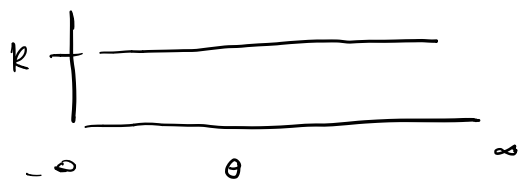
$$X_1, \ldots, X_n \mid \theta \overset{iid}{\sim} N(\theta, \sigma^2)$$

Assume improper prior on θ.

Assume that the prior on θ is constant over the real line,

$$p(\theta) \propto k, \quad -\infty < \theta < \infty$$



For conven, let $k = 1$. $p(\theta) \propto \underline{1}$.

Goal: Find $p(\theta \mid X_1, \ldots, X_n)$.

$$p(\theta \mid x_{1:n}) \propto p(x_{1:n} \mid \theta)\, p(\theta)$$ by slide 10

$\lambda = (\sigma^2)^{-1}$

$$\underset{\theta}{\propto} \prod_{i=1}^{n} \left\{ \sqrt{\frac{\lambda}{2\pi}} \exp\left(-\frac{\lambda}{2}(x_i - \theta)^2\right) \right\} \times 1$$

$\underline{(\pm \bar{x})}$

why? It leads to cancellation!

$$\underset{\theta}{\propto} \exp\left\{ -\frac{\lambda}{2} \sum_{i=1}^{n} (x_i - \theta)^2 \right\}$$

$$= \exp\left\{ -\frac{\lambda}{2} \sum_{i=1}^{n} (x_i - \bar{x} + \bar{x} - \theta)^2 \right\}$$

$$= \exp\left\{-\frac{\lambda}{2}\sum_{i=1}^{n}(x_i-\bar{x})^2\right\} \times \exp\left\{-\frac{\lambda}{2}\sum_{i=1}^{n}(\bar{x}-\theta)^2\right\}$$

$$\times\ \underbrace{2\exp\left\{-\frac{\lambda}{2}\sum_{i=1}^{n}(x_i-\bar{x})\underline{(\bar{x}-\theta)}\right\}}_{\parallel}$$

$$2\exp\left\{-\frac{\lambda}{2}(\bar{x}-\theta)\underbrace{\sum_{i=1}^{n}(x_i-\bar{x})}_{n\bar{x}-n\bar{x}=0}\right\}$$

$\sum x_i - n\bar{x}$ =

$$= 2e^0 = 2.$$

$$\underset{\theta}{\propto}\ \boxed{\underbrace{2\exp\left\{-\frac{\lambda}{2}\sum_{i=1}^{n}(x_i-\bar{x})^2\right\}}_{\text{constant!}}} \times \exp\left\{-\frac{\lambda n}{2}(\bar{x}-\theta)^2\right\}$$

$$\underset{\theta}{\propto}\ \exp\left\{-\frac{\lambda n}{2}(\bar{x}-\theta)^2\right\}$$

$$= \exp\left\{-\frac{\lambda n}{2}(\theta-\bar{x})^2\right\} \quad \text{Symmetry}$$

$$\theta \mid x_1,\dots,x_n \sim N\left(\bar{x},\ (\lambda n)^{-1}\right).$$