Classical Mechanics Tutorial I

Engineering Physics

Indian Institute of Information Technology, Allahabad

Calculus of Multivariable Functions

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Now consider an example of total derivative. Let us consider a function $f(x(t), y(t), t)$ as

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$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial t} = f_x\dot{x} + f_y\dot{y} + f_t \tag{6}$$

now you can calculate partial derivatives of f, i.e., f_x, f_y and f_z.

$$f_x = 2xyt + y^2 t \quad ; \quad f_y = x^2 t + 2xyt + t^2 \quad ; \quad f_t = x^2 y + xy^2 + 2yt \tag{7}$$

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If you divide by dt, you can recover the Eq.(6).

- Now let us look at an example of double partial derivative. Consider a function $f(x,y) = x^2y + xy^2$. Calculate $\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$, $\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)$ and $\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$?

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Calculate $f_{yx} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$ and check if they are equal?

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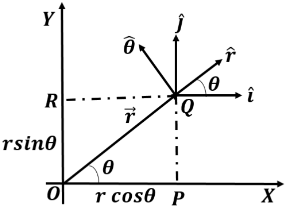

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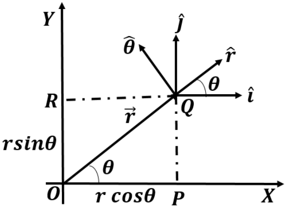

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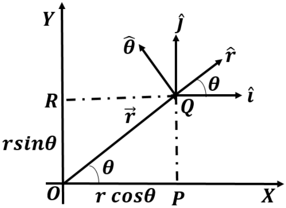

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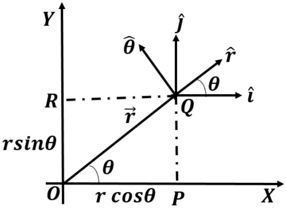

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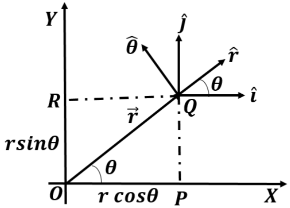

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- The Position vector is: $\vec{r} = x\hat{i} + y\hat{j} = r(\cos\theta\hat{i} + \sin\theta\hat{j})$

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Velocity construction:

- From the figure, we can write down the unit vectors for $\hat{r}$ and $\hat{\theta}$ in the direction of increasing θ and r respectively.

$$\hat{r} = \frac{\vec{r}}{|r|} = \cos\theta\hat{i} + \sin\theta\hat{j} \tag{16}$$

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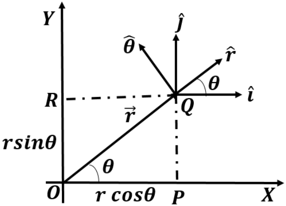

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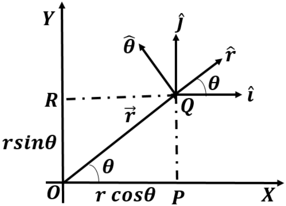

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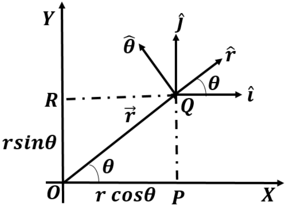

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$$\frac{d\hat{r}}{dt} = \dot{\hat{r}} = \frac{d\hat{r}}{d\theta}\frac{d\theta}{dt} = \hat{\theta}\dot{\theta} \tag{18}$$

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- Similarly one can also calculate

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$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(r\hat{r}) = \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt} \tag{20}$$

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} \tag{21}$$

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$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}\right) \tag{22}$$

$$= \left(\ddot{r} - r\dot{\theta}^2\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\theta} \tag{23}$$

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The term $\ddot{r}$ is *linear acceleration* in radial direction, $r\dot{\theta}^2$ is the *centripetal acceleration*, $\ddot{\theta}$ is the *acceleration* in the tangential direction, and $2\dot{r}\dot{\theta}$ is the *Coriolis acceleration*.

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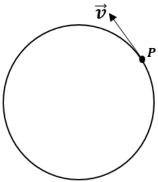
Acceleration construction: Consider an object P moving on a circular path with a uniform velocity and radius R. Prove that it'll always be attracted towards the center of the circle.



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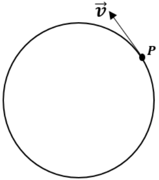

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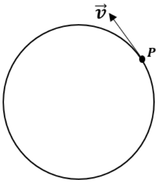

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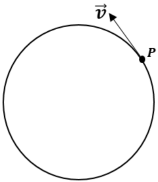

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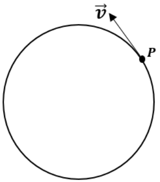

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- As the magnitude of velocity is constant, but due to change in the direction of velocity, it changes the direction and producing non-zero acceleration towards the center.

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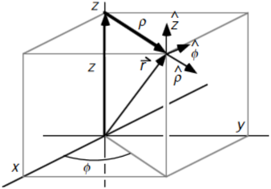

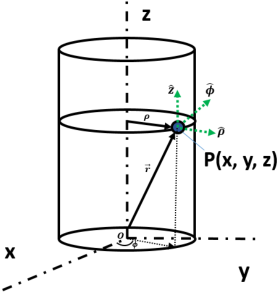

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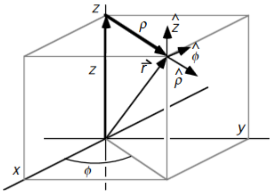

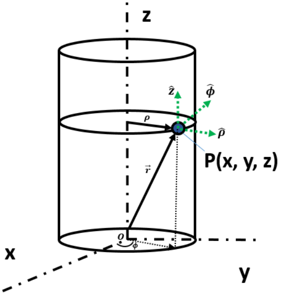

- The position vector for the point P is

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \tag{24}$$

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$$\vec{a} = \dot{\vec{v}} = \left(\ddot{\rho} - \rho\dot{\phi}^2\right) + \left(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}\right) + \ddot{z}\hat{z} \tag{30}$$

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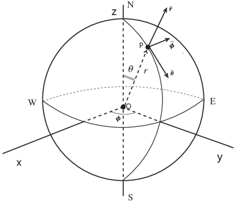

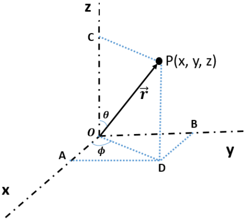

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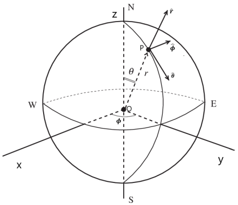

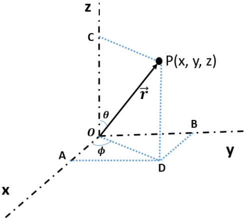

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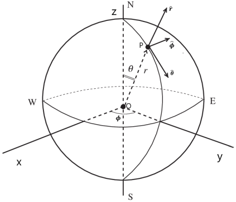

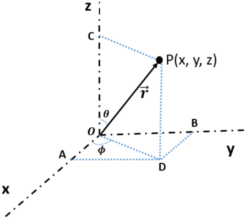

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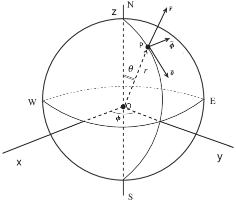

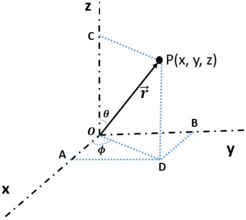

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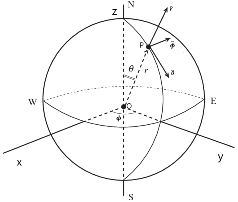

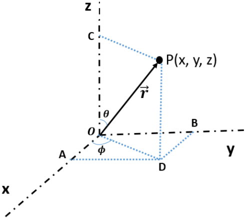

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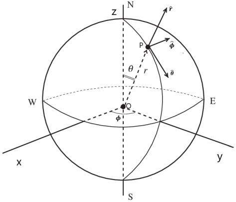

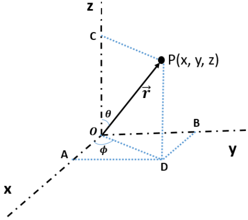

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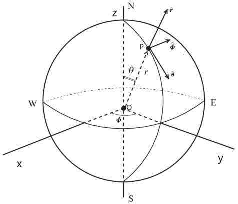

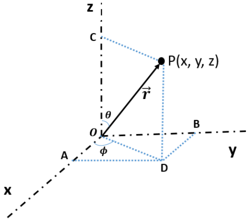

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- Find the expression for velocity & acceleration in spherical polar coordinate.