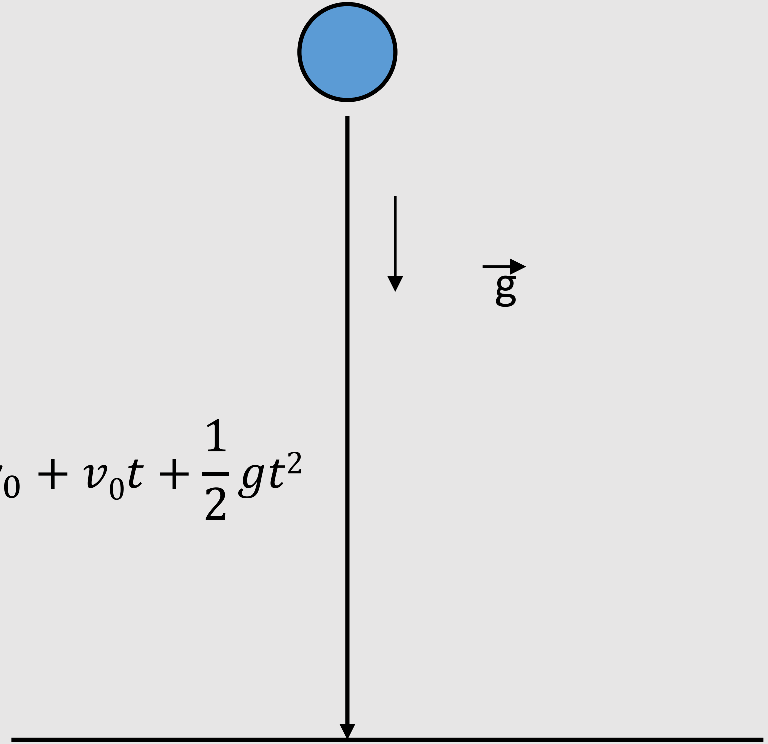
Lecture 4: Free Fall and Projectile Motion

1. Describe the effects of gravity on objects in motion.
2. Describe the motion of objects that are in free fall.
3. Calculate the position, time and velocity of objects in free fall
4. One and Two-Dimensional Vectors
5. Projectile Motion

Falling Objects

- Free fall is when an object is moving only under the influence of gravity
- In a vacuum all objects fall at same acceleration
- $g = 9.80\frac{m}{s^2}\,down$
- Any object thrown up, down, or dropped has this acceleration

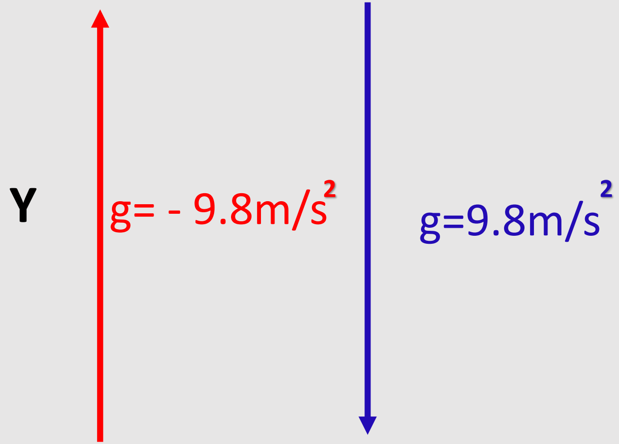


$$y = y_0 + v_0 t + \frac{1}{2} g t^2$$

We will use the same one-dimensional equations of motion only in a vertical motion. The only change is that the acceleration is $g=9.80m/s^2$ downward

Be care careful with the sign of g!!!!!!

$$y = y_0 + v_0 t + \frac{1}{2} g t^2$$



Practice

You drop a coin from the top of a 1000 m-story building. If you ignore air resistance,

1. How fast will it be falling right before it hits the ground?
2. How long does it take to hit the ground?

Givens: $v_0 = 0, v = ?, a = -9.80\frac{m}{s^2}, x_0 = 1000\ m, x = 0\ m$

Practice

A baseball is hit straight up into the air. If the initial velocity was 20 m/s,

1. How high will the ball go?
2. How long will it be until the catcher catches the ball at the same height it was hit?

$$x = 20.4\ m$$

$$t = 14.3\ s$$

Two-Dimensional Vectors

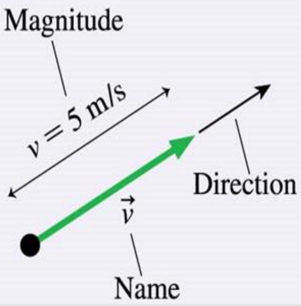
We will…

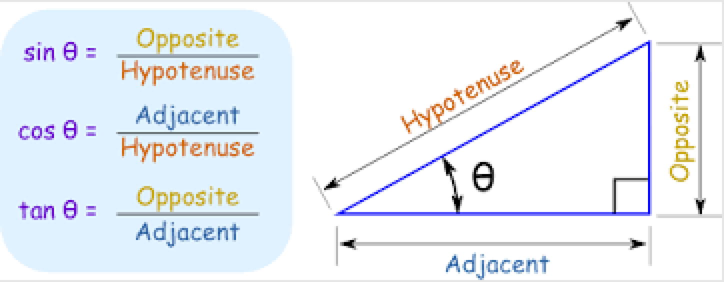
- see that motion in two dimensions consists of horizontal and vertical components.
- Understand the independence of horizontal and vertical vectors in two-dimensional motion.
- Understand the rules of vector addition, subtraction, and multiplication.
- Apply graphical methods of vector addition and subtraction.
- Understand the rules of vector addition and subtraction using analytical methods.
- Apply analytical methods to determine vertical and horizontal component vectors.
- Apply analytical methods to determine the magnitude and direction of a resultant vector.

Vectors

What is a vector?

A vector is a quantity with both a size—its magnitude—and a direction.



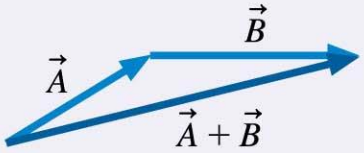


	0	30	45	60	90
Sin	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
Cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$

Addition and subtraction of Vectors

How are vectors added and subtracted?

Vectors are added "tip to tail." The order of addition does not matter. To subtract vectors, turn the subtraction into addition by writing $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$. The vector $-\vec{B}$ is the same length as $\vec{B}$ but points in the opposite direction.



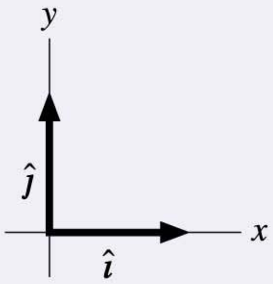
Unit Vectors

What are unit vectors?

Unit vectors define what we *mean* by the $+x$- and $+y$-directions in space.

- A unit vector has magnitude 1.
- A unit vector has no units.

Unit vectors simply point.



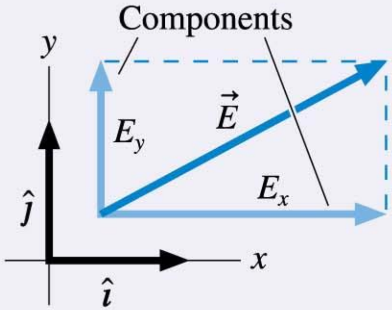
Vectors Components

What are components?

Components of vectors are the pieces of vectors parallel to the coordinate axes—in the directions of the unit vectors.
We write

$$\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath}$$

Components simplify vector math.



How are components used?

Components let us do vector math with algebra, which is easier and more precise than adding and subtracting vectors using geometry and trigonometry. Multiplying a vector by a number simply multiplies all of the vector's components by that number.

$$\vec{C} = 2\vec{A} + 3\vec{B}$$

means

$$\begin{cases} C_x = 2A_x + 3B_x \\ C_y = 2A_y + 3B_y \end{cases}$$

Practice

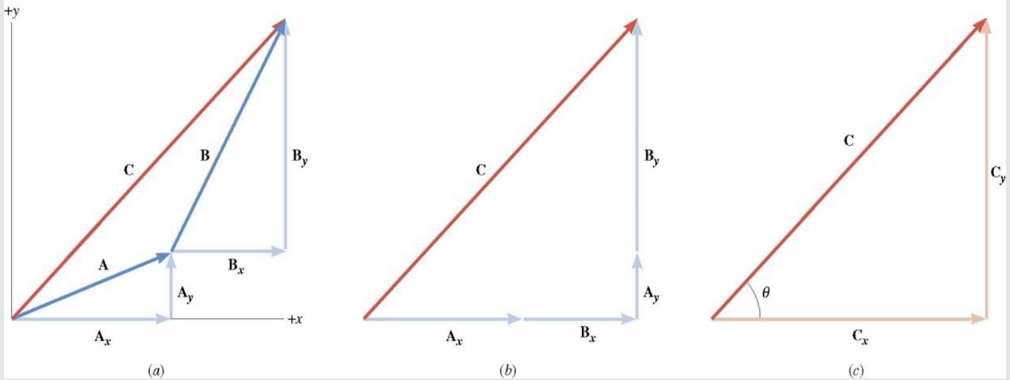
- Add the following vectors graphically. $\boldsymbol{A} = 2\sqrt{2}$ at 45° N of E, $\boldsymbol{B} = 2\sqrt{2}$ at 45° W of N.

Two-Dimensional Vectors

Vector Addition – Component Method

- Vectors can be described by their components in the x and y directions.
- To add vectors, you simply add the x-component and y-components to the resultant x and y components.

1. Find the components for the vectors to be added
2. Add the x-components
3. Add the y-components
4. Use the Pythagorean Theorem to find the magnitude of the resultant
5. Use $\tan^{-1}$ to find the direction (the direction is always found at the tail-end of the resultant)



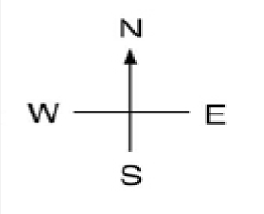
Two-Dimensional Vectors

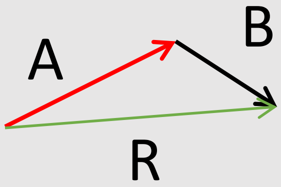
A jogger runs 145 m in a direction 20.0° east of north and then 105 m in a direction 35.0° south of east. Determine the magnitude and direction of the jogger's position from her starting point.

	x	y
A	$145\,m\,sin\,20° = \mathbf{49.6\,m}$	$145\,m\cos 20° = \mathbf{136.3\,m}$
B	$105\,m\cos 35° = \mathbf{86.0\,m}$	$-105\,m\sin 35° = \mathbf{-60.2\,m}$
R	$\mathbf{135.6\,m}$	$\mathbf{76.1\,m}$

$$R = \sqrt{(135.6\,m)^2 + (76.1m)^2} = 155.5\,m$$

$$\theta = \tan^{-1}\frac{76.1\,m}{135.6\,m} = 29.3°\,N\ of\ E$$





Projectile Motion

- Explain the properties of a projectile, such as acceleration due to gravity, range, maximum height, and trajectory.
- Determine the location and velocity of a projectile at different points in its trajectory.

Projectile Motion

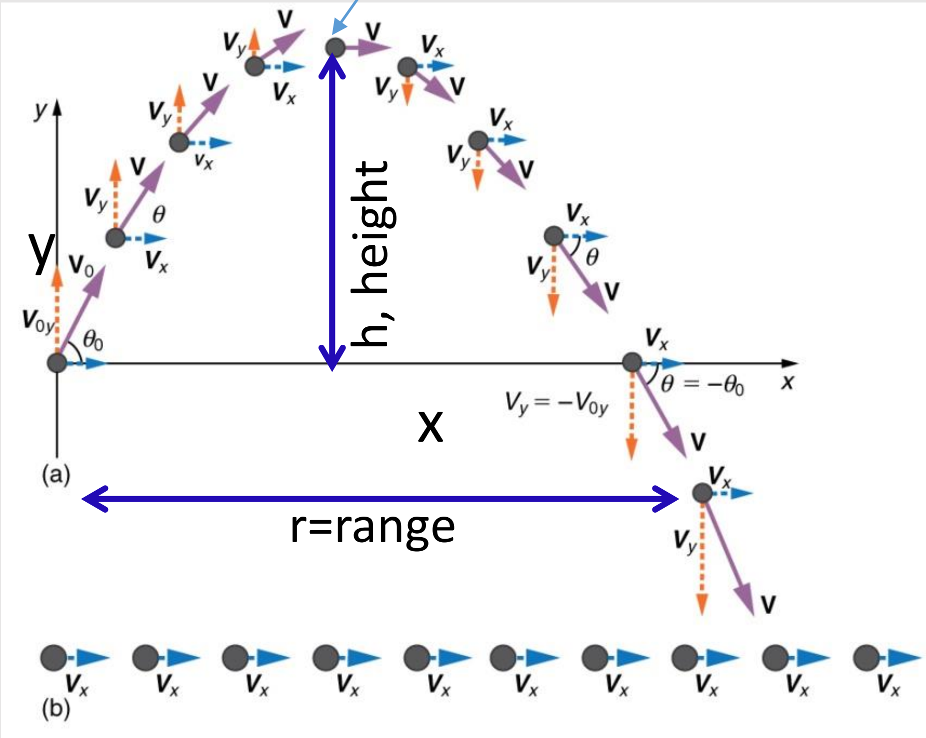
- Objects in flight only under the influence of gravity
- x and y components are independent and time is the only quantity that is the same
- x-component velocity constant

$$x = v_0 t + x0$$

- y-component changes because gravity pulling it down

$$y = \frac{1}{2} a_y t^2 + v_{0y} t + y_0$$

at this point Vy=0



Projectile Motion

If the starting and ending heights are the same, the distance the object goes **range, r,** and the maximum **height, h,** can be found with the equation

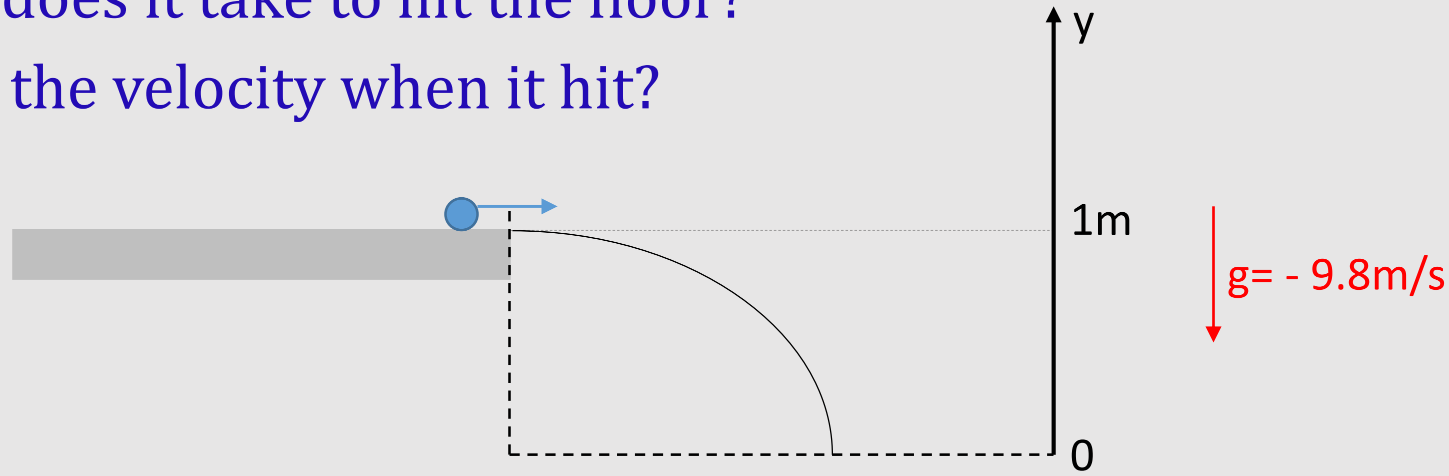
See details in class:

$$r = \frac{v_0^2 \sin 2\theta}{g} \quad , \quad h = \frac{(v_0 sin\theta)^2}{2g}$$

Practice

- A meatball with v = 5.0 m/s rolls off a 1.0 m high table.

1. How long does it take to hit the floor?
2. What was the velocity when it hit?



- Both x and y motion

X direction:

$$v_{0x} = 5.0\frac{m}{s}, t = 0.45\ s$$

$$v_x = 5.0\frac{m}{s}$$

Y direction:

$$v_{0y} = 0\frac{m}{s}, y_0 = 1.0\ m, t = 0.45\ s$$

$$y = 0\ m, a_y = -9.8\frac{m}{s^2},$$

$$v_y = a_y t + v_{0y}$$

$$v_y = -9.8\frac{m}{s^2}(0.45\ s) + 0\frac{m}{s}$$

$$v_y = -4.4\frac{m}{s}$$

$$v_R = \sqrt{\left(5.0\frac{m}{s}\right)^2 + \left(-4.4\frac{m}{s}\right)^2} = 6.7\frac{m}{s}$$

$$\theta = \tan^{-1}\frac{-4.4}{5.0} = -42°$$

$$\boxed{6.7\frac{m}{s}\ at\ 42°\ below\ \ horizontal}$$

Practice- homework

A truck (v = 11.2 m/s) turned a corner too sharp and lost part of the load. A falling box will break if it hits the ground with a velocity greater than 15 m/s. The height of the truck bed is 1.5 m.

Will the box break?

Answer: $v_R = 12.4\ m/s$ The box doesn't break

Practice- homework

- Rachid hits a ball at 35° with a velocity of 32 m/s.

1. How high did the ball go?
2. How long was the ball in the air?
3. How far did the ball go?

Answer:

$$y = 17\ m$$
$$t = 3.8\ s$$
$$x = 98\ m$$