

Goal: Find the Bayes rule.

- assume $\hat{\theta}$ estimates θ (r.v., unknown)
- assume data $x_1, \dots, x_n \rightarrow x_{1:n}$
- assume $l(\theta, \hat{\theta}) = (\theta - \hat{\theta})^2$ sq. error loss

To find the Bayes rule

→ minimize the posterior risk

① Find the posterior risk *← plug in my loss!*

$$\begin{aligned} r(\hat{\theta}, x_{1:n}) &\stackrel{\text{defn}}{=} E[l(\theta, \hat{\theta}) | x_{1:n}] \\ &= E[(\theta - \hat{\theta})^2 | x_{1:n}] \quad \text{expand the square} \\ &= E[\theta^2 - 2\theta\hat{\theta} + \hat{\theta}^2 | x_{1:n}] \quad \begin{array}{l} \hat{\theta} \text{ is est} \\ = (\text{known}) \\ \theta \text{ is random} \end{array} \\ &= \underbrace{E[\theta^2 | x_{1:n}] - 2\hat{\theta} E[\theta | x_{1:n}] + \hat{\theta}^2}_{\text{slide 9}} \end{aligned}$$

② Find the Bayes rule *0*

$$\begin{aligned} \frac{\partial r(\hat{\theta}, x_{1:n})}{\partial \hat{\theta}} &= \frac{\partial}{\partial \hat{\theta}} \left\{ \underbrace{E[\theta^2 | x_{1:n}]} - 2\hat{\theta} E[\theta | x_{1:n}] + \hat{\theta}^2 \right\} \\ &= 0 - 2E[\theta | x_{1:n}] + 2\hat{\theta} \quad \begin{array}{l} \text{set} \\ = 0 \\ \text{(solve for } \hat{\theta} \text{)} \end{array} \end{aligned}$$

1

$$\hat{\theta} = E[\theta | x_{1:n}]$$

$$X_1, \dots, X_n \stackrel{iid}{\sim} N(\mu, \sigma^2)$$

$$\mu \sim N(\theta, \tau^2).$$

X ~~is a random variable~~ $| \theta \sim \text{Bin}(n, \theta)$, $0 < \theta < 1$

$$p(x|\theta) = \binom{n}{x} \theta^x (1-\theta)^{n-x}$$
$$\propto \theta^x (1-\theta)^{n-x}$$