STA 360/602

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1. Let

$$
\begin{aligned}
Y|\theta &\sim \text{Exp}(\theta) \\
\theta &\sim \text{Gamma}(a, b).
\end{aligned}
$$

Assume that b is the scale parameter of the Gamma distribution for this problem. Suppose we have a new observation $\tilde{Y}|\theta \sim \text{Exp}(\theta)$, where conditional on θ, Y and $\tilde{Y}$ are independent. Show that

$$
p(\tilde{y}|y) = \frac{b(a+1)(by+1)^{a+1}}{(b\tilde{y} + by + 1)^{a+2}},
$$

where a is an integer. (Note that this is a valid density function that integrates to 1).

2. Suppose

$$
X_1, \ldots, X_n|\theta \overset{iid}{\sim} \text{Poisson}(\theta).
$$

   (a) Find Jeffreys' prior. Is it proper or improper?
   (b) Find $p(\theta|x_1, \ldots, x_n)$ under Jeffreys' prior.

3. Consider dose response models. The setup is the following: animals are tested for development of drugs or other chemical compounds. Someone administers various levels of doses to k batches of animals. The response variable is a dichotomous (binary) outcome. So, it might be alive or dead or maybe tumor or no tumor. Let x_i represent the data, n_i represent the number of animals receiving the ith dose, and y_i the number of positive outcomes for n_i animals.

   (a) Suppose that $y_i \overset{ind}{\sim} \text{Binomial}(n_i, \theta_i)$, where θ_i is the probability of death (or tumor) for the ith animal that receives dose x_i. The typical modeling the prior on θ_i is a logistic regression. That is, we suppose that $\text{logit}(\theta_i) = \alpha + \beta x_i$. Write out the likelihood in a simple form (it will contain a product).
   (b) Find Jeffreys' prior for (α, β). Also, write down the equations you need to solve for finding the posterior modes α and β under the uniform prior for α and β.