ECS647U / ECS773P
Bayesian Decision and Risk (BDRA)
Semester B, 2022

Coursework 2 –Bayes Theorem and Bayesian Learning

Deadline: Thursday 14th April 2022

This coursework is based on a fictitious virus called SARS-Cov-4 which causes a disease called Covid-24.

Question 1

> Despite assumptions to the contrary, it is possible to test positive using a PCR test, and be diagnosed as having Covid-24, even if someone has not been infected with SARS-Cov-4 and has no symptoms of the disease. The government's policy recommendation is that people testing positive should self isolate for 10 days. A number of factors might influence the reliability of the PCR test such as cross reactivity with other viruses, lab mix-ups and faulty/non-sterile testing equipment.

a) Calculate the posterior probability of a random person, with no Covid-24 symptoms, actually having SARS-Cov-4, $P(SARS\text{-}Cov\text{-}4 = True \mid PCR\ Test = Positive)$, given the following information [10]:

$P(SARS\text{-}Cov\text{-}4 = True) = 0.02$
$P(PCR\ Test = Positive \mid SARS\text{-}Cov\text{-}4 = True) = 0.9$
$P(PCR\ Test = Positive \mid SARS\text{-}Cov\text{-}4 = False) = 0.2$

b) Calculate the marginal probability of a false positive result [5]:

$P(False\ positive = Yes \mid PCR\ Test, SARS\text{-}Cov\text{-}4)$

c) Use the Binomial distribution to calculate the number of false positives, f, in the population, where $p = P(False\ positive = Yes \mid PCR\ Test, SARS\text{-}Cov\text{-}4)$. [5]

$$f \sim Binomial(p, n = 50\ milion)$$

d) What would the implications of random SARS-Cov-4 screening be on the self-isolation rate in an adult population of 50 million people subject to such screening? [5]

Question 2

This question is based on a fictitious therapeutic treatment and a fictitious vaccine for Covid-24.

Five groups of independent researchers in different countries have treated patients using a proposed therapeutic cure for severe cases of Covid-24. The data from these experiments is given below, where the number of patients in experiment i is n_i, the number of patients that died is x_i and p_i is the probability of a patient dying in a given experiment.

Experiment i	n_i	x_i	$p_i = \frac{x_i}{n_i}$
1	100	20	0.20
2	115	25	0.22
3	37	6	0.16
4	22	6	0.27
5	30	9	0.30
pooled	304	66	0.22

a) You are performing a meta-analysis to combine all of the data from these studies to assess the effectiveness of the therapeutic. To do so you must build a BN parameter learning model with the following configuration:

$$p_i \sim Beta(a, b, 0, 1)$$
$$a \sim Uniform(0, 200)$$
$$b \sim Uniform(0, 200)$$
$$x_i \sim Binomial(p_i, n_i)$$

Calculate $P(p \mid a, b, n_i, x_i, p_i)$ where p is the estimated probability of death for the therapeutic treatment. [15]

b) Current policy for treatment of Covid-24 is to rely on prior vaccination. Assuming if someone is already vaccinated there is a 20% probability of death if suffering from severe Covid-24. Calculate the probability than the therapeutic cure is better/worse than vaccination at reducing the probability of death from severe Covid-24. [5]

c) Would you recommend switching to use of the therapeutic or recommend continuing with vaccination policy? [5]

Notes:

- Where relevant use AgenaRisk to specify BNs and perform the necessary calculations. In your answer show the relevant probability distributions as screen shots and show the necessary summary statistics and probabilities needed for your answer.
- In Q1 assume 'no symptoms' is simply background information and plays no formal role in the probability evaluation.
- For all models use simulation settings: Max number of iterations = 50 and simulation convergence 0.001. Also use integer type nodes for Binomial distributions.