Indian Institute of Information Technology, Allahabad

Classical Mechanics Quiz, Engineering Physics (B.Tech. I Yr 2022)

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Time: 45 mins | Max mark: 20

Attempt all questions.

1. Consider a rope of mass m and length l which hangs over the edge of a table. The gravitational field of the earth is acting vertically downwards in the z-direction. Take generalized coordinates x = length of rope on the table, and z = length of the rope hanging down the edge of the table (the edge being the origin). Neglect the friction between the rope and the surface, and answer the following questions:

   (a) Identify the constraint on the motion of the rope and write down the Lagrangian for the system. Explain how you have obtained the kinetic and potential energies of the system. [4]

   (b) Find and solve the equation of motion for the end of the rope under the following initial conditions: at time $t = 0$ the rope is at rest and a part of length z_0 is hanging over the edge. [3]

   (c) What is the velocity of the rope at the moment when it leaves the surface of the table? [1]

   (d) Show that the time τ it takes the rope to slide off the table if it starts from rest is [2]

$$-\dot{p} = \frac{\partial H}{\partial x} = 0 \qquad \tau = \sqrt{\frac{l}{g}} \ln\left(\frac{l}{z_0} + \sqrt{\frac{l^2}{z_0^2} - 1}\right) \qquad H : \frac{\dot{x}^2}{2} + \frac{\dot{y}^2}{2}$$

2. (a) Write down the infinitesimal squared displacement between two points of a plane in plane polar coordinates and find the shortest path between those points using plane polar coordinates. [6]

   (b) Find out the Hamiltonian for the Lagrangian $L = \frac{\dot{x}^2}{2} + \frac{\dot{y}^2}{2} - x\dot{y}$ and identify the conserved quantities. [4]