# Conceptual and Multiple Choice Exercises

**Select the correct answer(s). There could be one answer or multiple correct answers for a problem.**

1. What is the minimal condition that must be satisfied in order to use an improper prior in Bayesian inference?
   (a) The posterior distribution must be continuous
   (b) The prior distribution must be symmetric
   (c) The prior distribution must have finite mean and variance
   (d) The posterior distribution must be proper
2. Which of the following is not a meaningful prior elicitation approach?
   (a) Expert opinion
   (b) Prior centered weakly at the MLE
   (c) Point mass at the MLE
   (d) Flat uninformative prior
3. Which of the following is a model diagnostics tool?
   (a) posterior predictive checks
   (b) trace plots
   (c) the ergodic theorem
   (d) hypothesis testing
4. The effective sample size in an MCMC procedure can be interpreted as the number of dependent Monte Carlo samples necessary to give the same precision as the samples from MCMC.
   (a) True
   (b) False
5. If a conjugate prior is available for a sampling distribution (model), we should always adopt the conjugate prior for easy computation.
   (a) True

(b) False

6. Suppose $\theta^*$ is a new data point and $\theta^s$ is the current state of your Metropolis sampler. Which are symmetric proposal (jumping) distributions?

   (a) $J(\theta^* \mid \theta^s) = Normal(0, \delta^2)$.
   (b) $J(\theta^* \mid \theta^s) = Uniform(\theta^s - \delta, \theta^s + \delta)$
   (c) $J(\theta^* \mid \theta^s) = Normal(\theta^s, \delta^2)$.
   (d) None of the above are symmetric.

7. The burn-in period in MCMC is theoretically justified.

   (a) True
   (b) False

8. We frequently report the posterior mean as our Bayes estimator because it

   (a) minimizes the posterior expected linear loss
   (b) minimizes the posterior expected quadratic loss
   (c) maximizes the posterior expected linear loss
   (d) maximizes the posterior expected quadratic loss

9. Let $y_1, \dots, y_n$ be a random iid sample from an exponential distribution, where

$$p(y_i \mid \theta) = \theta e^{-\theta y_i}.$$

   Suppose the prior is

$$p(\theta) \propto \theta^{\alpha-1} \exp -\beta\theta.$$

   Write the posterior mean as a weighted average of the prior mean and $\hat{\theta}$, which is an estimate of the parameter of $\theta$ that is only based upon the data. That is, write

$$E(\theta \mid y_1, \dots, y_n) = a \times \text{prior mean} + b \times \hat{\theta}.$$

   What are $a$ and $b$?

   (a) $a = \frac{\beta}{\beta + \sum_i y_i}$ $\quad b = \frac{\sum_i y_i}{\beta + \sum_i y_i}$
   (b) $a = \frac{\alpha}{\alpha + \sum_i y_i}$ $\quad b = \frac{\sum_i y_i}{\alpha + \sum_i y_i}$
   (c) $a = \frac{\alpha}{\beta + \sum_i y_i}$ $\quad b = \frac{\sum_i y_i + \alpha}{\beta + \sum_i y_i}$
   (d) $a = \frac{\beta}{\beta + \alpha + \sum_i y_i}$ $\quad b = \frac{\sum_i y_i}{\beta + \alpha + \sum_i y_i}$

10. Rejection sampling is fairly efficient as the dimension of the problem gets larger.

   (a) True
   (b) False