

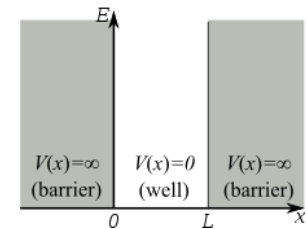
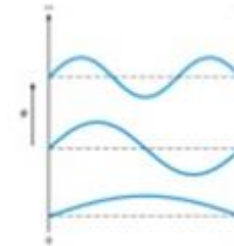
Semiconductors: Introduction, Energy Bands in conductors, semiconductors, insulators, intrinsic and extrinsic semiconductor, Carrier transport in semiconductor: diffusion current, drift current, mobility and resistivity. Generation and recombination of carriers in semiconductors. Thermal Noise, Shot Noise. Electrons and Holes in semiconductors: Silicon crystal structure, Donors and acceptors in the band model, electron effective mass, Density of states, Thermal equilibrium, Fermi-Dirac distribution function for electrons and holes, Fermi energy. Equilibrium distribution of electrons & holes. The n-p product and the intrinsic carrier concentration at extremely high and low temperatures, Variation of Fermi energy with doping concentration and temperature.

Motion and Recombination of Electrons and Holes: Carrier drift: Electron and hole mobilities, Mechanism of carrier scattering, Drift current and conductivity. Carrier diffusion: diffusion current, Total current density, relation between the energy diagram and potential, electric field, Einstein relationship between diffusion coefficient and mobility.

# Infinite Square-Well Potential

- The simplest such system is that of a particle trapped in a box with infinitely hard walls that the particle cannot penetrate. This potential is called an infinite square well and is given by

$$V(x) = \begin{cases} \infty & x \leq 0, x \geq L \\ 0 & 0 < x < L \end{cases}$$



- Clearly the wave function must be zero where the potential is infinite,

there will be infinitely many  $E_n$  eigenvalues with their matching  $\psi_n$  where  $n \frac{\lambda}{2} = L \quad n = 1, 2, 3, \dots$

- Where the potential is zero inside the box, the Schrödinger wave

equation becomes  $\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi = -k^2\psi$  where  $k = \sqrt{2mE/\hbar^2}$ ,  $k = 2\pi/\lambda$

- The general solution is  $\psi(x) = A \sin kx + B \cos kx$   
 $B = 0$ , as just the sin term will do

$$\sin(k_n x) = \frac{e^{ik_n x} - e^{-ik_n x}}{2i}$$

$$k_n = 2\pi/\lambda_n$$

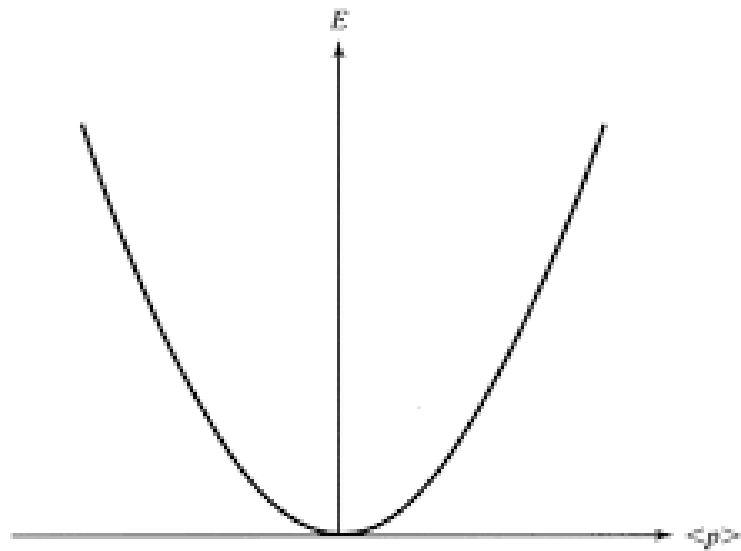
Note that this wave function is real because we are considering a standing wave, that is the sum of two moving waves

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \quad \psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

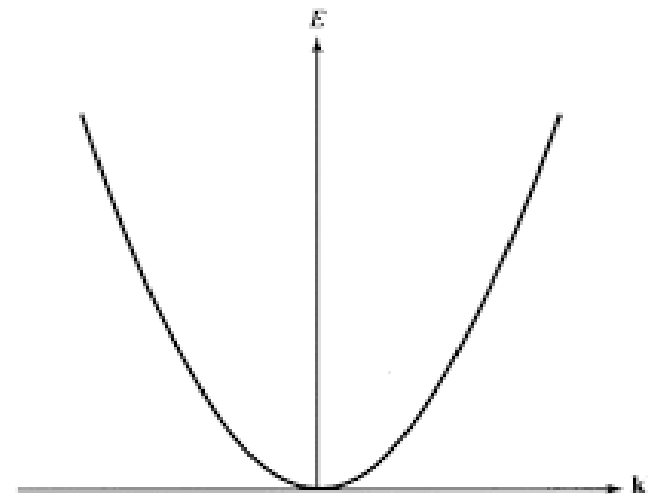
$$E_n = \frac{\hbar^2}{2m} \left( \frac{n\pi}{L} \right)^2$$

$$k = \frac{n\pi}{L} \text{ and } n = 1, 2, 3, \dots$$

$$E_n = \frac{\hbar^2}{2m} (k)^2$$

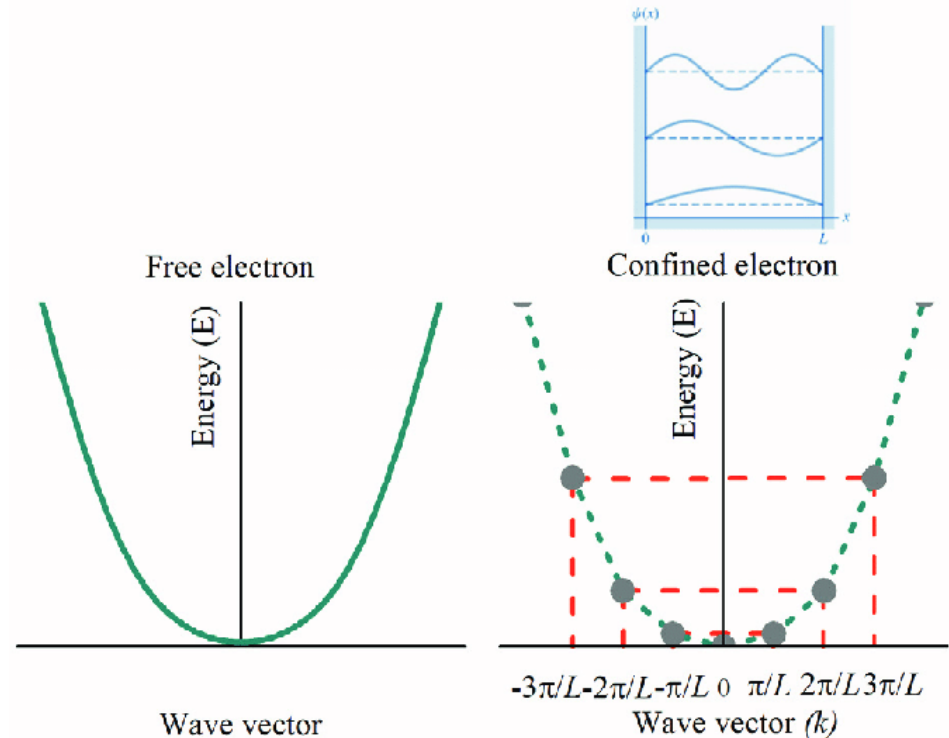


Energy-momentum relationship for a free particle.



Energy-momentum relationship for a free particle.

Recall, a free particle has  $E \sim k^2$ . Instead of being continuous in  $k^2$ ,  $E$  is discrete in  $n^2$ ! I.e the energy value (and thus, wavelengths/ $k$ ) of a confined electron are quantized (take on only certain values). Note that as the dimension of the energy well increases, the spacing between discrete energy levels (and discrete  $k$  values) reduces. In the infinite crystal, a continuum same as our free particle solution is obtained.



- Fixed boundary conditions  $\psi(x)=0$  and  $\psi(L)=0$
- Boundary conditions are fixed and the probability density function is periodic in  $x$ .

Standing Waves

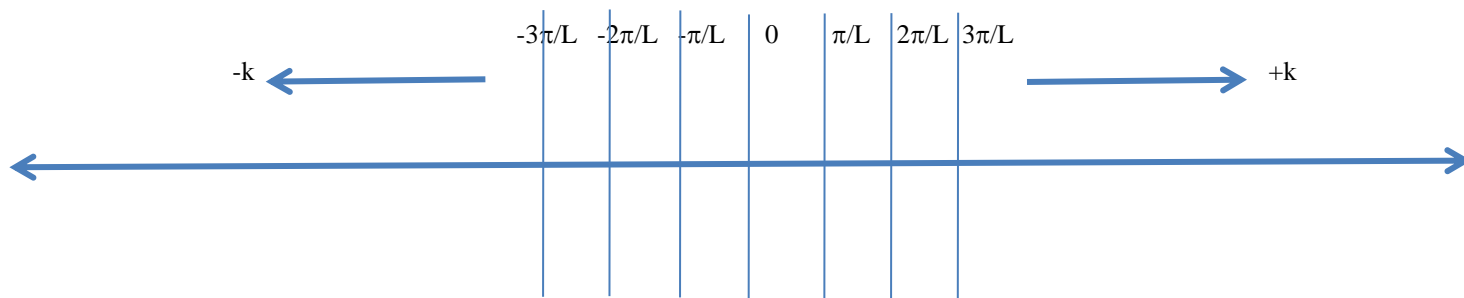
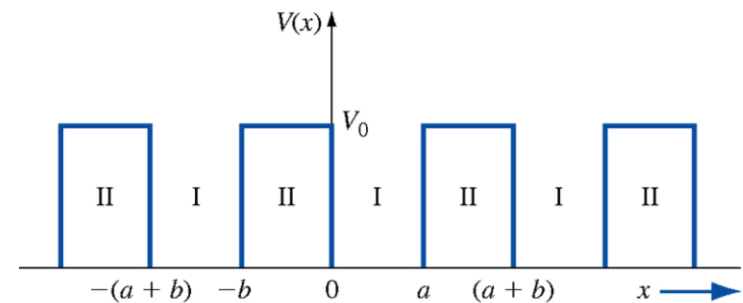
$$\psi(x) = \psi(L)$$

Dr. Pramod Kumar, Associate Prof.

# Boundary Conditions

- With the condition that all the components of  $k$  (periodic boundary condition) take the values  $k=\pm 2n \pi/L$ ,  $n=0,\pm 1,\pm 2,\dots,\pm N/2$  (upper limit to  $n$ )
- $k$  should satisfy the periodicity of lattice, i.e.  $k(x+L)=k(x)$ , where  $L$  is lattice periodicity. This can be verified for  $k=2\pi/L$

$$\begin{aligned}\psi &= e^{ik(x+L)} = e^{i\frac{2\pi(x+L)}{L}} \\ &= e^{i\frac{2\pi x}{L}} e^{i2\pi} = e^{i\frac{2\pi x}{L}} \\ \psi(x+L) &= \psi(x)\end{aligned}$$



- If  $a$  is the spacing between the fixed nuclei, and  $L$  is the characteristic length of our potential well, then  $N=L/a$ , Now  $k=\pm 2n \pi/L=\pm 2(N/2) \pi/L=\pm 2(L/2a) \pi/L=\pm \pi/a$

# Group Velocity and Effective Mass

- What effect does this E-k square law relation ship have on electron velocity and mass?

The group velocity of a wave is

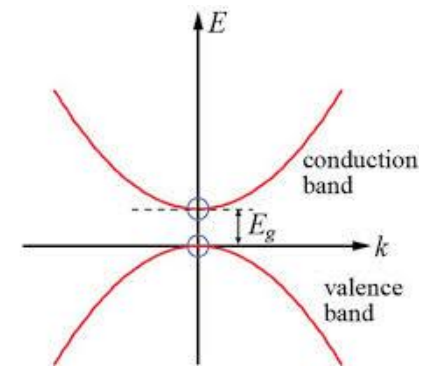
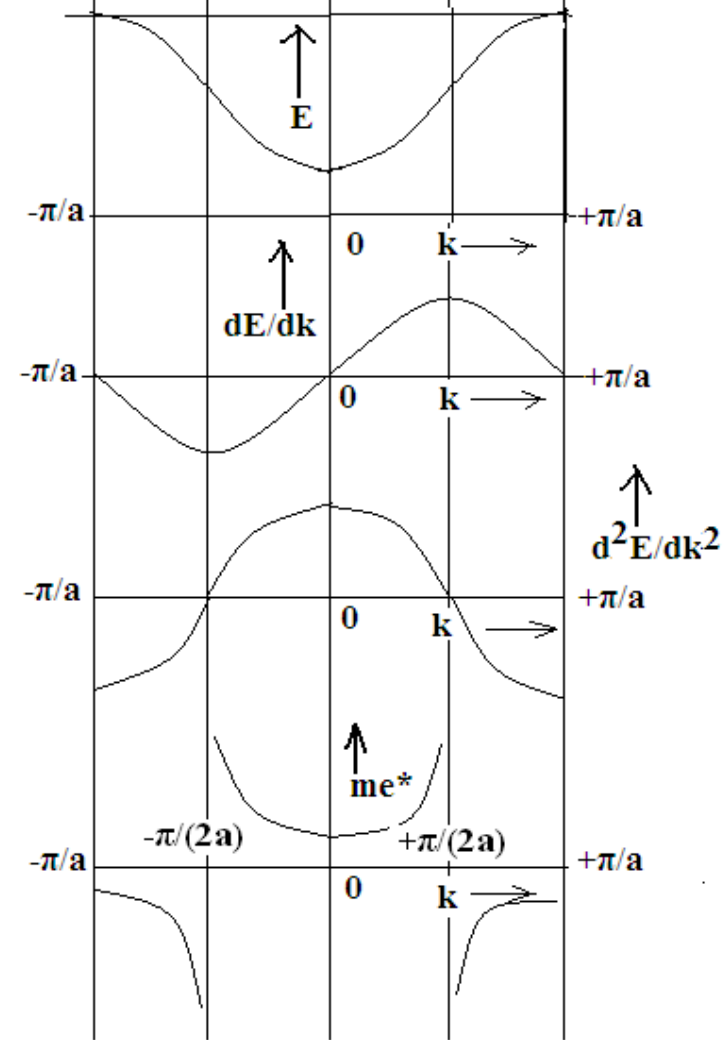
$$v_g = \frac{dE}{dp} = \frac{1}{\hbar} \frac{dE}{dk}$$

- So the speed of an electron in the direction defined by p is found from the slope of the E-k diagram. Similarly, since

$$E_n = \frac{\hbar^2}{2m} (k)^2$$

$$(1/\hbar^2) (d^2E/dk^2) = 1/m^*$$

- So the effective mass of an electron is related To the local inverse curvature of the E-k diagram.
- The sign of effective mass is determined directly from the sign of the curvature of the E-k curve.
- Particle (electron) sitting near the minimum have a  $+m^*$
- Particle (hole) sitting near the maximum have a  $-m^*$
- A negative  $m^*$  implies that a particle will go “**the wrong way**” when an external force applied.



**Goal: show that an electron behaves like a particle with mass**

$$m^* = \hbar^2(d^2E/dK^2)^{-1}$$

- Recall that the electron energy is related to the frequency of the electron wave

$$E = \hbar\omega$$

- and the group velocity of the wave is the velocity of the electron

$$v_g = d\omega/dk = 1/\hbar dE/dk$$

- The acceleration of a particle is given by the time-derivative of its velocity:

$$\begin{aligned}
 a &= dv_g/dt = d/dt(d\omega/dk) \\
 &= d/dk(d\omega/dk)dk/dt \\
 &= (1/\hbar^2) d/dk(d\hbar\omega/dk)(d(\hbar k)/dt) \\
 &= (1/\hbar^2) (d^2E/dk^2)(d(\hbar k)/dt)
 \end{aligned}$$

This is the term we are looking to show is:

$$(1/\hbar^2) (d^2E/dk^2) = 1/m^*$$



# What is $d(\hbar k)/dt$ ?

If we apply an external force on the electron, for example an electric field ( $F_{\text{ext}}=qE$ ), then we will do work on the electron:

$$\begin{aligned} dW_e &= F_{\text{ext}} dx = F_{\text{ext}} (v_g dt) \dots \text{since } v_g = dx/dt = d\omega/dk \\ &= F_{\text{ext}} (d\omega/dk) dt \end{aligned}$$

**Doing work on the electron increases its energy**

$$\begin{aligned} dW_e &= F_{\text{ext}} (d\omega/dk) dt = dE \\ &= (dE/dk) dk \\ &= [d(\hbar\omega)/dk] dk \\ &= \hbar (d\omega/dk) dk \end{aligned}$$

$$\text{therefore: } F_{\text{ext}} dt = \hbar dk$$

$$\text{or } F_{\text{ext}} = d(\hbar k)/dt$$

*note:* since  $F=d(\text{mv})/dt$ ,

$\hbar k$  is called the “crystal momentum”

# Finally...

$$a = (1/\hbar^2) (d^2E/dk^2) (d(\hbar k)/dt)$$

and

$$F_{\text{ext}} = d(\hbar k)/dt$$

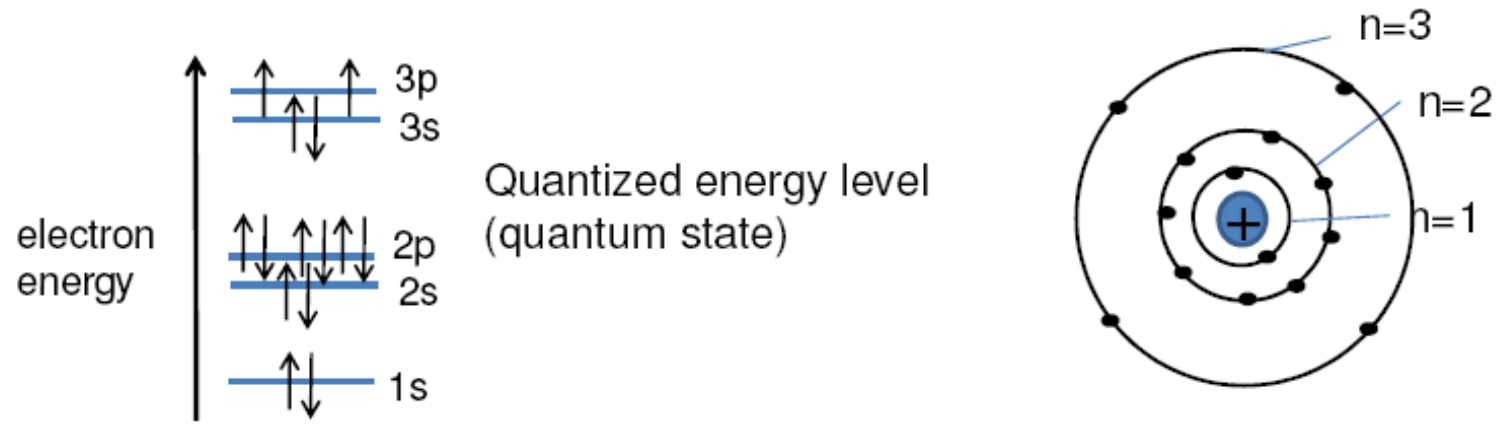
gives us

$$a = (1/m^*) F_{\text{ext}} \quad \text{or} \quad F_{\text{ext}} = m^* a$$

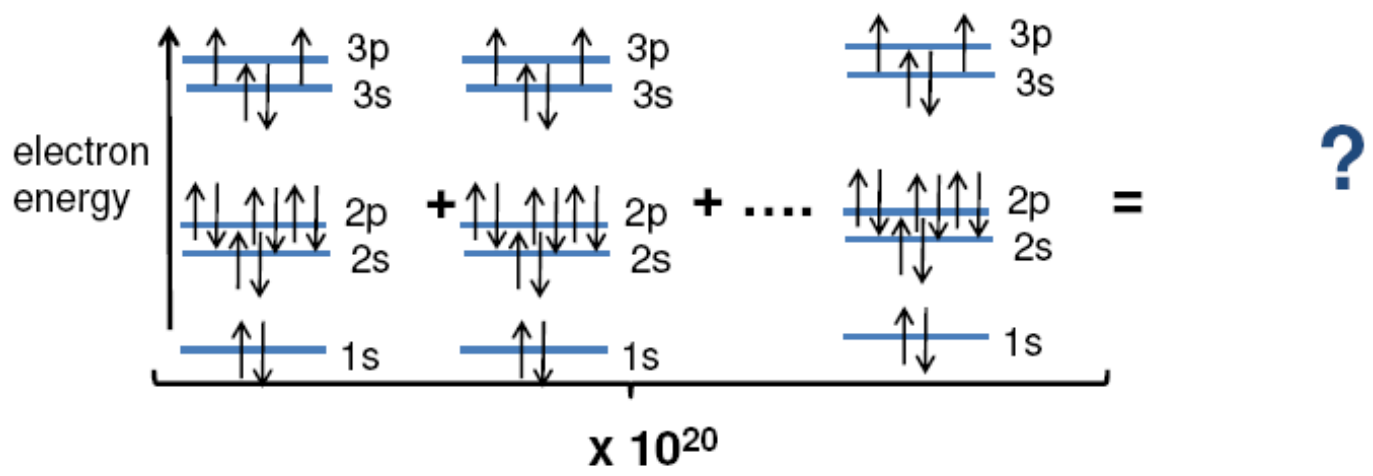
$$\text{Where } m^* = [(1/\hbar^2) (d^2E/dk^2)]^{-1} = \hbar^2 (d^2E/dk^2)^{-1}$$

# Formation of Band

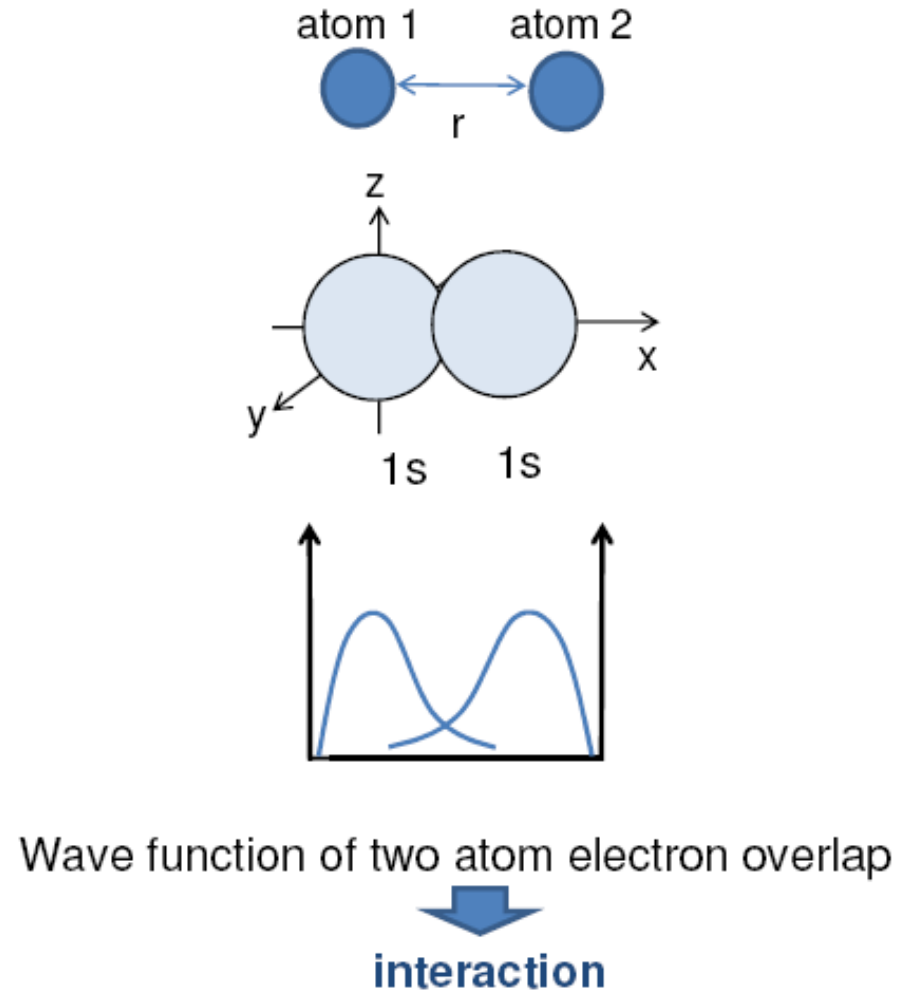
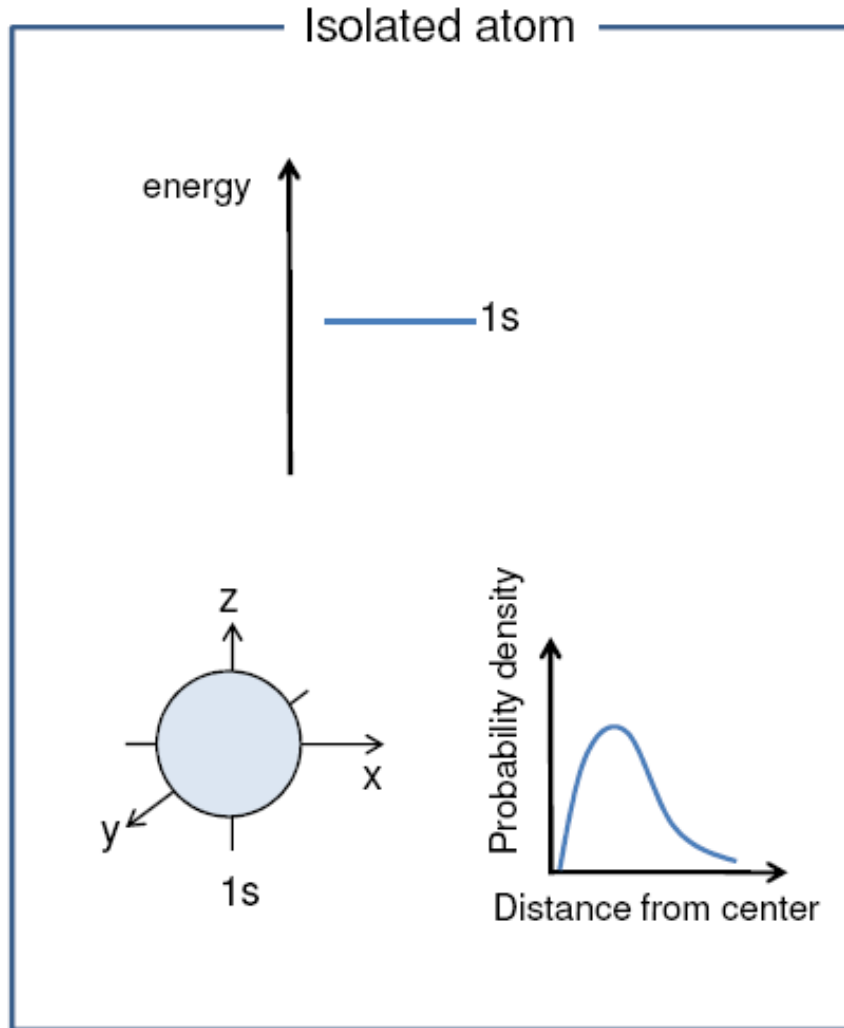
Isolated single atom (ex; Si)

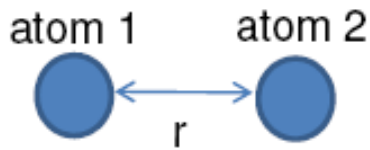


Crystal ( $\sim 10^{20}$  atom)



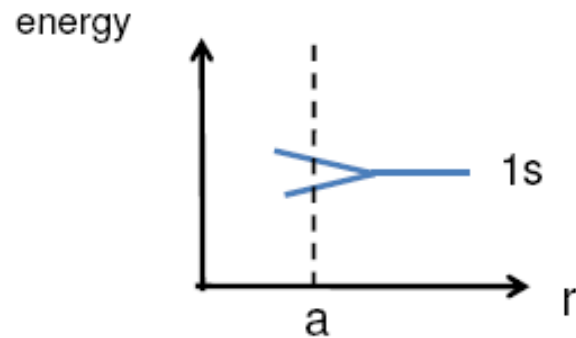
# What happen if 2 identical atoms approach each other ?





➤ When the atoms are far apart ( $r = \infty$ ), electron from different atoms can occupy same energy level.

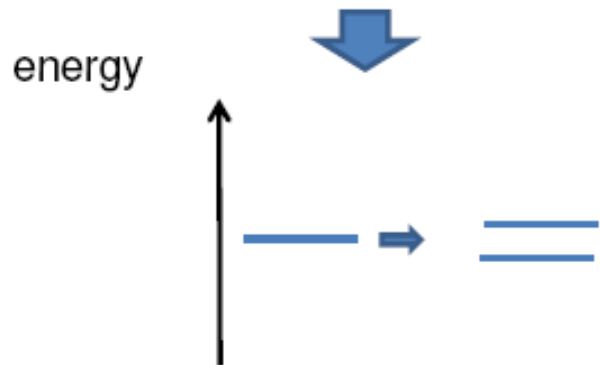
$$E_{1s, \text{atom 1}} = E_{1s, \text{atom 2}}$$



➤ As the atoms approach each other, **energy level splits**

a ; equilibrium interatomic distance

$$E_{1s, \text{atom 1}} \neq E_{1s, \text{atom 2}}$$

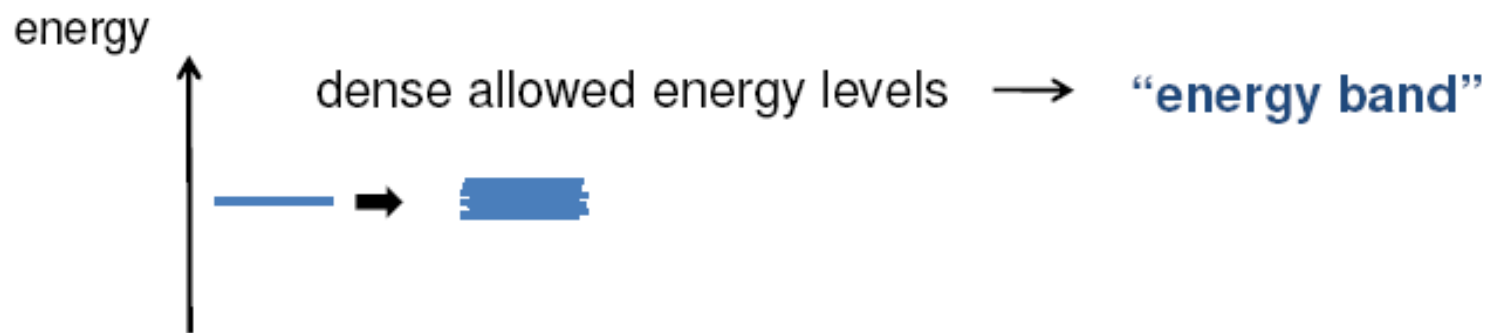
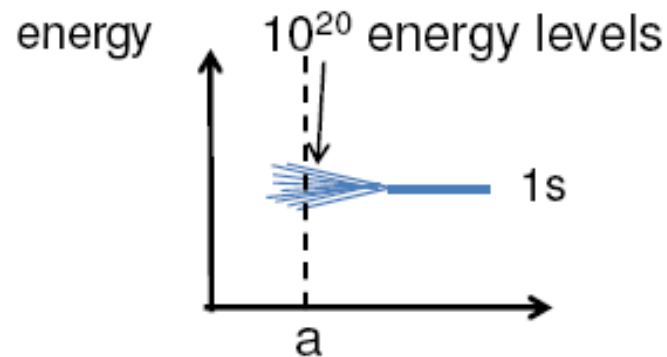


- interaction between two overlap wave function
- Consistent with Pauli exclusion principle

# Regular periodic arrangement of atom (crystal)

ex:  $10^{20}$  atoms

Total number of quantum states  
do not change when forming a  
system (crystal)



Consider

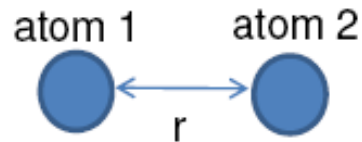
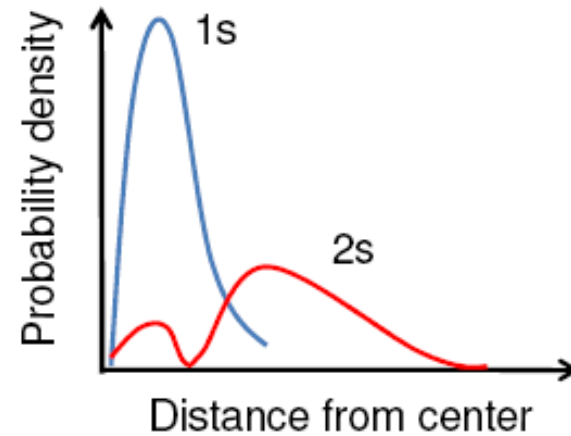
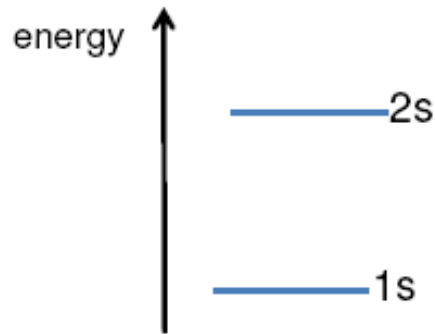


- $10^{20}$  energy state
- Energy states are equidistant

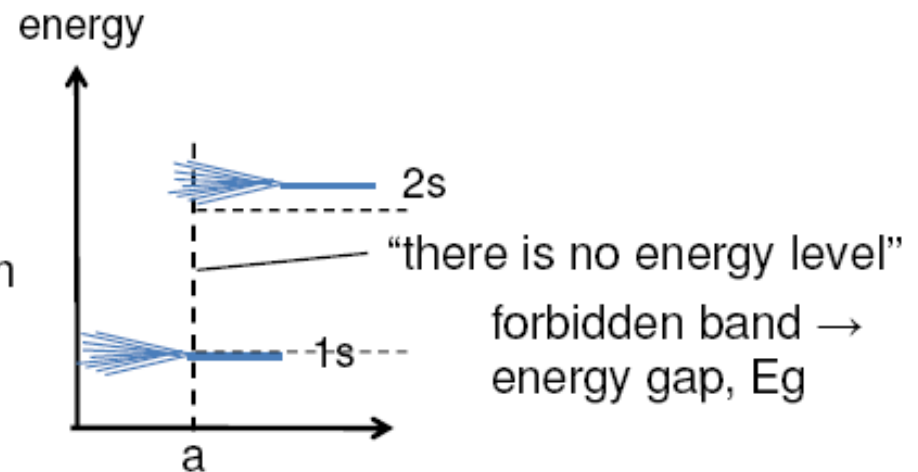


Energy states are separated by  $1/10^{20} \text{ eV} = \mathbf{10^{-20} \text{ eV}}$

(Almost) **continuous energy states** within energy band



➤ As the atoms are brought together, electron from 2s will interact. Then electron from 1s.



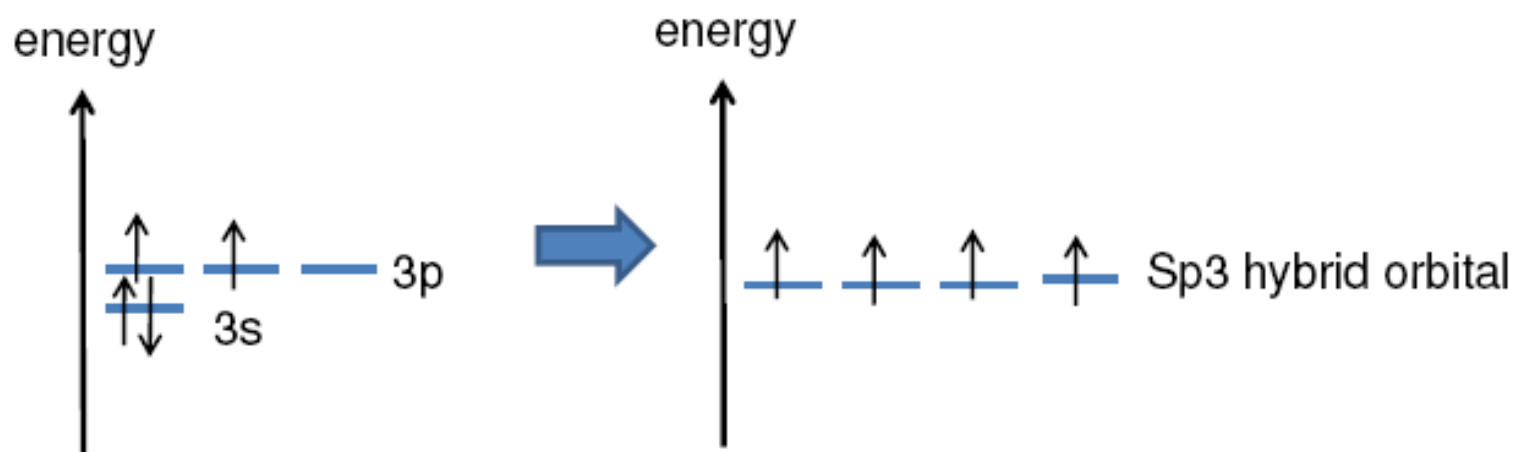


Ex;

Si:  $1s(2), 2s(2), 2p(6), 3s(2), 3p(2)$  14 electrons

Tightly bound to  
nucleus

Involved in  
chemical reactions



Reform 4 equivalent states  $\rightarrow$  4 equivalent bond (symmetric)



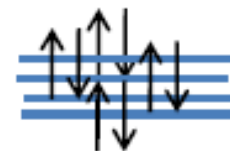
energy



energy

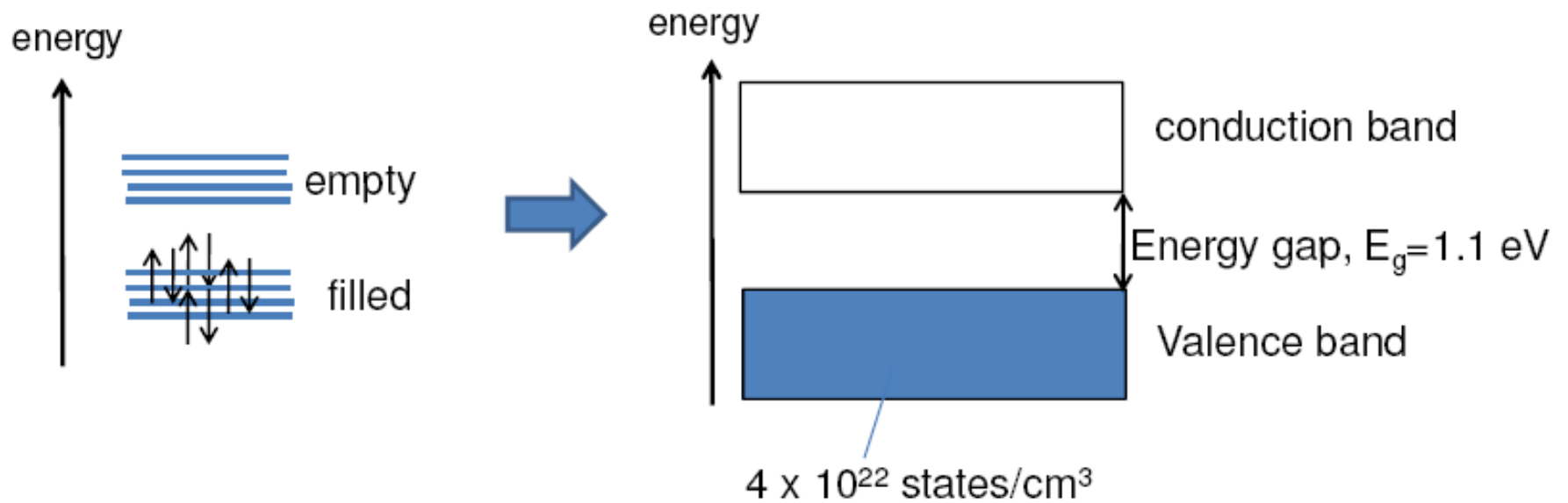


empty

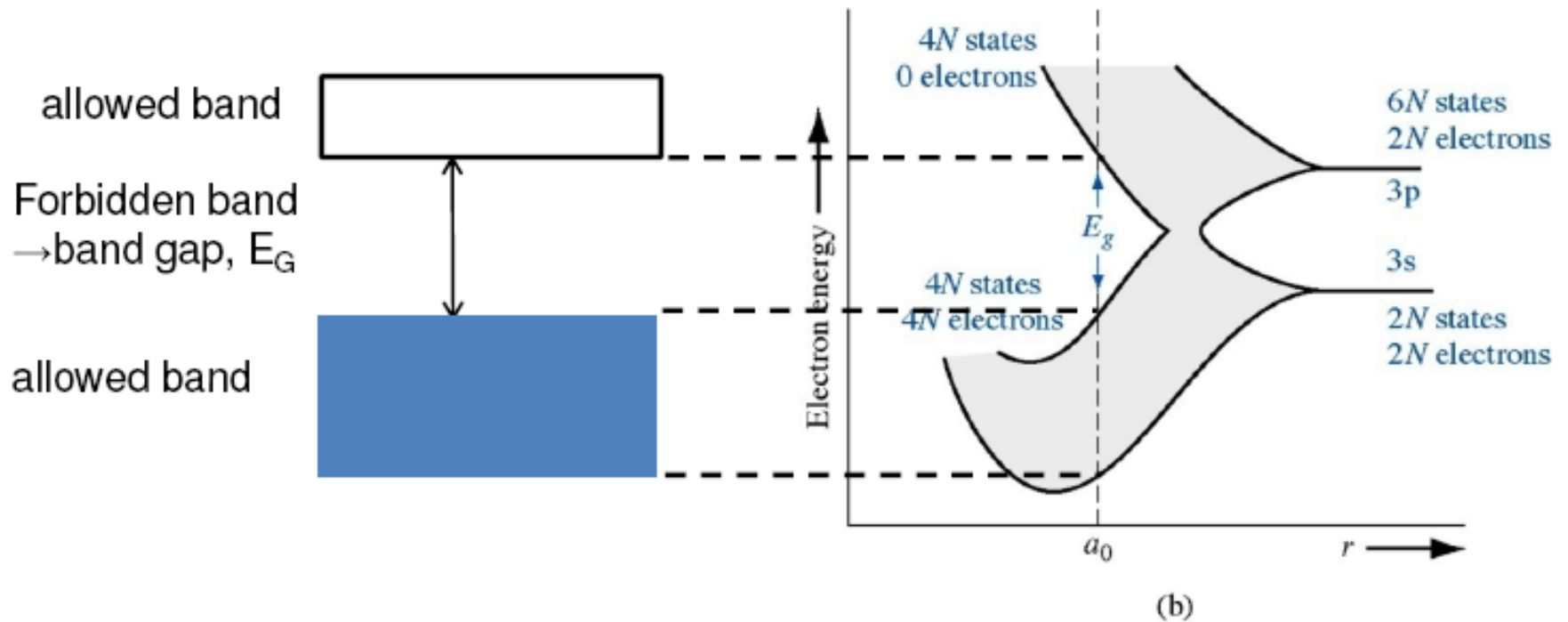


filled

## Si crystal ( $10^{22}$ atoms/cm<sup>3</sup>)



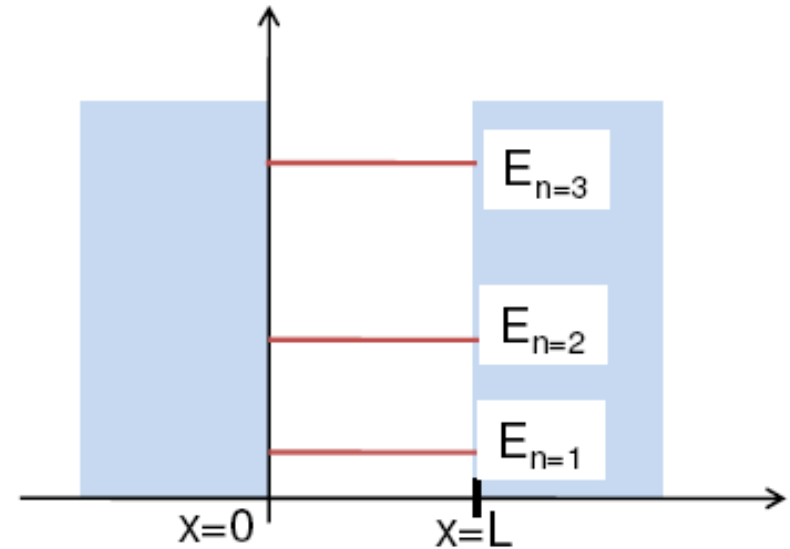
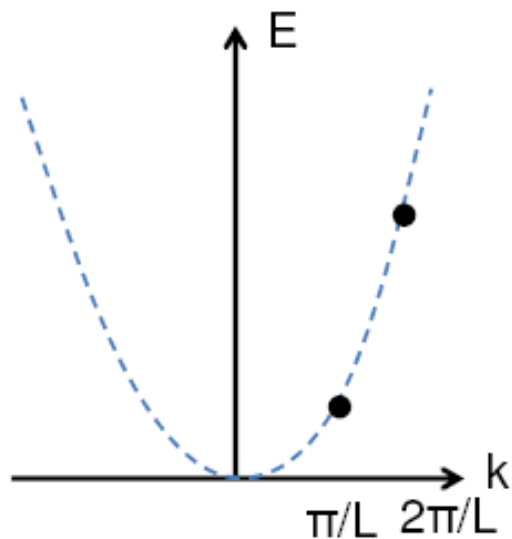
Actual band structure ← “calculated by quantum mechanics”



## E-k diagram for electron in quantum well

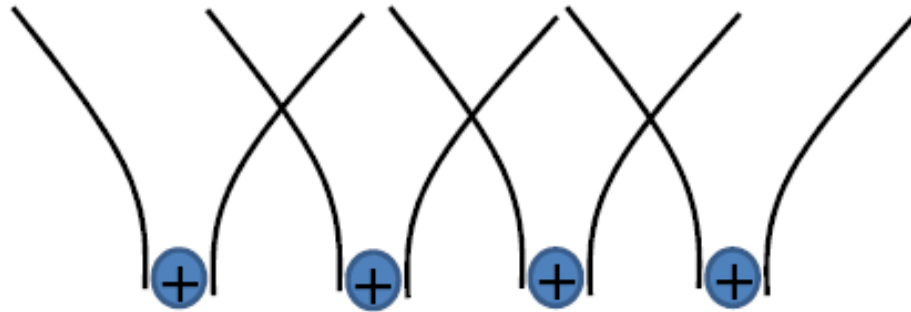
$$E = \frac{\hbar^2}{2m} \left( \frac{\pi}{L} \right)^2 n^2$$

$$E = \frac{\hbar^2 k^2}{2m} \quad k = \left( \frac{\pi}{L} \right) n$$



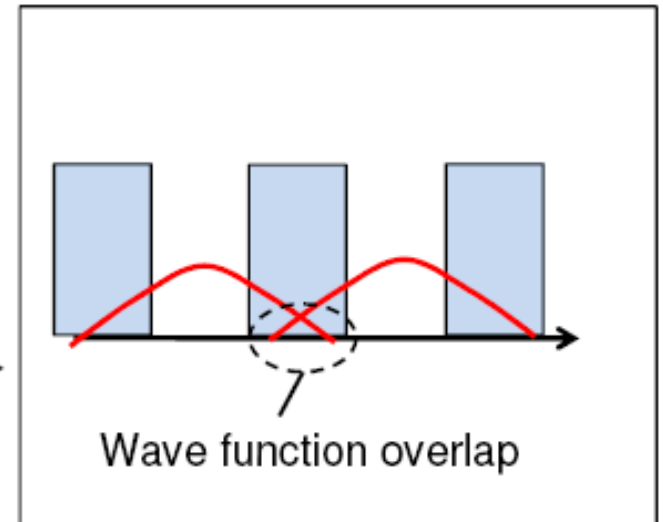
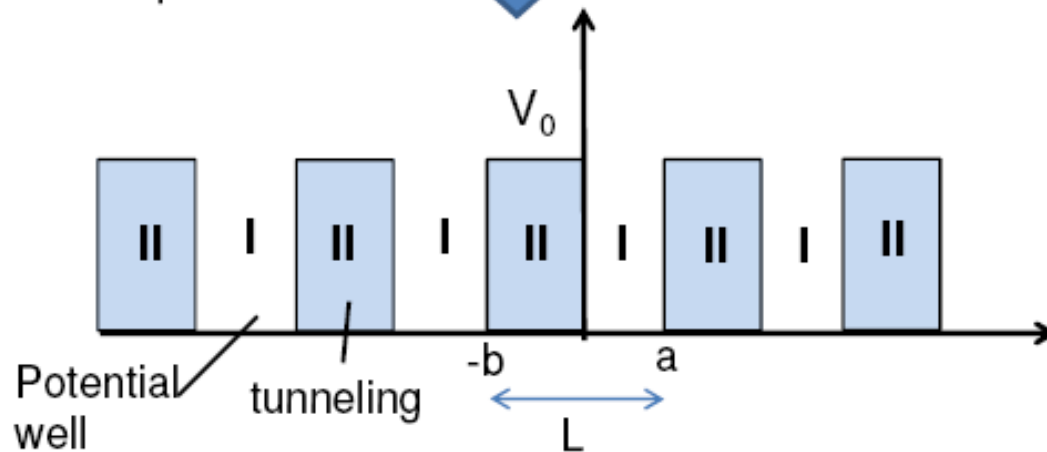
E-k diagram for electron in crystal? ➡ The Kronig-Penney Model

# The Kronig-Penney Model



$$V(r) = \frac{-e^2}{4\pi\epsilon_0 r}$$

Periodic potential



Determine a relationship between  $k$ ,  $E$  and  $V_0$

The Schrödinger time-independent wave equation for the motion of an electron along X-direction is given by:

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} [E - V(x)] \psi = 0 \quad 1$$

The energies and wave functions of electrons associated with this model can be calculated by solving time-independent one-dimensional Schrödinger's wave equations for the two regions I and II as shown in Fig

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E\psi = 0 \quad \text{for } 0 < x < a \quad 2 \qquad \frac{d^2\psi}{dx^2} + \alpha^2\psi = 0 \quad 6$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} [E - V_0] \psi = 0 \quad \text{for } -b < x < 0 \quad 3 \qquad \frac{d^2\psi}{dx^2} - \beta^2\psi = 0 \quad 7$$

We define two real quantities (say)  $\alpha$  and  $\beta$  such that:

$$\alpha^2 = \frac{2mE}{\hbar^2} \quad 4 \quad \text{and} \quad \beta^2 = \frac{2m}{\hbar^2} (V_0 - E) \quad 5$$

According to Bloch's theorem, the wavefunction solution of the Schrödinger equation when the potential is periodic [ $V(x+a)=V(x)$ ] and to make sure the function  $u(x)$  is also continuous and smooth, can be written as:

$$\psi(x) = e^{ikx}u(x). \quad 8$$

Where  $u(x)$  is a periodic function which satisfies  $u(x+a) = u(x)$ .

$$\frac{d^2u}{dx^2} + 2ik\frac{du}{dx} + (\alpha^2 - k^2) = 0 \quad 0 < x < a \quad 9$$

$$\frac{d^2u}{dx^2} + 2ik\frac{du}{dx} - (\beta^2 + k^2) = 0 \quad -b < x < 0 \quad 10$$

Solutions of  
above these  
two  
equations

$$u_1 = Ae^{i(\alpha-k)x} + Be^{-i(\alpha+k)x} \quad 0 < x < a \quad 11$$

$$u_2 = Ce^{(\beta-ik)x} + De^{-(\beta+ik)x} \quad -b < x < 0 \quad 12$$



## Boundary conditions

$$u_1(0) = u_2(0) \quad 13 \quad \longrightarrow$$

$$A + B = C + D \quad 17$$

$$\left(\frac{du_1}{dx}\right)_{x=0} = \left(\frac{du_2}{dx}\right)_{x=0} \quad 14 \quad \longrightarrow$$

$$\beta C - \beta D = i\alpha A - i\alpha B \quad 18$$

$$u_1(a) = u_2(-b) \quad 15 \quad \longrightarrow$$

$$Ae^{i(\alpha-k)a} + Be^{-i(\alpha+k)a} = Ce^{-(\beta-ik)b} + De^{(\beta+ik)b} \quad 19$$

$$\left(\frac{du_1}{dx}\right)_{x=a} = \left(\frac{du_2}{dx}\right)_{x=-b} \quad 16 \quad \longrightarrow$$

$$i(\alpha-k)Ae^{i(\alpha-k)a} - i(\alpha+k)Be^{-i(\alpha+k)a} = (\beta-ik)Ce^{-(\beta-ik)b} - (\beta+ik)De^{(\beta+ik)b} \quad 20$$

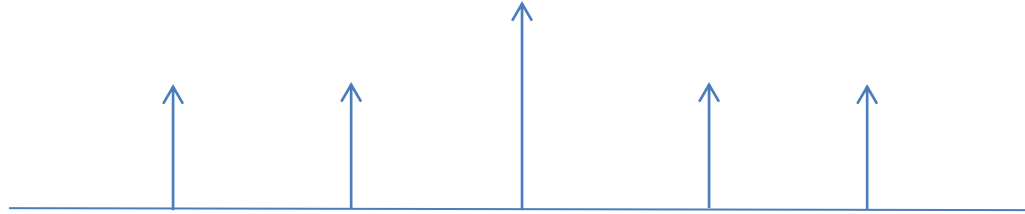
For a nontrivial solution to the above four equations for A,B,C, and D the determinant of their coefficients must vanish.

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ \beta & -\beta & i\alpha & -i\alpha \\ e^{(ik-\beta)b} & e^{(ik+\beta)b} & e^{i\alpha(\alpha-k)} & e^{i\alpha(\alpha+k)} \\ (\beta-ik)e^{(ik-\beta)b} & -(\beta+ik)e^{(ik+\beta)b} & i(\alpha-k)e^{i\alpha(\alpha-k)} & -i(\alpha+k)e^{i\alpha(\alpha+k)} \end{vmatrix} = 0 \quad 21$$

$$\frac{\beta^2 - \alpha^2}{2\alpha\beta} \sinh(\beta b) \sin(\alpha a) - \cosh(\beta b) \cos(\alpha a) = \cos(a+b)k \quad 22$$

$$\frac{\beta^2 - \alpha^2}{2\alpha\beta} \sinh(\beta b) \sin(\alpha a) - \cosh(\beta b) \cos(\alpha a) = \cos(a + b)k \quad 22$$

*If  $V_0 \rightarrow \infty$  i.e.  $b \rightarrow 0$*

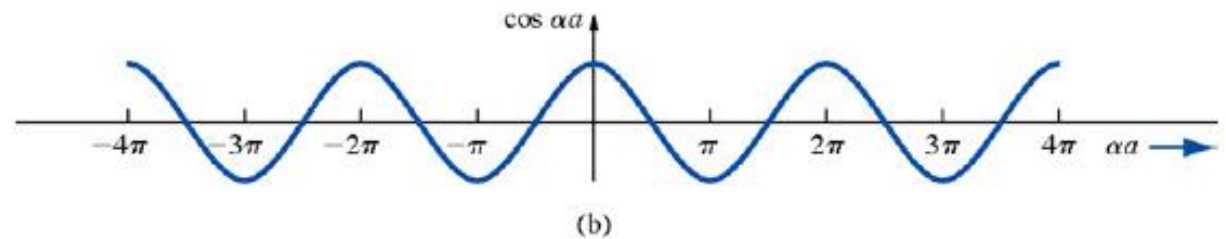
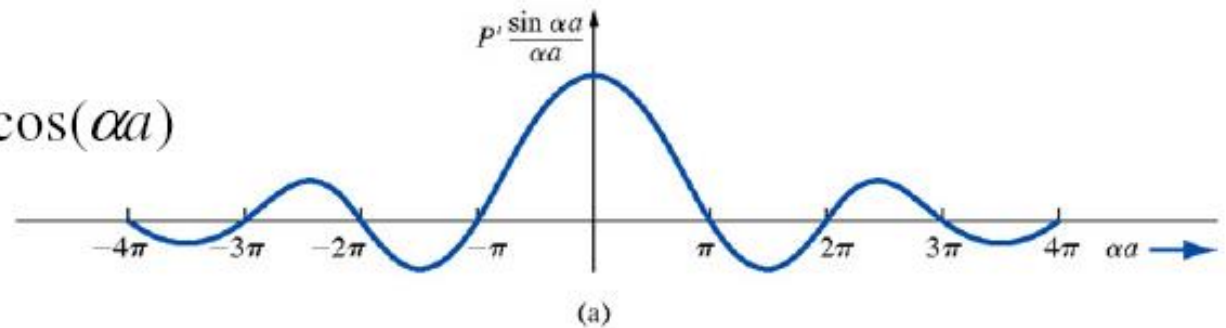


Thus eq. 22 reduces to

$$P \frac{\sin(\alpha a)}{\alpha a} + \cos(\alpha a) = \cos(ka), \quad \text{where } P = \frac{mV_0 b a}{\hbar^2} \quad 23$$

Left side

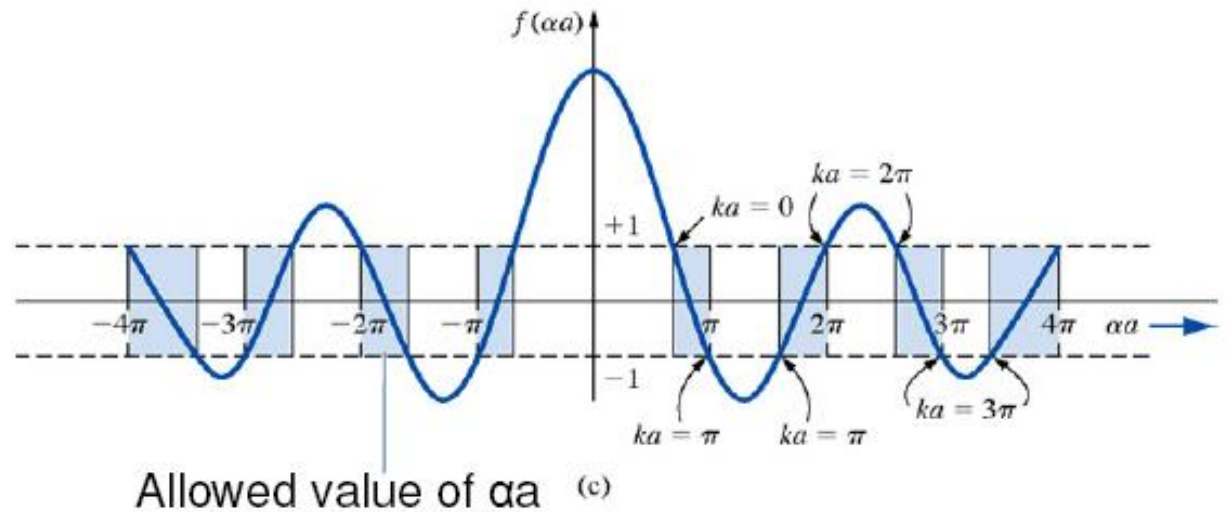
$$f(\alpha a) = P' \frac{\sin(\alpha a)}{\alpha a} + \cos(\alpha a)$$



Right side

$$f(\alpha a) = \cos(ka)$$

Value must be  
between -1 and 1

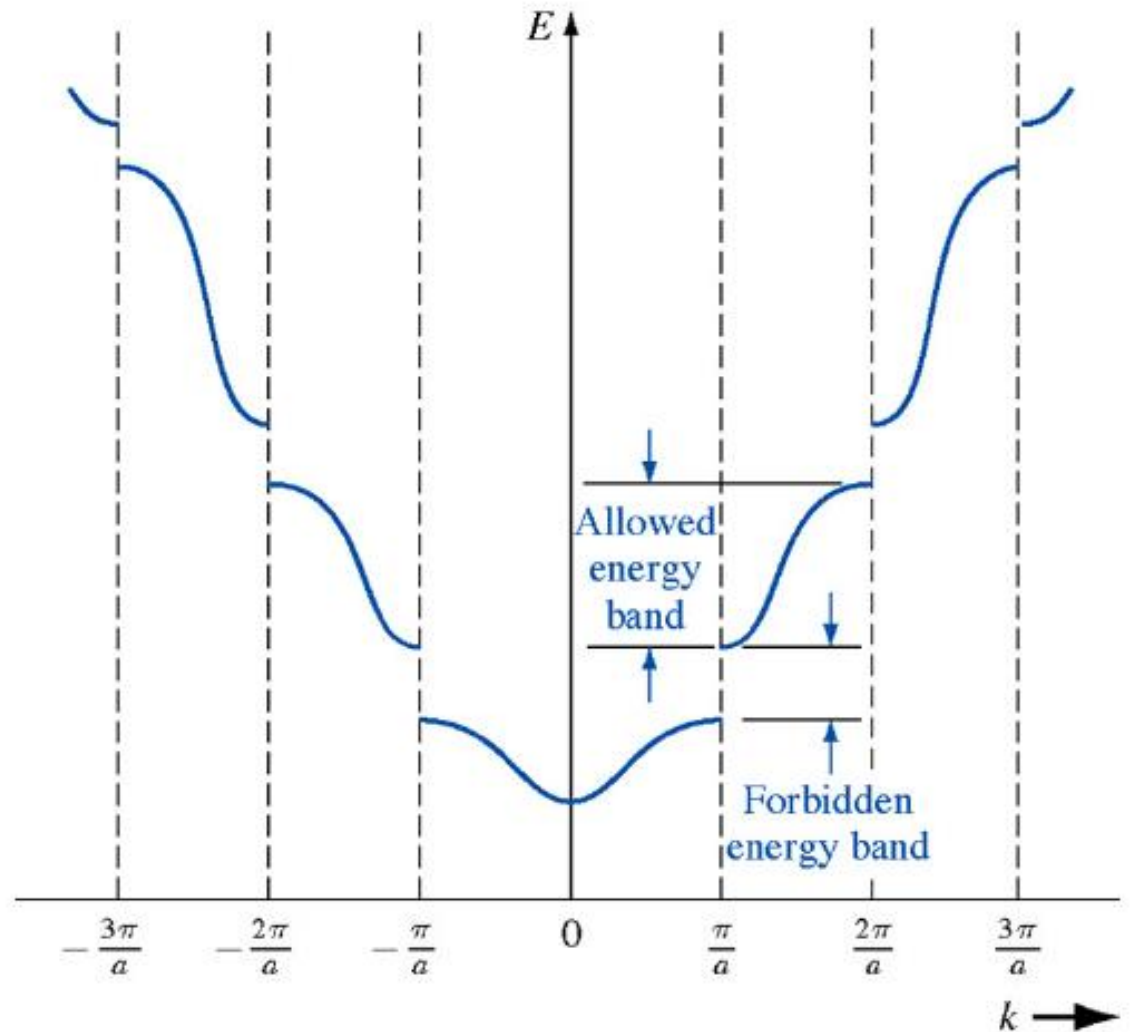


$$\alpha^2 = \frac{2mE}{\hbar^2}$$

$$E = \frac{\alpha^2 \hbar^2}{2m}$$

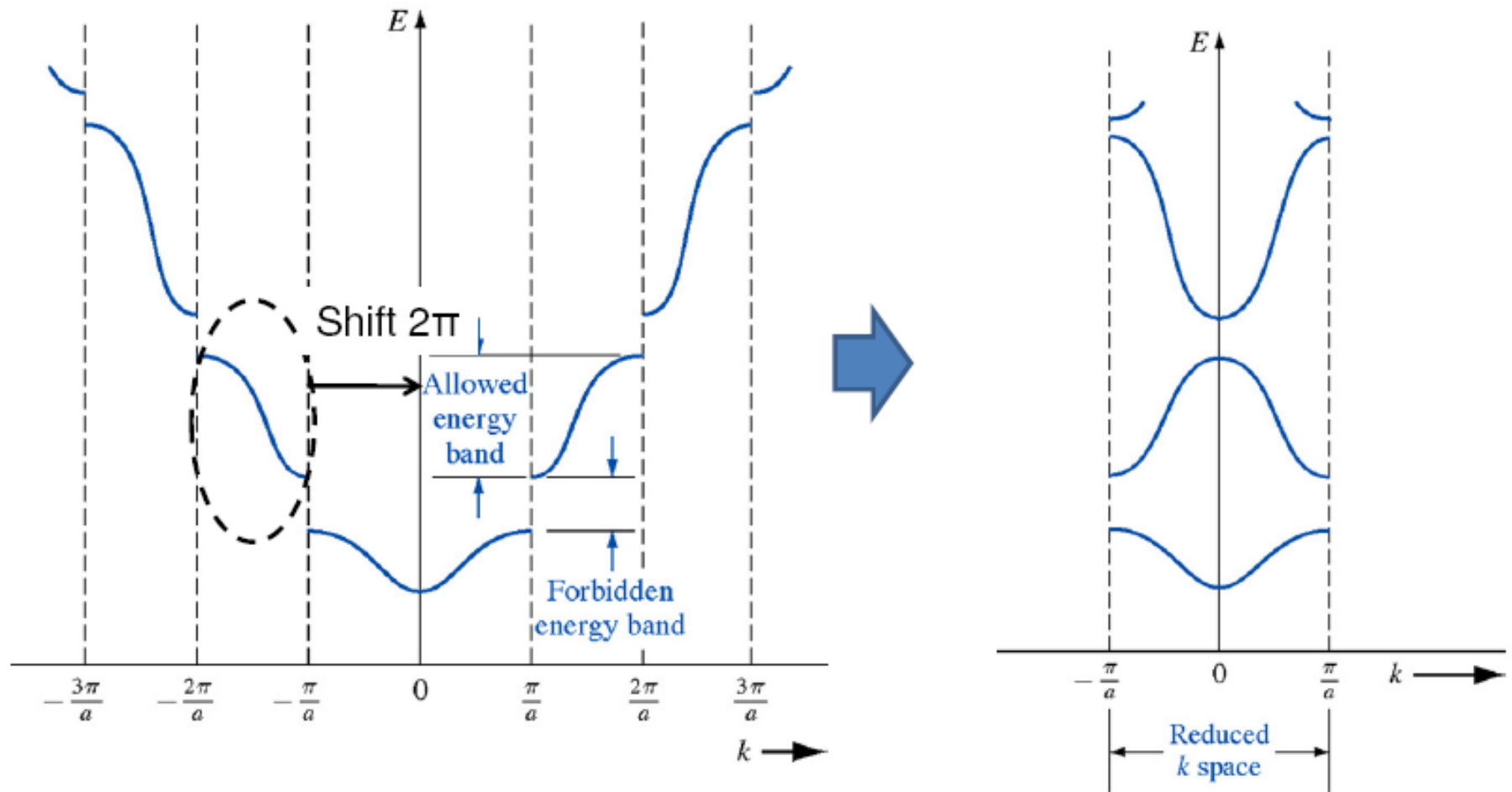
Plot E-k →

Discontinuity of E

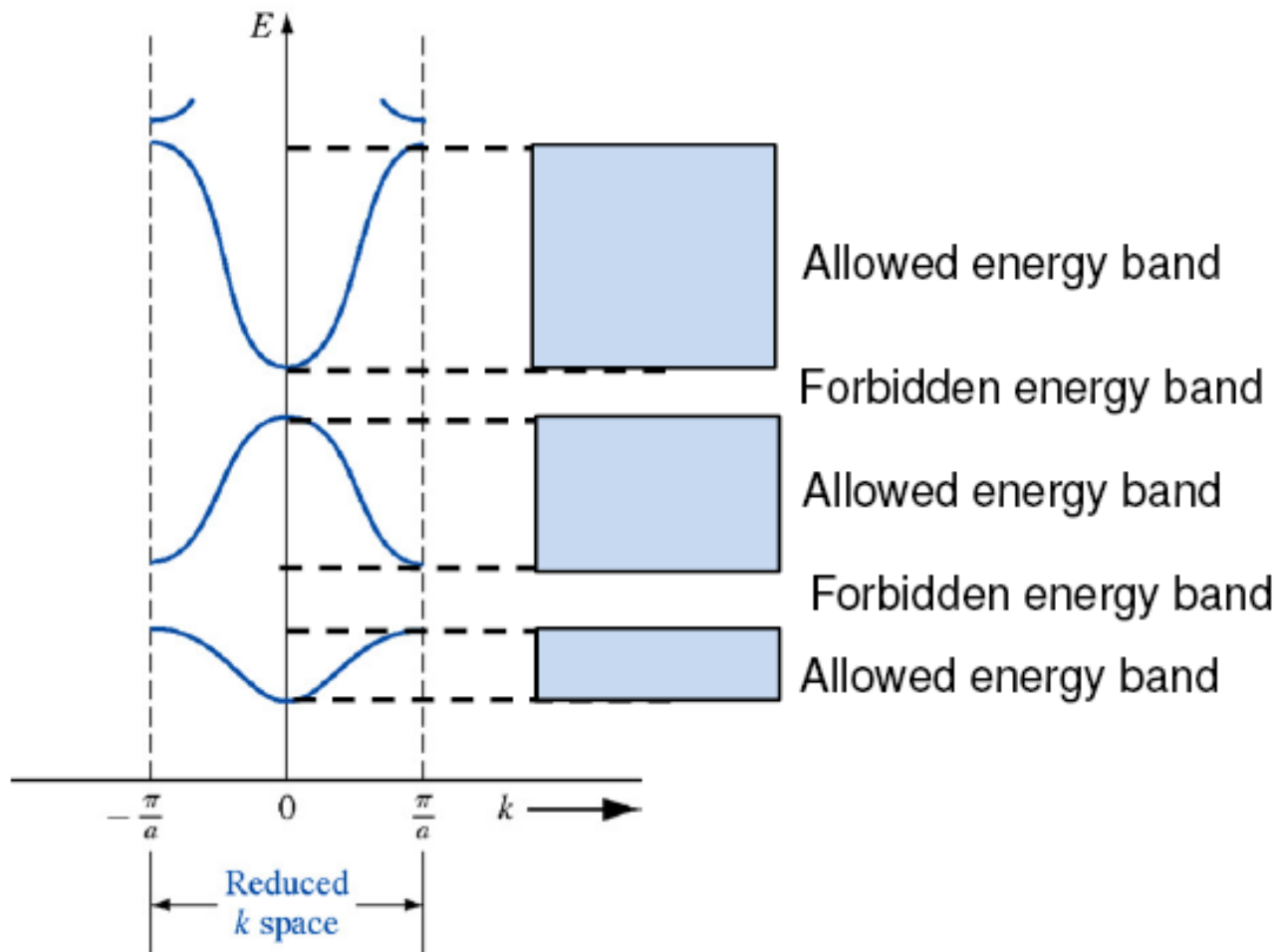


Right side

$$f(\alpha a) = \cos(ka) = \cos(ka + 2n\pi) = \cos(ka = 2n\pi)$$

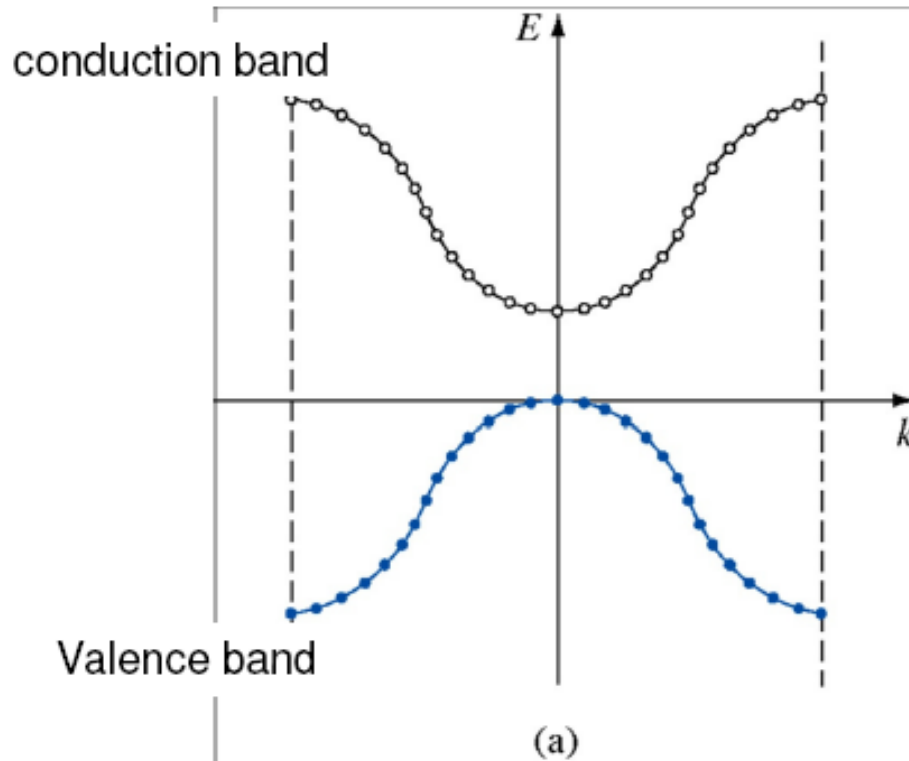


From the Kronig-Penney Model (1 dimensional periodic potential function)

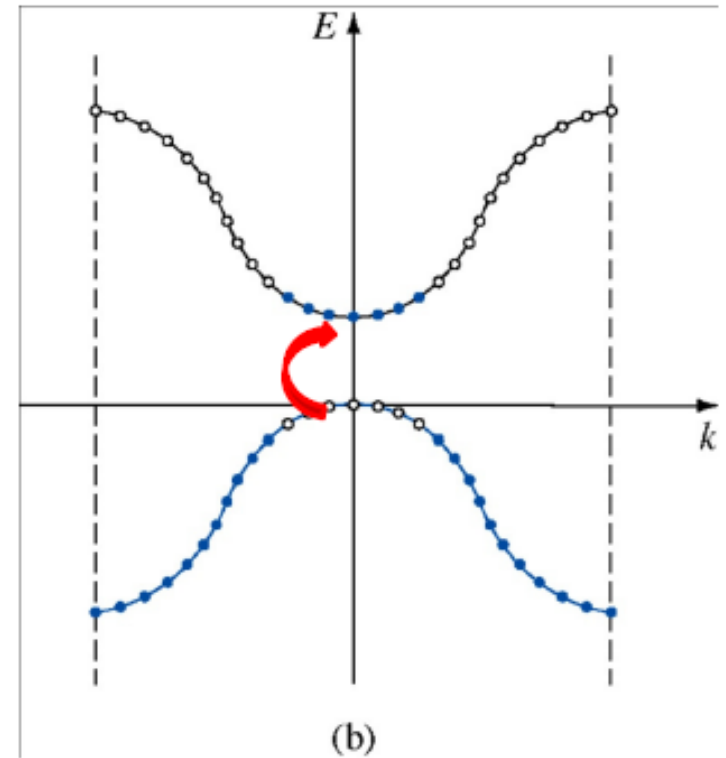


First Brillouin zone

## E versus k energy band



$T = 0 \text{ K}$

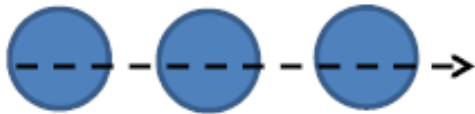


$T > 0 \text{ K}$

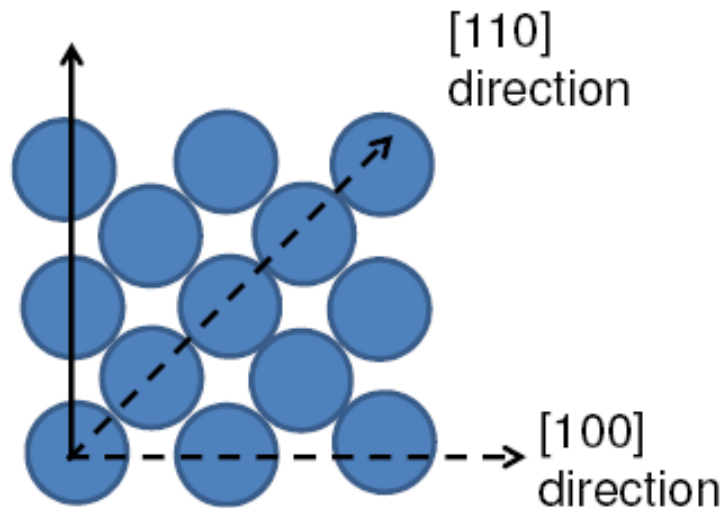
When no external force is applied, electron and “empty state” distributions are symmetrical with  $k$

## Extension to three dimensions

1 dimensional model (kronig-Penney Model)



1 potential pattern



Different direction



Different potential patterns

**E-k diagram is given by a function of the direction in the crystal**



## E-k diagram of Si

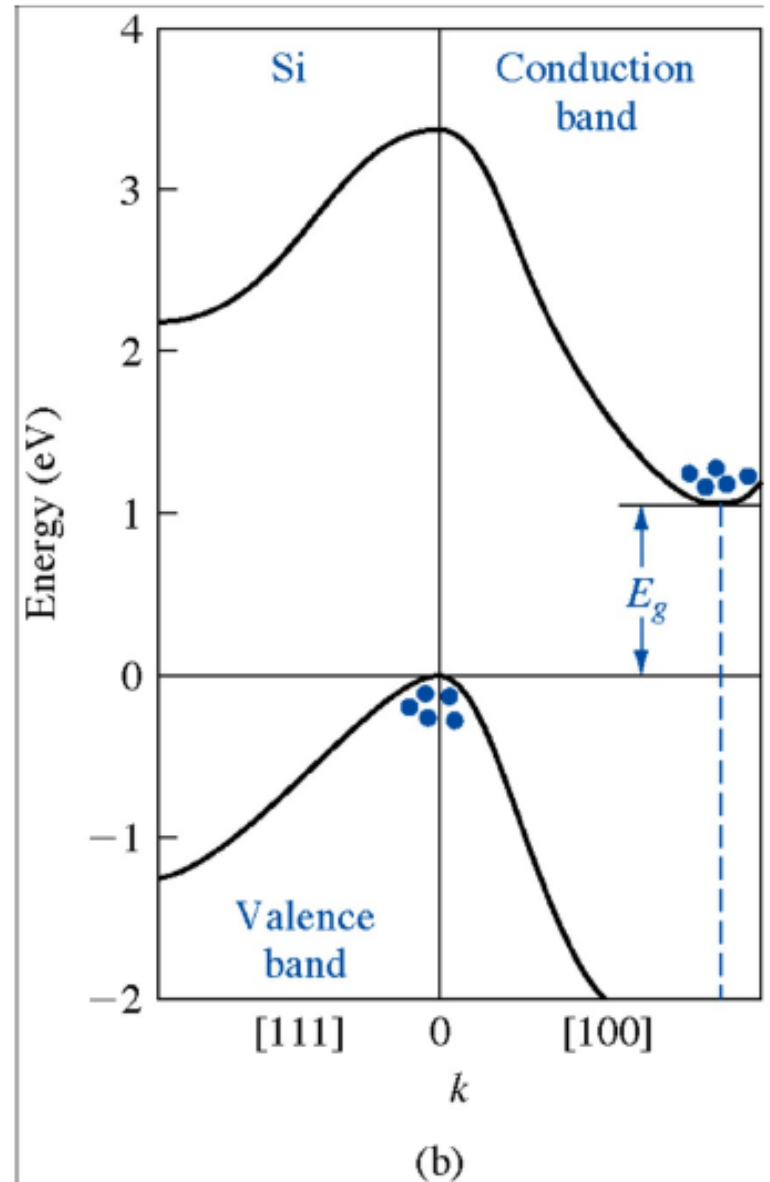
- **Energy gap**; Conduction band minimum – valence band maximum

$$E_g = 1 \text{ eV}$$

- **Indirect bandgap**;  
Maximum valence band and minimum conduction band do not occur at the same  $k$



Not suitable for optical device application (laser)



## E-k diagram of GaAs

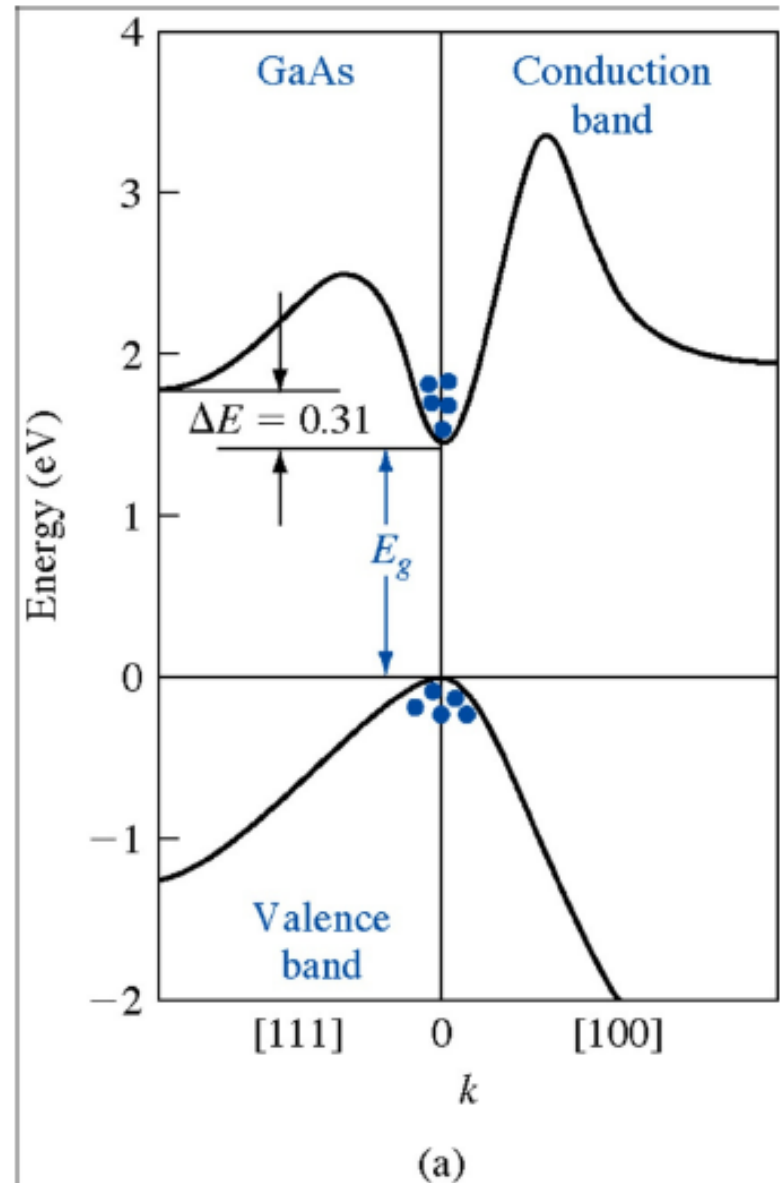
➤  $E_g = 1.4 \text{ eV}$

➤ Direct band gap



suitable for optical device application  
(laser)

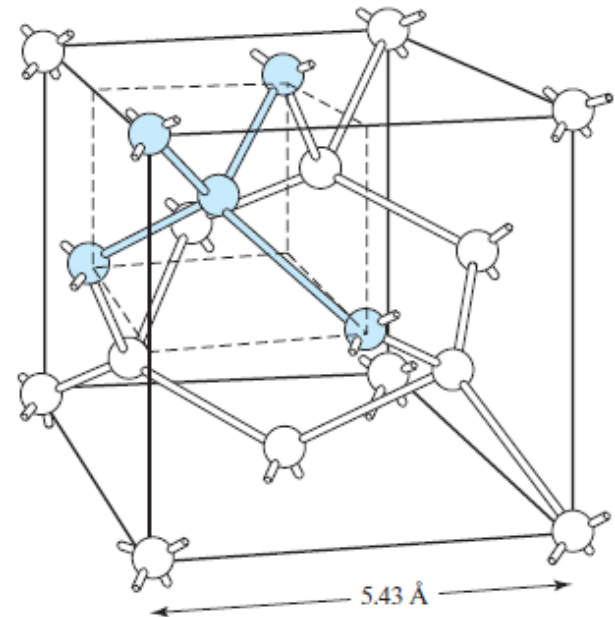
➤ Smaller effective mass than Si.  
(curvature of the curve)



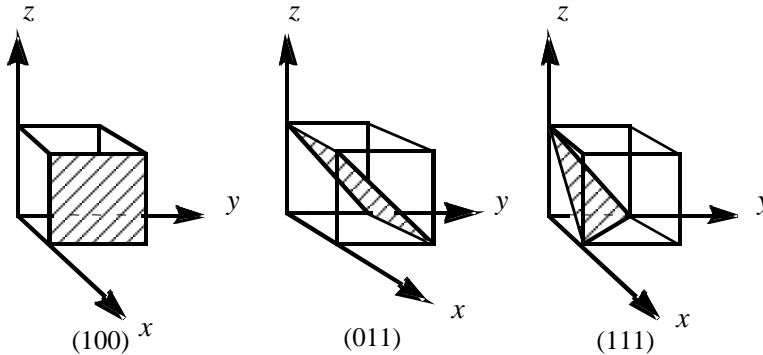
# Electrons and Holes in Semiconductors

## 1.1 Silicon Crystal Structure

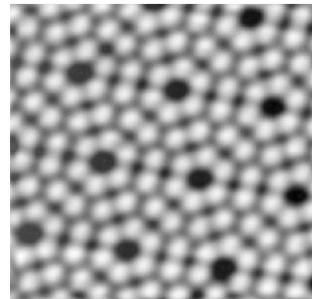
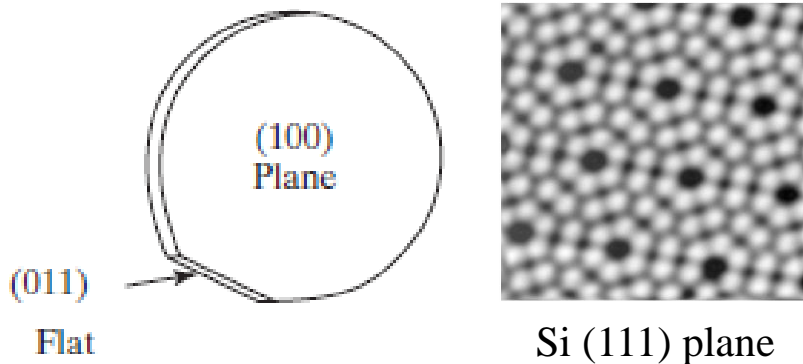
- $1s^2 2s^2 2p^6 3s^2 3p^2$
- ***Unit cell*** of silicon crystal is cubic.
- ***Each Si atom has 4 nearest neighbors.***
- The length of the unit cell, e.g., 5.43 Å



# Silicon Wafers and Crystal Planes



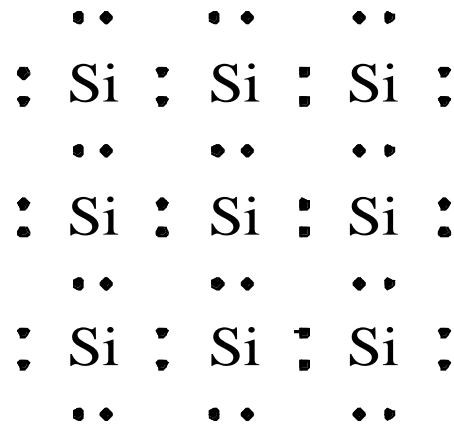
- The standard notation for crystal planes is based on the cubic unit cell.



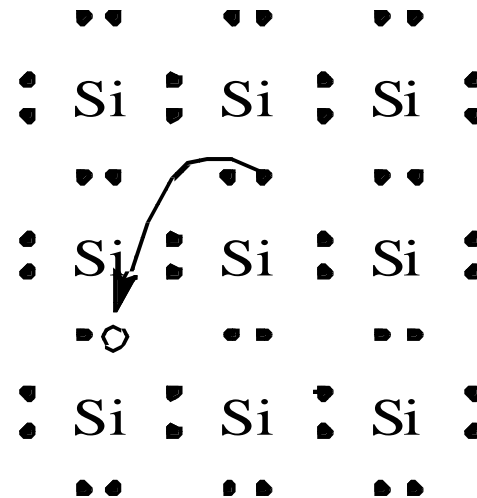
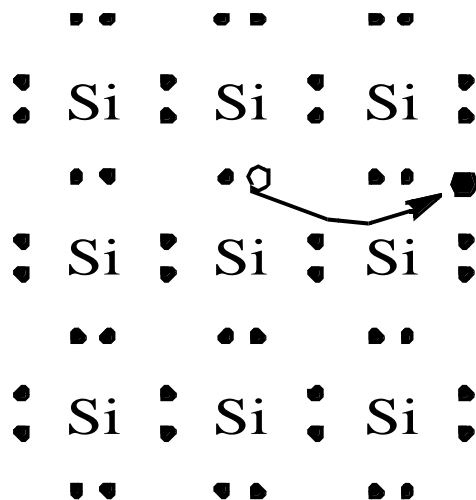
Si (111) plane

- Silicon wafers are usually cut along the (100) plane with a flat or notch to help orient the wafer during IC fabrication.

## 1.2 Bond Model of Electrons and Holes

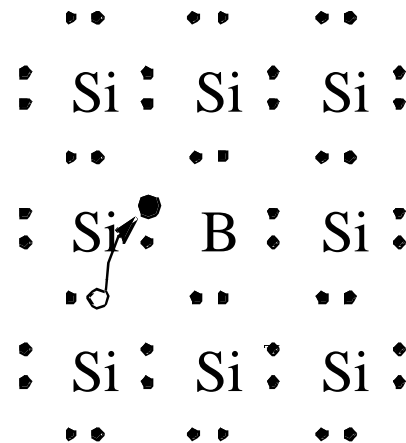
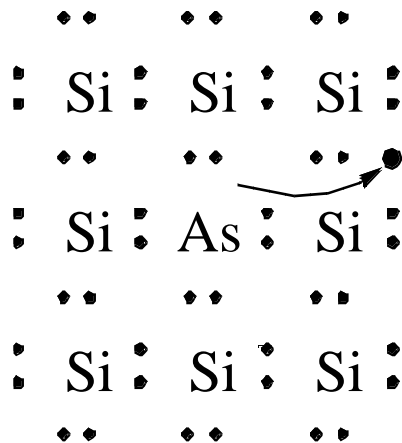


- Silicon crystal in a two-dimensional representation.



- When an electron breaks loose and becomes a *conduction electron*, a *hole* is also created.

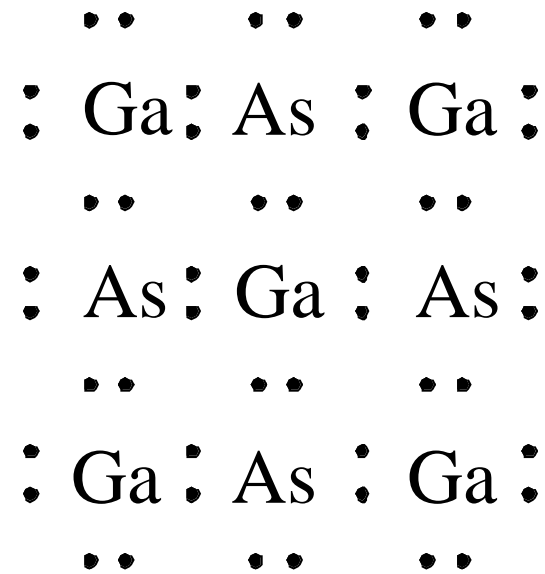
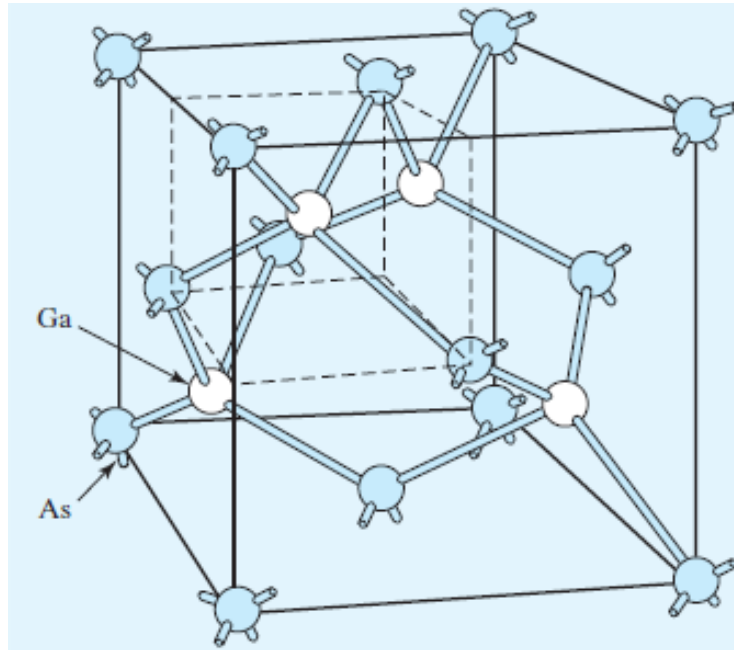
# Dopants in Silicon



- As, a Group V element, introduces conduction electrons and creates ***N-type silicon***, and is called a ***donor***.
- B, a Group III element, introduces holes and creates ***P-type silicon***, and is called an ***acceptor***.
- Donors and acceptors are known as dopants. Dopant ionization energy ~50meV (very low).

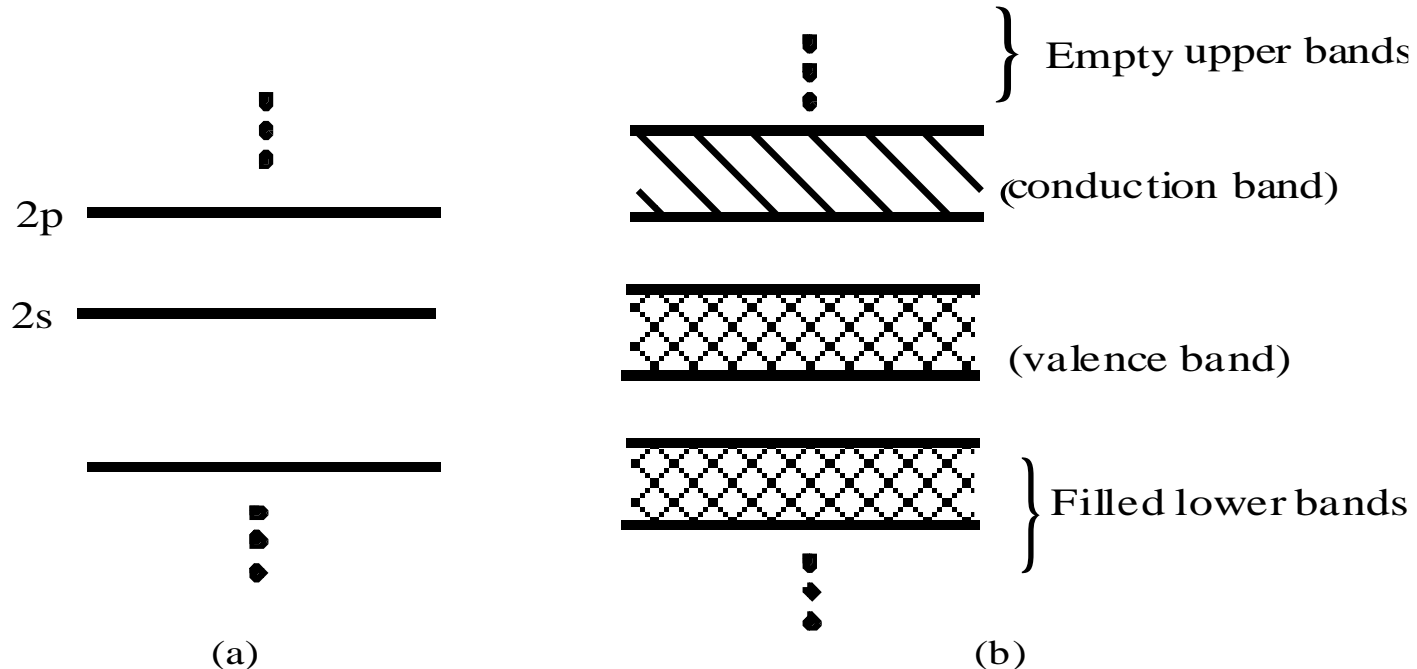
Hydrogen: 
$$E_{ion} = \frac{m_0 q^4}{8\epsilon_0^2 h^2} = 13.6 \text{ eV}$$

# ***GaAs, III-V Compound Semiconductors, and Their Dopants***



- GaAs has the same crystal structure as Si.
- GaAs, GaP, GaN are III-V compound semiconductors, important for optoelectronics.
- Which group of elements are candidates for donors? acceptors?

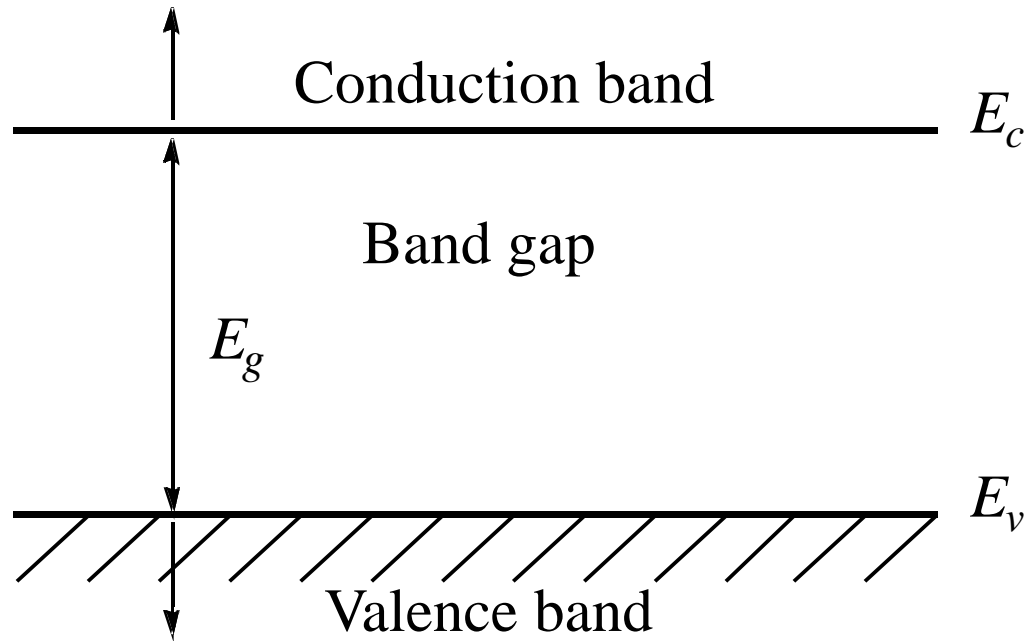
## 1.3 Energy Band Model



- Energy states of Si atom (a) expand into energy bands of Si crystal (b).
- The lower bands are filled and higher bands are empty in a semiconductor.
- The highest filled band is the *valence band*.
- The lowest empty band is the *conduction band*.

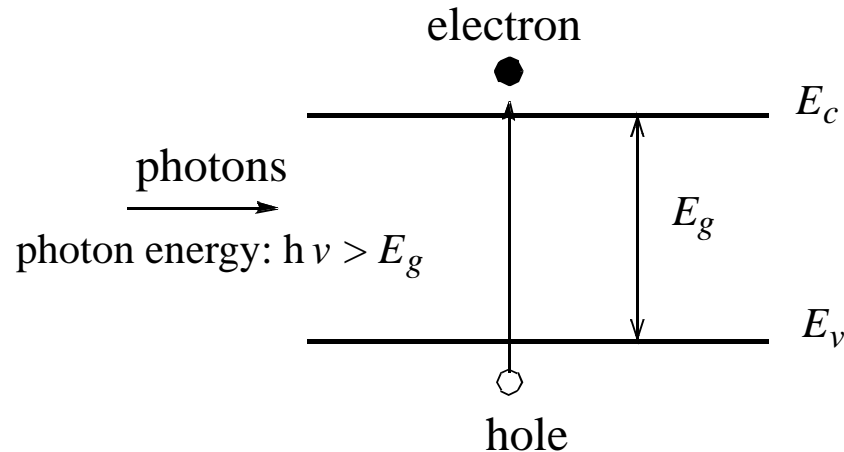


### 1.3.1 Energy Band Diagram



- ***Energy band diagram*** shows the bottom edge of conduction band,  $E_c$ , and top edge of valence band,  $E_v$ .
- $E_c$  and  $E_v$  are separated by the ***band gap energy***,  $E_g$ .

# Measuring the Band Gap Energy by Light Absorption

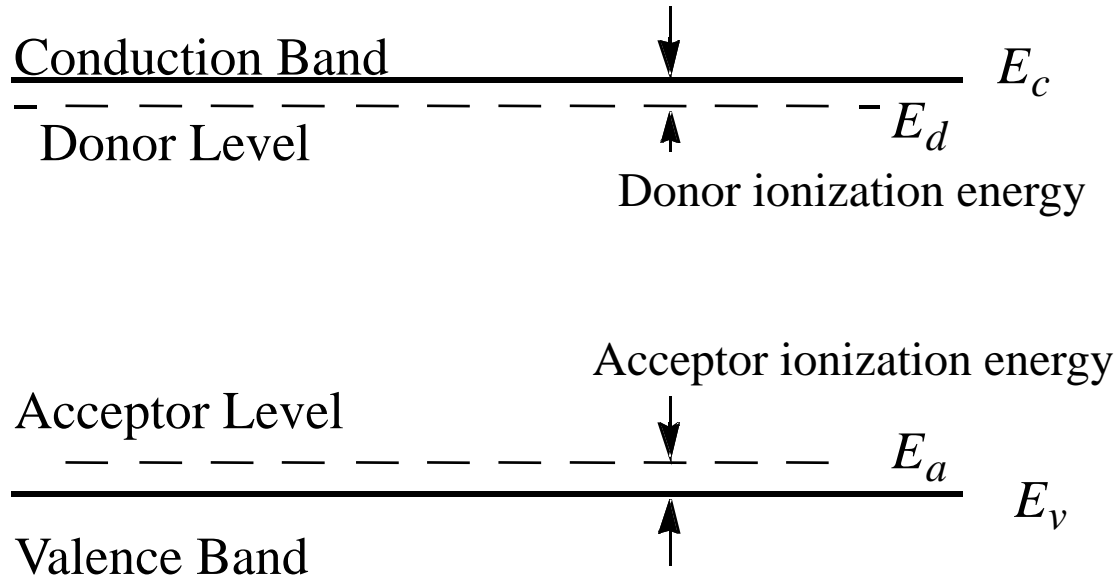


- $E_g$  can be determined from the minimum energy ( $h\nu$ ) of photons that are absorbed by the semiconductor.

**Bandgap energies of selected semiconductors**

Semi-conductor	InSb	Ge	Si	GaAs	GaP	ZnSe	Diamond
$E_g$ (eV)	0.18	0.67	1.12	1.42	2.25	2.7	6

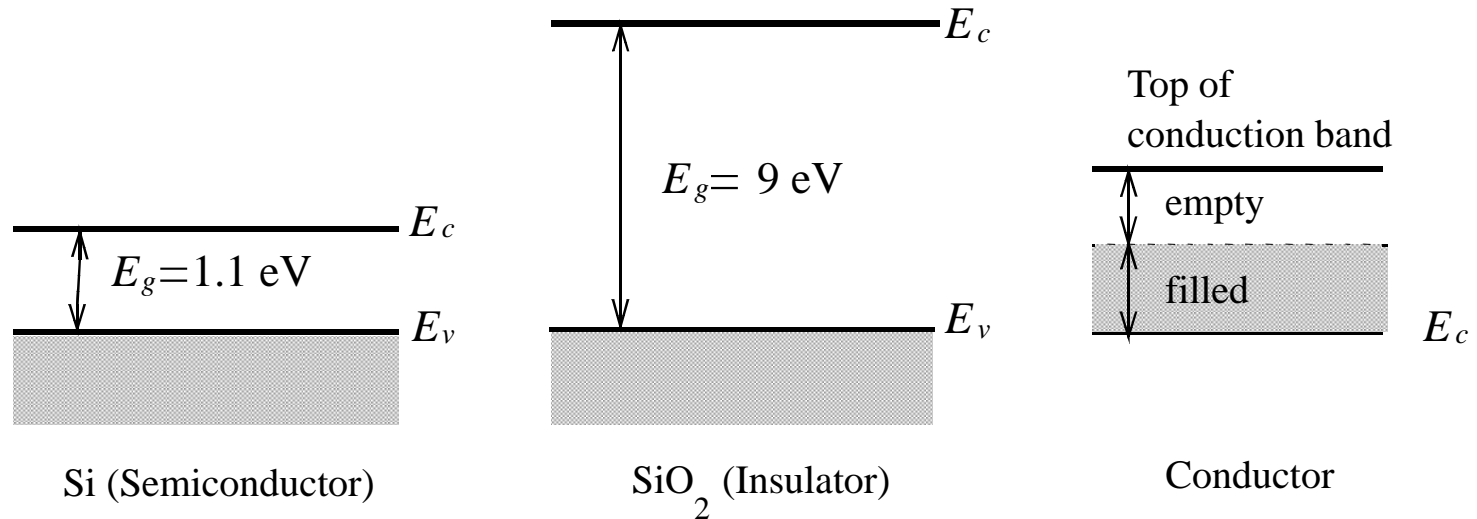
## 1.3.2 Donor and Acceptor in the Band Model



### Ionization energy of selected donors and acceptors in silicon

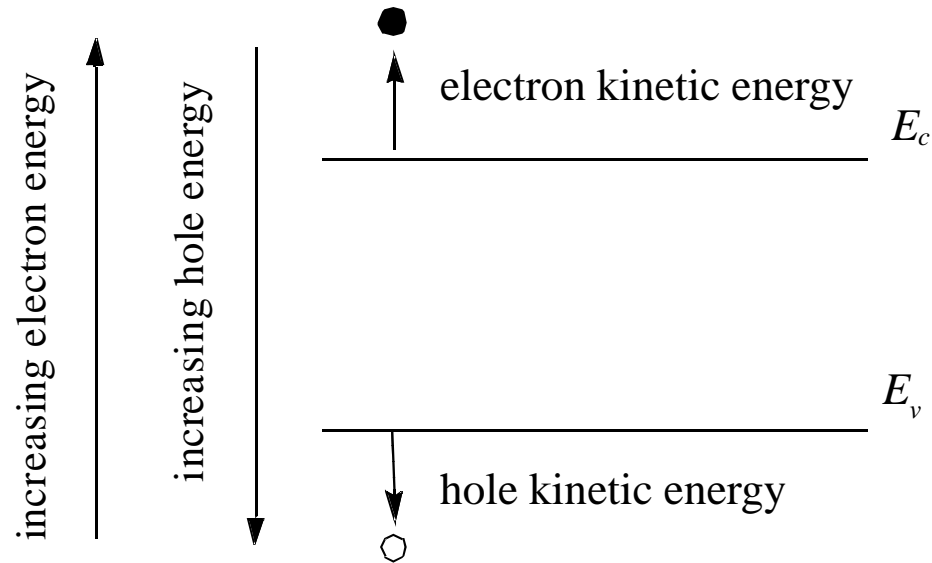
	Donors			Acceptors		
Dopant	Sb	P	As	B	Al	In
Ionization energy, $E_c - E_d$ or $E_a - E_v$ (meV)	39	44	54	45	57	160

## 1.4 Semiconductors, Insulators, and Conductors



- Totally filled bands and totally empty bands do not allow current flow. (Just as there is no motion of liquid in a totally filled or totally empty bottle.)
- Metal conduction band is half-filled.
- Semiconductors have lower  $E_g$ 's than insulators and can be doped.

## 1.5 Electrons and Holes



- Both electrons and holes tend to seek their lowest energy positions.
- Electrons tend to fall in the energy band diagram.
- Holes float up like bubbles in water.

Without the crystal lattice, the hole cannot exist. It is an artifact of the periodic potential ( $E_p$ ) created by the crystal.

## 1.5.1 Effective Mass

In an electric field,  $\mathbf{E}$ , an electron or a hole accelerates.

$$a = \frac{-q\mathcal{E}}{m_n} \quad \text{electrons}$$

$$a = \frac{q\mathcal{E}}{m_p} \quad \text{holes}$$

### Electron and hole effective masses

	Si	Ge	GaAs	InAs	AlAs
$m_n/m_0$	0.26	0.12	0.068	0.023	2
$m_p/m_0$	0.39	0.3	0.5	0.3	0.3

## 1.5.2 How to Measure the Effective Mass

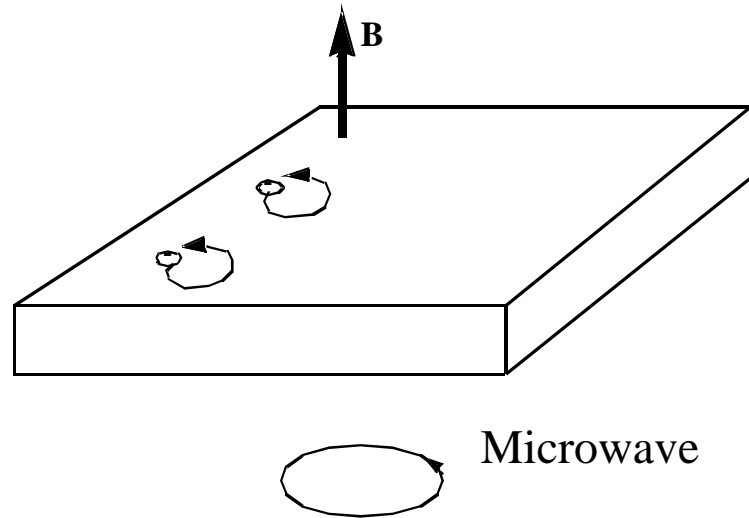
### Cyclotron Resonance Technique

Centripetal force = Lorentzian force

$$\frac{m_n v^2}{r} = qvB$$

$$v = \frac{qBr}{m_n}$$

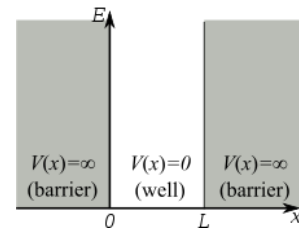
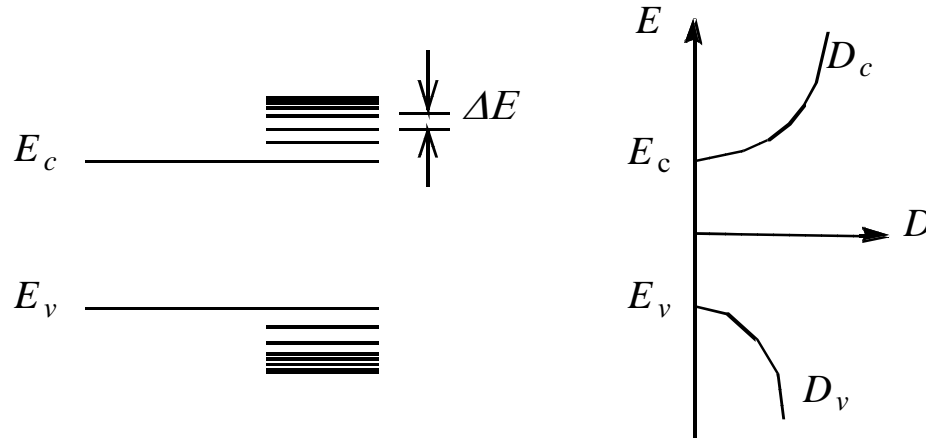
$$f_{cr} = \frac{v}{2\pi r} = \frac{qB}{2\pi m_n}$$



- $f_{cr}$  is the Cyclotron resonance frequency.
- It is independent of  $v$  and  $r$ .
- Electrons strongly absorb microwaves of that frequency.
- By measuring  $f_{cr}$ ,  $m_n$  can be found.

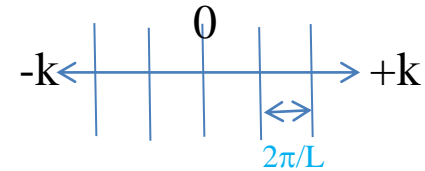


# 1.6 Density of States



$$k = \pm 2n \pi / L, n = 0, \pm 1, \pm 2, \dots$$

$$E_n = \frac{\hbar^2}{2m} (k)^2$$



$$= 2\pi \backslash L$$

$$= (L/2\pi)$$

$$D_c(E) \equiv \frac{\text{number of states in } \Delta E}{\Delta E \cdot \text{volume}} \left( \frac{1}{\text{eV} \cdot \text{cm}^3} \right)$$

$$D_c(E) \equiv \frac{8\pi m_n \sqrt{2m_n (E - E_c)}}{h^3}$$

$$D_v(E) \equiv \frac{8\pi m_p \sqrt{2m_p (E_v - E)}}{h^3}$$

$$N(E) = 2k (L/2\pi)$$

$$k = (2mE/\hbar^2)^{1/2}$$

$$N(E) = 2 (2mE/\hbar^2)^{1/2} (L/2\pi)$$

$$g(E) = dN(E)/dE \\ = L/\pi (2m/\hbar^2)^{1/2} E^{-1/2}$$

# Density of States

Consider a 3D free electron ( $V=0$ ) volume of length  $L$  and volume  $V$ .

$$\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \quad E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \quad k = \frac{n\pi}{L} \text{ and } n = 1, 2, 3, \dots$$

Inside this volume, in  $k$  space allowed states are uniformly distributed with a spacing  $= 2\pi/L$

The volume (in  $k$  space) per state is  $= (2\pi/L)^3$

The number of states per unit volume is thus  $= (L/2\pi)^3$

$$L^3 = V, \text{ so } = V/8\pi^3$$

Number of states within the volume is  $N(E) = (4/3)\pi k^3 * V/8\pi^3$

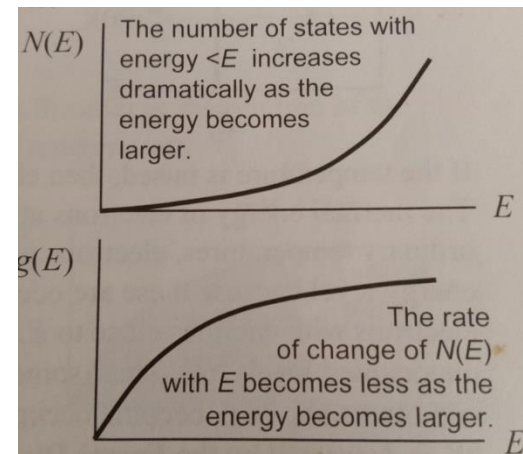
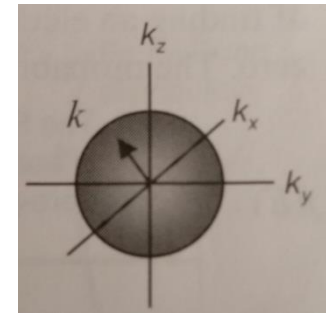
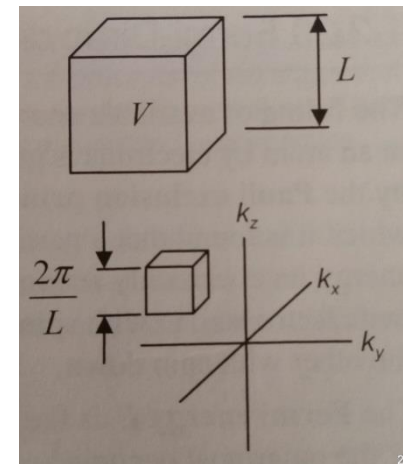
But  $k^2 = 2mE/\hbar^2$

$$k^3 = (2mE/\hbar^2)^{3/2}$$

And so  $N(E) = (4/3)\pi * V/8\pi^3 (2mE/\hbar^2)^{3/2}$

$$g(E) = dN(E)/dE = (3/2)(2m/\hbar^2) (4/3)\pi * V/8\pi^3 (2mE/\hbar^2)^{1/2}$$

$$= V/2\pi^2 (2m/\hbar^2)^{1/2} (E)^{1/2}$$



The density of states is zero at the bottom of the well as well as for negative energies.

The same analysis also applies to electrons in a semiconductor. The effective mass takes into account the effect of the periodic potential on the electron. The minimum energy of the electron is the energy at the bottom of the conduction band,  $E_c$ , so that the density of states for electrons in the conduction band is given by:

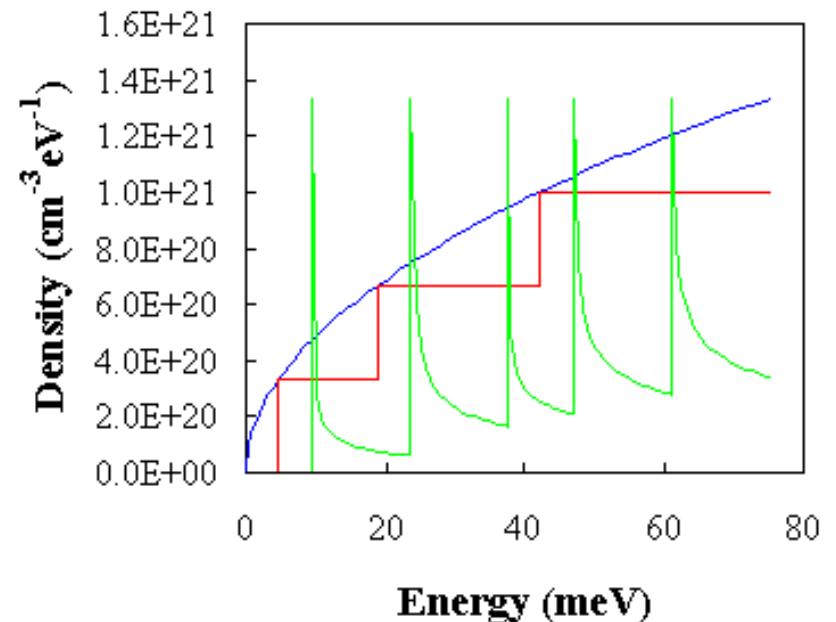
$$g_c(E) = \frac{8\pi\sqrt{2}}{h^3} m^{*3/2} \sqrt{E - E_c}, \text{ for } E \geq E_c$$

$$g_c(E) = 0, \text{ for } E < E_c$$

$$g_{c,3D} = \frac{dN_{3D}}{dE} = \frac{8\pi\sqrt{2}}{h^3} m^{*3/2} \sqrt{E - E_{\min}}, \text{ for } E \geq E_{\min}$$

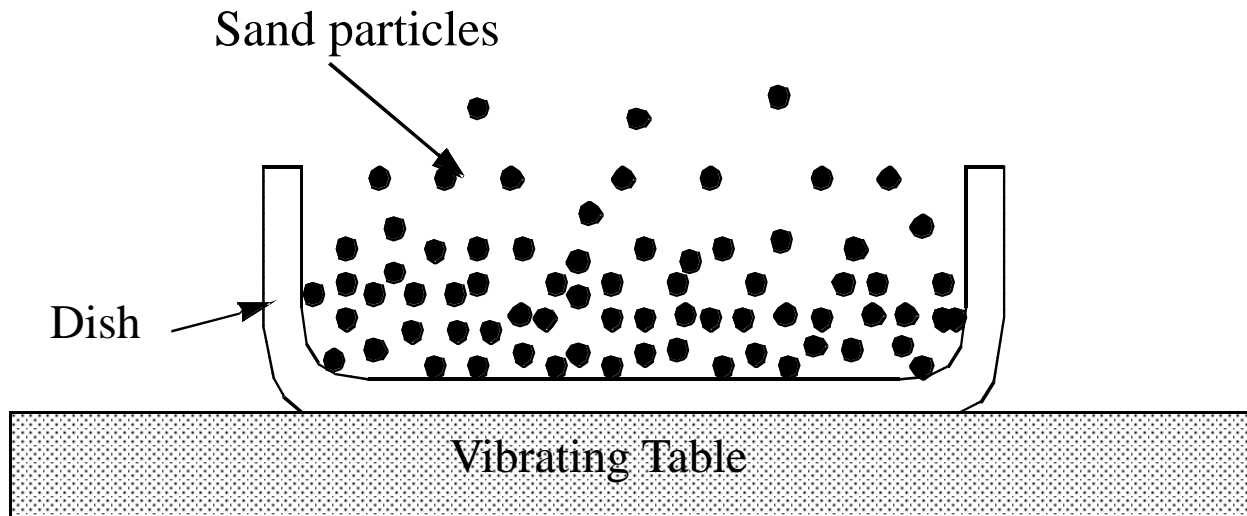
$$g_{c,2D} = \frac{dN_{2D}}{dE} = \frac{4\pi m^*}{h^2}, \text{ for } E \geq E_{\min}$$

$$g_{c,1D} = \frac{dN_{1D}}{dE} = \sqrt{\frac{2\pi m^*}{h^2}} \frac{1}{\sqrt{E - E_{\min}}}, \text{ for } E \geq E_{\min}$$



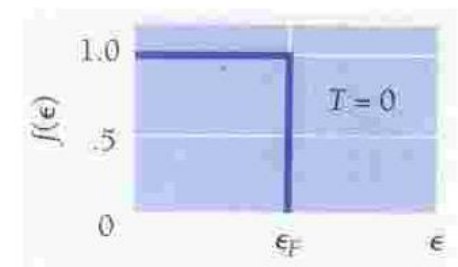
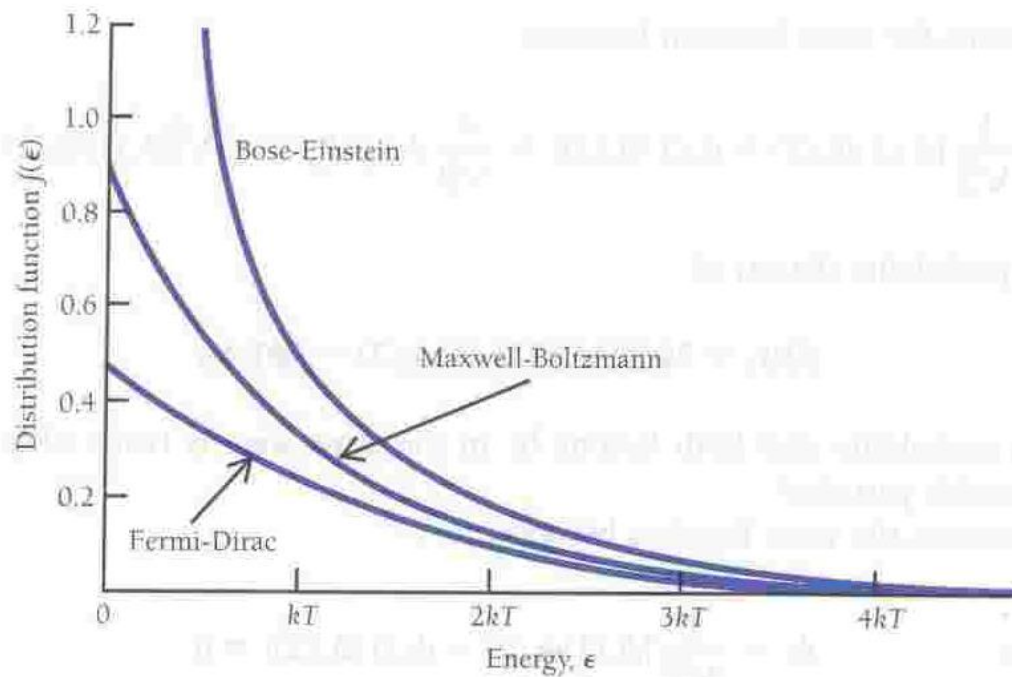
# ***1.7 Thermal Equilibrium and the Fermi Function***

## **1.7.1 An Analogy for Thermal Equilibrium**

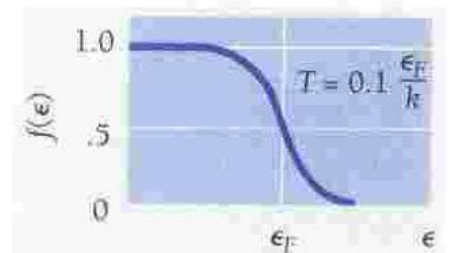


- There is a certain probability for the electrons in the conduction band to occupy high-energy states under the agitation of thermal energy.

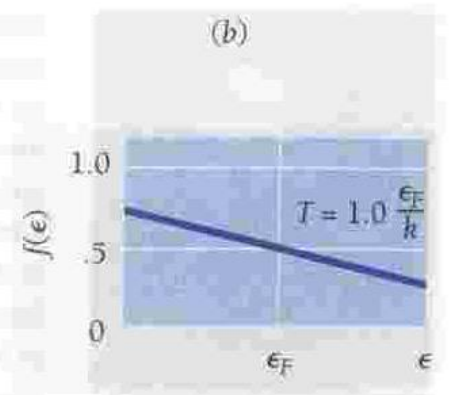
	Maxwell-Boltzmann	Bose-Einstein	Fermi-Dirac
Applies to systems of	Identical, distinguishable particles	Identical, indistinguishable particles that do not obey exclusion principle	Identical, indistinguishable particles that obey exclusion principle
Category of particles	Classical	Bosons	Fermions
Properties of particles	Any spin, particles far enough apart so wave functions do not overlap	Spin 0, 1, 2, ...; wave functions are symmetric to interchange of particle labels	Spin $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$ ; wave functions are antisymmetric to interchange of particle labels
Examples	Molecules of a gas	Photons in a cavity; phonons in a solid; liquid helium at low temperatures	Free electrons in a metal; electrons in a star whose atoms have collapsed (white dwarf stars)
Distribution function (number of particles in each state of energy $\epsilon$ at the temperature $T$ )	$f_{MB}(\epsilon) = Ae^{-\epsilon/kT}$	$f_{BE}(\epsilon) = \frac{1}{e^{\epsilon/kT} - 1}$	$f_{FD}(\epsilon) = \frac{1}{e^{(\epsilon - \epsilon_F)/kT} + 1}$
Properties of distribution	No limit to number of particles per state	No limit to number of particles per state; more particles per state than $f_{MB}$ at low energies; approaches $f_{MB}$ at high energies	Never more than 1 particle per state; fewer particles per state than $f_{MB}$ at low energies; approaches $f_{MB}$ at high energies



(a)



(b)



(c)

$$f(E) = \frac{1}{1 + e^{(E - E_f)/kT}}$$

$$T = 0, \epsilon < \epsilon_F: \quad f_{FD}(\epsilon) = \frac{1}{e^{(\epsilon - \epsilon_F)/kT} + 1} = \frac{1}{e^{-\infty} + 1} = \frac{1}{0 + 1} = 1$$

$$T = 0, \epsilon > \epsilon_F: \quad f_{FD}(\epsilon) = \frac{1}{e^{(\epsilon - \epsilon_F)/kT} + 1} = \frac{1}{e^{\infty} + 1} = 0$$

## 1.7.2 Fermi Function–The Probability of an Energy State Being Occupied by an Electron

$$f(E) = \frac{1}{1 + e^{(E-E_f)/kT}}$$

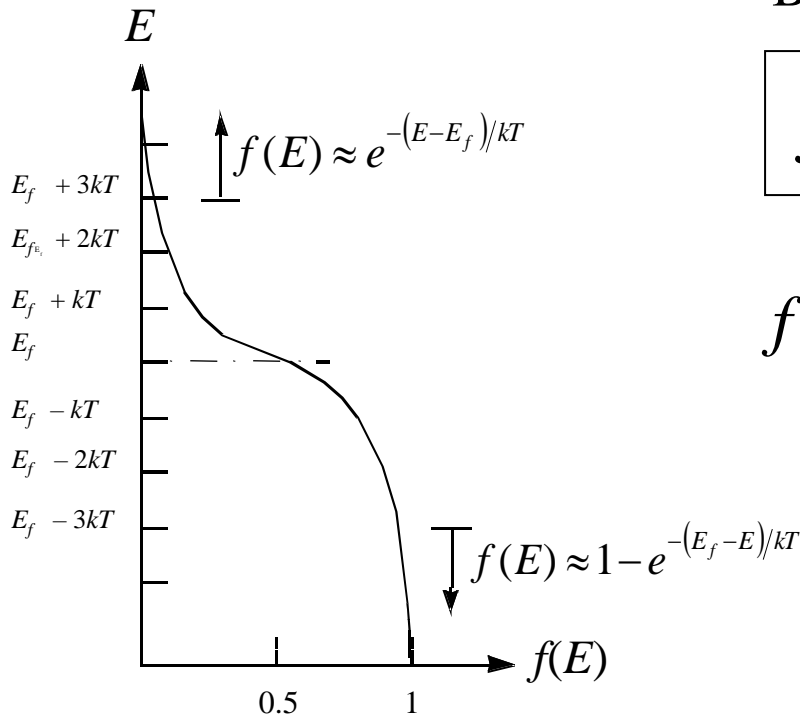
$E_f$  is called the *Fermi energy* or the *Fermi level*.

Boltzmann approximation:

$$f(E) \approx e^{-(E-E_f)/kT} \quad E - E_f \gg kT$$

$$f(E) \approx 1 - e^{-(E_f-E)/kT} \quad E - E_f \ll -kT$$

***Remember: there is only one Fermi-level in a system at equilibrium.***

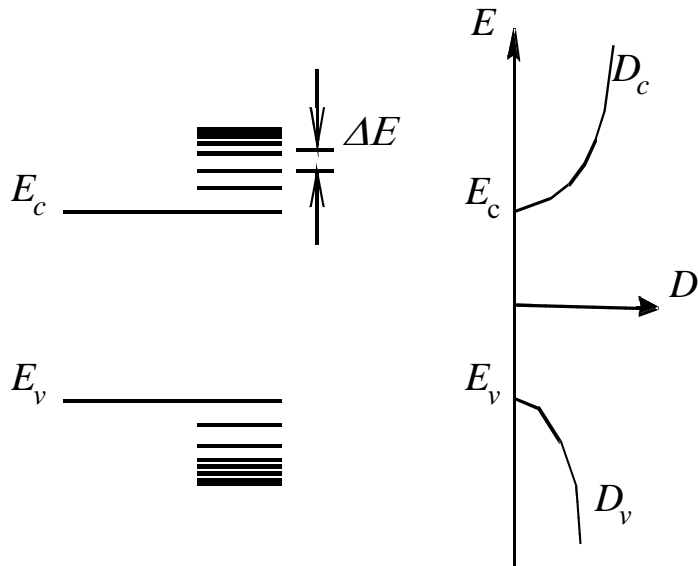


# 1.8 Electron and Hole Concentrations

## 1.8.1 Derivation of $n$ and $p$ from $D(E)$ and $f(E)$

$$n = \int_{E_c}^{\text{top of conduction band}} f(E) D_c(E) dE$$

$$n = \int_{E_c}^{\infty} D(E) f(E) dE$$



$$D(E) = \frac{1}{2\pi^2} \left( \frac{2m^*}{\hbar^2} \right)^{\frac{3}{2}} (E - E_c)^{1/2}$$

$$f(E) = \frac{1}{1 + e^{((E-E_F)/kT)}}$$

If  $E - E_F \gg kT$

$$f(E) = e^{-(E-E_F)/kT}$$

$$n = \int_{E_c}^{\infty} \frac{1}{2\pi^2} \left( \frac{2m^*}{\hbar^2} \right)^{\frac{3}{2}} (E - E_c)^{1/2} e^{\left( \frac{-(E-E_F)}{kT} \right)} dE$$



$$n = \frac{1}{2\pi^2} \left( \frac{2m^*}{\hbar^2} \right)^{\frac{3}{2}} e^{(E_F - E_C)/kT} \int_{E_C}^{\infty} (E - E_C)^{1/2} e^{-(E - E_C)/kT} dE$$

$$x = \frac{E - E_C}{kT}$$

$$n = \frac{1}{2\pi^2} \left( \frac{2m^*}{\hbar^2} \right)^{\frac{3}{2}} e^{(E_F - E_C)/kT} \int_{E_C}^{\infty} (xkT)^{1/2} e^{(-x)} kT dx$$

$$n = \frac{1}{2\pi^2} \left( \frac{2m^*kT}{\hbar^2} \right)^{\frac{3}{2}} e^{(E_F - E_C)/kT} \int_{E_C}^{\infty} (x)^{1/2} e^{(-x)} dx$$

$$\Gamma = \int_0^{\infty} e^{-x} x^{1/2} dx = \left( \frac{3}{2} \right) = \frac{\sqrt{\pi}}{2}$$

$$n = \frac{1}{4} \left( \frac{2m^*kT}{\pi\hbar^2} \right)^{\frac{3}{2}} e^{(E_F - E_C)/kT} = N_c e^{(E_F - E_C)/kT}$$

$$N_c = \frac{1}{4} \left( \frac{2m_e^*kT}{\pi\hbar^2} \right)^{\frac{3}{2}}$$

## ***Electron and Hole Concentrations***

$$n = N_c e^{-(E_c - E_f)/kT}$$

$$N_c \equiv 2 \left[ \frac{2\pi m_n kT}{h^2} \right]^{3/2}$$

$$p = N_v e^{-(E_f - E_v)/kT}$$

$$N_v \equiv 2 \left[ \frac{2\pi m_p kT}{h^2} \right]^{3/2}$$

$N_c$  is called the *effective density of states (of the conduction band)* .

$N_v$  is called the *effective density of states of the valence band*.

**Remember:** the closer  $E_f$  moves up to  $N_c$ , the larger  $n$  is; the closer  $E_f$  moves down to  $E_v$ , the larger  $p$  is.

**For Si,**  $N_c = 2.8 \times 10^{19} \text{ cm}^{-3}$  and  $N_v = 1.04 \times 10^{19} \text{ cm}^{-3}$ .

**For intrinsic semiconductor**  $n = p$

$$N_C e^{(E_F - E_C)/kT} = N_V e^{(E_V - E_F)/kT}$$

$$\frac{1}{4} \left( \frac{2m_n^* kT}{\pi \hbar^2} \right)^{\frac{3}{2}} e^{(E_F - E_C)/kT} = \frac{1}{4} \left( \frac{2m_p^* kT}{\pi \hbar^2} \right)^{\frac{3}{2}} e^{(E_V - E_F)/kT}$$

$$\left( \frac{m_n^*}{m_p^*} \right)^{\frac{3}{2}} = \frac{e^{(E_V - E_F)/kT}}{e^{(E_F - E_C)/kT}}$$

$$\frac{3}{2} \ln \left( \frac{m_n^*}{m_p^*} \right) = \frac{e^{(E_V - E_F)/kT}}{e^{(E_F - E_C)/kT}} = \frac{2E_F - (E_C + E_V)}{kT}$$

$$\frac{3}{4} kT \ln \left( \frac{m_n^*}{m_p^*} \right) + \frac{E_C + E_V}{2} = E_F$$

$m_n^* = m_p^*$

$$\frac{E_C + E_V}{2} = E_F$$

Dr. Pramod Kumar, Associate Prof.

**The Fermi level lies in the middle of the forbidden gap**

# The Mass Action Law

For intrinsic semiconductor  $n = p$

$$n_i^2 = n * p = \frac{1}{4} \left( \frac{2m^* kT}{\pi \hbar^2} \right)^{\frac{3}{2}} e^{(E_F - E_C)/kT} * \frac{1}{4} \left( \frac{2m_p^* kT}{\pi \hbar^2} \right)^{\frac{3}{2}} e^{(E_V - E_F)/kT}$$

$$n_i = \frac{1}{4} \left( \frac{2kT}{\pi \hbar^2} \right)^{\frac{3}{2}} (m_n^* m_p^*)^{3/4} e^{(E_V - E_F + E_F - E_C)/2kT}$$

$$n_i = \frac{1}{4} \left( \frac{2kT}{\pi \hbar^2} \right)^{\frac{3}{2}} (m_n^* m_p^*)^{3/4} e^{(E_V - E_C)/2kT}$$

$$n_i = \frac{1}{4} \left( \frac{2kT}{\pi \hbar^2} \right)^{\frac{3}{2}} (m_n^* m_p^*)^{3/4} e^{-E_g/2kT}$$

## 1.8.2 The Fermi Level and Carrier Concentrations

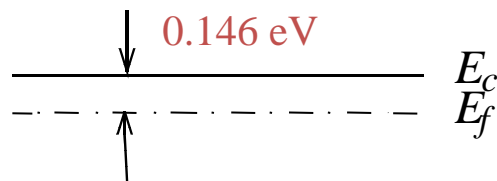
Where is  $E_f$  for  $n = 10^{17} \text{ cm}^{-3}$ ? And for  $p = 10^{14} \text{ cm}^{-3}$ ?

**Solution:** (a)  $n = N_c e^{-(E_c - E_f)/kT}$

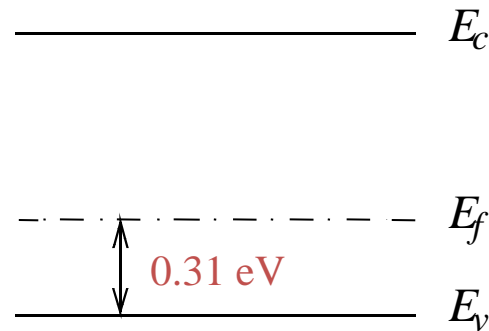
$$E_c - E_f = kT \ln(N_c/n) = 0.026 \ln(2.8 \times 10^{19} / 10^{17}) = 0.146 \text{ eV}$$

(b) For  $p = 10^{14} \text{ cm}^{-3}$ , from Eq.(1.8.8),

$$E_f - E_v = kT \ln(N_v/p) = 0.026 \ln(1.04 \times 10^{19} / 10^{14}) = 0.31 \text{ eV}$$

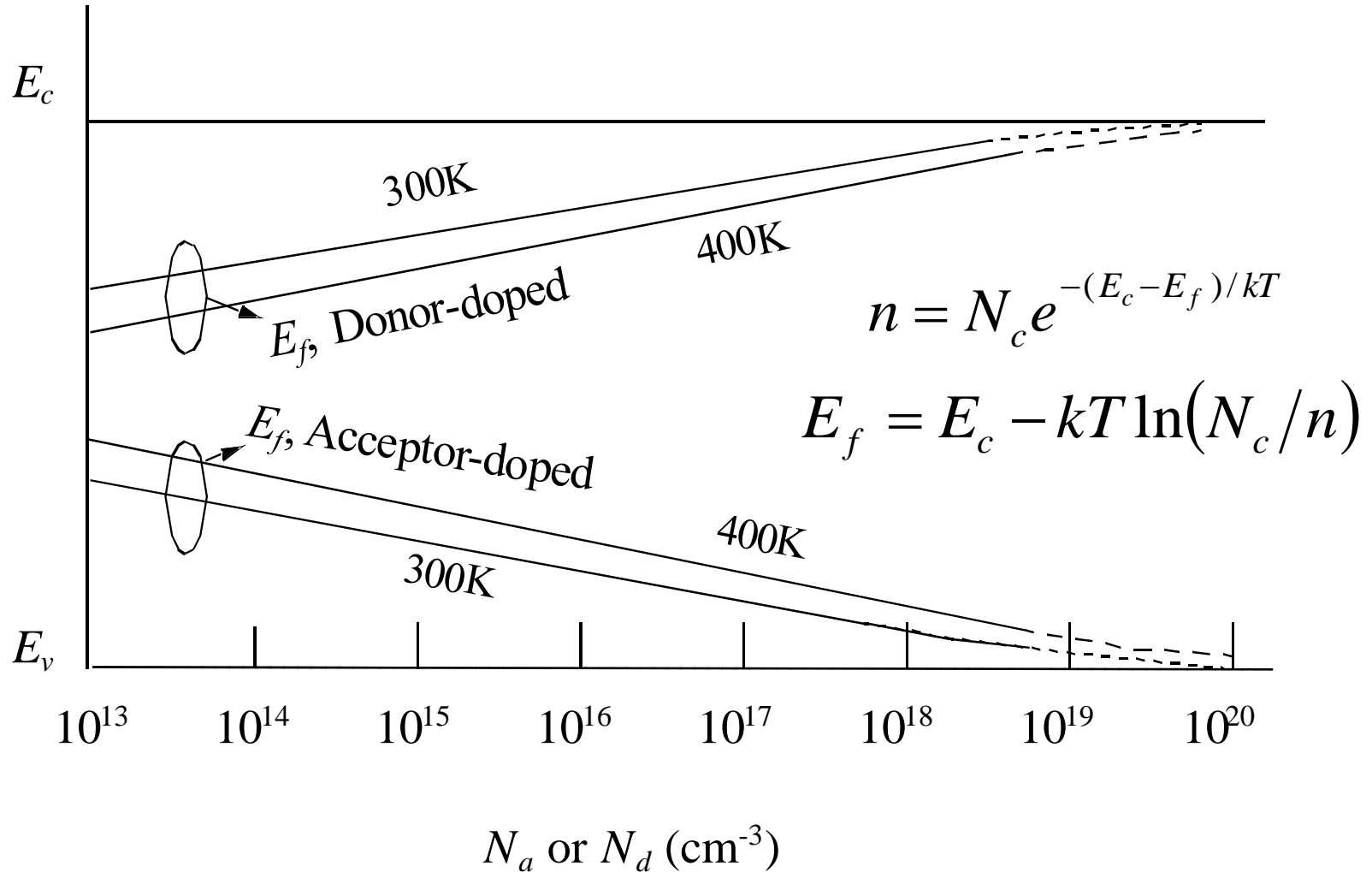


(a)



(b)

## 1.8.2 The Fermi Level and Carrier Concentrations



### 1.8.3 The $np$ Product and the Intrinsic Carrier Concentration

Multiply  $n = N_c e^{-(E_c - E_f)/kT}$  and  $p = N_v e^{-(E_f - E_v)/kT}$

$$np = N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT}$$

$$np = n_i^2$$

$$n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$$

- In an intrinsic (undoped) semiconductor,  $n = p = n_i$ .
- $n_i$  is the ***intrinsic carrier concentration***,  $\sim 10^{10} \text{ cm}^{-3}$  for Si.

## **EXAMPLE: Carrier Concentrations**

**Question:** What is the hole concentration in an N-type semiconductor with  $10^{15} \text{ cm}^{-3}$  of donors?

**Solution:**  $n = 10^{15} \text{ cm}^{-3}$ .

$$p = \frac{n_i^2}{n} \approx \frac{10^{20} \text{ cm}^{-3}}{10^{15} \text{ cm}^{-3}} = 10^5 \text{ cm}^{-3}$$

After increasing  $T$  by  $60^\circ\text{C}$ ,  $n$  remains the same at  $10^{15} \text{ cm}^{-3}$  while  $p$  increases by about a factor of 2300 because  $n_i^2 \propto e^{-E_g/kT}$ .

**Question:** What is  $n$  if  $p = 10^{17} \text{ cm}^{-3}$  in a P-type silicon wafer?

**Solution:**

$$n = \frac{n_i^2}{p} \approx \frac{10^{20} \text{ cm}^{-3}}{10^{17} \text{ cm}^{-3}} = 10^3 \text{ cm}^{-3}$$



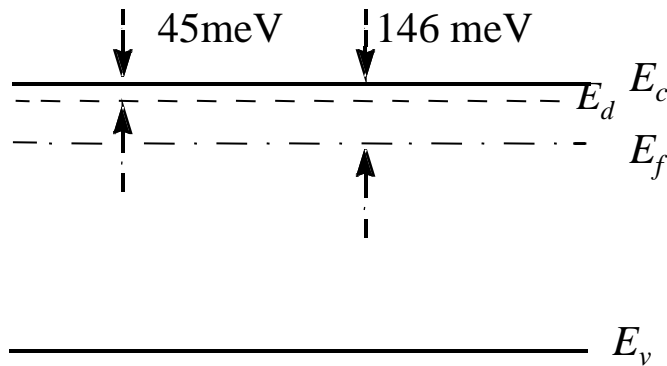
## 1.9 General Theory of $n$ and $p$

### **EXAMPLE: Complete ionization of the dopant atoms**

$N_d = 10^{17} \text{ cm}^{-3}$ . What fraction of the donors are not ionized?

**Solution:** First assume that all the donors **are** ionized.

$$n = N_d = 10^{17} \text{ cm}^{-3} \Rightarrow E_f = E_c - 146 \text{ meV}$$



$$\text{Probability of not being ionized} \approx \frac{1}{1 + \frac{1}{2} e^{(E_d - E_f)/kT}} = \frac{1}{1 + \frac{1}{2} e^{((146 - 45) \text{ meV})/26 \text{ meV}}} = 0.04$$

Therefore, it is reasonable to assume complete ionization, i.e.,  $n = N_d$ .

## ***1.9 General Theory of $n$ and $p$***

Charge neutrality:  $n + N_a = p + N_d$

$$np = n_i^2$$

$$p = \frac{N_a - N_d}{2} + \left[ \left( \frac{N_a - N_d}{2} \right)^2 + n_i^2 \right]^{1/2}$$

$$n = \frac{N_d - N_a}{2} + \left[ \left( \frac{N_d - N_a}{2} \right)^2 + n_i^2 \right]^{1/2}$$

## ***1.9 General Theory of on $n$ and $p$***

**I.**  $N_d - N_a \gg n_i$  (i.e., N-type)

$$n = N_d - N_a$$

$$p = n_i^2 / n$$

If  $N_d \gg N_a$  ,  $n = N_d$  and  $p = n_i^2 / N_d$

**II.**  $N_a - N_d \gg n_i$  (i.e., P-type)

$$p = N_a - N_d$$

$$n = n_i^2 / p$$

If  $N_a \gg N_d$  ,  $p = N_a$  and  $n = n_i^2 / N_a$

## EXAMPLE: Dopant Compensation

What are  $n$  and  $p$  in Si with (a)  $N_d = 6 \times 10^{16} \text{ cm}^{-3}$  and  $N_a = 2 \times 10^{16} \text{ cm}^{-3}$  and (b) additional  $6 \times 10^{16} \text{ cm}^{-3}$  of  $N_a$ ?

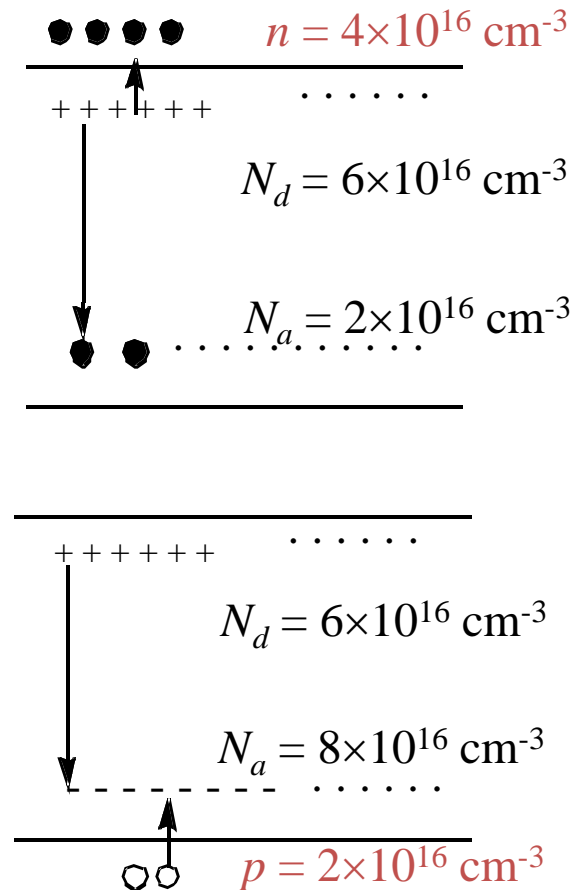
(a)  $n = N_d - N_a = 4 \times 10^{16} \text{ cm}^{-3}$

$$p = n_i^2 / n = 10^{20} / 4 \times 10^{16} = 2.5 \times 10^3 \text{ cm}^{-3}$$

(b)  $N_a = 2 \times 10^{16} + 6 \times 10^{16} = 8 \times 10^{16} \text{ cm}^{-3} > N_d$

$$p = N_a - N_d = 8 \times 10^{16} - 6 \times 10^{16} = 2 \times 10^{16} \text{ cm}^{-3}$$

$$n = n_i^2 / p = 10^{20} / 2 \times 10^{16} = 5 \times 10^3 \text{ cm}^{-3}$$



## 1.11 Chapter Summary

Energy band diagram. Acceptor. Donor.  $m_n$ ,  $m_p$ .  
Fermi function.  $E_f$ .

$$n = N_c e^{-(E_c - E_f)/kT}$$

$$p = N_v e^{-(E_f - E_v)/kT}$$

$$n = N_d - N_a$$

$$p = N_a - N_d$$

$$np = n_i^2$$