Indian Institute of Information Technology, Allahabad

Quiz 2: Engineering Physics **B. Tech.-1st Semester, 2023**

Full Marks: 30 **Time: 45 Mins.**

Instruction: All questions are compulsory. A scientific calculator is allowed.

Part A **(1 x 10 = 10)**

Q1. What is the major equipment of capacitance and Permittivity measurement experiment?

Ans. Parallel plate Capacitor and perspex/polythene.

The kit consists of a reed relay switch with its a.c. supply, integrated circuit current amplifier and 0-100 micrometer housed in a cabinet. In addition a pair of capacitor plates 0.3m x 0.3m, a Perspex Sheet, one standard capacitor, and a set of perspex/polythene spacers are provided with the kit.

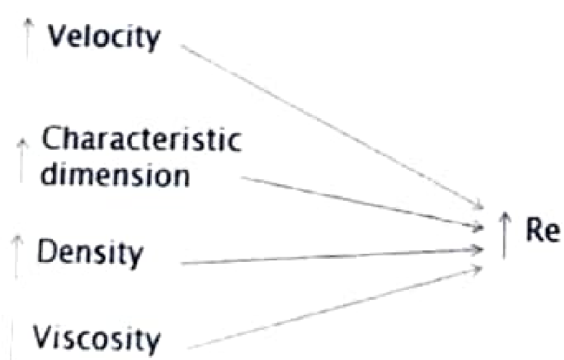
Q2. Why fringes are circular in the Newton's ring experiment?

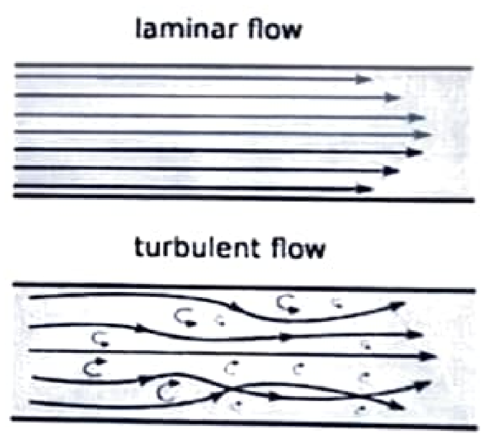
Ans. The fringes are circular due to the fact that air film is symmetrical about the point of contact.

Q3. Define Reynold number and explain different kinds of flow of liquid.

Ans. Mathematically, Reynolds number is:

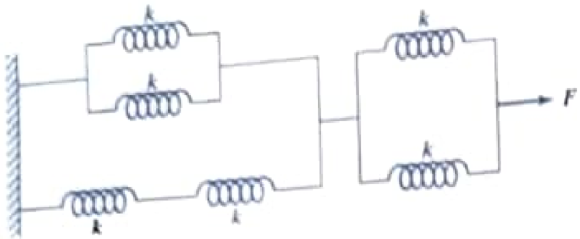
$$Re = \frac{\text{inertia forces}}{\text{viscous forces}} = \frac{\rho \,.\, V \,.\, D}{\mu}$$





Reynolds number, in fluid mechanics, is a criterion of whether fluid (liquid or gas) flow is absolutely steady (streamlined, or laminar) or on the average steady with small unsteady fluctuations (turbulent). Whenever the Reynolds number is less than about 2,000, flow in a pipe is generally laminar, whereas, at values greater than 2,000, flow is usually turbulent. Actually, the transition between laminar and turbulent flow occurs not at a specific value of the Reynolds number but in a range usually beginning between 1,000 to 2,000 and extending upward to between 3,000 and 5,000.

Q4. Calculate the resultant spring constant for the following arrangement:



Ans. K= 10k/9

Q5. In the grating spectrum, which colour least deviates and why?

Ans. Violet colour least deviates. $d \sin \theta = n\lambda$

Q6. Calculate the Hall voltage when B=5A/m, I=2A, w=5cm and n=10^{20}.

Ans. $V_H = \frac{BI}{new} = \frac{5\times2}{10^{20}\times1.6\times10^{-19}\times0.05} = 12.5\,V$

Q7. Suppose the terminal velocity of a falling sphere in a liquid is v. If the radius of the sphere is doubled then find the terminal velocity of the sphere by which it will fall.

Ans. 4v

Q8. What is polarization and how many types of polarization existing in a material (give only name).

Ans. Polarization is a phenomenon in which the dipoles inside the material are aligned to the applied external electric field.

Types of polarization in materials: Electronic polarization, Ionic polarization, etc.

Q9. What is the value of Planck's constant in erg-second?

Ans. $6.625 \times 10^{-27} erg \cdot s$

Q10. Which quantity do you measure by four probe experiment and how?

Ans. We study the temperature dependence behaviour of the resistivity and measure the band gap of the material using the following formula:

$\rho = A\exp(E_g/2k_BT)$

where, E_g=Band gap in eV.

Part B (2 x 10 = 20)

Q1. What is the probability of finding a particle in the state Φ_1 if the wavefunction Ψ is given by $\Psi = \text{Cos}\theta\, \Phi_1 + \text{Sin}\theta\, \Phi_2$? Where Φ_1 and Φ_2 are normalized energy eigenfunctions. Is Ψ normalized?

Ans. $\text{Cos}^2\theta$. Yes since $\text{Cos}^2\theta+\text{Sin}^2\theta=1$.

Q2. Consider a particle of mass m in a 1-D box with the following potential

$$V(x)=\infty \text{ for } |x|\geq a$$

$$=0 \text{ for } |x|<a .$$

Write down the normalization constant for the energy eigenfunction. What is the energy eigenvalue?

Ans. $1/\sqrt{a}$; $E_n=\frac{n^2\pi^2\hbar^2}{8ma^2}$

Q3. Find out the Fermi energy expression for a 2-dimensional free electron gas using the density of states.

Ans. $E_F = \frac{\pi\hbar^2 N}{m^* L^2}$ where m* is the electron's effective mass. L is the dimension of the 2D box. Give full marks if one uses m instead.

Using $\int_0^{E_F} \rho(E)dE = N$

Q4. Consider a particle of mass m in a 1-D box. Find out the probability of finding a particle between 0 to a/2 (the length of the box is a) in the ground state.

Ans. 1/2

Q5. An electron is confined in a three-dimensional infinite potential well of sides equal to L. Find the excitation energy between the ground state to the first excited state.

Ans. $\Delta E = \frac{3\pi^2\hbar^2}{2mL^2}$

Q6. Find the relation between density of state and energy for 1D materials.

Ans. dn/dE proportional to $E^{-1/2}$.

Sol. $\rho(E)_{2D} = \frac{m^* L^2}{\pi\hbar^2}$

$\therefore N = \int_0^{E_F} \rho(E)\, dE = \frac{m^* E_F L^2}{\pi\hbar^2}$

# of states

$\therefore E_F = \frac{\pi\hbar^2 N}{m^* L^2}$

Q7. Consider the conduction band of a crystal whose energy vs wave vector relation E(k) along some direction in k-space is given by the following expression:

$$E(k) = E_o + E_1 cos[\alpha(k - k_o)]$$

Note that this band has a minimum at k = k_o. Calculate the electron effective mass near the minimum of the band.

Sol.

$$\frac{dE(k)}{dk} = -\alpha E_1 \sin[\alpha(k - k_o)]$$

$$\Rightarrow \frac{d^2E(k)}{dk^2} = -\alpha^2 E_1 \cos[\alpha(k - k_o)]$$

At $k = k_o$, $\Rightarrow \frac{d^2E(k)}{dk^2} = -\alpha^2 E_1$

Then,

$$m^* = \frac{\hbar^2}{\frac{d^2E(k)}{dk^2}} = \frac{-\hbar^2}{\alpha^2 E_1}$$

Q8. A particular metal has 10^{22} electrons per cubic centimeter. Calculate the Fermi energy and the Fermi velocity (at 0 K).

Sol.: The Fermi energy is the highest occupied energy state at 0 K and is given by

$$E_F = \frac{\hbar^2}{2m_o}(3\pi^2 n)^{\frac{2}{3}}$$

$$\Rightarrow E_F = \frac{(1.05 \times 10^{-34}\, J - s)^2[3\pi^2(10^{28}\, m^{-3})]^{\frac{2}{3}}}{2(9.1 \times 10^{-31}\, kg)}$$

$$= 2.75 \times 10^{-19}\, J$$
$$= 1.72\, eV$$

The Fermi velocity is:

$$v_F = \frac{\hbar}{m_o}(3\pi^2 n)^{\frac{1}{3}}$$

$$\Rightarrow v_F = \frac{1.05 \times 10^{-34}\, J - s[3\pi^2(10^{28}\, m^{-3})]^{\frac{1}{3}}}{(9.1 \times 10^{-31}\, kg)}$$
$$= 7.52 \times 10^5\, m/_s$$

Q9. A beam of electrons is incident on a barrier 6.00 eV high and 0.200 nm wide. Find the energy they should have if 1.00 percent of them are to get through the barrier.

Approximate transmission probability $T = e^{-2k_2 L}$

That gives $$k_2 = \frac{1}{2L}\ln\left(\frac{1}{T}\right) = \frac{1}{2(0.2\times10^{-9}m)}\ln(100) = 1.15\times10^{10}m^{-1}$$

And,

$$E = V - \frac{(\hbar k_2)^2}{2m} \qquad \text{since } k_2 = \frac{\sqrt{2m(V-E)}}{\hbar}$$

$$= 6.00\,eV - \frac{[(1.054\times10^{-34}J.s)(1.15\times10^{10}m^{-1})]^2}{\left(2(9.1\times10^{-31}\,kg)\left(1.6\times10^{-19}\frac{J}{eV}\right)\right)}$$

$$= 0.95\,eV$$

Q10. The density of bcc iron is 7.9×10^3 kg/m³ and atomic weight is 56. Calculate the size of the unit cell and atomic diameter of iron atom.

Sol.: The relation for density of unit cell is:

$$a^3 = \frac{Mn}{N\rho}$$

where a (is lattice constant) = ?
M (is atomic weight) =56
n (is number of atom per unit cell)= 2
N (is Avogadro number)= 6.02×10^{26}
ρ (is density) = $7.9\times10^3\ kg/m^3$

$$\Rightarrow a^3 = \frac{56\times2}{6.02\times10^{26}\times7.9\times10^3} = 23.55\times10^{-30}m^3$$

$$a = 2.866\ \text{Å}$$

Atomic diameter of bcc iron atom= $2r = \frac{a\sqrt{3}}{2}$

$$= 2.866\times\frac{\sqrt{3}}{2} = 2.482\ \text{Å}$$

Sol. :- $V_{\text{single-state}} = \frac{\pi}{L}$ (K-space volume)

$V_{\text{line}} = K$

$$N = \frac{KL}{\pi} = \frac{L}{\pi}\frac{\sqrt{2m^*E}}{\hbar}$$

$$\therefore \frac{dN}{dE} \propto E^{-1/2}$$