$$\frac{1}{\theta_2^2}\sum_{i=1}^{m}(x_i-\theta_1)^2 = \frac{m}{\theta_2}$$

$$\frac{\theta_2^2}{\theta_2} = \frac{1}{m}\sum_{i=1}^{m}(x_i-\theta_1)^2$$

$$\Rightarrow \theta_2 = \frac{1}{n}\sum_{i=1}^{m}(x_i-\theta_1)^2$$

Sample variance

2. $x_1, x_2, \dots, x_m$ be a random sample from $B(m, \theta)$ distribution, where $\theta \in \Theta = (0,1)$ is unknown and 'm' is a known +'ve integer. Compute θ using MLE.

We are given Binomial distribution.

$$L(\theta) = \prod_{i=1}^{n}\binom{m}{x_i}\theta^{x_i}(1-\theta)^{m-x_i}$$

Taking natural log,

$$\ln(L(\theta)) = \sum_{i=1}^{n}\left[\ln\binom{m}{x_i} + x_i\ln(\theta) + (m-x_i)\ln(1-\theta)\right]$$

Diff. w.r.t. θ

$$\frac{\partial}{\partial\theta}\ln L(\theta) = \sum_{i=1}^{m}\left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta}\right) = 0$$

Solving for θ

$$\sum_{i=1}^{n}\frac{x_i}{\theta} = \sum_{i=1}^{n}\frac{m-x_i}{1-\theta}$$

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Parameter Estimation

1. $(x_1, x_2, \ldots)$ be a random sample of size n taken from a normal population, mean = θ and variance = θn. Find MLE of these 2 Parameters.

Function will be [Normal Population Distribution]

$$L(\theta_1, \theta_2) = \prod_{i=1}^{m} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{\left(\left(\frac{x_i - \mu}{2\sigma^2}\right)^2\right)}$$

Take natural log on both sides

$$\ln(L(\theta_1, \theta_2)) = \sum_{i=1}^{m} \left(\frac{-(x_i - \mu)^2}{2\sigma^2} - \frac{1}{2}\ln(2\pi\sigma^2) \right)$$

To find the MLE, differentiate log-likelihood of n w.r.t θ_1 & θ_2.

~~To find the MLE, differentiate log-likelihood~~

$$\frac{d}{d\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1} \left(\frac{x_i - \mu}{\sigma^2} \right) = 0$$

This implies,

$$\sum_{i=1}^{m} x_i - n\mu = 0$$

$$\frac{\theta_1}{\mu} = \frac{1}{n} \sum_{i=1}^{m} x_i$$

For θ_2,

$$\frac{d}{d\theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{m} \left(-\frac{(x_i - \theta_i)^2}{2(\theta_2)^2} + \frac{1}{2\theta_2} \right) = 0$$

$$\Rightarrow \sum_{i=1}^{m} \frac{(x_i - \theta_1)^2}{\theta_2^2} - \frac{m}{\theta_2} = 0$$

$$\sum_{i=1}^{n} x_i(1-\theta) = \sum_{i=1}^{n} (m - x_i)\theta$$

$$\cancel{\theta(}\ \theta \sum_{i=1}^{n} x_i = m \sum_{i=1}^{n} \theta$$

$$\boxed{\theta = \frac{1}{m} \sum_{i=1}^{n} x_i}$$

MLE of θ is sample mean of observation