an empty state in a nearly filled band
(c) A fictitious particle that is really just an empty state in a nearly empty band
(d) An impurity (in small concentration) in the crystal lattice

Que. 4 Consider a 1D bandstructure given by $E(k_x) = \hbar v_F |k_x|$, where v_F is the velocity. What is the effective mass?
(a) $m^* = \hbar v_F$ (b) $m^* = m_o$ (c) $m^* = \infty$ **(d) not really defined**

Que. 5 The force on a particle with an effective mass of m^* could be written as "$F = m^* \vec{a}$, but the effective mass depends on the bandstructure. More generally, how is the force defined?
(a) $\vec{F} = \vec{L} \times \vec{r}$ (b) $\vec{F} = m^* v^2$ **(c)** $\vec{F} = \hbar \frac{d\vec{k}}{dt}$ (d) $\vec{F} = \hbar^2 k\vec{k}$

Que. 6 For a bandstructure with $E(k_x, k_y) = \frac{\hbar^2 k_x^2}{2m^*} + \frac{\hbar^2 k_y^2}{2m^*}$ what is the shape of the constant energy "surface"?
(a) a line **(b) a circle** (c) an ellipse (d) a sphere

Que. 7 Silicon, Germanium, and Gallium Arsenide have different bandstructures. Which of the following is true?
(a) The conduction bands for them are similar in shape.
(b) The valence bands for them are similar in shape.
(c) Si and GaAs have similar conduction bands but different valence bands.
(d) Ge and GaAs have similar conduction bands but different valence bands.

Que. 8 Which of the three semiconductors, Ge, Si, and GaAs, has a direct bandgap?
(a) Ge (b) Si **(c) GaAs** (d) Ge and Si

Que. 9 The operator $\left(x + \frac{d}{dx}\right)$ has the eigen value α. The corresponding wavefunction is:
(a) $\psi_o exp^{\alpha x - \frac{x^2}{2}}$ (b) $\psi_o exp^{\alpha x - \frac{x^3}{2}}$ (c) $\psi_o exp^{\alpha x^2 - \frac{x^2}{2}}$ (d) $\psi_o exp^{\alpha x + \frac{x^2}{2}}$

Que. 10 Density of states of free electrons in a solid moving with an energy 0.1 eV is given by 2.15×10^{21} $eV^{-1}cm^{-3}$. The density of states (in $eV^{-1}cm^{-3}$) for electrons moving with an energy of 0.4 eV will be:
(a) 1.07×10^{21} (b) 1.52×10^{21} (c) 3.04×10^{21} (d) 4.30×10^{21}

Que. 11 In an infinite potential well of size L, what is the probability of finding a particle between L/4 to 3L/4 in the ground state?
(a) $\frac{1}{4} + \frac{1}{\pi}$ **(b)** $\frac{1}{2} + \frac{1}{\pi}$ (c) $\frac{1}{\pi} - \frac{1}{4}$ (d) $1 - \frac{\pi}{4}$

Que. 12 Which one of the following is a valid wavefunction?
(a) e^{-x} over $(-\infty, \infty)$ (b) e^{x} over $(-\infty, \infty)$ (c) $\cos x$ over $(0, \infty)$ **(d) e^{-x} over $[0, \infty)$**

Que. 13 A particle of mass "m" is trapped in an infinite potential well of size "a". The probability of finding the trapped particle between $x = 0$ and $x = a/n$ when it is in the n^{th} state is:
(a) $\frac{1}{n}$ (b) $\frac{2}{n}$ (c) $\frac{1}{2n}$ (d) 0

Que. 14 The wavefunction of a particle, confined to move in an infinite potential well of size L is $\psi(x) = \dfrac{\sqrt{30}x(x-L)}{L^k}$. The value of k is:
(a) 0 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ **(d)** $\frac{5}{2}$

Que. 15 If dispersion relation of an electron is $E_k = -\beta \sin(ka)$, therefore effective mass m^* of the electron with momentum $k = \frac{\pi}{2a}$ is:
(a) $\dfrac{\hbar^2}{2\beta k}$ **(b)** $\dfrac{\hbar^2}{\beta k^2}$ (c) $\dfrac{\hbar}{2\beta^2 k^2}$ (d) 0

Que. 16 If the $E = pc$, the corresponding density of the state D(E) of the electron (in 3D) is proportional to:
(a) $\dfrac{E^3}{2}$ (b) $E^{1.5}$ (c) $E^{1/2}$ **(d)** E^2

Que. 17 The lowest energy of an electron confined to move in a one dimensional potential well of length 0.5 Å is:
(a) 150.7 eV (b) 250.7 eV (c) 350.7 eV (d) 450.7 eV

Que. 18 A particle is confined in an infinite potential well. Let the wavefunction of the particle is given by: $\psi(x) = -\frac{2}{\sqrt{5}}\phi_o + \frac{1}{\sqrt{5}}\phi_1$, where ϕ_o and ϕ_1 are the eigen functions of the ground state and the first excited state, respectively. What is the probability of finding the particle in the first excited stated after the measurement?
(a) $\frac{1}{\sqrt{5}}$ **(b)** $\frac{1}{5}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{4}{5}$

Que. 19 The dispersion relation of phonons in a solid is given by: $\omega^2(k) = \omega_o^2(3 - \cos k_x a - \cos k_y a - \cos k_z a)$. The velocity of the phonons at large wavelength is:
(a) $\dfrac{\omega_o a}{\sqrt{3}}$ (b) $a\omega_o$ (c) $\sqrt{3}a\omega_o$ **(d)** $\dfrac{\omega_o a}{\sqrt{2}}$

Que. 20 A particle is confined to the region $0 < x < L$, If the particle is in the lowest energy state then the probability of finding the particle in the region $0 < x < L/4$ is:
(a) $\frac{1}{4} - \frac{1}{2\pi}$ (b) $\frac{1}{4} + \frac{1}{2\pi}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

★ Subjective Questions ★

Que. 1 A particle is bounded in a 1-d box (with origin at the left corner) of length L in the 2nd excited state. For this particle calculate:
(a) Uncertainty in position **(b)** Uncertainty in momentum
(c) The uncertainty product and check for its validity. [10]

Sol.: The wavefunction for the particle in 1-d box of lenght L in the 2nd excited state, i.e. for $n = 3$, will be:

$$\psi_3 = \sqrt{\frac{2}{L}} \sin \frac{3\pi x}{L} \qquad [1]$$

****Note**:** If the wavefunction is written incorrectly, no need to check the question further.

(a) To find the uncertainty in position:

$$\begin{aligned} \langle x \rangle &= \int_{-\infty}^{\infty} \psi^*(x)\ \hat{x}\ \psi(x)\ dx \\ &= \int_0^L \frac{2}{L}\ x\ sin^2 \frac{3\pi x}{L}\ dx\ =\ \frac{L}{2}. \end{aligned} \qquad [1]$$

Also,

$$\begin{aligned} \left\langle x^2 \right\rangle &= \int_{-\infty}^{\infty} \psi^*(x)\ x^2\ \psi(x)\ dx \\ &= \int_0^L \frac{2}{L}\ x^2\ sin^2 \frac{3\pi x}{L}\ dx \\ &= \frac{L^2}{3} - \frac{L^2}{18\pi^2}. \end{aligned} \qquad [1]$$

Then, uncertainty in position is given by:

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = L\ \sqrt{\frac{1}{12} - \frac{1}{18\pi^2}} \qquad [1]$$

(b) The momentum operator is $\hat{p}_x = -i\hbar \frac{d}{dx}$.

$$\begin{aligned} \langle p_x \rangle &= \int_{-\infty}^{\infty} \psi^*(x)\ \left[-i\hbar \frac{d}{dx} \right]\ \psi(x)\ dx \\ &= -\left(\frac{2}{L}\right)\ (i\hbar)\ \frac{3\pi}{L} \int_0^L sin\frac{3\pi x}{L}\ cos\frac{3\pi x}{L}\ dx\ =\ 0. \end{aligned} \qquad [1]$$

and,

$$\langle p_x^2 \rangle = \int_{-\infty}^{\infty} \psi^*(x) \left[-\hbar^2 \frac{d^2}{dx^2}\right] \psi(x)\, dx$$

$$= \left(\frac{2}{L}\right) (\hbar^2) \left(\frac{3\pi}{L}\right)^2 \int_0^L sin^2 \frac{3\pi x}{L}\, dx \;=\; \frac{9\pi^2\hbar^2}{L^2}. \qquad [1]$$

Then, uncertainty in momentum is given by:

$$\Delta p_x = \sqrt{\langle p_x^2 \rangle - \langle p_x \rangle^2} = \frac{3\pi\hbar}{L}. \qquad [1]$$

(c) The uncertainty product is $\Delta x\, \Delta p_x$.
Here,

$$\Delta x\, \Delta p_x = 3\pi\hbar \sqrt{\frac{1}{12} - \frac{1}{18\pi^2}} \approx 2.63\hbar. \qquad [2]$$

This value of uncertainty product is more than $\hbar/2$. Thus, the uncertainty principle holds true. [1]

Que. 2 The energy dispersion relation for a solid is given as: $E(k) = E_o - C \cdot \cos k_x a \cdot \cos k_y a$. Calculate:
(a) Band width of the band. (b) Density of state near $(0,0)$.
(c) Effective mass m^* at $\left(0, \frac{\pi}{a}\right)$ and $(0,0)$. (d) Velocity of the electrons. [10]

Sol.: (a) Top of the band for the given energy lies at $(0, \frac{\pi}{a})$ or $(\frac{\pi}{a}, 0)$ where the energy is maximum, while the minimum of it lies at $(0,0)$. So,

$$E_{top} = E_{(0,\frac{\pi}{a})} = E_o + C$$
$$E_{bottom} = E_{(0,0)} = E_o - C$$

Then Band width is:

$$\Delta E = E_{top} - E_{bottom}$$
$$= E_o + C - (E_o - C)$$
$$= 2C. \qquad [2]$$

(b) Expanding the cosines to simplify the $E(k)$ equation:

$$E(k) = E_o - C \left[1 - \frac{k_x^2 a^2}{2} + ...\right] \left[1 - \frac{k_y^2 a^2}{2} + ...\right]$$
$$= E_o - C \left[1 - \frac{a^2}{2}(k_x^2 + k_y^2) + \frac{k_x^2 k_y^2 a^4}{4} + ...\right]$$

Neglecting the higher order terms, because near $(0,0)$ $k_x, k_y << 1$.

$$\Rightarrow\ E(k) = E_o - C + C\ \frac{a^2}{2}(k_x^2 + k_y^2)$$
$$= E_o - C + C\ \frac{k^2 a^2}{2} \qquad [2]$$

Then,

$$\frac{dE(k)}{dk}\ =\ \frac{2Ca^2k}{2}\ =\ Ca^2k \qquad [1]$$

Density of states per unit energy range in 2-d near $(0,0)$ is:

$$D(E) = 2\left(\frac{L}{2\pi}\right)^2\ 2\pi k\frac{dk}{dE}$$
$$= \frac{L^2}{\pi}k\frac{1}{Ca^2k}$$

And, density of states per unit volume per unit energy range is:

$$g(E) = \frac{1}{\pi Ca^2} \qquad [2]$$

(c) Effective mass is given by:

$$m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$

Since, $\left.\frac{d^2E}{dk^2}\right|_{(0,\frac{\pi}{a})} = -Ca^2$ and $\left.\frac{d^2E}{dk^2}\right|_{(0,0)} = Ca^2$

So,

$$\Rightarrow\ m^*_{(0,\frac{\pi}{a})} = -\frac{\hbar^2}{Ca^2}$$
$$\text{and}\qquad m^*_{(0,0)} = \frac{\hbar^2}{Ca^2} \qquad [2]$$

(d) Velocity of the electrons is given by:

$$v = \frac{1}{\hbar}\frac{dE}{dk}$$

Then, Velocity of electrons in the band is:

$$v = \frac{Ca^2k}{\hbar} \qquad [1]$$