Exercise

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Consider the following Exponential model for an observation x:

$$p(x|a,b) = ab\exp(-abx)\mathbb{1}(x > 0)$$

and suppose the prior is

$$p(a,b) = \exp(-a-b)\mathbb{1}(a,b > 0).$$

You want to sample from the posterior $p(a,b|x)$. Find the conditional distributions needed for implementing a Gibbs sampler.

Solution

The Gibbs sampler consists of alternately sampling from $a|b,x$ and $b|a,x$. First note that the joint p.d.f. is

$$p(x,a,b) = ab\exp(-abx - a - b)\mathbb{1}(a,b,x>0).$$

Thus,

$$p(a|b,x) \underset{a}{\propto} p(x,a,b) \underset{a}{\propto} a\exp(-abx-a)\mathbb{1}(a>0) = a\exp(-(bx+1)a)\mathbb{1}(a>0) \underset{a}{\propto} \text{Gamma}(a \mid 2,\, bx+1).$$

Therefore, $p(a|b,x) = \text{Gamma}(a \mid 2,\, bx+1)$ and by symmetry, $p(b|a,x) = \text{Gamma}(b \mid 2,\, ax+1)$.