ME512/ME6106: Mobile Robotics

Robotics Terminology

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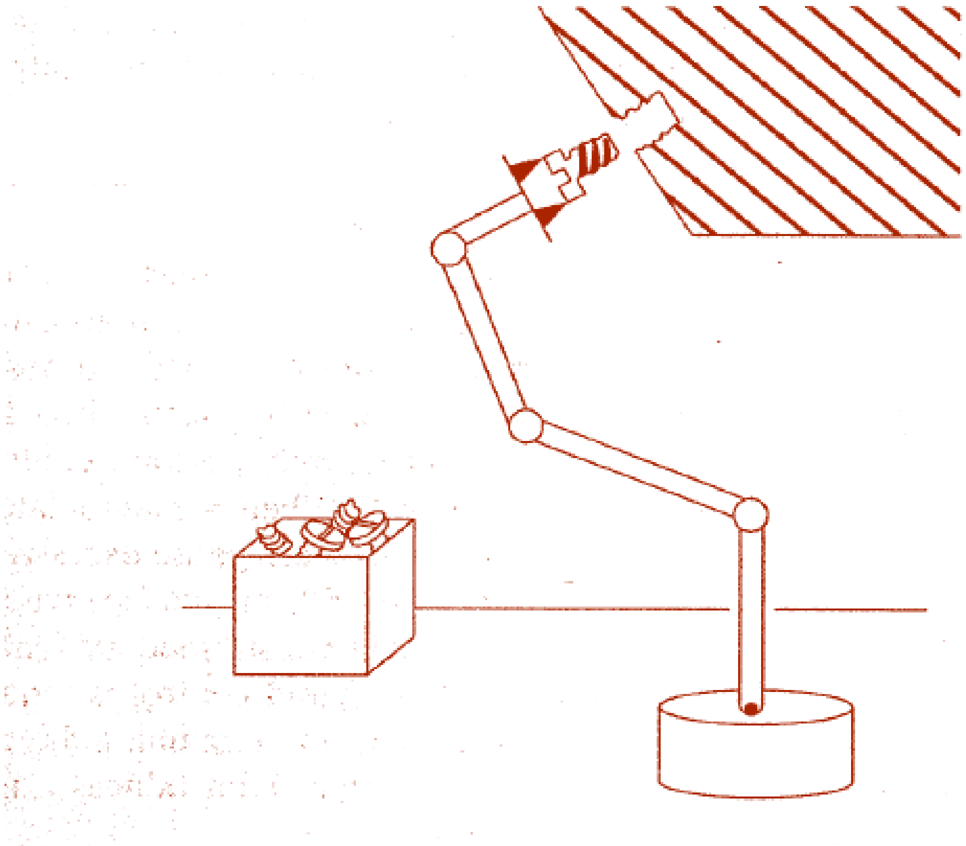
Parts of Stationary Robot

- Mechanisms (Mechanical Structure)
- End Effecters
- Tools
- Controllers
- Actuators
- Sensors
- Programming Interface

A typical task



What do we need?

- How to move the robot through space to complete the specified task
- What torques and forces to apply on the joints?

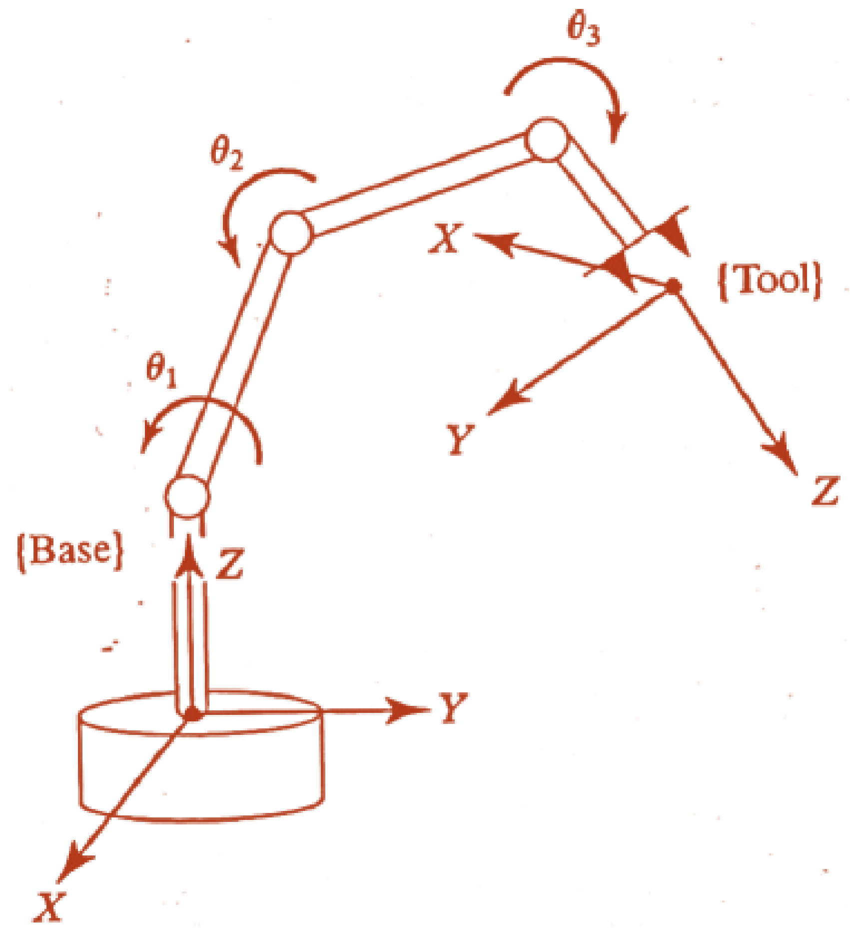
Modeling

- ❑ The underlying mathematical model that describes the robot behavior

- ✓ Will be needed to support off line programming
- ✓ Will be needed to design robots
- ✓ Selecting motors, link lengths and cross sections

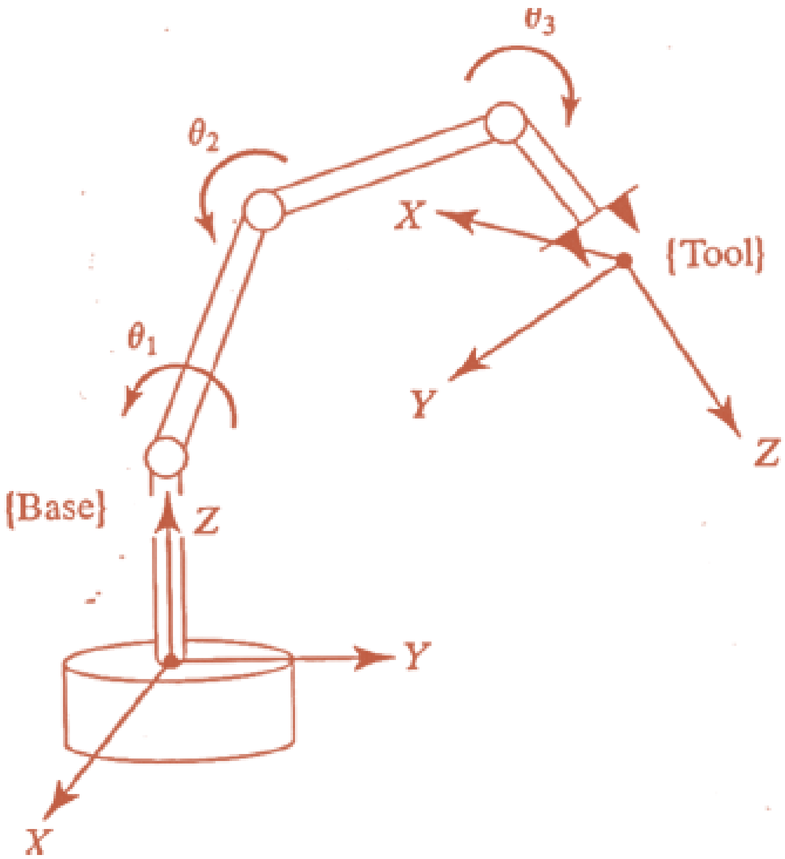
Forward Kinematics

- Given joint parameters, determine the final end effecter location



Inverse Kinematics

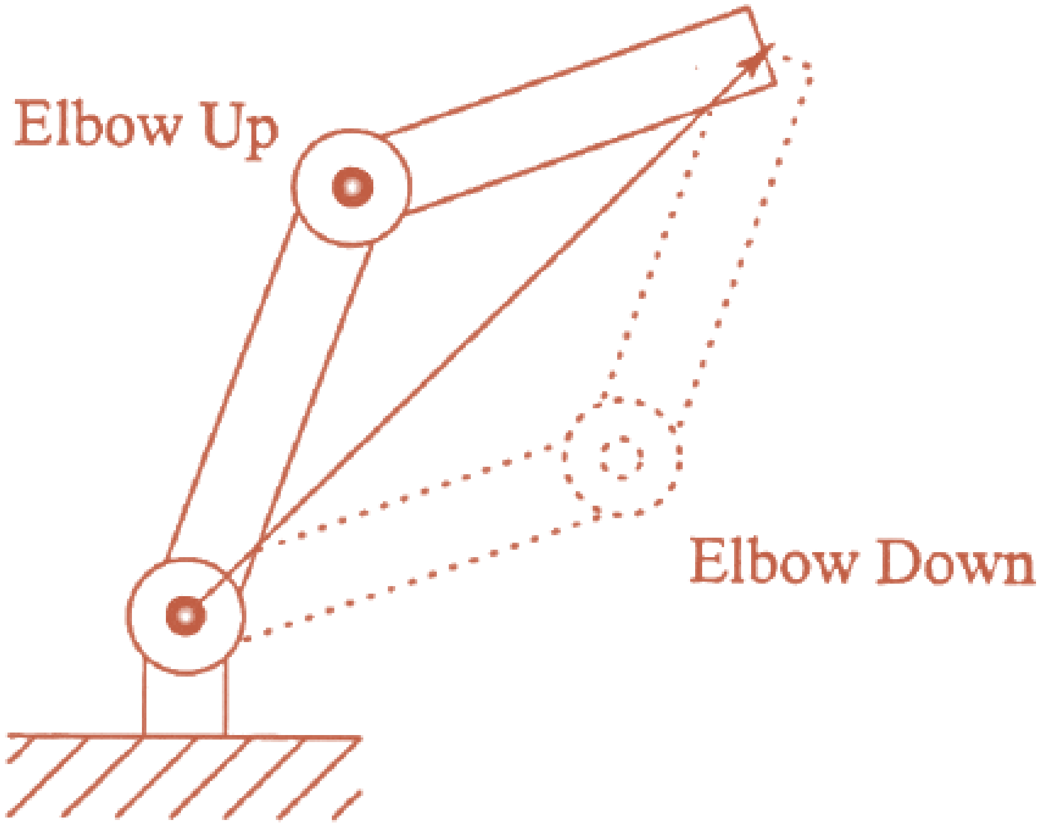
- Given desired end effecter position and orientation determine the joint parameters



Inverse Kinematics Solutions

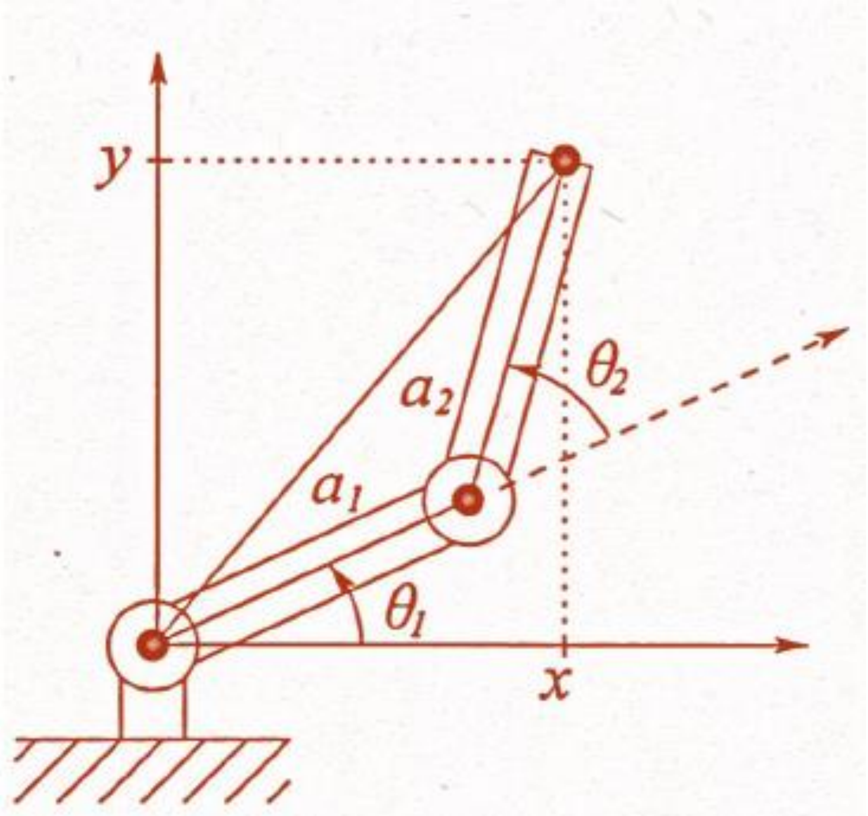
- Inverse kinematics may produce
  - One solution
  - Multiple solution
  - No solution

Multiple Solution Case



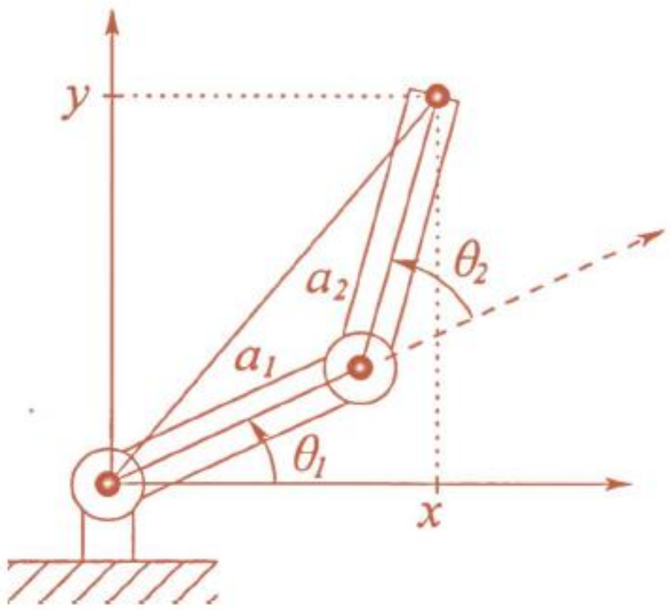
No Solution Case

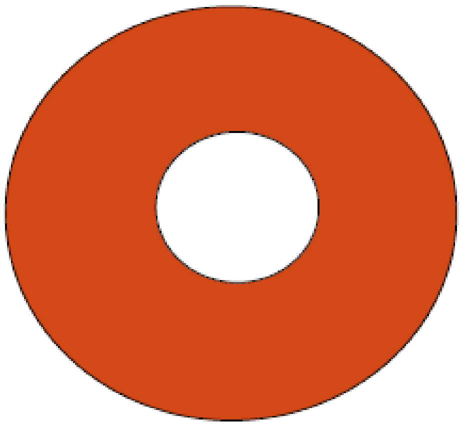
The selected location is outside the robot workspace



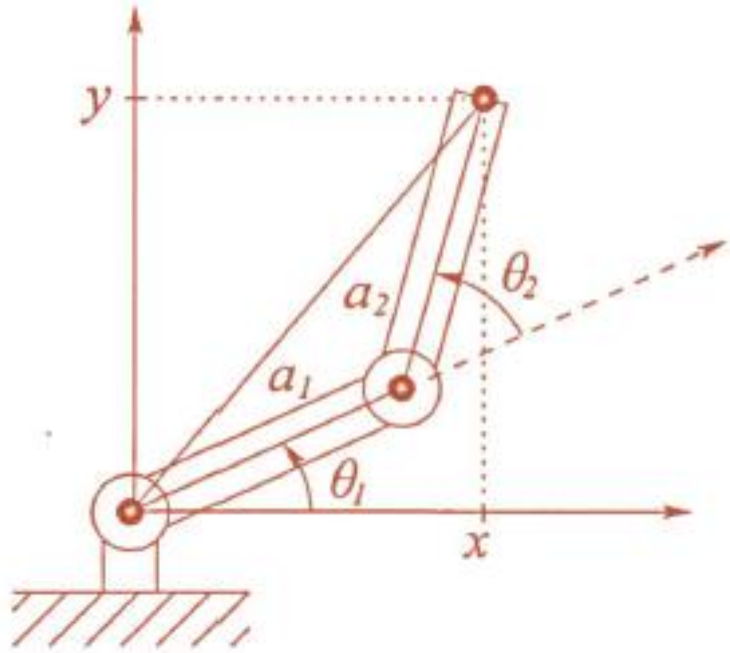
Workspace

The set of locations that can be reached by the robot





Example

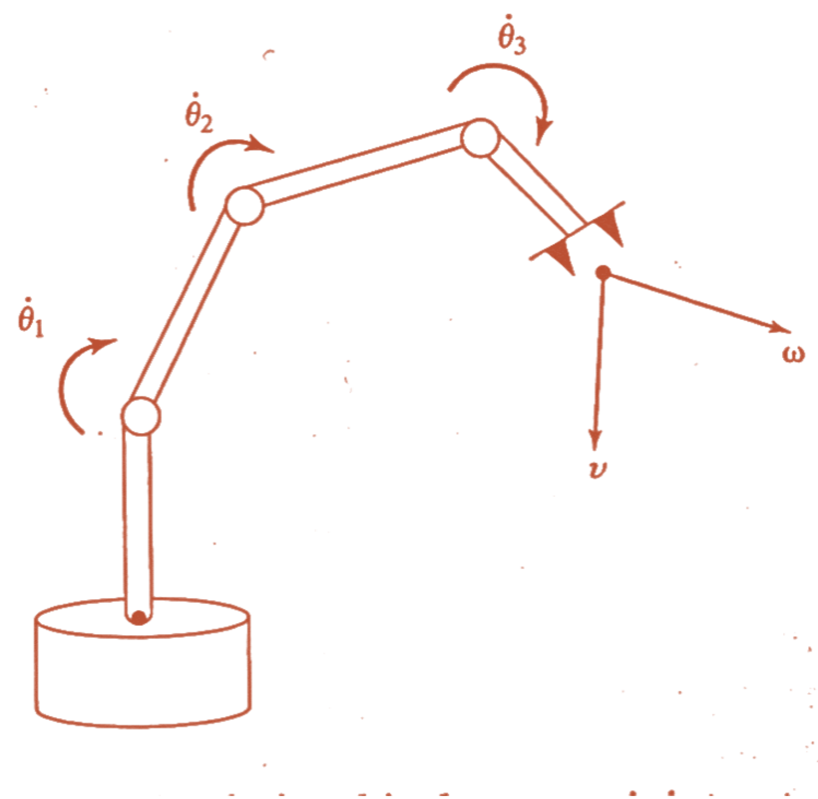


$$x = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2)$$

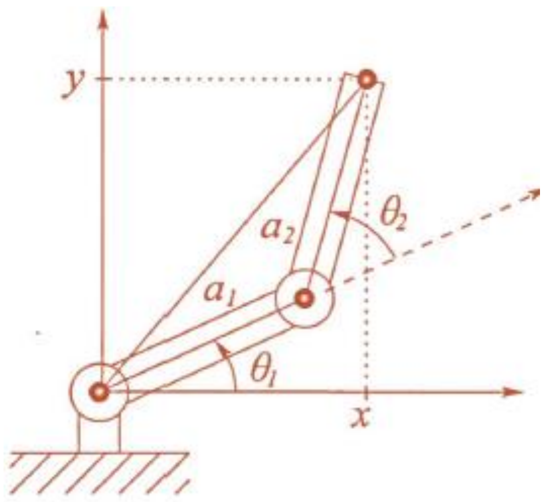
$$y = a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2)$$

Jacobian: Relating Velocities with Joint Velocities

- Given joint velocities determine the desired end effecter velocities



Example



$$x = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2)$$

$$y = a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2)$$

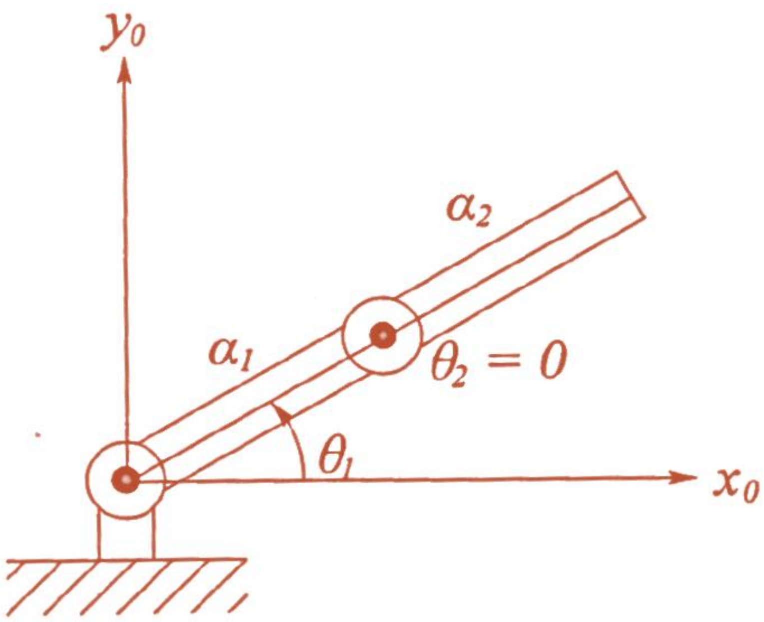
$$\Rightarrow \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} -a_1 \sin\theta_1 - a_2 \sin(\theta_1 + \theta_2) & -a_2 \sin(\theta_1 + \theta_2) \\ a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2) & a_2 \cos(\theta_1 + \theta_2) \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}$$

Jacobian

$$\dot{x} = J\dot{\Theta}$$

Achieving Desired End Effecter Velocities

- Jacobian matrix needs to be inverted to determine the joint velocities to achieve the desired end effecter velocity
- This is not always possible
- Matrix may not be invertible



$$\dot{\Theta} = J^{-1}\dot{x}$$

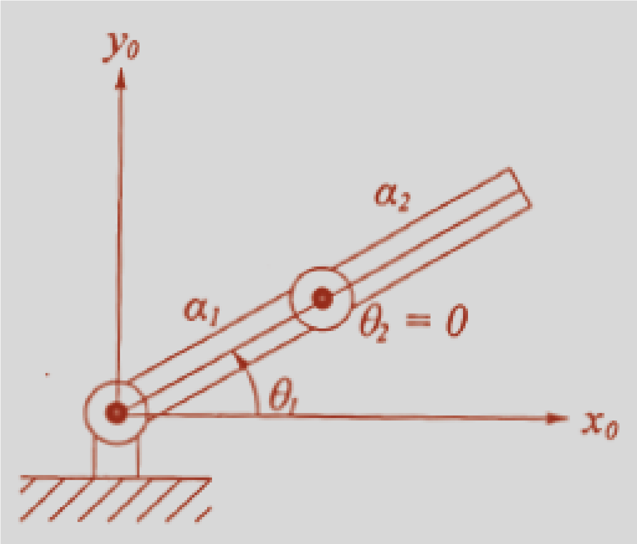
Singularity

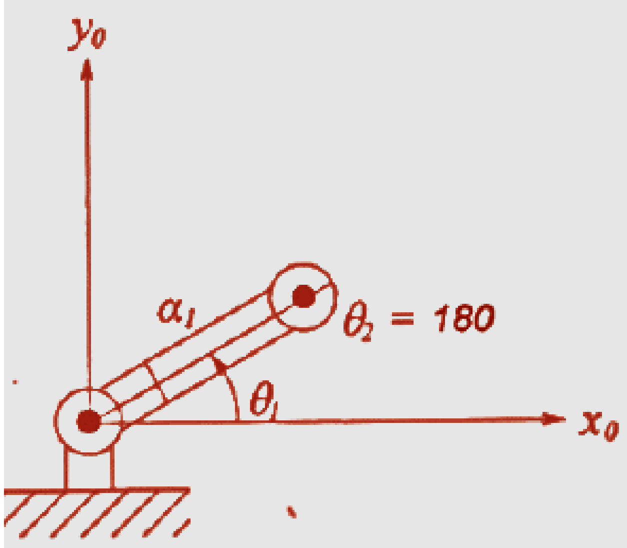
$$\dot{\Theta} = J^{-1}\dot{x}$$

Can it be guaranteed that J is always invertible?

$$\det(J) = a_1 a_2 \sin(\theta_2)$$

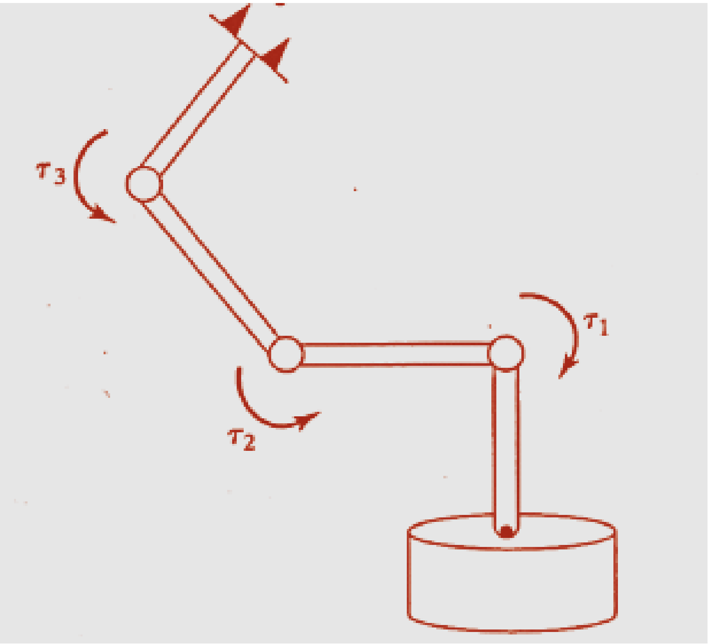
At θ_2=0 or 180 degrees J can't be inverted





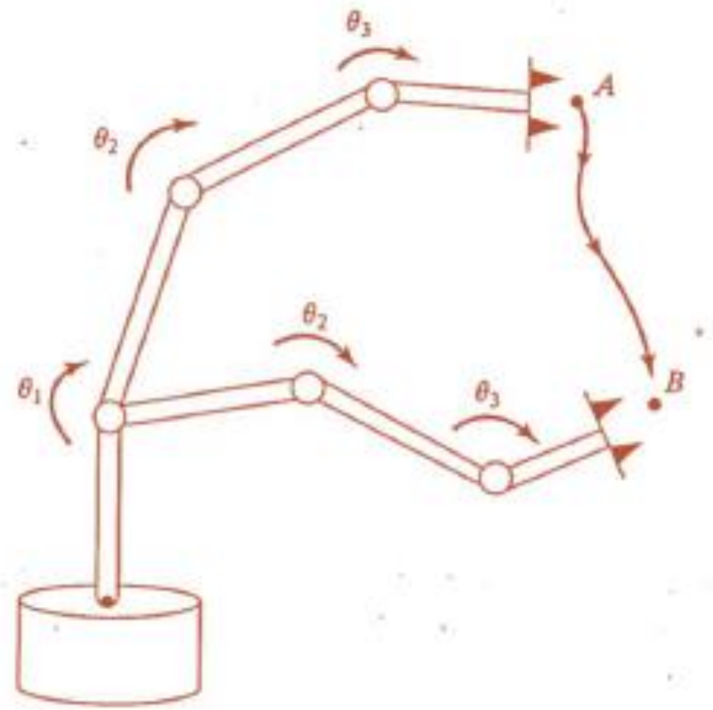
Dynamics

What forces and torque need to be applied to joints to achieve the desired velocities and accelerations ?



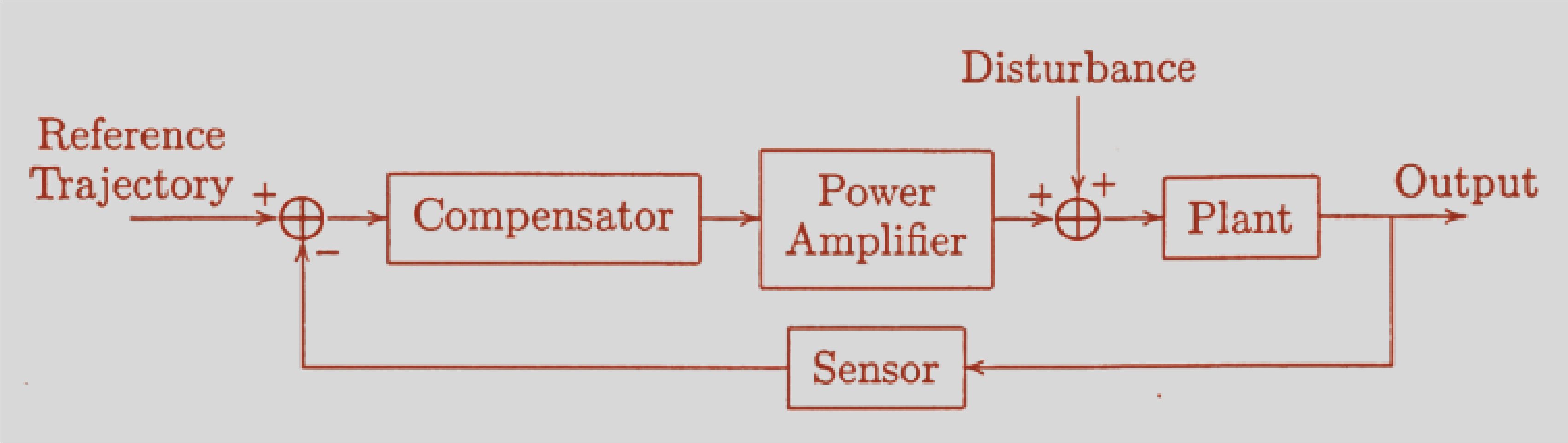
Trajectory Generation

How to trace a path through the space at the specified velocities



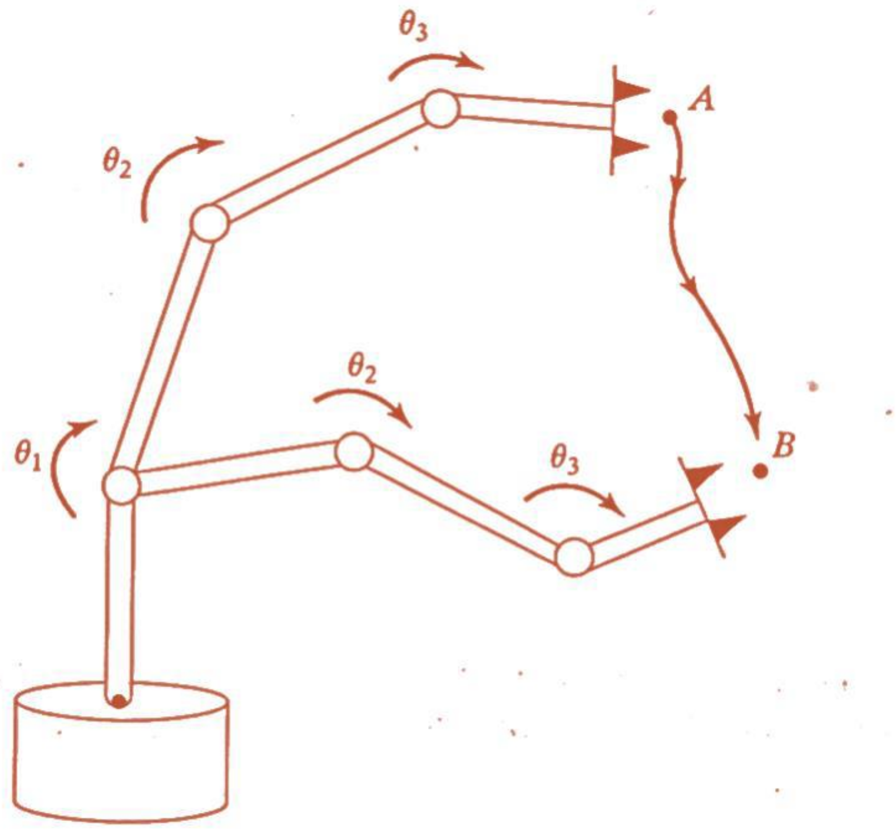
Open Loop vs Closed Loop Control

- If we have a perfect model, then we can just position motors at the desired location with no feedback
- But this does not work in practice



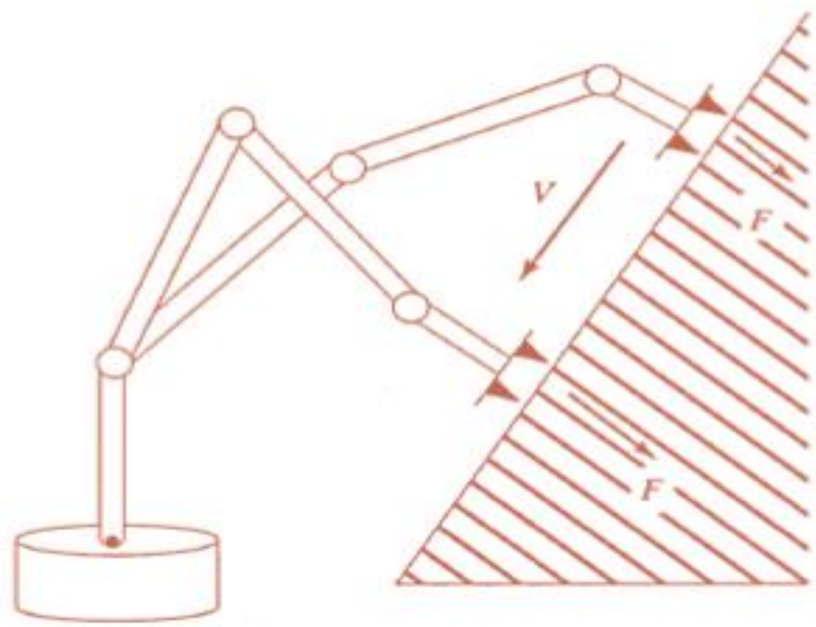
Position Control

- How to compensate for errors and inaccuracies



Force Control

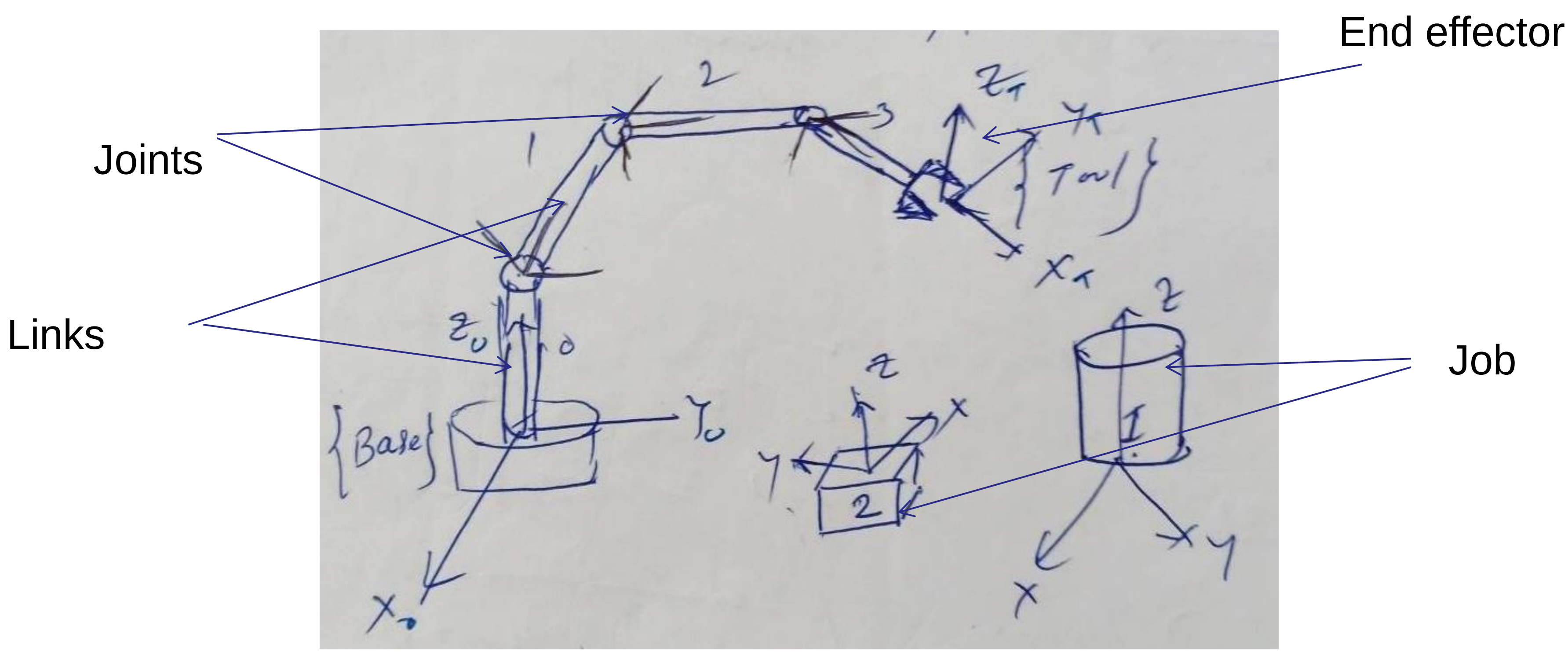
- Control force to ensure that robot can handle delicate objects and move more constrained surfaces



Description of position and orientation

❖Concerned with the location of the objects in three dimensional space

❖**Objects:** links, joints, end effector/tool and job/workpiece



How to describe and manipulate position and orientation mathematically?

Mathematical fundamentals for robot kinematics

Mapping:

- Changing the description of a point (or value) in space from one frame to another frame.

Different cases of mapping

(1) Second frame is rotated w.r.t the first, origin of both frames is same (changing of orientation)

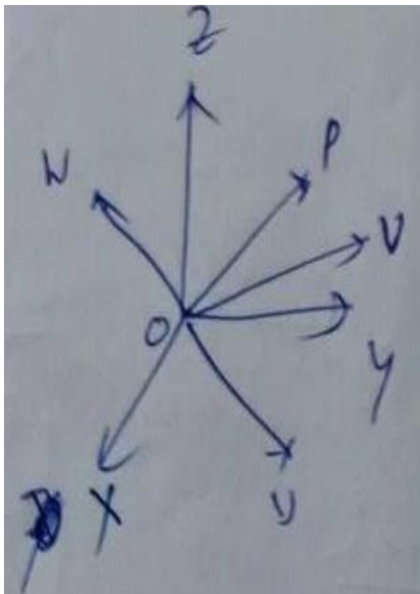
(2) Second frame is moved away from the first. Axes of both frames remain parallel (change of position)

(3) Second frame is rotated w.r.t the first and move away from it (change of position and orientation)

Mathematical fundamentals for robot kinematics

Mapping:

(1) Second frame is rotated w.r.t the first, origin of both frames is same (changing of orientation)



$$\{1\} \rightarrow \{x\ y\ z\}$$

$$\{2\} \rightarrow \{u\ v\ w\}$$

$$^1\vec{p} = {}^1p_x\,\hat{x} + {}^1p_y\,\hat{y} + {}^1p_z\,\hat{z}$$

$$^2\vec{p} = {}^2p_u\,\hat{u} + {}^2p_v\,\hat{v} + {}^2p_w\,\hat{w}$$

$$^1p_x = \hat{x}\cdot{}^2\vec{p}$$

$$= \hat{x}\cdot\left({}^2p_u\,\hat{u} + {}^2p_v\,\hat{v} + {}^2p_w\,\hat{w}\right)$$

$$^1p_y = \hat{y}\cdot\left({}^2p_u\,\hat{u} + {}^2p_v\,\hat{v} + {}^2p_w\,\hat{w}\right)$$

$$^1p_z = \hat{z}\cdot\left({}^2p_u\,\hat{u} + {}^2p_v\,\hat{v} + {}^2p_w\,\hat{w}\right)$$

Mathematical fundamentals for robot kinematics

Mapping:

$$\begin{pmatrix} {}^1p_x \\ {}^1p_y \\ {}^1p_z \end{pmatrix} = \begin{bmatrix} \hat{x}\cdot\hat{u} & \hat{x}\cdot\hat{v} & \hat{x}\cdot\hat{w} \\ \hat{y}\cdot\hat{u} & \hat{y}\cdot\hat{v} & \hat{y}\cdot\hat{w} \\ \hat{z}\cdot\hat{u} & \hat{z}\cdot\hat{v} & \hat{z}\cdot\hat{w} \end{bmatrix} \begin{bmatrix} {}^2p_u \\ {}^2p_v \\ {}^2p_w \end{bmatrix}$$

$$\boxed{{}^1\vec{p} = {}^1R_2\, {}^2\vec{p}}$$

Relation of rotation matrix

$$^{2}\vec{p} = {}^{2}R_{p}\,{}^{1}\vec{p} \qquad {}^{2}R_{1} = ({}^{1}R_{2})^{T}$$

$$^{1}\vec{p} = {}^{1}R_{2}\,{}^{2}\vec{p} \Rightarrow {}^{2}\vec{p} = ({}^{1}R_{2})^{-1}\,{}^{1}p$$

$$^{2}R_{1} = ({}^{1}R_{2})^{-1}$$

$$({}^{1}R_{2})^{T} = ({}^{1}R_{2})^{-1}$$

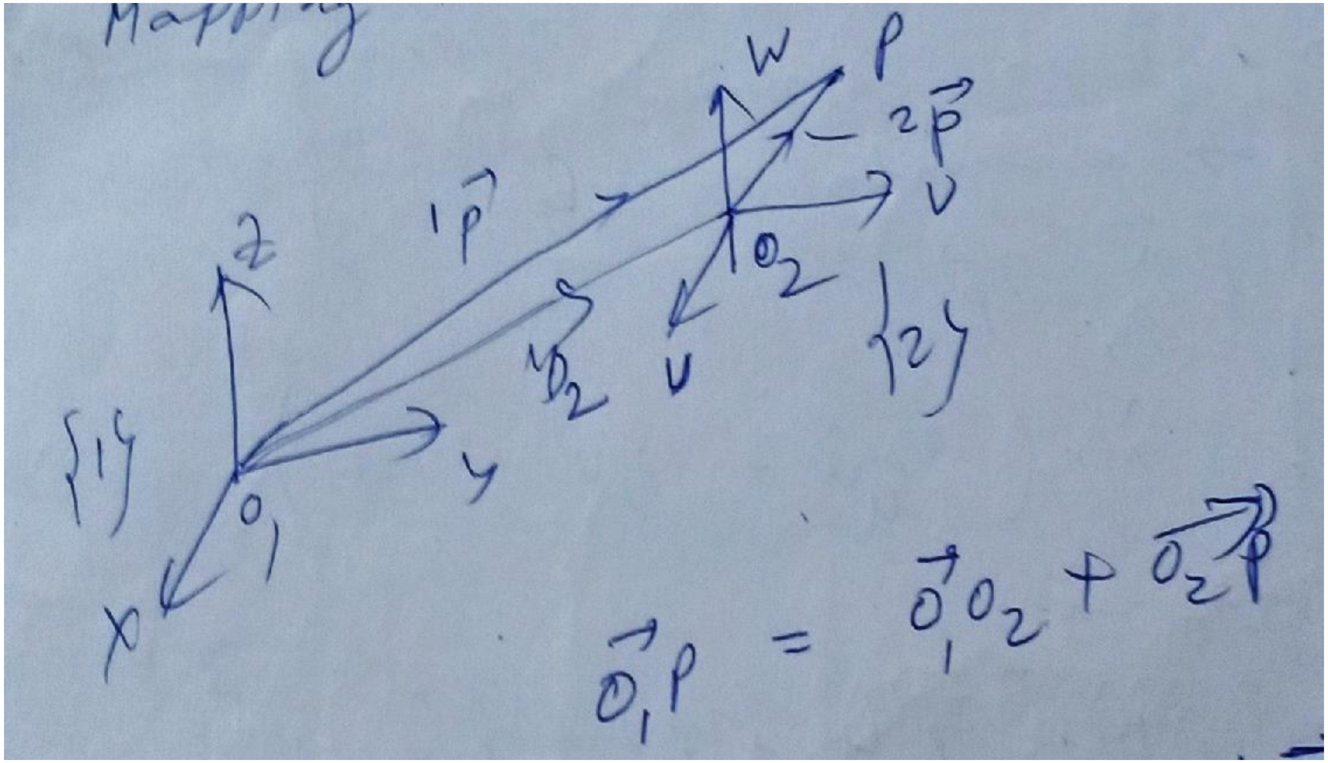
$$^{1}R_{2}({}^{1}R_{2})^{T} = I$$

$$\boxed{RR^{T} = I}$$

Mathematical fundamentals for robot kinematics

Mapping:

(2) Second frame is moved away from the first. Axes of both frames remain parallel (change of position)



Mathematical fundamentals for robot kinematics

$${}^{1}\vec{p} = \cancel{{}^{2}\vec{p} + {}^{1}\vec{D}_{2}} \quad {}^{1}\vec{D}_{2} + {}^{2}\vec{p}$$

$${}^{1}p_x = {}^{2}p_u + d_x$$

$${}^{1}p_y = {}^{2}p_v + d_y$$

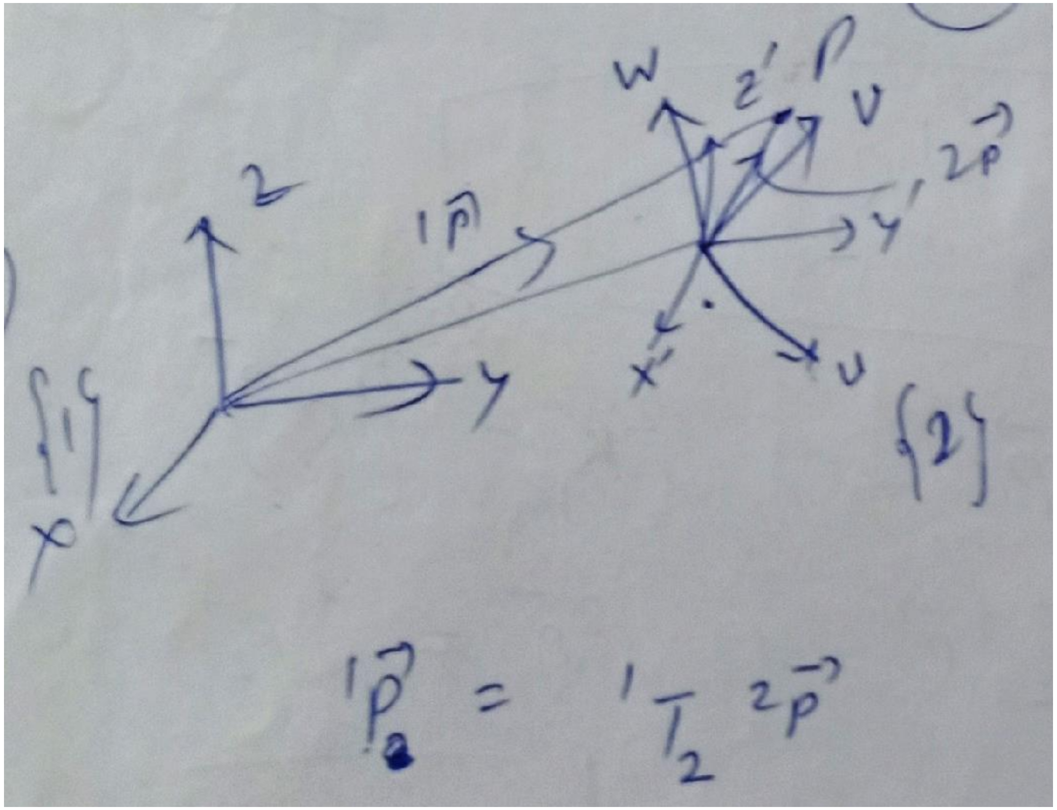
$${}^{1}p_z = {}^{2}p_w + d_z$$

$$\begin{pmatrix} {}^{1}p_x \\ {}^{1}p_y \\ {}^{1}p_z \\ 1 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} {}^{2}p_u \\ {}^{2}p_v \\ {}^{2}p_w \\ 1 \end{pmatrix}$$

Mathematical fundamentals for robot kinematics

Mapping:

(3) Second frame is rotated w.r.t the first and move away from it (change of position and orientation)



Mathematical fundamentals for robot kinematics

Mapping:

(3) **Second frame is rotated w.r.t the first and move away from it (change of position and orientation)**

$$
{}^{1}T_{2} = \left[\begin{array}{ccc|c}
\hat{x}.\hat{u} & \hat{x}.\hat{v} & \hat{x}.\hat{w} & d_x \\
\hat{y}.\hat{u} & \hat{y}.\hat{v} & \hat{y}.\hat{w} & d_y \\
\hat{z}.\hat{u} & \hat{z}.\hat{v} & \hat{z}.\hat{w} & d_z \\
\hline
0 & 0 & 0 & 1
\end{array}\right]
$$

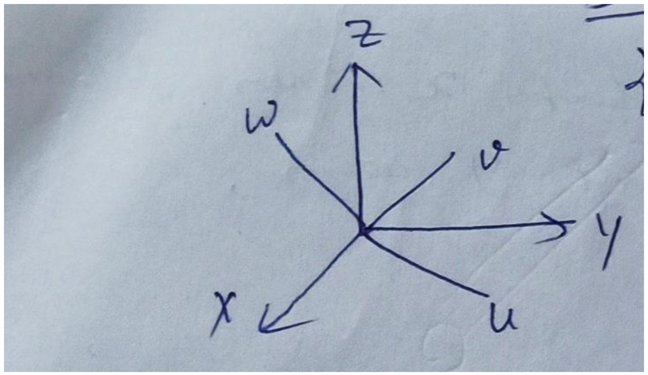
Translation vector

rotation matrix (3×3)

(Perspective transform (1×3))

(Scale factor)

Rotation matrix:



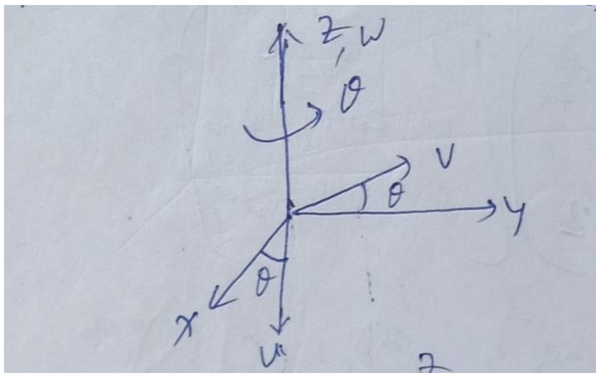
$\{x\ y\ z\} \rightarrow \{1\}$

$\{u\ v\ w\} \rightarrow \{2\}$

$$
{}^{1}R_{2} = \begin{bmatrix} \hat{x}\cdot\hat{u} & \hat{x}\cdot\hat{v} & \hat{x}\cdot\hat{w} \\ \hat{y}\cdot\hat{u} & \hat{y}\cdot\hat{v} & \hat{y}\cdot\hat{w} \\ \hat{z}\cdot\hat{u} & \hat{z}\cdot\hat{v} & \hat{z}\cdot\hat{w} \end{bmatrix}
$$

Fundamental rotation matrices

(1) Rotation about z axis



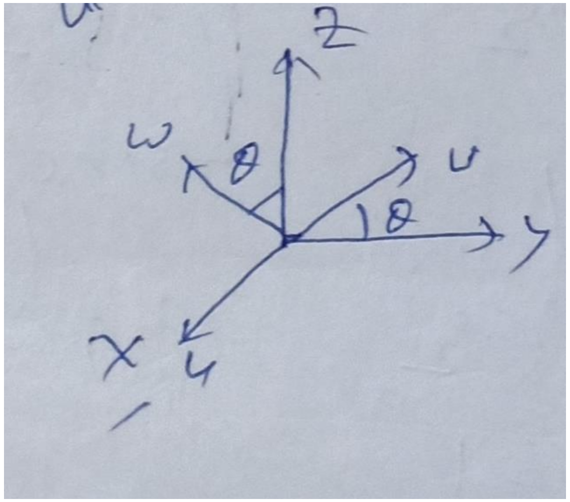
$$\begin{bmatrix} \hat{x}\cdot\hat{u} & \hat{x}\cdot\hat{v} & \hat{x}\cdot\hat{w} \\ \hat{y}\cdot\hat{u} & \hat{y}\cdot\hat{v} & \hat{y}\cdot\hat{w} \\ \hat{z}\cdot\hat{u} & \hat{z}\cdot\hat{v} & \hat{z}\cdot\hat{w} \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & \cos(\theta+90^\circ) & \cos 90^\circ \\ \cos(90-\theta) & \cos(\theta) & \cos 90^\circ \\ \cos 90^\circ & \cos 90^\circ & \cos 0^\circ \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Fundamental rotation matrices

(1) Rotation about x axis



$$R_x = \begin{bmatrix} \cos 0^\circ & \cos 90^\circ & \cos 90^\circ \\ \cos 90^\circ & \cos\theta & \cos(90^\circ+\theta) \\ \cos 90^\circ & \cos(90^\circ-\theta) & \cos(\theta) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Problem#1

Frame {2} is rotated w.r.t frame {1} about the x axis by angle of 60°. The position of the origin of frame {2} as ~~shoes~~ seen from frame {1} is $^1D_2 = \{7.0, 5.0, 7.0\}^T$

Obtain the transformation matrix $^1T_2$ which described frame {1} w.r.t {2}.

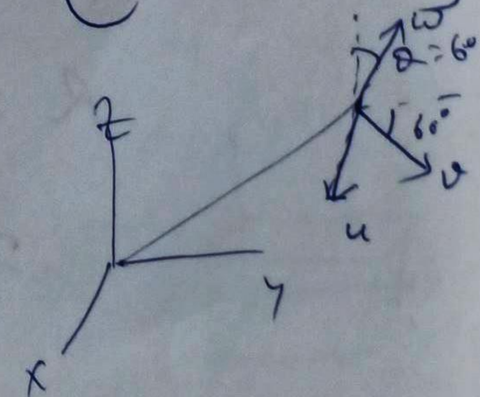
Also find the description of point P in frame {1} if $^2\vec{p} = [2.0, 4.0, 6.0]^T$

solution

$$^1T_2 = \left[\begin{array}{c|c} ^1R_2 & ^1P_2 \\ \hline 0\;0\;0 & 1 \end{array}\right]$$

$$^1R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 60^\circ & -\sin 60^\circ \\ 0 & \sin 60^\circ & \cos 60^\circ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & -0.866 \\ 0 & 0.866 & 0.5 \end{bmatrix}$$

$$^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 7.0 \\ 0 & 0.5 & -0.866 & 5.0 \\ 0 & 0.866 & 0.5 & 7.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Mathematical fundamentals for robot kinematics

$$ {}^{1}\vec{p} = {}^{1}T_{2}\, {}^{2}\vec{p} \qquad = \begin{bmatrix} 9.0 & 1.84 & 13.46 \end{bmatrix}^{T} $$

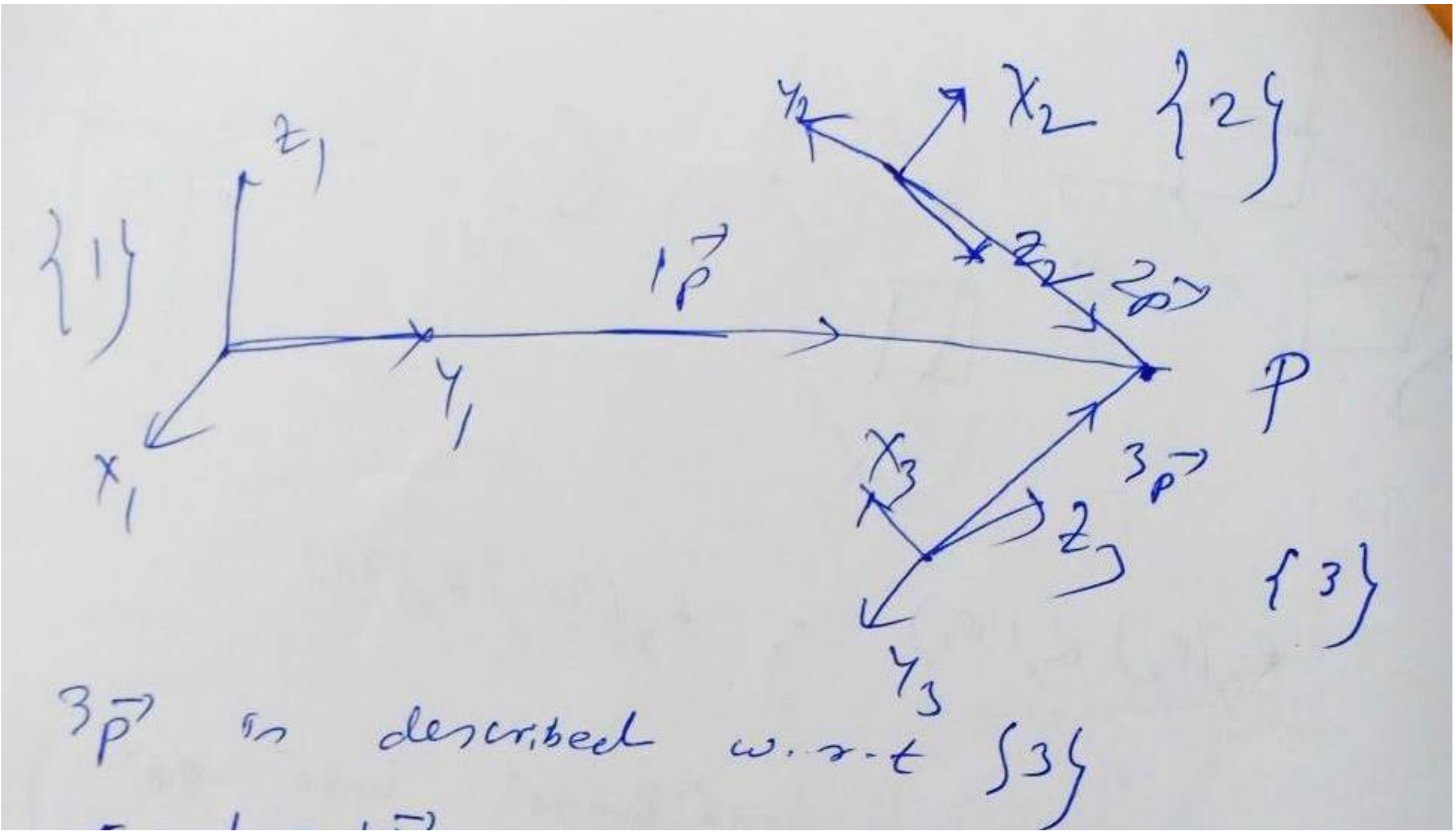
Where,

$$ {}^{1}T_{2} = \begin{bmatrix} 1 & 0 & 0 & 7.0 \\ 0 & 0.5 & -0.866 & 5 \\ 0 & 0.866 & 0.5 & 7.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}\vec{p} = \begin{bmatrix} 2.0, & 4.0, & 6.0 \end{bmatrix}^{T} $$

Compound or composite transformation

❖The point P is defined w.r.t frame 3, obtain the transformation matrix between frame 1 and 3.



Compound or composite transformation

❖The point P is defined w.r.t frame 3, obtain the transformation matrix between frame 1 and 3.

$$ {}^2\vec{p} = {}^2T_3 \, {}^3\vec{p} \qquad\qquad {}^1\vec{p} = {}^1T_2 \, {}^2\vec{p} $$

$$ {}^1\vec{p} = \left({}^1T_3\right) {}^3\vec{p} \qquad\qquad = \left({}^1T_2 \, {}^2T_3\right) {}^3\vec{p} $$

$$ \boxed{{}^1T_3 = {}^1T_2 \, {}^2T_3} $$

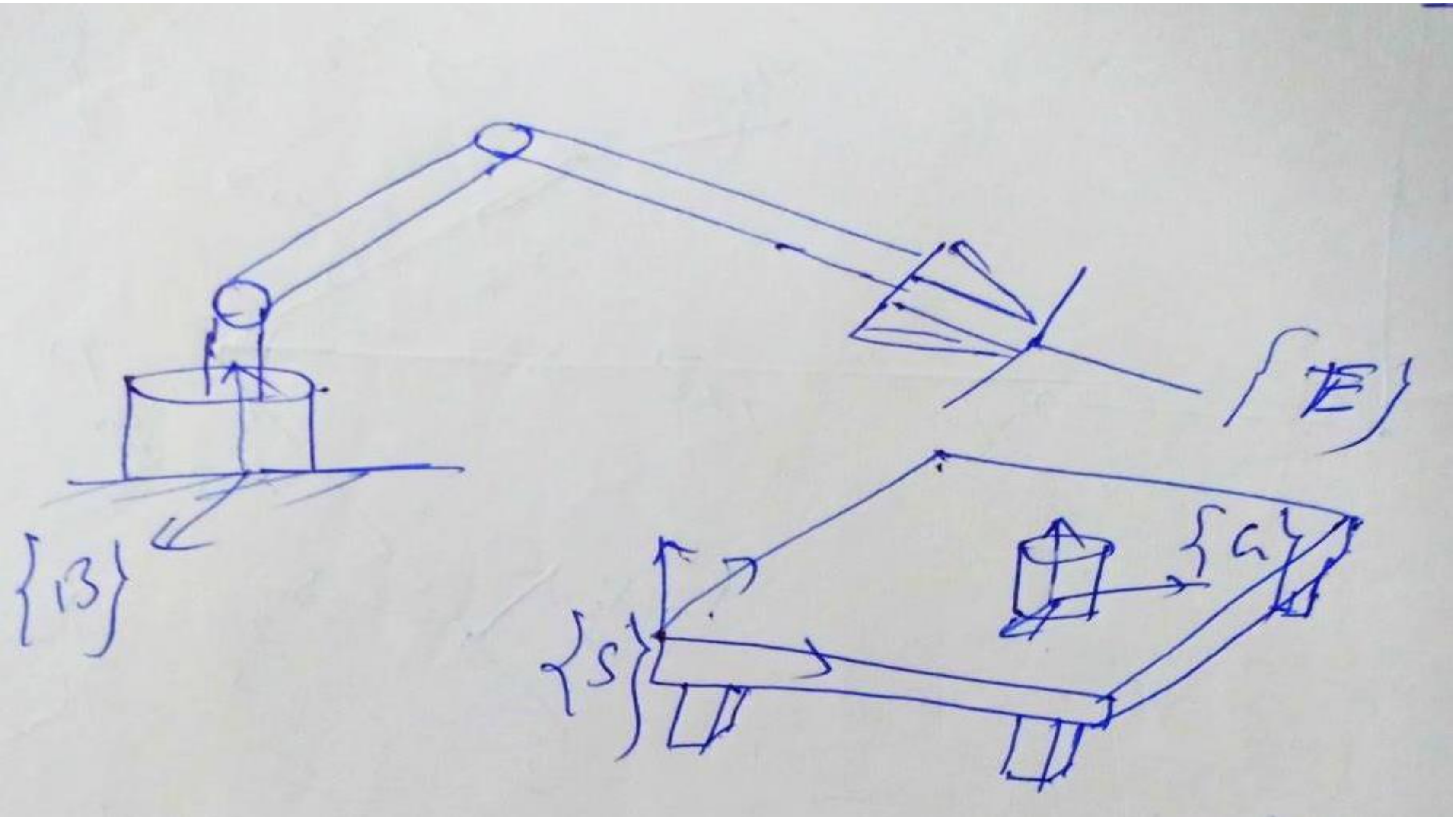
So $$ {}^1T_i = {}^1T_2 \, {}^2T_3 \cdots {}^jT_{j+1} \cdots {}^{i-1}T_i $$

or in general from frame $\{i\}$ to fram $\{j\}$

$$ {}^jT_i = {}^jT_{j+1} \, {}^{j+1}T_{j+2} \cdots {}^{i-1}T_i \qquad i > j $$

Composite transformation in manipulator

❖To find the position and orientation of manipulator end effector w.r.t. object.



Composite transformation in manipulator

❖To find the position and orientation of manipulator end effector w.r.t. object.

$$^{B}T_{E} = {}^{B}T_{1}\,{}^{2}T_{3} \cdots {}^{n}T_{E}$$

$n \rightarrow$ no. of joint (D.O.F)

$\rightarrow$ Known.

$^{B}T_{S} \rightarrow$ known (between $\{B\}$ & $\{S\}$)

$^{S}T_{G}$ = known (between $\{S\}$ & $\{G\}$)

Composite transformation in manipulator

❖ To find the position and orientation of manipulator end effector w.r.t. object.

Find the position and orientation of ~~tool frame~~ end effector $\{E\}$ w.r.t $\{G\}$

$${}^{E}T_{G}$$

$${}^{E}T_{G} = {}^{E}T_{B} \cdot {}^{B}T_{S} \cdot {}^{S}T_{G}$$

$$\boxed{{}^{E}T_{G} = \left({}^{B}T_{E}\right)^{-1} \cdot {}^{B}T_{S} \cdot {}^{S}T_{G}}$$