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ECE 514
Final Report

Simulating Random Variables
In this project, we simulated 3 different distributions using Matlab's built in function and the acceptance rejection method.
We were given:
- Normal (mean = 2, variance = 2)
- Uniform [2,4] with mean = 3
- Exponential (lambda = 2)

We were asked to see the result for each distribution with varying values in length of X. Thus each distribution had 3 different X. X with 100 values, 1000 values and 10000 values.

<u>Matlab inbuilt routine:</u>
For the matlab inbuilt, we use randn, rand, and exprnd to create a normal distribution, uniform distribution, and exponential distribution. Using the inbuilt function and specifying the parameters we are able to create the distributions.

<u>Rejection method:</u>
In the rejection method, we simulated random X values. Once these values were simulated we took out any values that did not fit within our desired curve For example, if the desired curve was a normal curve values that fell inside that curve were kept and every other random X variable was rejected.

Parameters :

Name ▲	Value
a	0
b	4
c	0.8862
exp_mean	2.4655
exp_var	0.1767
i	3
n	10001
normal_mean	1.9794
normal_var	1.0224
T	[100,1000,10000]
U1	1.6061
U2	0.5638
uni_mean	3.0088
uni_var	0.3263
X	*1x10000 double*

As we can see that there is a deviation from the theoretical mean and variance from the sample mean and variance
However, when we increase the N from 100 to 1000 to 10,000 the mean and variance come closer to the theoretical mean and variance.

These differences are due to the difference between sample and mean and population mean.

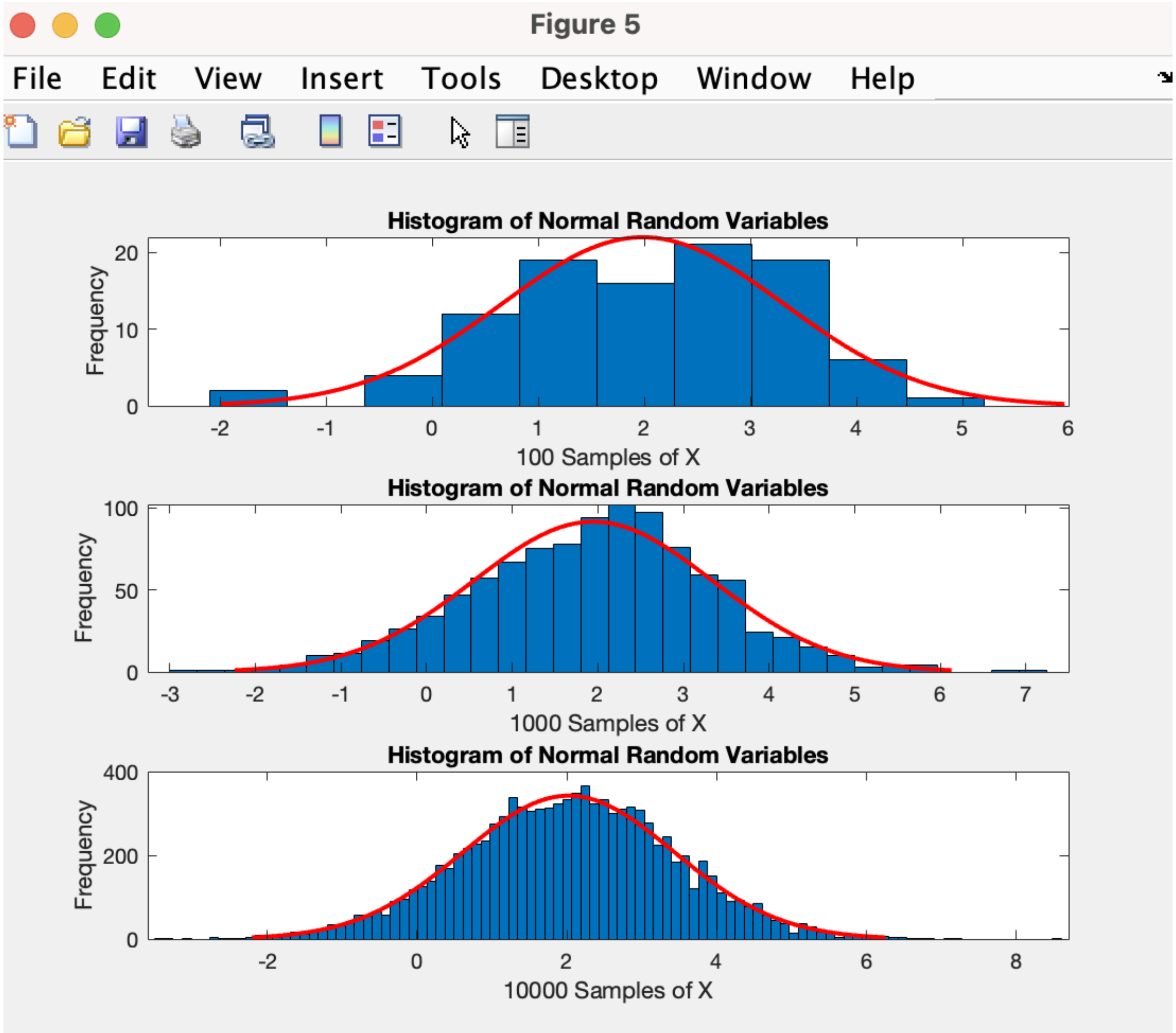
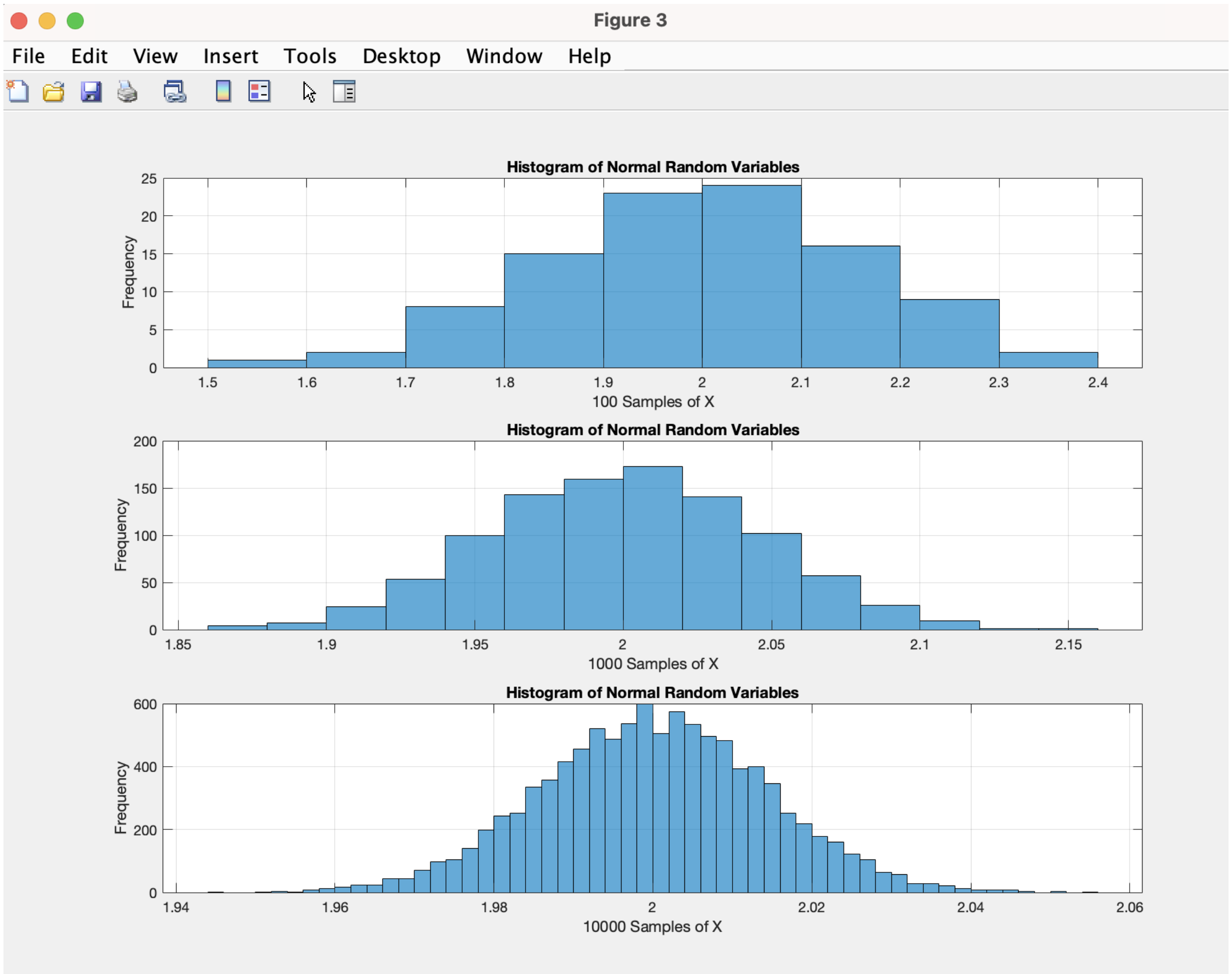


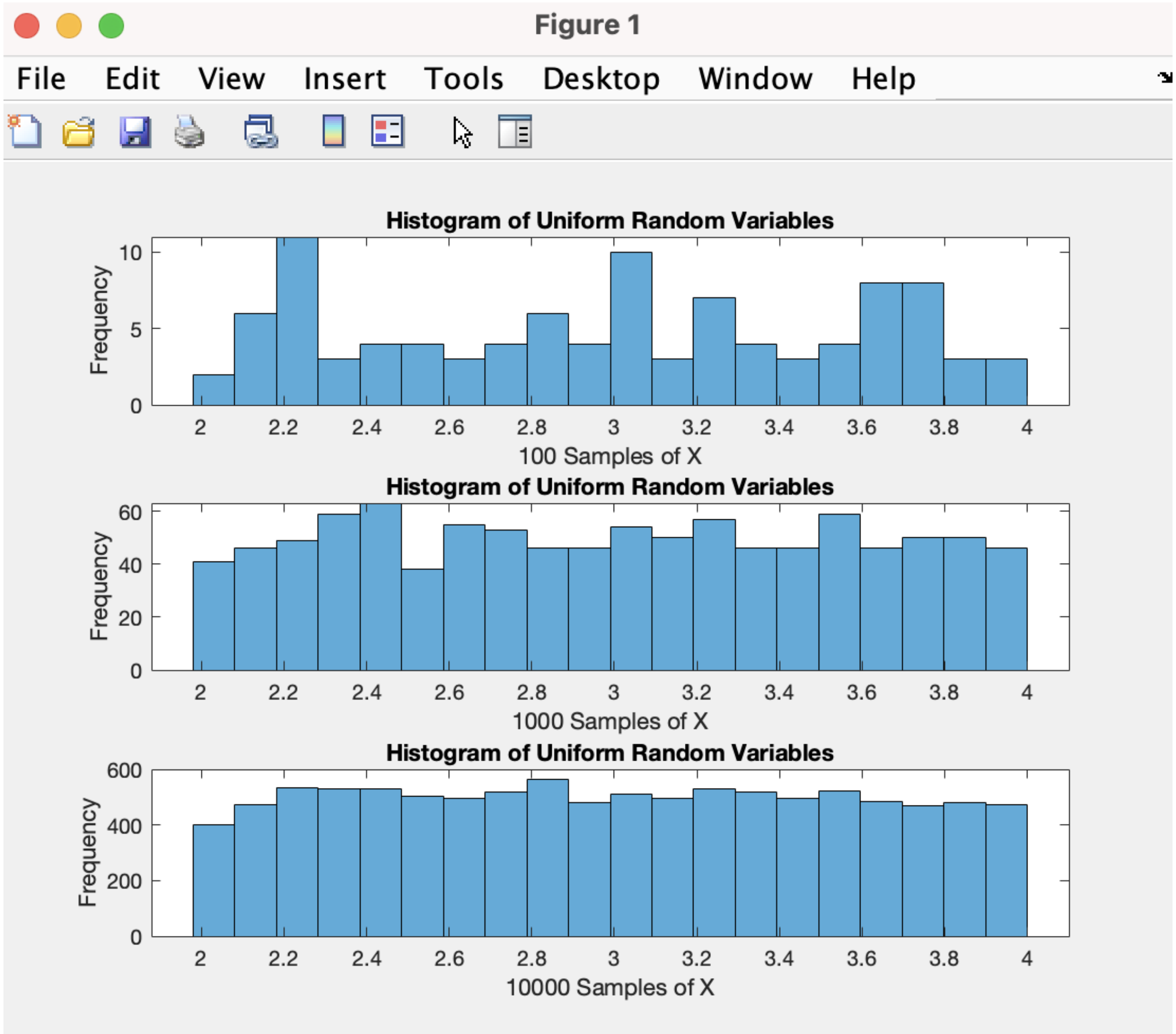
Figure 5
File Edit View Insert Tools Desktop Window Help
Histogram of Normal Random Variables
Frequency
100 Samples of X
Histogram of Normal Random Variables
Frequency
1000 Samples of X
Histogram of Normal Random Variables
Frequency
10000 Samples of X

Rejection Method

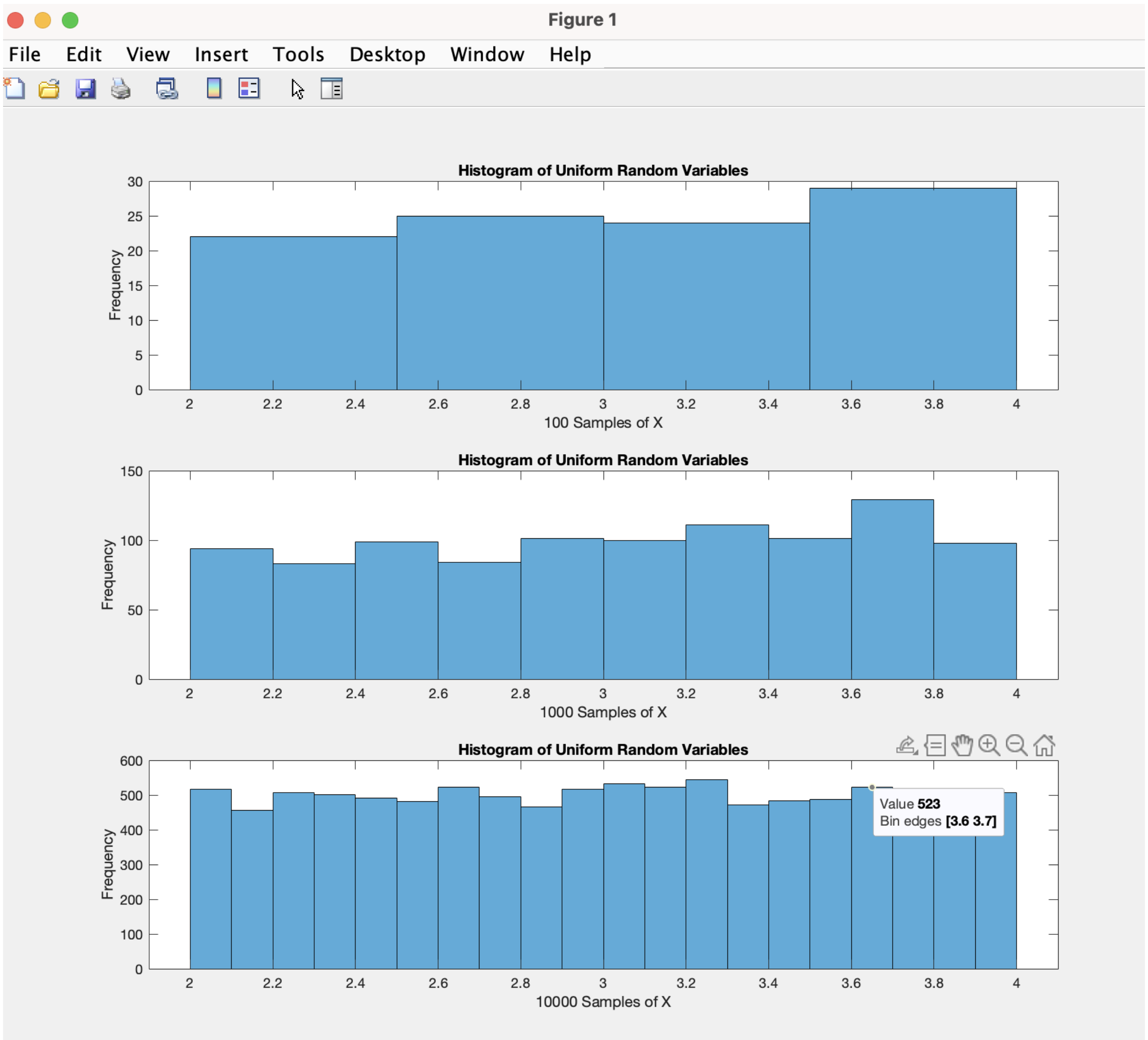


Parameters For Uniform :

Inbuilt Routine

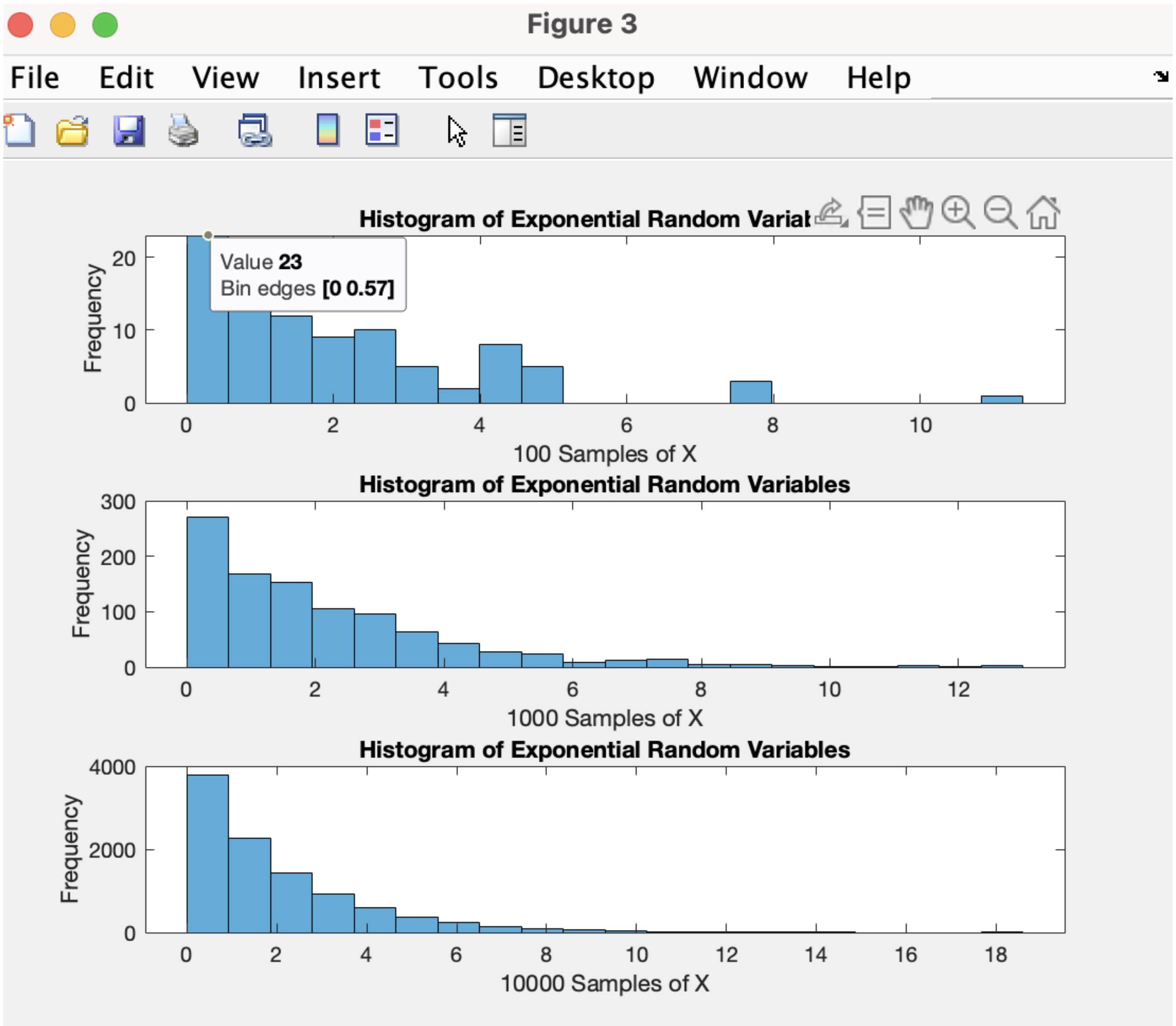


Rejection Method:



Parameters For Exponential :

Inbuilt Routine:



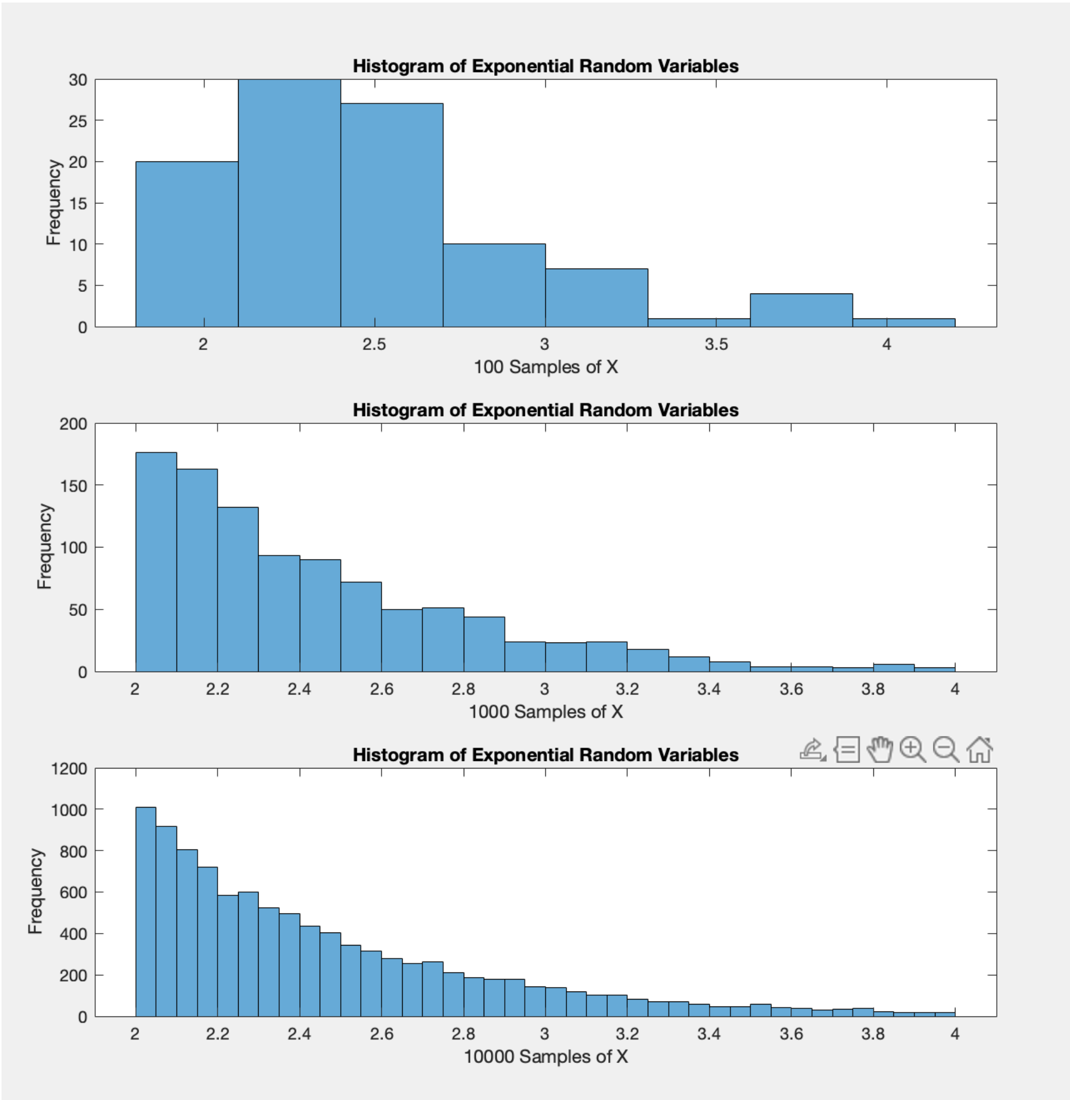
Rejection Method:

In the rejection method, we have used f(x) as our target distribution and another distribution g(x) to get our samples for our target distribution. All samples under the curve are accepted and others are rejected.

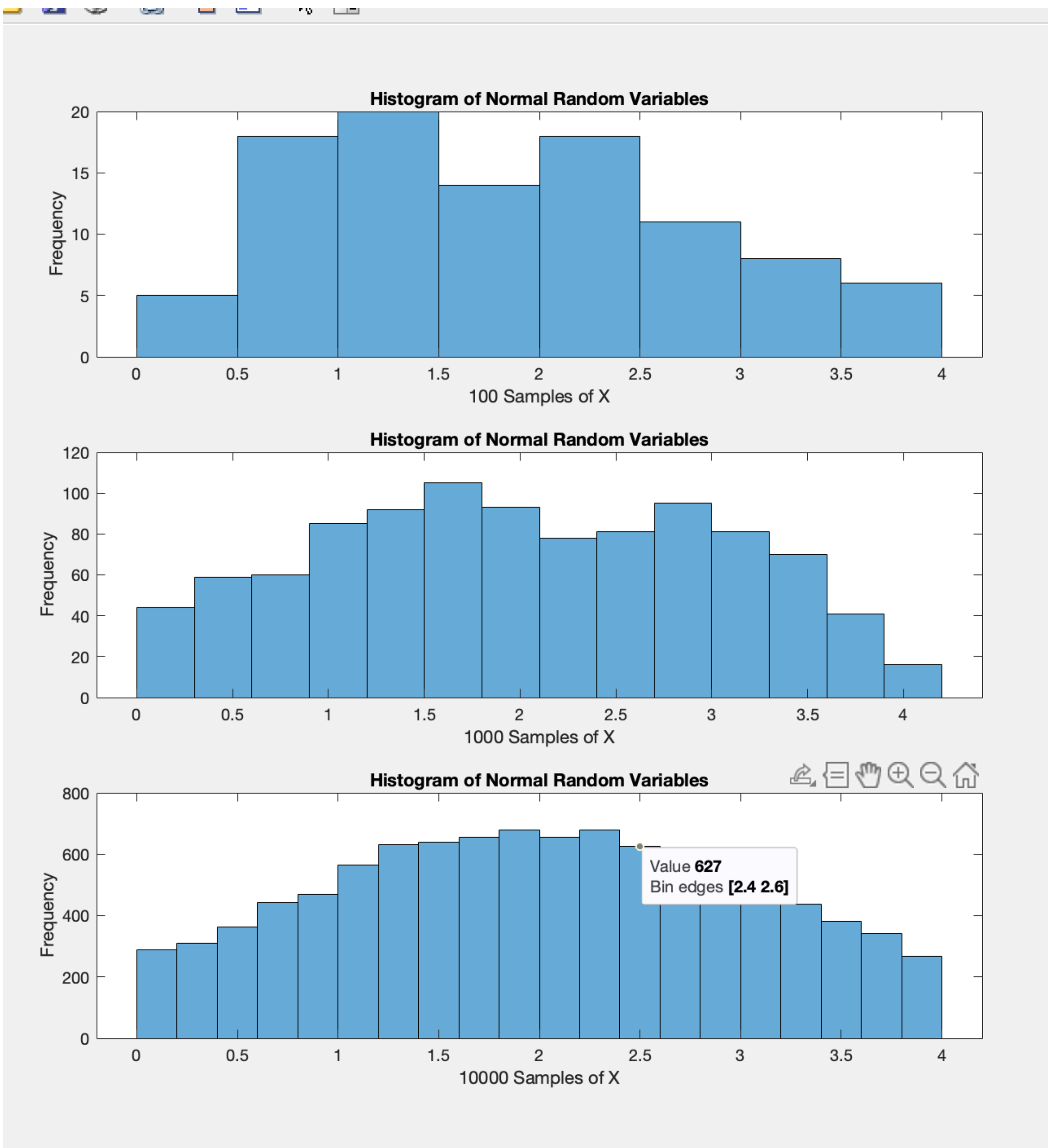
$$\frac{f(y)}{g(y)} \leq c \quad \text{(for all y)}$$

Show that the following method generates a random variable with density function $f(x)$.

- Generate Y having density g.
- Generate a random number U from Uniform $(0, 1)$.
- If $U \leq \frac{f(Y)}{cg(Y)}$, set $X = Y$. Otherwise, return to step 1.



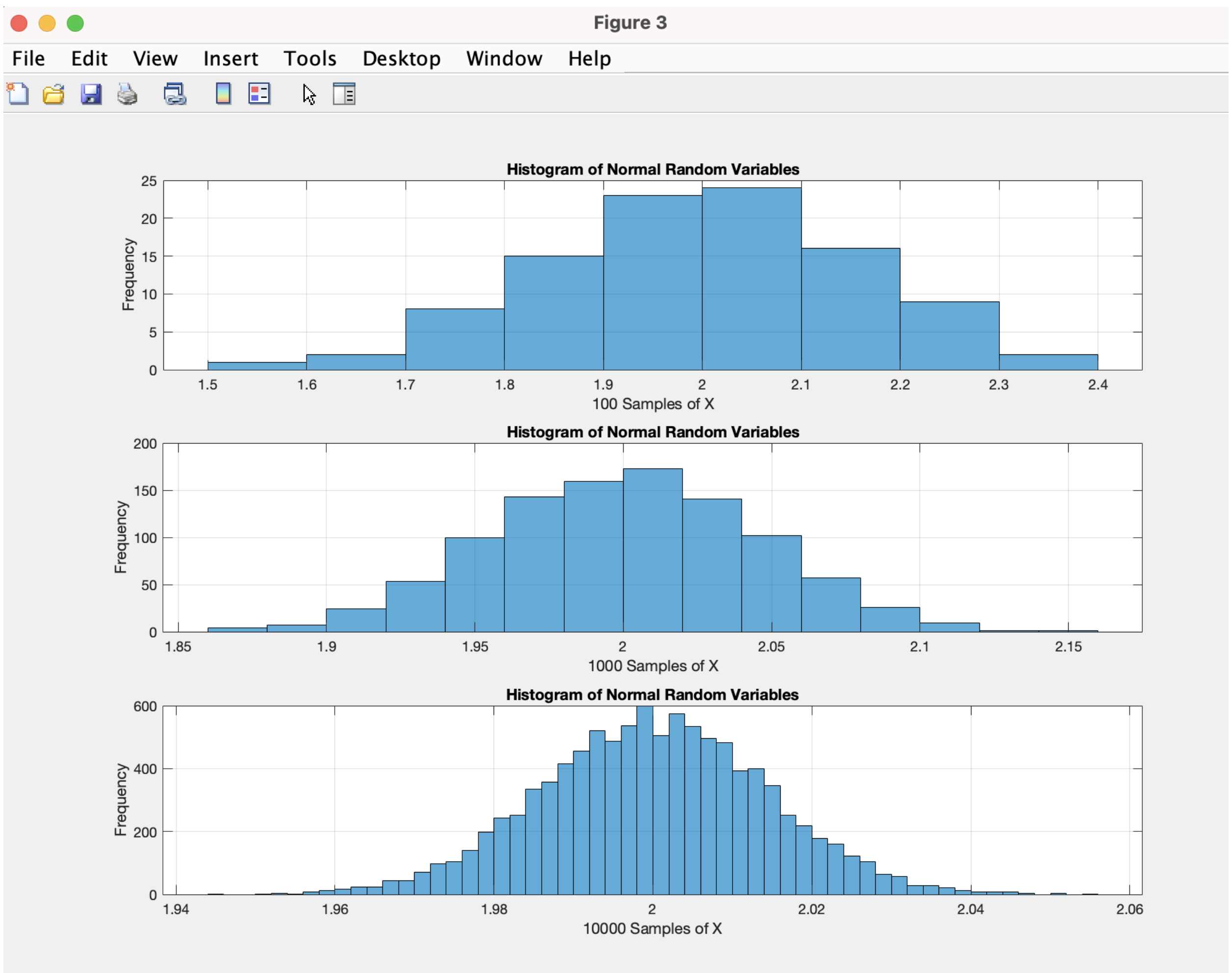
Rejection Method for Uniform , Gaussian and Exponential



Transformation of Uniform, Normal and Exponential

From the transformation we can see that the PDF that matches closely with all the means of the distributions is the Normal PDF.
Hence this satisfies the central limit theorem where taking means of any distribution and plotting them for N tends to infinity gives us the pdf for a normal distribution.

As T = 100 and then 1000 and then 10,000 we can see the pdf slowly approaching Normal distribution.



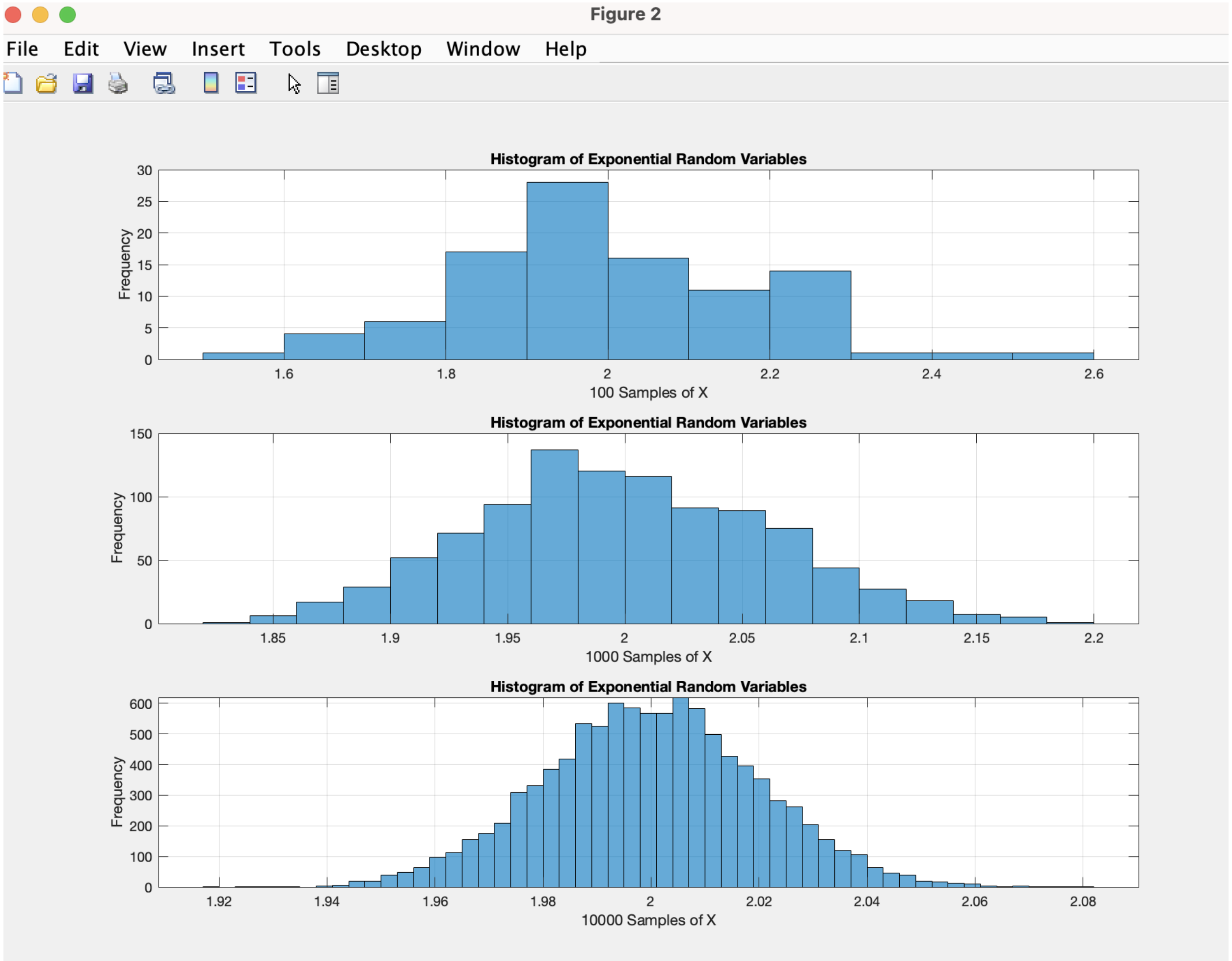


Figure 2
File Edit View Insert Tools Desktop Window Help
Histogram of Exponential Random Variables
Frequency
100 Samples of X
Histogram of Exponential Random Variables
Frequency
1000 Samples of X
Histogram of Exponential Random Variables
Frequency
10000 Samples of X

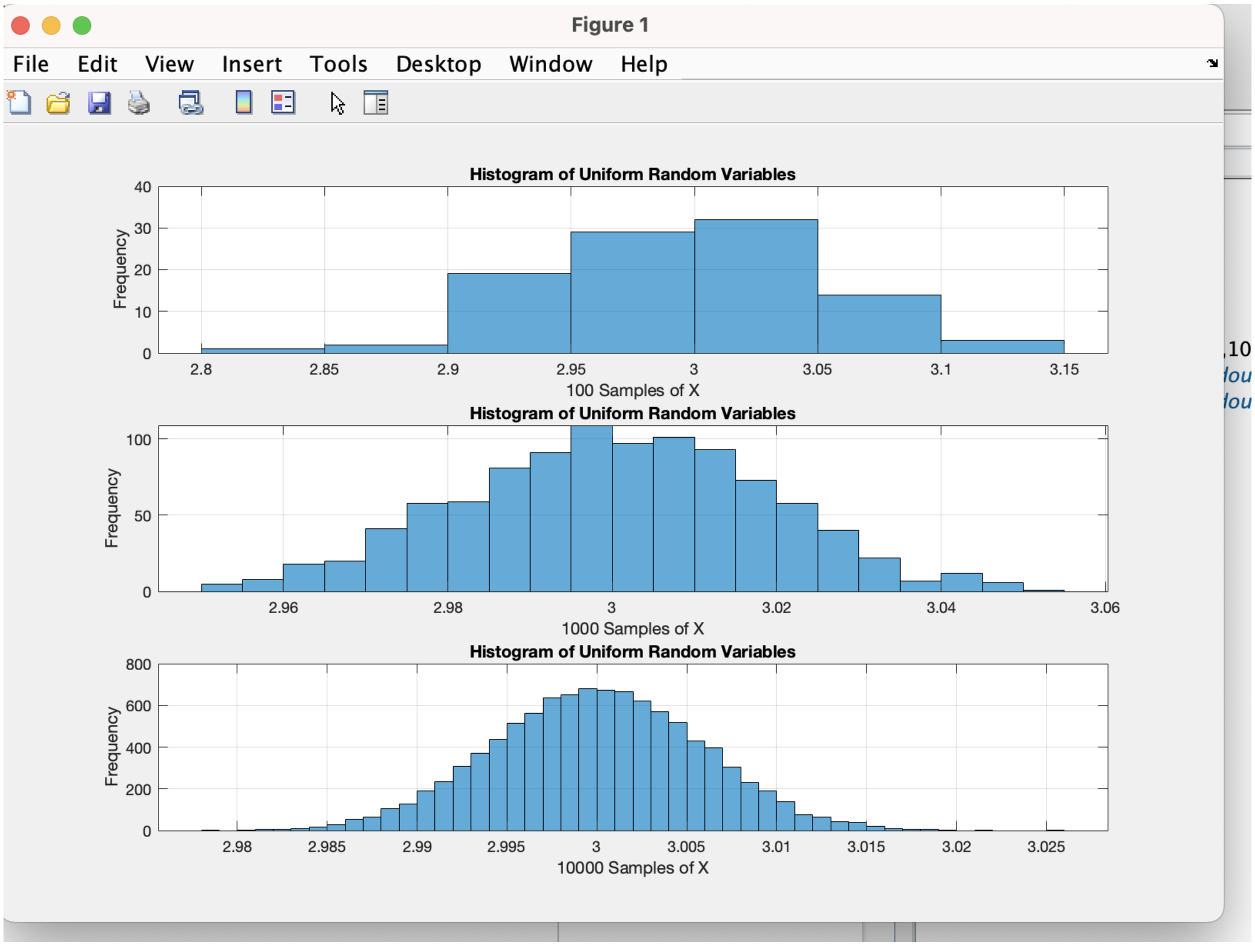


Figure 1
File Edit View Insert Tools Desktop Window Help
Histogram of Uniform Random Variables
Frequency
100 Samples of X
Histogram of Uniform Random Variables
Frequency
1000 Samples of X
Histogram of Uniform Random Variables
Frequency
10000 Samples of X

Convergence GUI

Here we have created a GUI to show that the difference between sample mean and population mean tends to zero when N tends to infinity.

If an experiment is repeated M times "independently" under essentially identical conditions, and if event A occurs k times, then as M increases, the ratio k/M approaches a fixed limit, namely the probability P (A) of A.

Taking this experiment :

$X_n = Y_n = {}_n\, {}_{i=1} Y_i \longrightarrow X=0$,where the random variables Y_i are iid N (0, 1). We use M = 500 realizations, consider ε = 0.05, and take n_{max} = 2000.

I have plotted 3 distribution of Normal, Uniform, Exponential and shown the following convergence:

Convergence in distribution.
Convergence in Mean squared
Almost sure convergence
And Convergence in Probability.

We write $X_n \xrightarrow{P} X$ and say that the sequence $(X_n)_{n\in\mathbb{N}}$ converges in probability to X if

$$\forall\epsilon > 0, \qquad p_n = P[\omega; |X_{n,\omega} - X_\omega| > \epsilon] \underset{n\to\infty}{\longrightarrow} 0. \qquad (1)$$

We write $X_n \xrightarrow{a.s.} X$ and say that the sequence $(X_n)_{n\in\mathbb{N}}$ converges almost surely to X if

$$P\left[\omega; \lim_{n\to\infty} X_{n,\omega} = X_\omega\right] = 1. \qquad (2)$$

For a real number $r > 0$, we write $X_n \xrightarrow{r} X$ and say that the sequence $(X_n)_{n \in \mathbb{N}}$ converges to X in the rth mean if

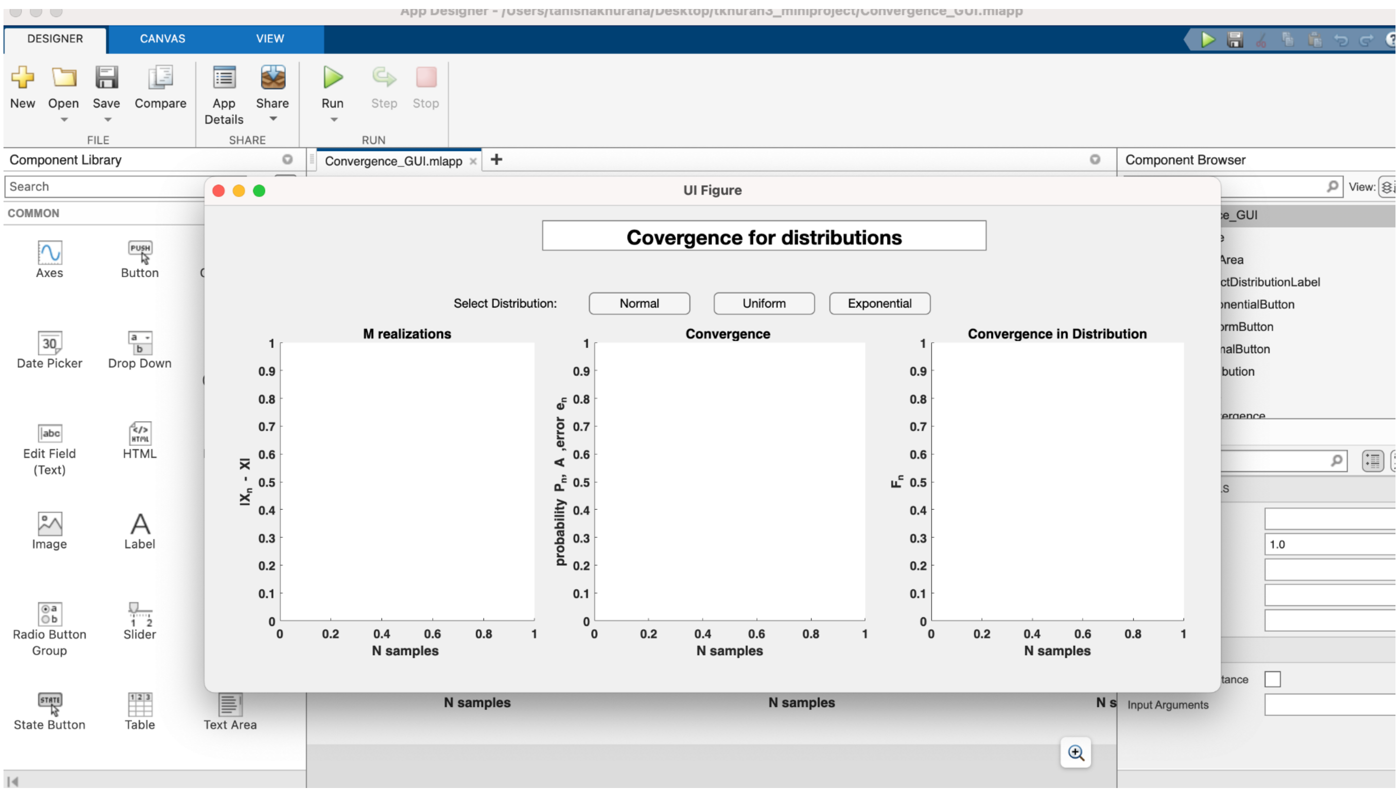
$$e_{n,r} = E|X_n - X|^r \underset{n \to \infty}{\longrightarrow} 0. \tag{3}$$

We write $X_n \xrightarrow{L} X$ and say that the sequence $(X_n)_{n \in \mathbb{N}}$, with distribution functions $(F_n)_{n \in \mathbb{N}}$, converges to X in law if

$$l_n(t) = |F_n(t) - F(t)| \underset{n \to \infty}{\longrightarrow} 0 \tag{4}$$

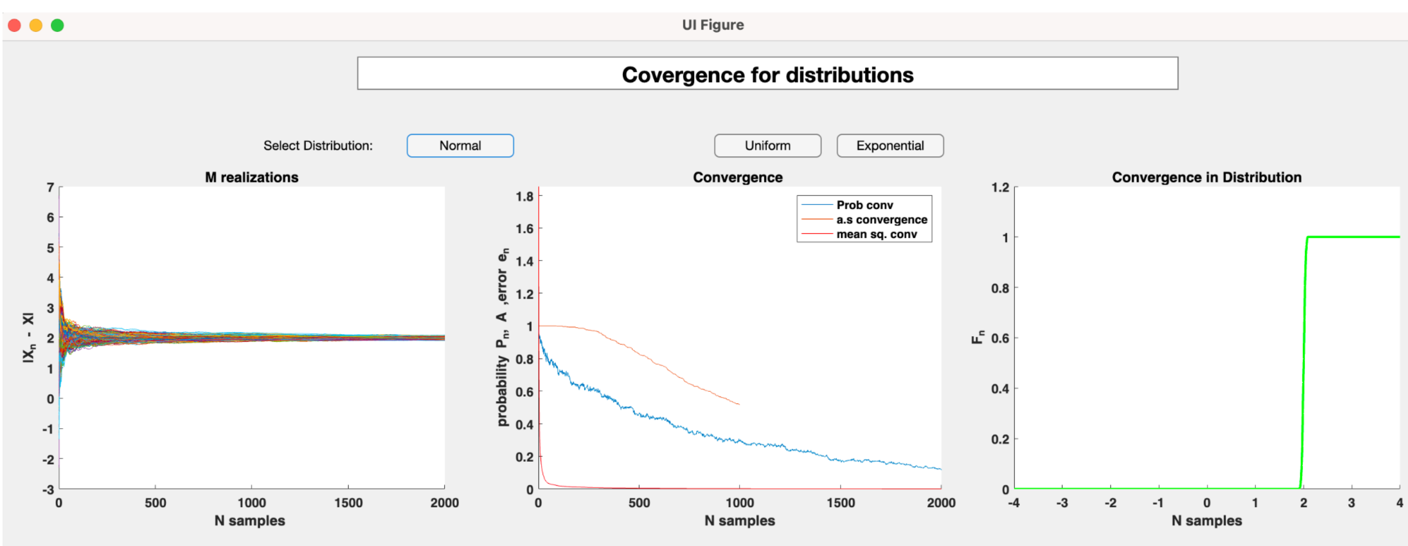
I have used a T array of MxN realizations where N = 2000 and M = 500 thus 500 realizations of 2000 samples are used.
Thus, we can see that YT converges to mean which is 3 for uniform and 2 for Normal and 2 for exponential.

And error tends to 0 and distribution is the same and the probability for almost surely tends to 1.



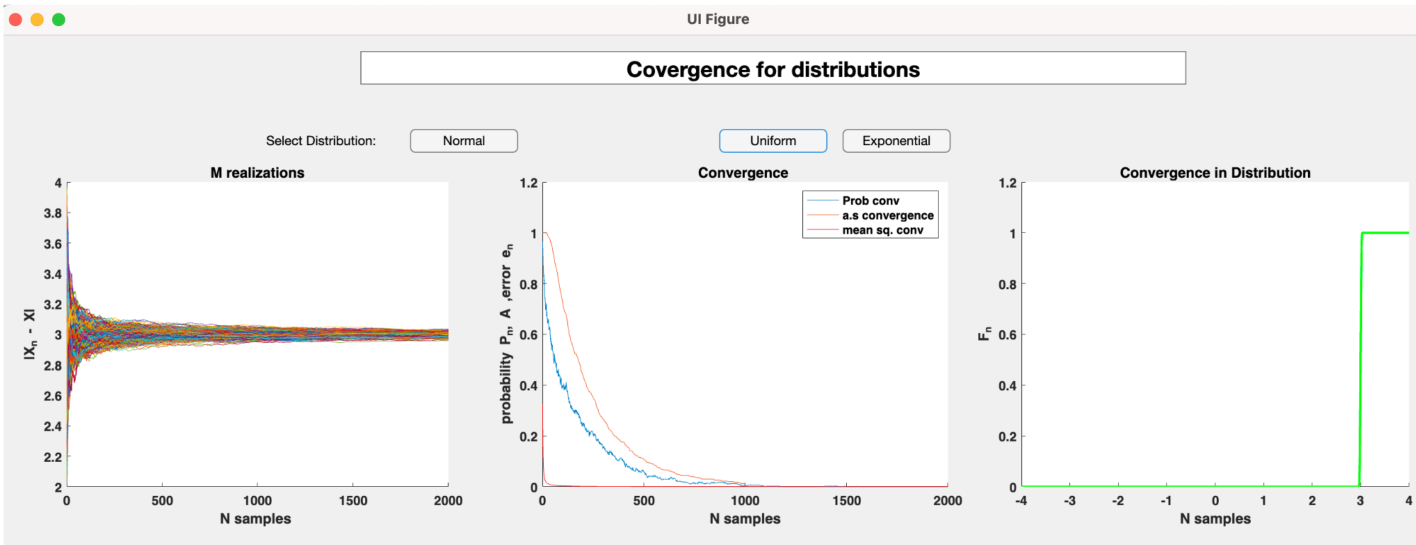
UI Figure
Covergence for distributions
Select Distribution:
Normal
Uniform
Exponential
M realizations
Convergence
Convergence in Distribution
N samples

Normal -

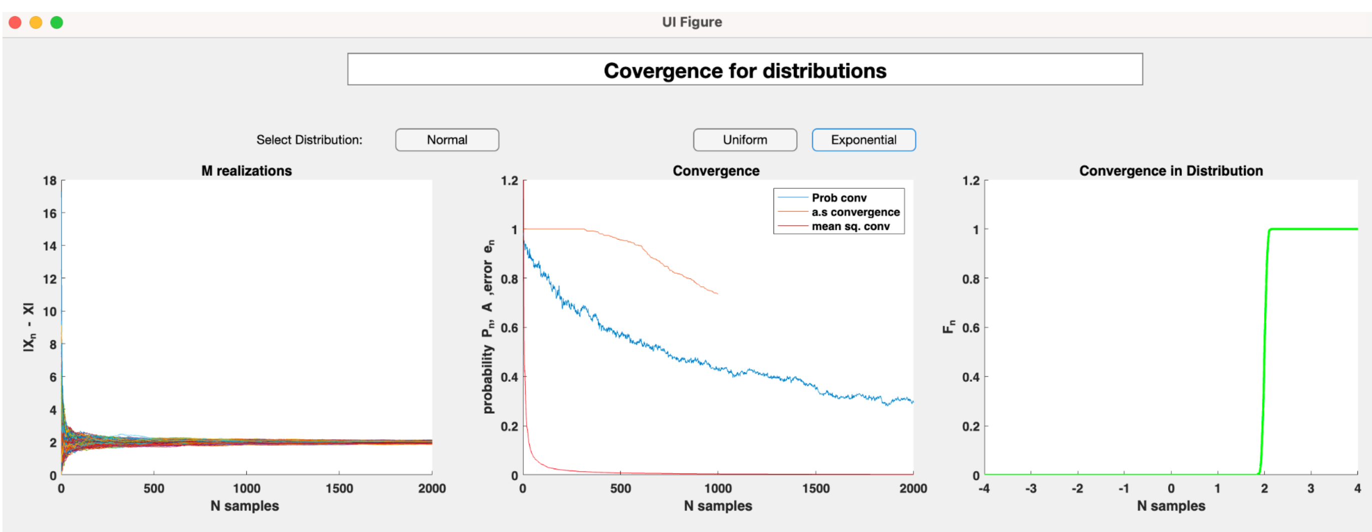


UI Figure
Covergence for distributions
Select Distribution:
Normal
Uniform
Exponential
M realizations
Convergence
Convergence in Distribution
Prob conv
a.s convergence
mean sq. conv
N samples

Uniform -



Exponential -



Part -2

In this part of the project we were asked to make a covariance matrix with the following conditions

$$f_{XY}(x,y) = \begin{cases} x + \frac{3}{2}y^2 & ,0 < x, y \leq 1 \\ 0 & ,Otherwise \end{cases}$$

To the correlation and covariance matrices of U.

Step 1: Calculate the pdf of X
Step 2: Calculate the expected values of X
Step 3: Calculate the expected values of X^2
Step 4: Calculate the variance using formula E[X^2]-E[X]
Step 5: Now calculate the covariance using the integral
Ste

$$cov_{x,y} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{N - 1}$$

We have found out the `Covariance Matrix and Correlation along with the Cholesky`

Command Window

```
[ 0.0764, -0.0104]
[-0.0104,   0.076]

[0.4167, 0.3542]
[0.3542, 0.4667]

[ 0.0739, -0.0095]
[-0.0095,  0.0765]
```