ME512/ME6106: Mobile Robotics

Mathematical fundamentals for robot kinematics

Dr. Karali Patra
Professor
Department of Mechanical Engineering
IIT Patna
Room no. 111, Block 3
Email: kpatra@iitp.ac.in
Whatsapp: 7295834306

Mathematical fundamentals for robot kinematics

Mapping:

- Changing the description of a point (or value) in space from one frame to another frame.

Different cases of mapping

(1) Second frame is rotated w.r.t the first, origin of both frames is same (changing of orientation)

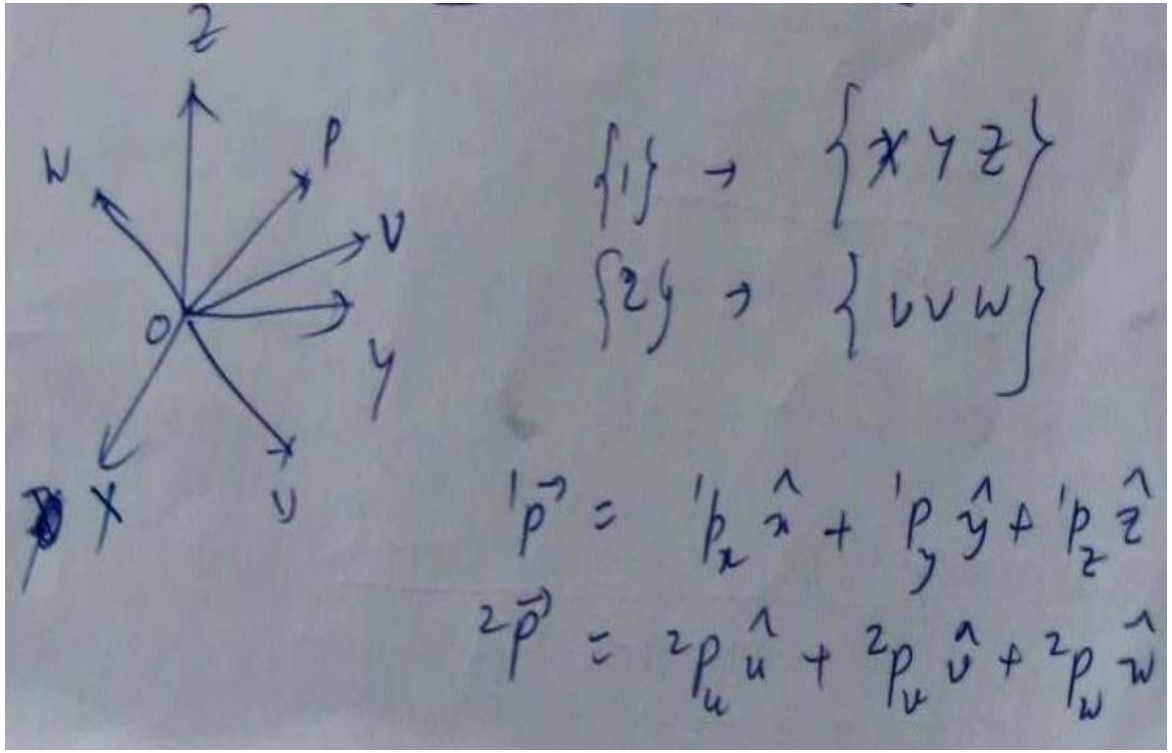
(2) Second frame is moved away from the first. Axes of both frames remain parallel (change of position)

(3) Second frame is rotated w.r.t the first and move away from it (change of position and orientation)

Mathematical fundamentals for robot kinematics

Mapping:

(1) Second frame is rotated w.r.t the first, origin of both frames is same (changing of orientation)



$$^1p_x = \hat{x} \cdot {}^2\vec{p}$$

$$= \hat{x} \cdot ({}^2p_u\hat{u} + {}^2p_v\hat{v} + {}^2p_w\hat{w})$$

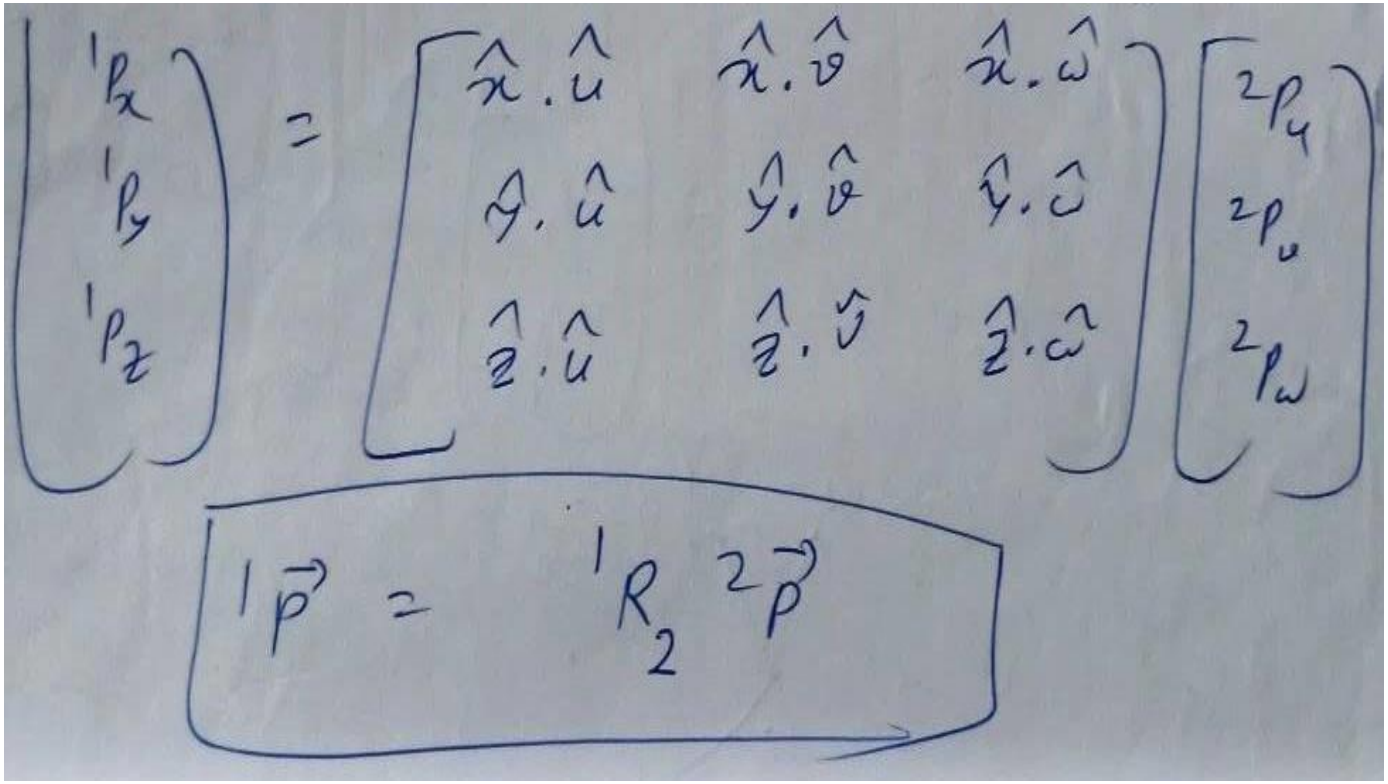
$$= \hat{x}.$$

$$^1p_y = \hat{y} \cdot ({}^2p_u\hat{u} + {}^2p_v\hat{v} + {}^2p_w\hat{w})$$

$$^1p_z = \hat{z} \cdot ({}^2p_u\hat{u} + {}^2p_v\hat{v} + {}^2p_w\hat{w})$$

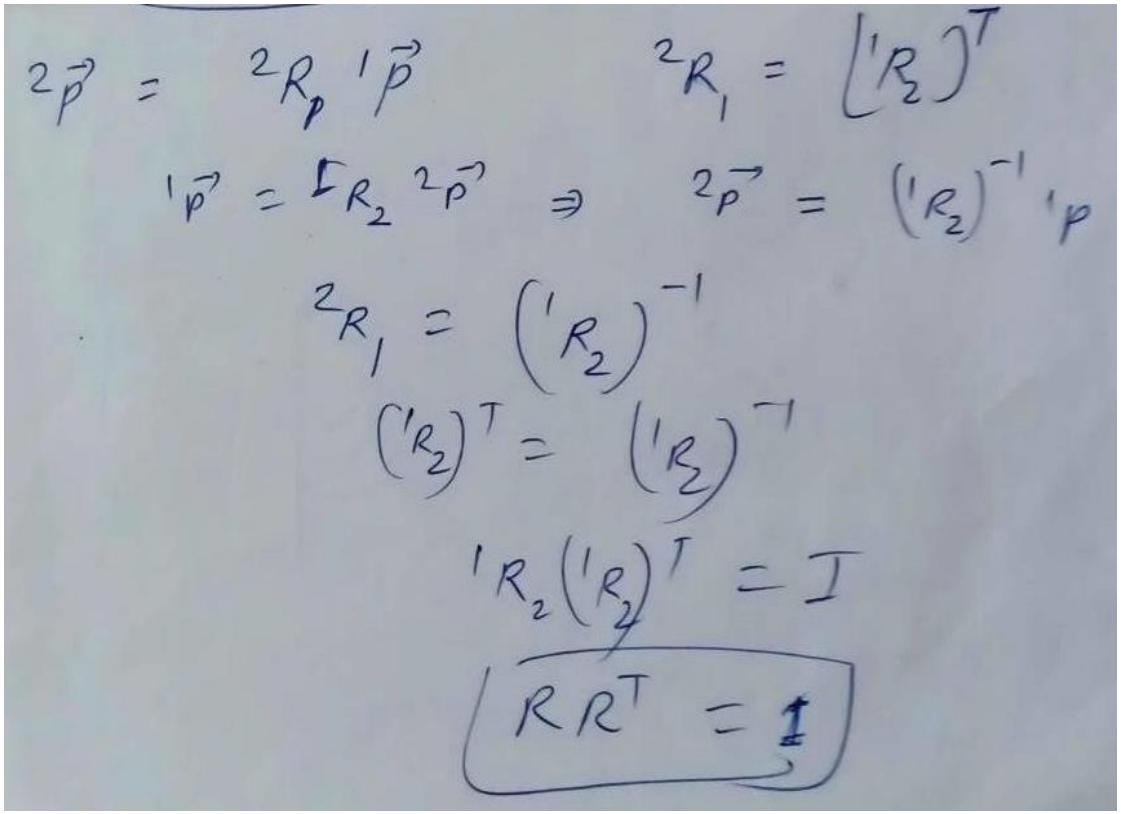
Mathematical fundamentals for robot kinematics

Mapping:



Mathematical fundamentals for robot kinematics

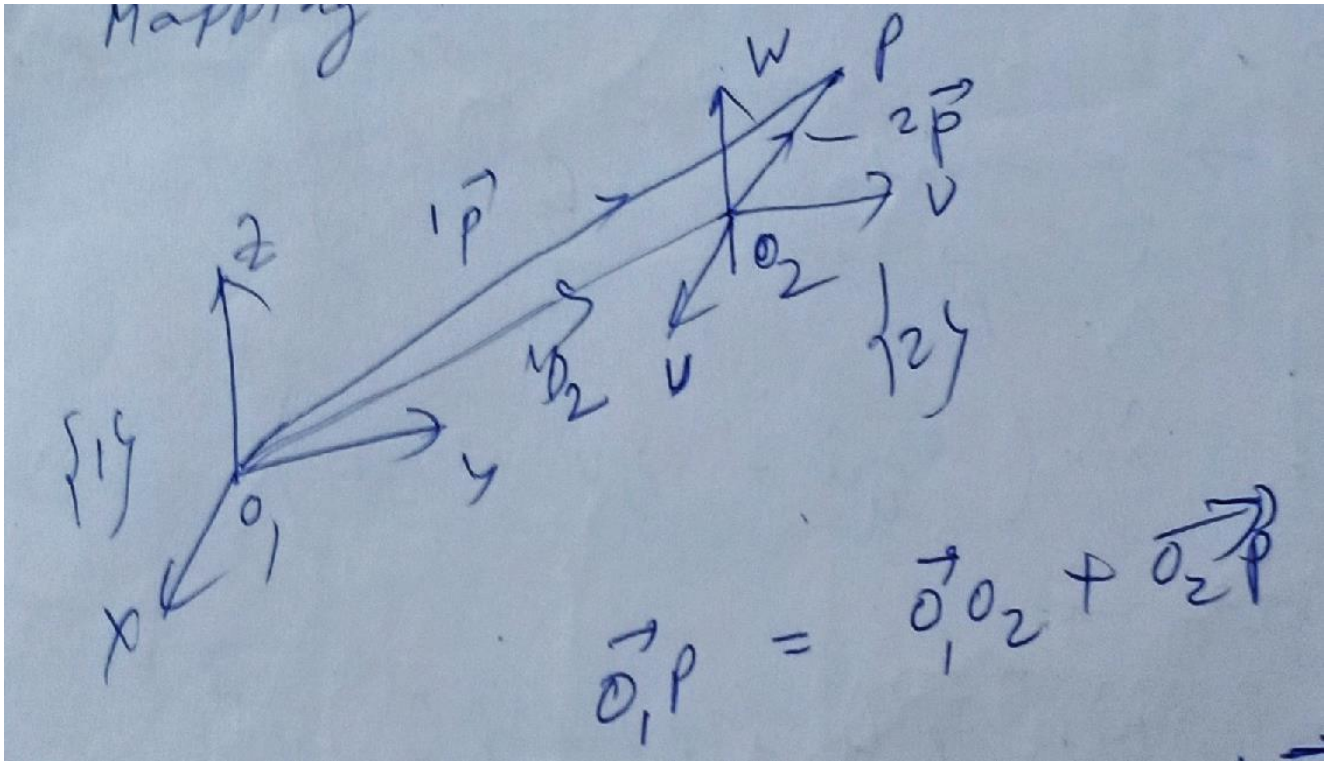
Relation of rotation matrix



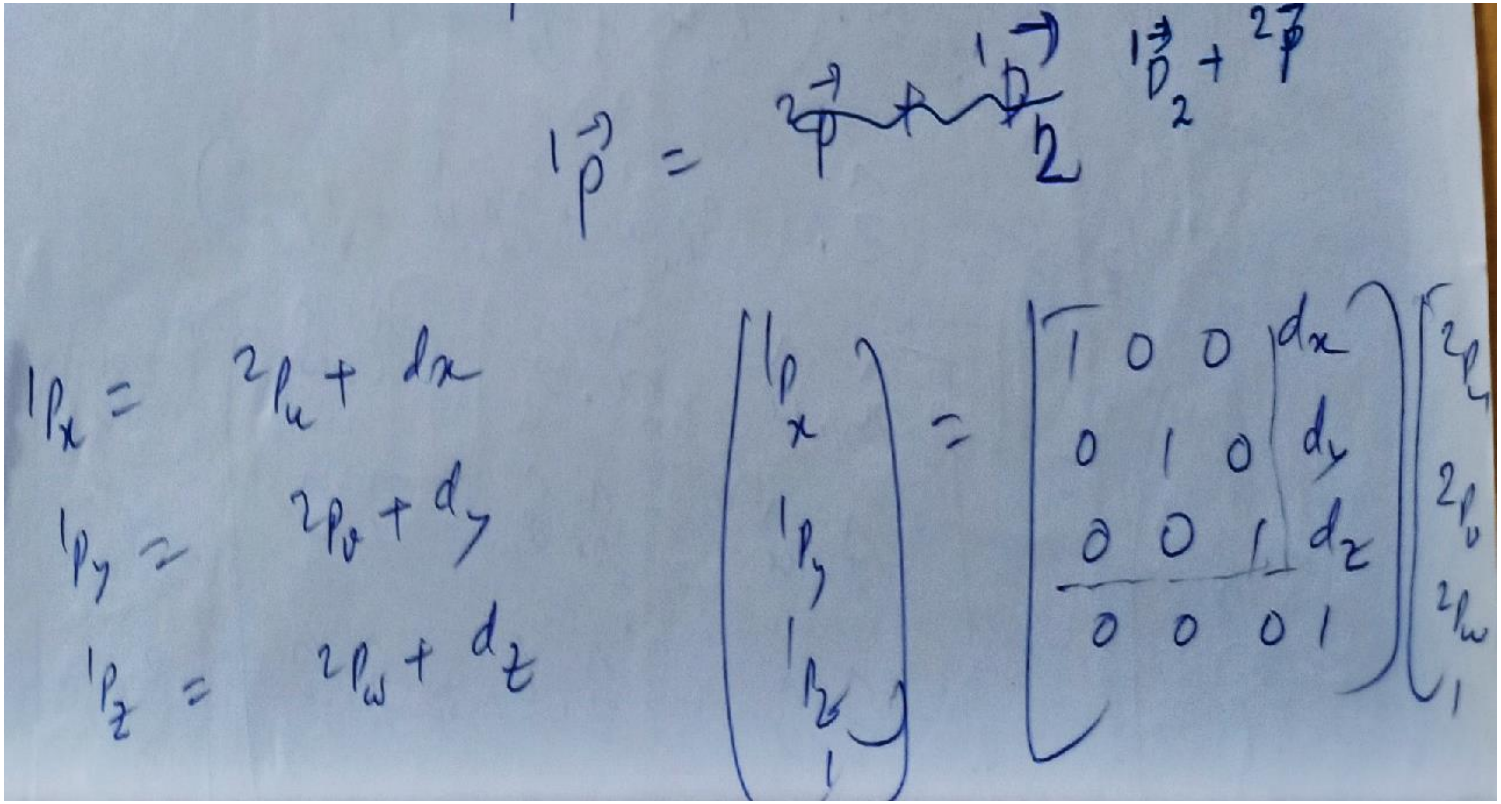
Mathematical fundamentals for robot kinematics

Mapping:

(2) Second frame is moved away from the first. Axes of both frames remain parallel (change of position)



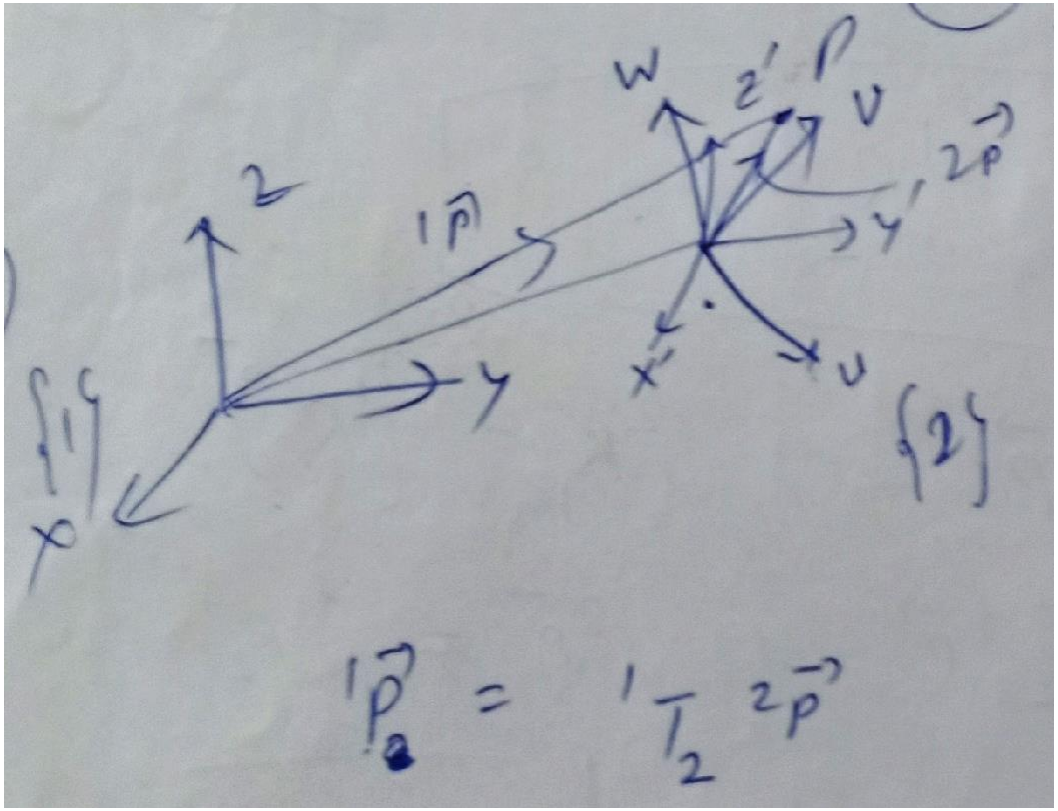
Mathematical fundamentals for robot kinematics



Mathematical fundamentals for robot kinematics

Mapping:

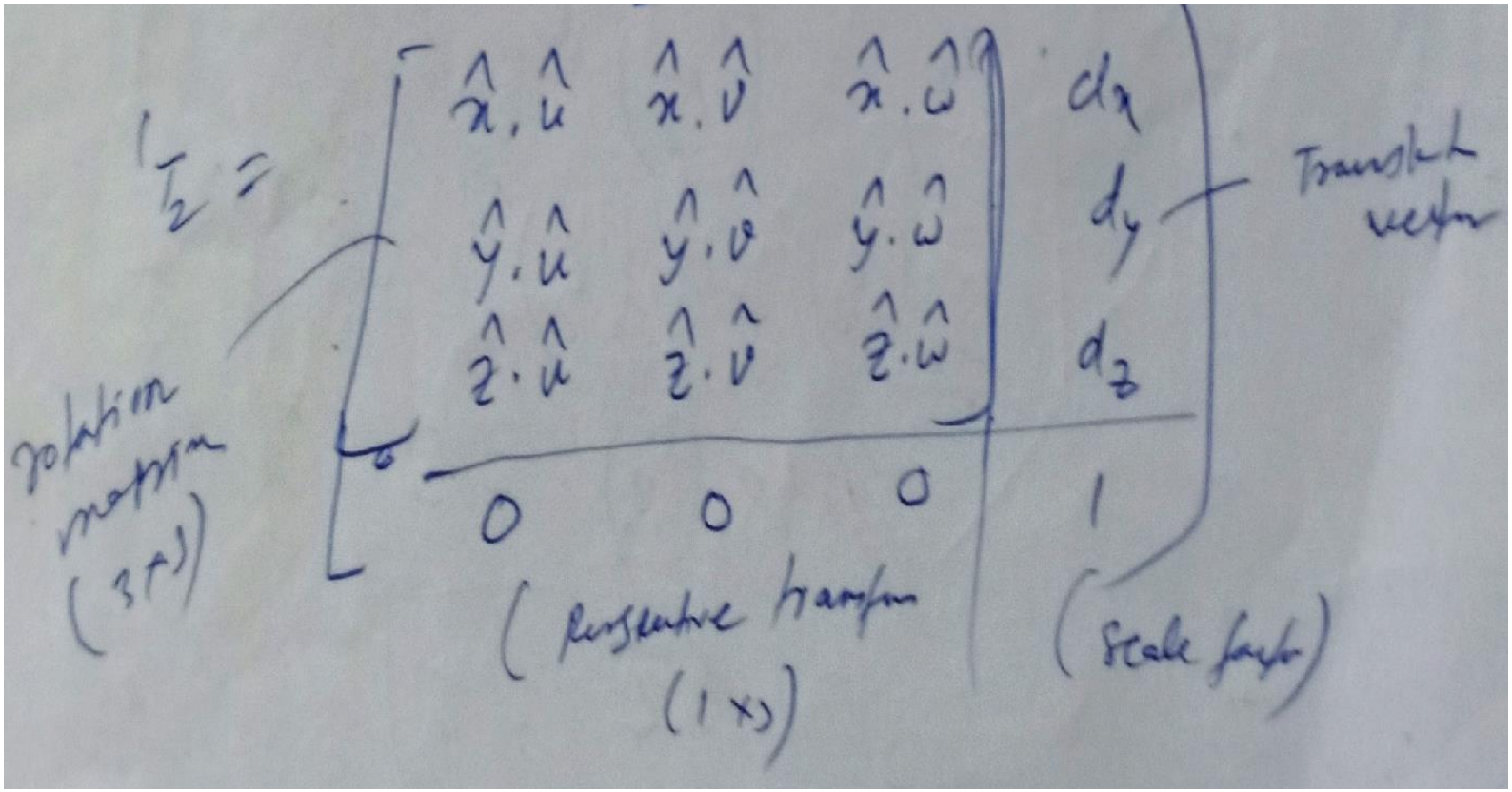
(3) Second frame is rotated w.r.t the first and move away from it (change of position and orientation)



Mathematical fundamentals for robot kinematics

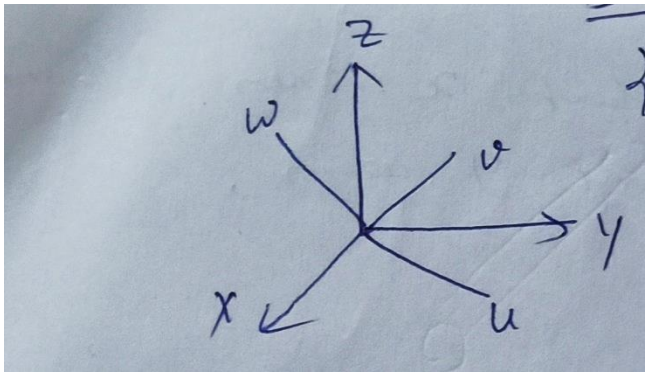
Mapping:

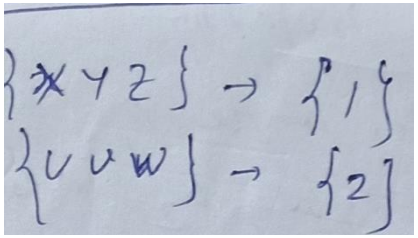
(3) Second frame is rotated w.r.t the first and move away from it (change of position and orientation)

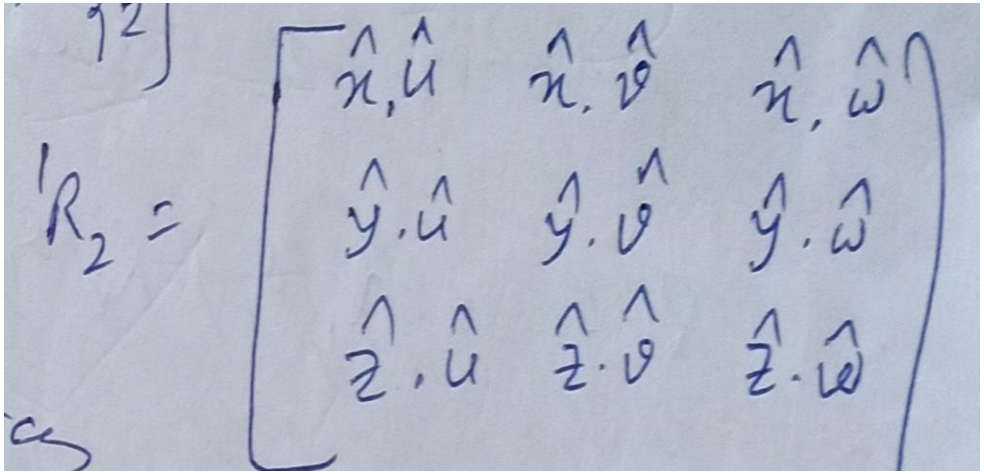


Mathematical fundamentals for robot kinematics

Rotation matrix:



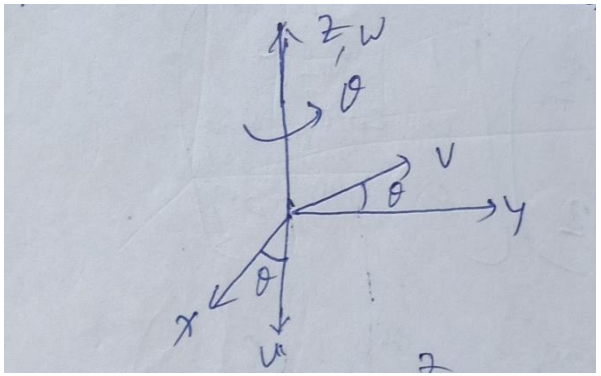




Mathematical fundamentals for robot kinematics

Fundamental rotation matrices

(1) Rotation about z axis



$$R_z(\theta) = \begin{bmatrix} \cos\theta & \cos(\theta+90^\circ) & \cos 90^\circ \\ \cos(90-\theta) & \cos(\theta) & \cos 90^\circ \\ \cos 90^\circ & \cos 90^\circ & \cos 0^\circ \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

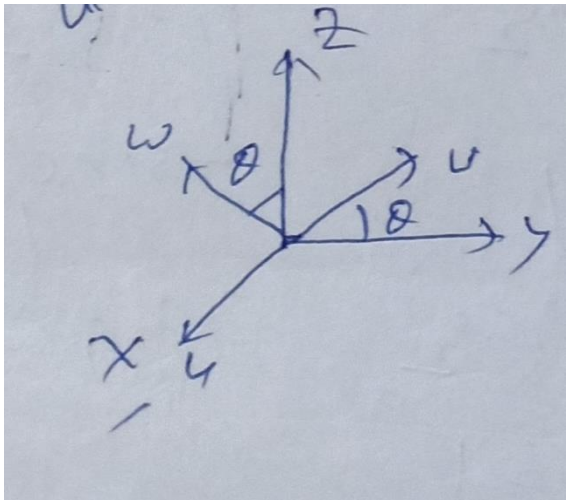
$$\begin{bmatrix} \hat{x}\cdot\hat{u} & \hat{x}\cdot\hat{v} & \hat{x}\cdot\hat{w} \\ \hat{y}\cdot\hat{u} & \hat{y}\cdot\hat{v} & \hat{y}\cdot\hat{w} \\ \hat{z}\cdot\hat{u} & \hat{z}\cdot\hat{v} & \hat{z}\cdot\hat{w} \end{bmatrix}$$

Mathematical fundamentals for robot kinematics

Fundamental rotation matrices

(1) Rotation about x axis

R_x =



$$\begin{bmatrix} \cos 0^\circ & \cos 90^\circ & \cos 90^\circ \\ \cos 90^\circ & \cos\theta & \cos(90^\circ+\theta) \\ \cos 90^\circ & \cos(90^\circ-\theta) & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Mathematical fundamentals for robot kinematics

Problem#1

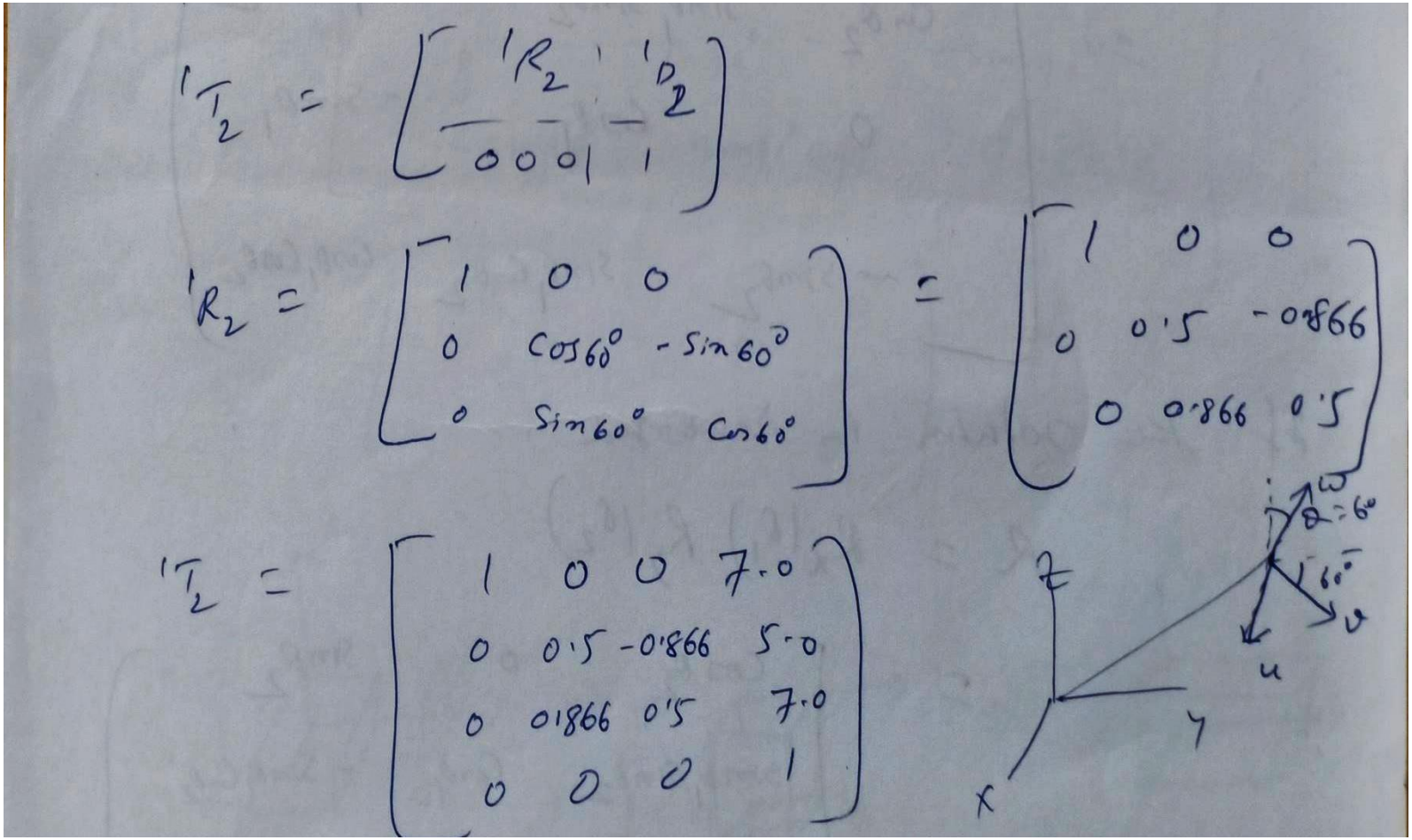
Frame {2} is related w.r.t frame {1} about the x axis by angle of 60°. The position of the origin of frame {2} as seen from frame {1} is ${}^1D_2 = \{7.0, 5.0, 7.0\}^T$

Obtain the transformation matrix ${}^1T_2$ which describes frame {1} w.r.t {2}.

Also find the description of point P in frame {1} if ${}^2\vec{P} = [2.0, 4.0, 6.0]^T$

Mathematical fundamentals for robot kinematics

solution



Mathematical fundamentals for robot kinematics

$$ {}^{1}\vec{p} = {}^{1}_{2}T \; {}^{2}\vec{p} $$

$$ = \begin{bmatrix} 9.0 & 1.84 & 13.46 \end{bmatrix}^T $$

Where,

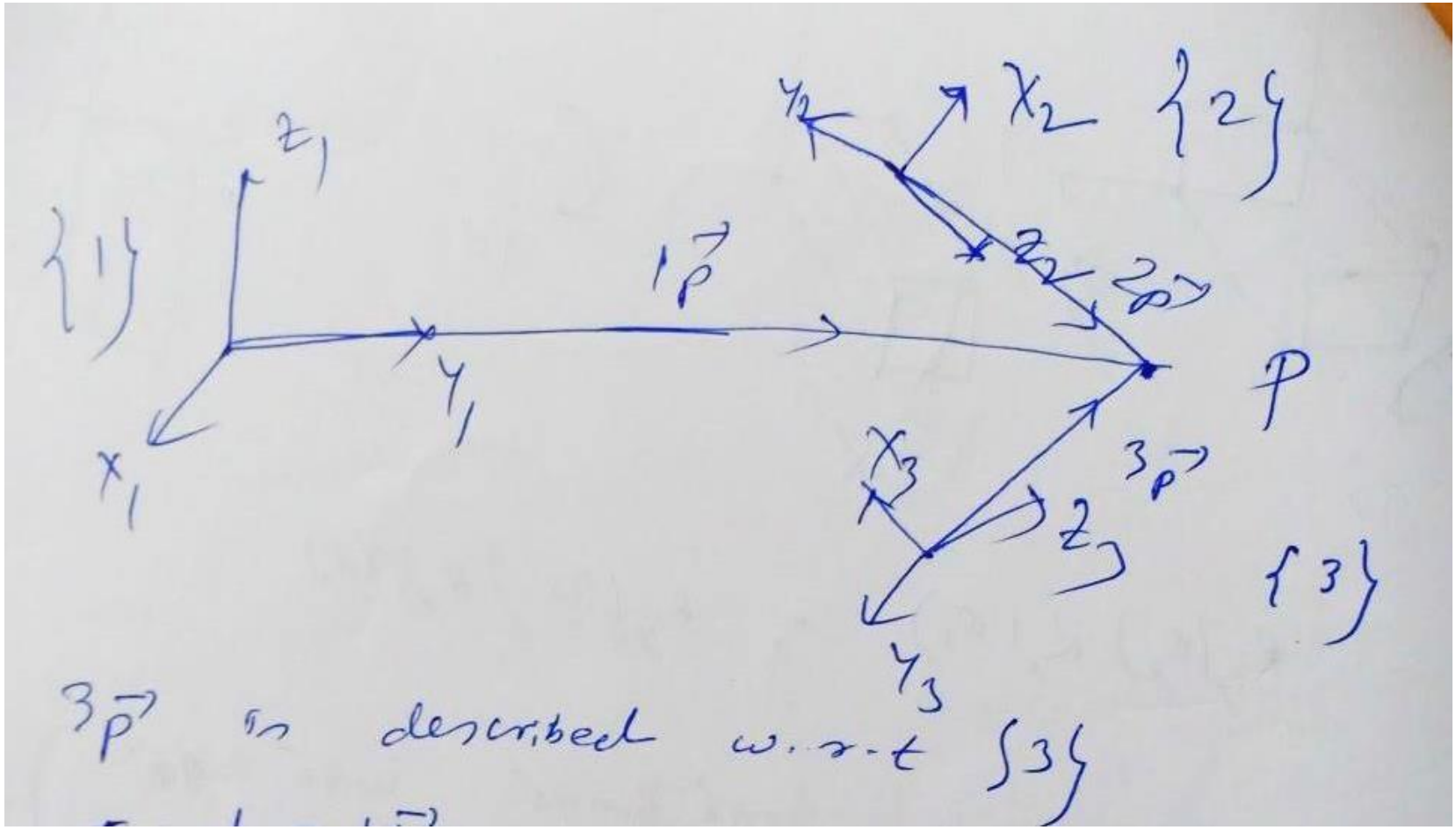
$$ {}^{1}_{2}T $$

$$ = \begin{bmatrix} 1 & 0 & 0 & 7.0 \\ 0 & 0.5 & -0.866 & 5 \\ 0 & 0.866 & 0.5 & 7.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}\vec{p} = \begin{bmatrix} 2.0, & 4.0, & 6.0 \end{bmatrix}^T $$

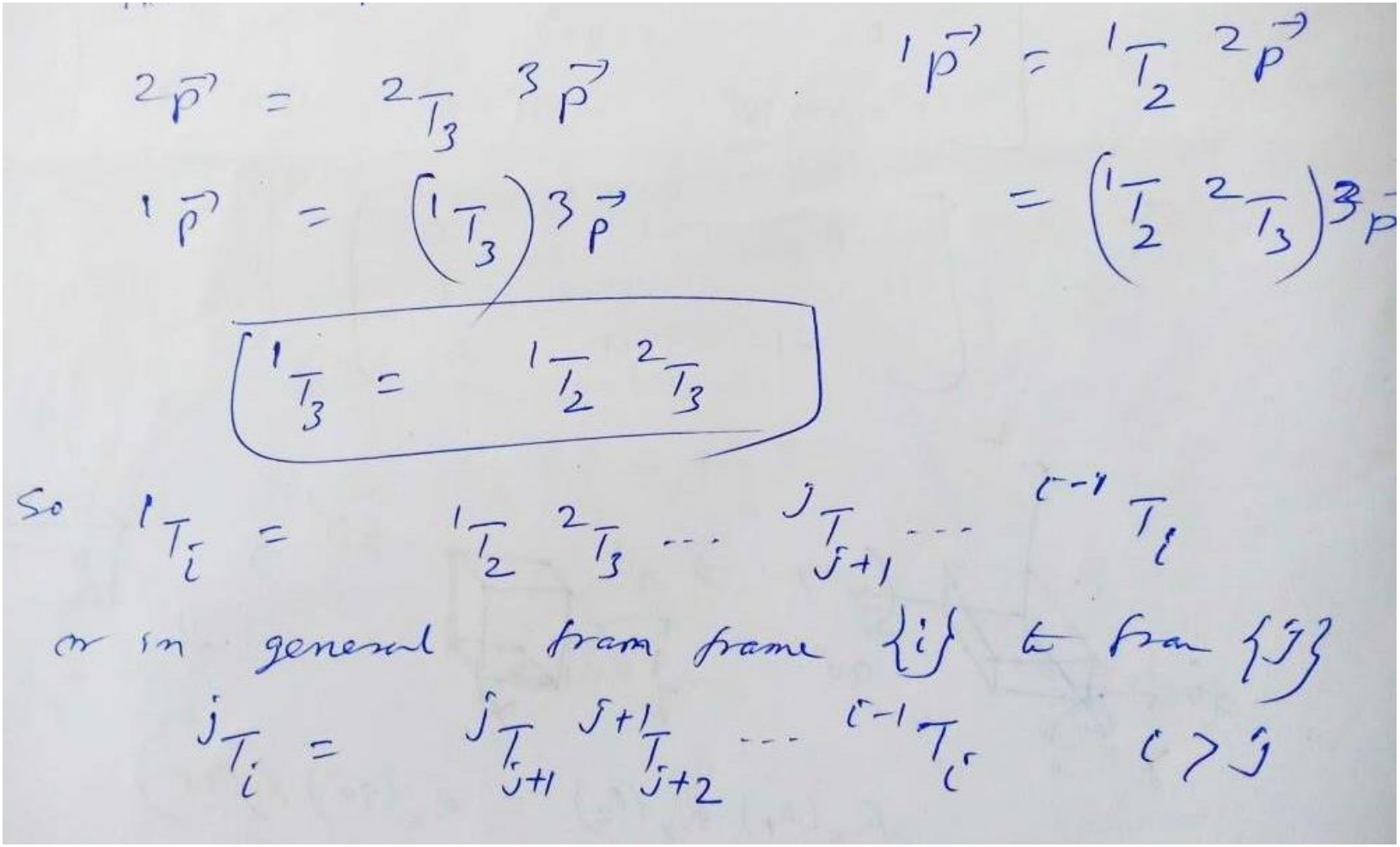
Compound or composite transformation

❖The point P is defined w.r.t frame 3, obtain the transformation matrix between frame 1 and 3.



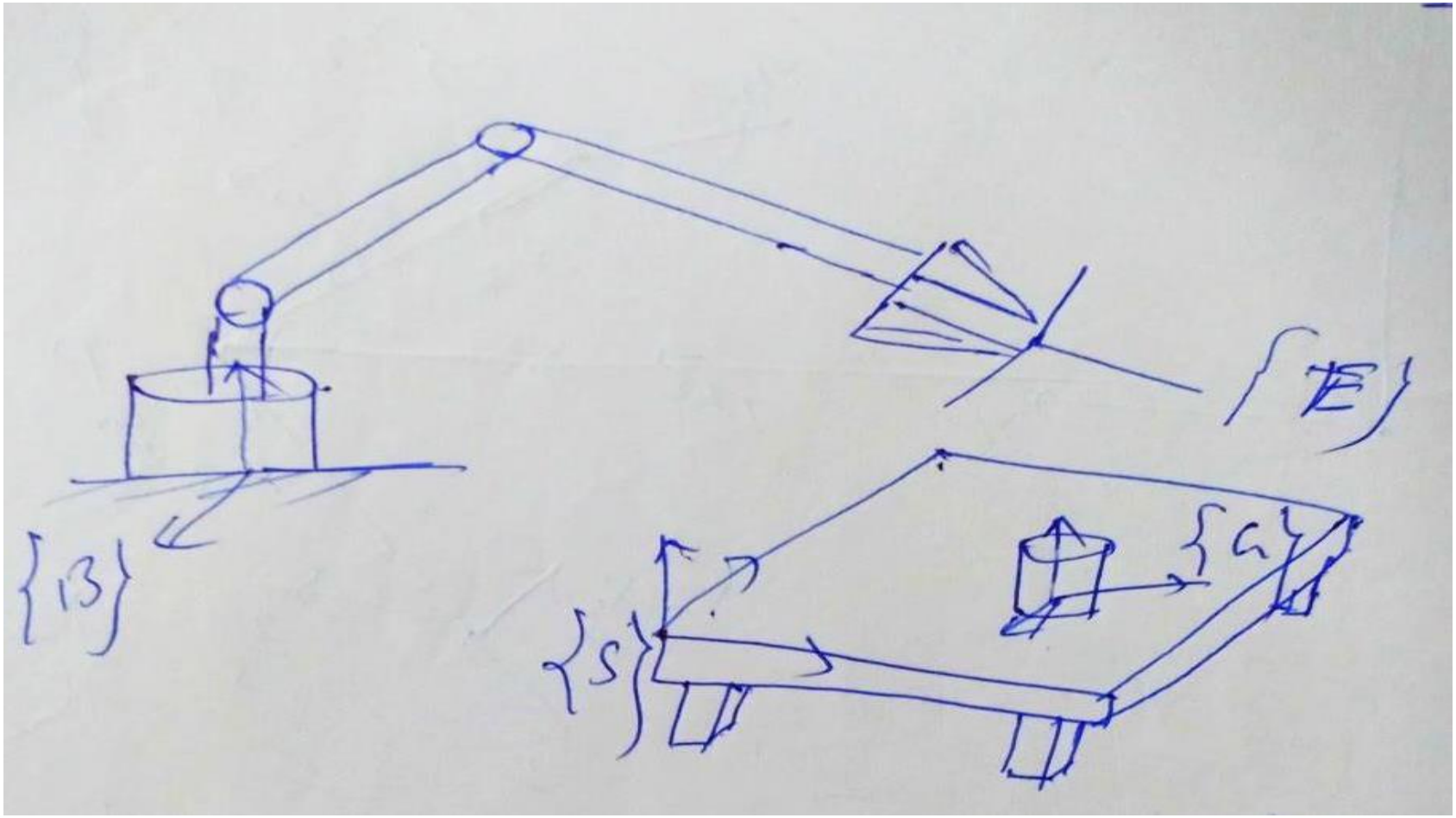
Compound or composite transformation

❖The point P is defined w.r.t frame 3, obtain the transformation matrix between frame 1 and 3.



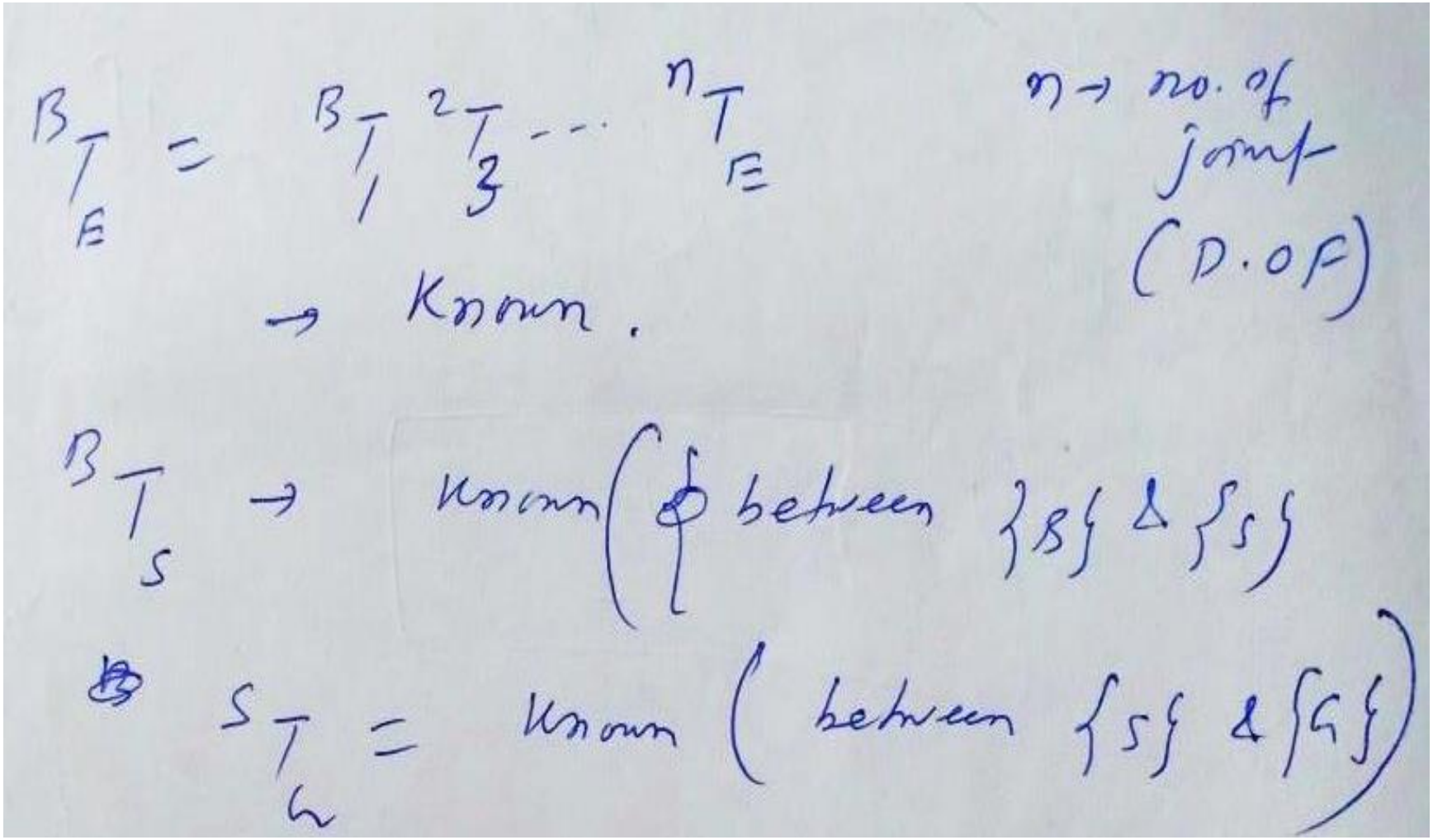
Composite transformation in manipulator

❖To find the position and orientation of manipulator end effector w.r.t. object.



Composite transformation in manipulator

❖To find the position and orientation of manipulator end effector w.r.t. object.



Composite transformation in manipulator

❖To find the position and orientation of manipulator end effector w.r.t. object.