Indian Institute of Information Technology, Allahabad

Mid Sem Question Paper

B. Tech. 1st Semester

Course Name: Engineering Physics **Course Coordinator: Dr. Akhilesh Tiwari**

Course code: BS-AS-EGP102 **Exam Date: 18/10/23** **Max. marks: 25**

Note: Use of a non-programmable scientific calculator is allowed.

1. Consider a surface of revolution, generated by taking a curve passing between two fixed endpoints $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the xy plane and revolving around the x-axis. Use the variational principle and obtain the curve for which the surface area is a minimum. [5]

2. Suppose a planet of mass m moves around a star of mass M in an elliptical orbit due to the gravitational force between them.

   a) Construct the Lagrangian for the system.
   b) Find out the Hamiltonian. Identify the conserved quantity. [2+3]

3. The phase velocity of deep-water waves is given by $v^2 = \frac{g\lambda}{2\pi} + \frac{2\pi\sigma}{\rho\lambda}$, where $g = 9.8\ m/s^2, \rho = 1000\ kg/m^3$ and $\sigma = 7.2 \times 10^{-2} Nm$ (σ is the Surface tension of water).

   (a) Determine the value (λ_0) of the wavelength of the waves which do not disperse in water. What are the phase and group velocities at this wavelength?

   (b) Show that for wavelength $\lambda << \lambda_0; v_g = 1.5v$ and $\lambda >> \lambda_0; v_g = v/2$, where v_g is the group velocity. [6]

4. (a) Find the normalization constant A for the wave function $\psi(x,t) = A\ e^{\frac{i}{\hbar}(px - Et)}$, where the range of x is $-\infty$ to ∞. What would be $\langle x \rangle$?

   (b) Let $\psi(x,t) = C\ e^{-ax + ibt}, (0 < x < \infty)$, where a, b are real constants. Normalize the wave function and find $\langle x \rangle$. [1+2]

5. (a) Let us consider the wave function of a 3-D square well potential with dimensions $L_x = a, L_y = b, L_z = c$. What will be the normalized wave function for the system? If $b = 2a$ and $c = 2a$, then find the energies of the two lowest states.

   (b) Consider the wave function given by the linear superposition of two stationary states of a 1-D particle of mass m in a box,

$$\psi(x,t) = \frac{1}{\sqrt{a}}\left[\sin\left(\frac{\pi x}{a}\right) e^{-i\omega t} + \sin\left(\frac{2\pi x}{a}\right) e^{-4i\omega t}\right], \quad with\ \omega = \frac{\pi^2\hbar}{2ma^2}$$

   Find the expectation value of the position and show it oscillates with a frequency 3ω. [3+3]