Tutorial - 4

(1) $T(n) = 3T(n/2) + n^2$

$a = 3,\ b = 2 \quad , \quad f(n) = n^2$

$n^{\log_b^a} = n^{\log_2^3}$

Comparing $n^{\log_2^3}$ and n^2

$n^{\log_2^3} < n^2$ (case 3)

$\therefore$ according to

(2) $T(n) = \cancel{4}\, T(n/2) +$

$a = 1,\ b = 2$

$n^{\log_2^1} = n^0 = 1$

$1 < 2^n$ (case 3)

$\therefore$ according to

(3) $T(n) = 4T(n/2) + n^2$

$a = 4,\ b = 2$

$n^{\log_b^a} = n^{\log_2^4} = n^2 = f(n)$ (case 2)

$\therefore$ according to

(4) $T(n) = 16T(n/4) + n$

$a = 16,\ b = 4$

$n^{\log_b^a} = n^{\log_4^{16}} = n^2$

$n^2 > f(n)$ (case 1)

(5) $T(n) = 16T(n/4) + n$

$a = 16$, $b = 4$

$n^{\log_b^a} = n^{\log_4^{16}} = n^2$, $f(n) = n$

$n^2 > f(n)$ (case 1)

$T(n) = \theta(n^2)$

(6) $T(n) = 2T(n/2) + n\log n$

$a = 2$, $b = 2$,

$f(n) = n\log n$

$n^{\log_b^a} = n^{\log_2^2} = n$

Now $f(n) > n$

$\therefore$ According to

(7) $T(n) = 2T(\frac{n}{2}) + \frac{n}{\log n}$

$a = 2$, $b = 2$, $f(n) = \frac{n}{\log n}$

$n^{\log_b^a} = n^{\log_2^2} = n$

$n > f(n)$

$\therefore$ According to

(8) $T(n) = 2T(\frac{n}{4}) + n^{0.51}$

$a = 2$, $b = 4$, $f(n) = n^{0.51}$

$n^{\log_b^a} = n^{\log_4^2} = n^{0.5}$

$n^{0.5} < f(n)$

$\therefore$ According to

(9) $T(n) = 0.5T(\frac{n}{2}) + \frac{1}{n}$

$\therefore$ $a < 1$

(10) $T(n) = 16T(\frac{n}{4}) + n!$

$a = 16, b = 4, f(n) = n!$

$n^{\log^a} = n^{\log^{16}} = n^2$

$n^2 < n!$

$\therefore$ According to

(11) $T(n) = sqrt(n) + (n/2) + \log^n$

$\therefore$

(12) $T(n) = 4T(n/2) + \log^n$

$a = 4, b = 2, f(n) = \log^n$

$n^{\log_b^a} = n^{\log_2^4} = n^2$

$n^2 > f(n)$

$\therefore$ According to

(13) $T(n) = 3T(\frac{n}{2}) + n$

$a = 3, b = 2, f(n) = n$

$n^{\log_b^a} = n^{\log_2^3} = n^{1.58}$

$n^{1.58} > f(n)$.

$\therefore$ According to

(14) $T(n) = 3T(\frac{n}{3}) + \sqrt{n}$

$a = 3, b = 3, f(n) = \sqrt{n}$

$n^{\log_b^a} = n^{\log_3^3} = n$

$n > \sqrt{n}$

$\therefore$ According to

(15) $T(n) = 4T(n/2) + cn$

$a = 4, b = 2, f(n) = c*n$

$n^{\log_b a} = n^{\log 4} = n^2$

$\frac{n}{2} > c*n$

$\therefore$ According to

(16) $T(n) = 3T(n/4) + n\log n$

$a = 3, b = 4, f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n\log n$

$\therefore$ According to

(17) $T(n) = 3T(n/3) + n/2$

$a = 3, b = 3, f(n) = \frac{n}{2}$

$n^{\log_b a} = n^{\log_3 3} = n$

$Q(n) = Q(\frac{n}{2})$

$\therefore$ According to

(18) $T(n) = 6T(n/3) + n^2\log n$

$a = 6, b = 3, f(n) = n^2\log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{1.63} < n^2\log n$

$\therefore$ According to

(19) $T(n) = 4T(n/2) + n\log n$

$a = 4, b = 2, f(n) = n\log n$

$n^{\log_b a} = n^{\log_2 4} = n2$

$n^2 > n/\log n$

$\therefore$ According to

(Q0) $T(n) = 64T(n/8) - n^2 \log n$

Master's theoram is not applicable as $f(n)$ is not increasing function.

(Q1) $T(n) = 7T(n/3) + n^2$

$a = 7, b = 3, f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

$n^{1.7} < n^2$

$\therefore$ According to

(Q2) $T(n) = T(n/2) + n(2 - \cos n)$

Masters theoram is n't applicable since regularly condition is violated in case 3.