$$p(x_{1:n} \mid a, b) = \prod_{i=1}^{n} p(x_i \mid a, b)$$

$$= \prod_{i=1}^{n} \left[ab \exp(-ab x_i) \right] \qquad \text{dist. the } \prod$$

$$= (ab)^n \exp\left\{ -ab \sum_{i=1}^{n} x_i \right\}.$$

Remark / obs: likelik. is symm about a, b so if I find

$p(a \mid x, b)$ then I will know $\underline{p(b \mid x, a)}$ by symmetry.

$$p(a \mid \underbrace{x_{1:n}, b}_{\text{constant}}) \underset{a}{\propto} p(a, b, x_{1:n})$$

r.v.

$$= \underbrace{p(x_{1:n} \mid a, b)}\, p(a, b)$$

$$= \underbrace{(ab)^n \exp\left\{ -ab \sum_{i=1}^{n} x_i \right\}}_{\text{Likelih.}} \times \exp\{-a-b\}$$

$$I(x_1 > 0) \cdots I(x_n > 0)\, I(a, b > 0)$$

$$= a^n \boxed{b^n} \exp\left\{ -ab \sum_{i=1}^{n} x_i \right\} \times e^{-a} \times \boxed{e^{-b}}$$

$$\underset{a}{\propto} a^n \exp\left\{ -ab \sum_{i=1}^{n} x_i - a \right\}$$

so symm,

Due to ... we can immed write $b \mid x_{1:n}, a$

$$= a^{\underline{n+1-1}} \exp\left\{ -a \underline{\left(b \sum_{i=1}^{n} x_i + 1\right)} \right\}$$

$$a \mid x_{1:n}, b \sim \text{Gamma}\left(n+1,\ b\sum x_i + 1\right)$$

$$b \mid x_{1:n}, a \overset{\text{by symm.}}{\sim} \text{Gamma}\left(n+1,\ a\sum x_i + 1\right).$$

For practice you could derive $b \mid x_{1:n}, a$.