Lecture 11 (22/08/25)

ME512/ME6106: Mobile Robotics

Robot Kinematics

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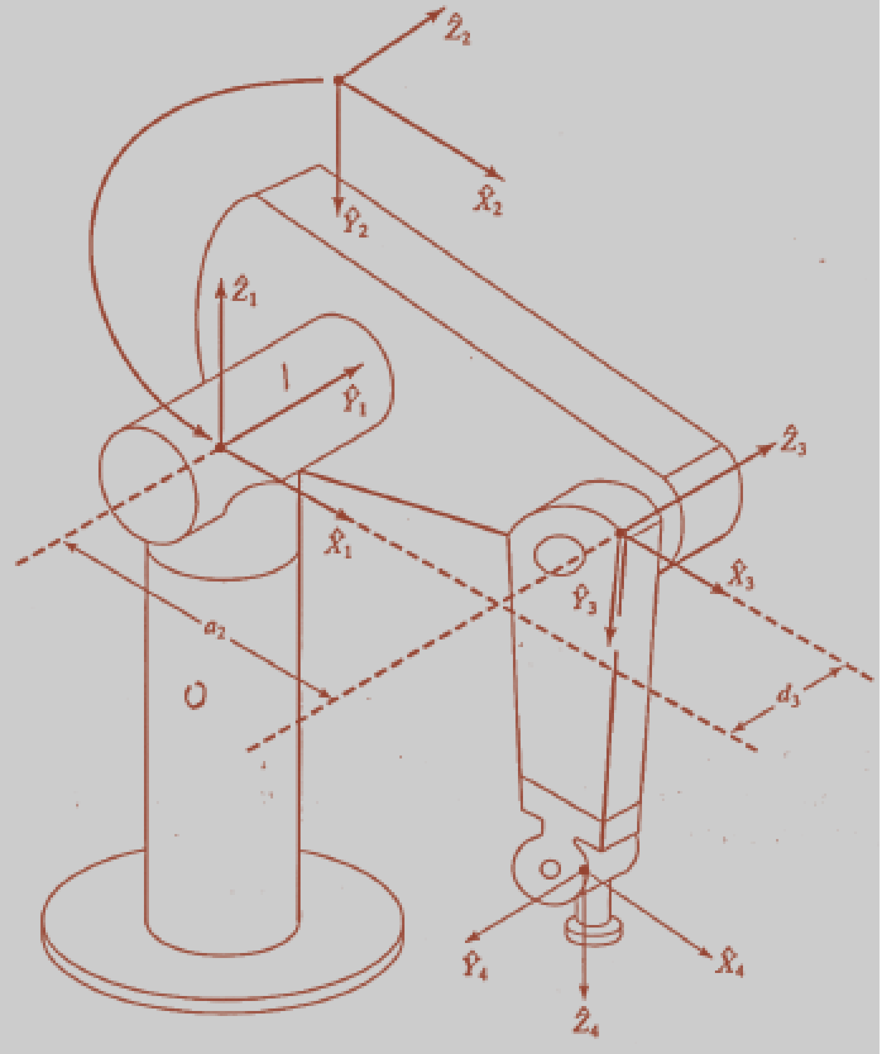
Manipulator kinematics



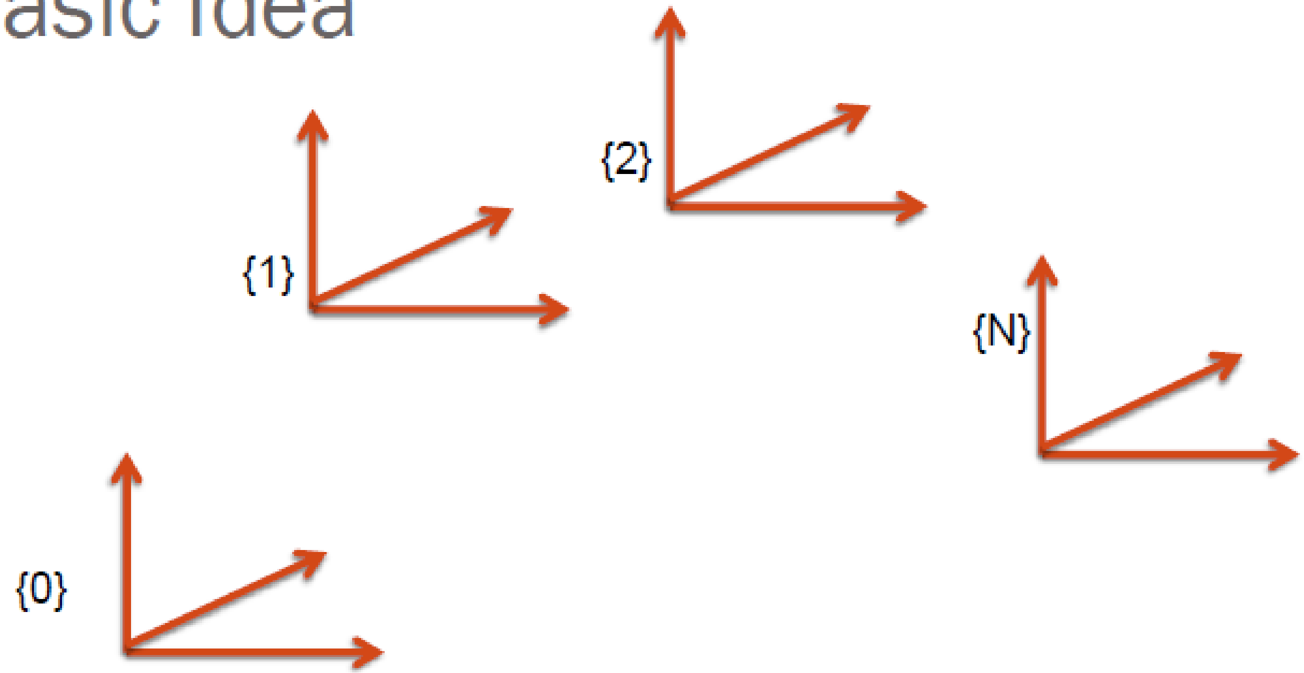
- ❑ Goal: Describe end effecter frame in terms of base frame
- ➢ Location and orientation of end effecter frame depends upon the joint values

Strategy

- Attach frames to base and end effecter
- Attach frames to links
- Develop frame to frame transforms
  - In terms of joint parameters and link parameters
- Compute transform to express end effecter frame in terms of base frame
  - Use composition rules

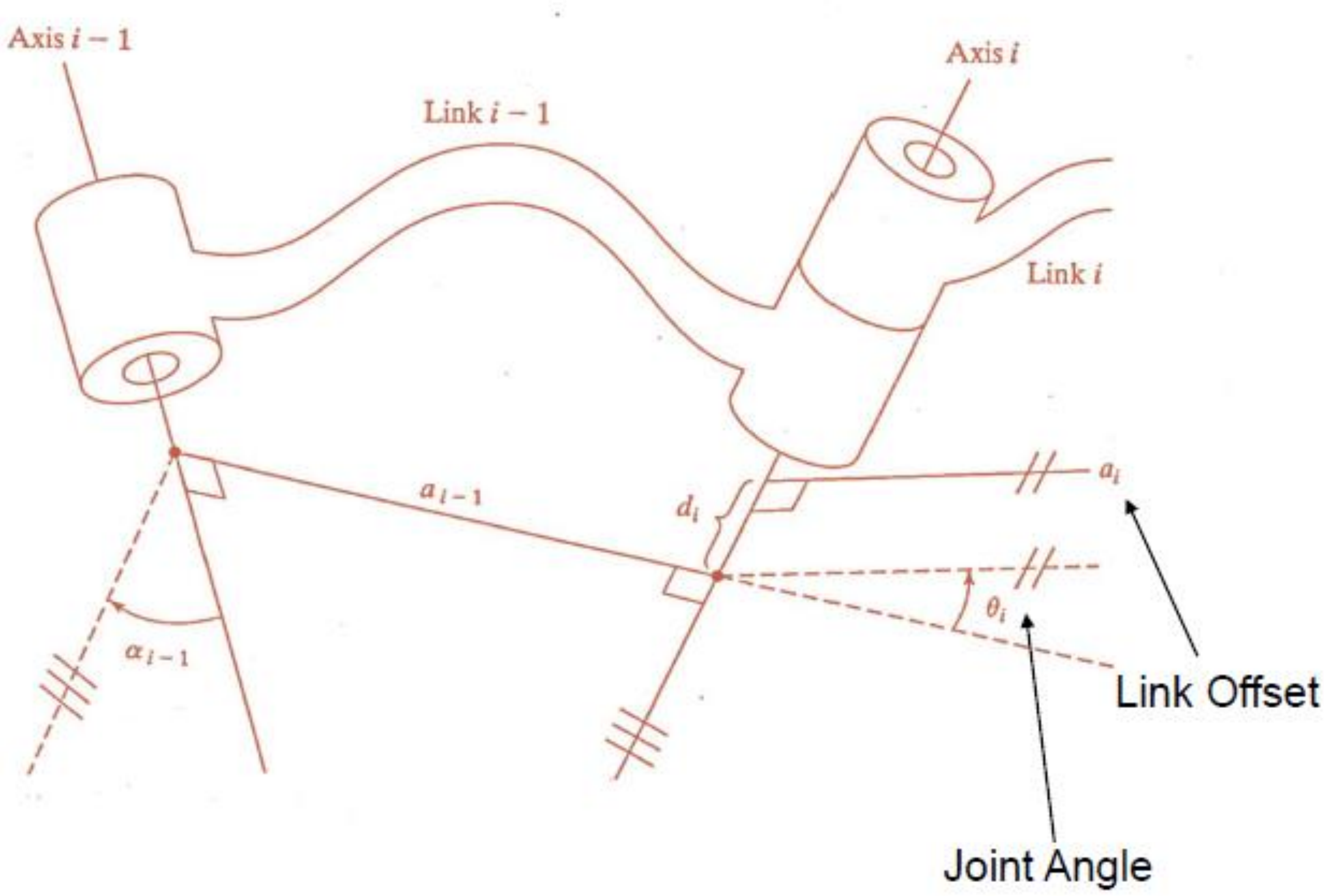
Strategy

Basic Idea

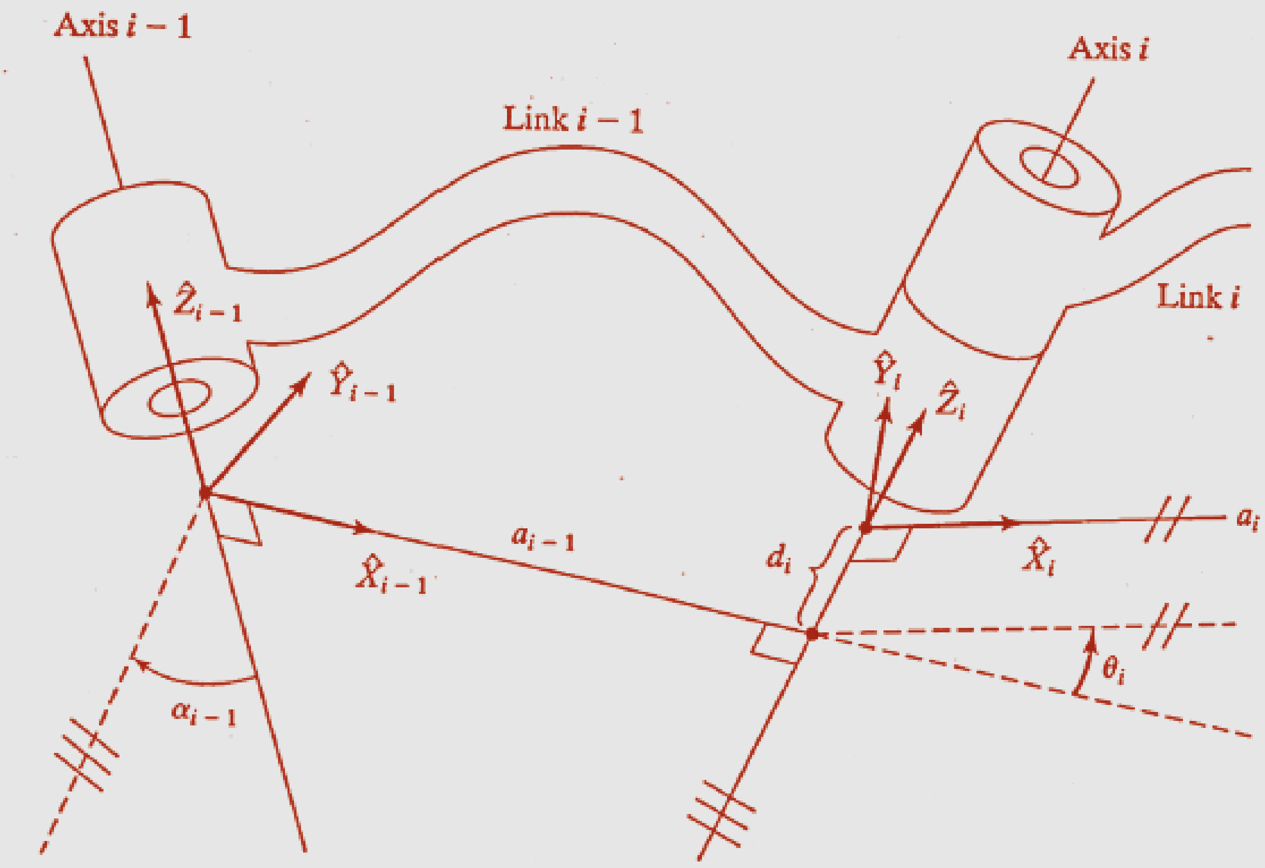


$$ {}^{0}_{N}T = {}^{0}_{1}T \, {}^{1}_{2}T \ldots {}^{N-1}_{N}T $$

Joint Nomenclature



Attaching Frames to Links



Intermediate Links

First and Last Links

- Frame {0} is located such that it coincides with Frame {1} when joint variable 1 is zero
- For last joint (nth)
  - Revolute joint: Choose frame such that it aligns with the previous frame when joint angle is zero and it produces zero link offset
  - Prismatic joint: Choose frame such that it leads to maximum number of linkage parameters to be zero

Summary of Link Frame Attachment

Identify joint axes and draw infinite lines through them

Identify common perpendicular or point o intersection as applicable

- between pairs of subsequent axes

Assign $\hat{Z}_i$ axis pointing along i^{th} joint axis

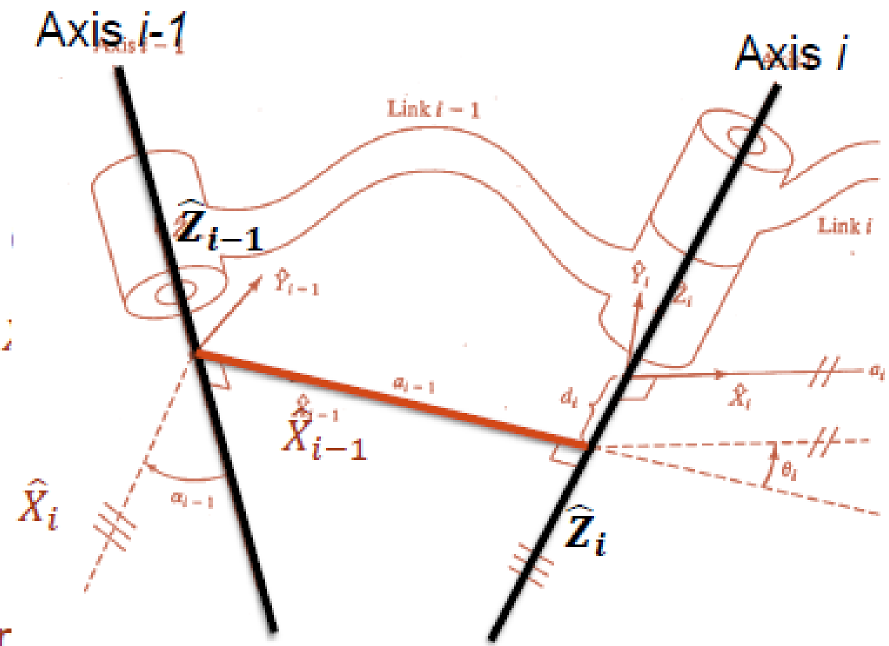
Assign $\hat{X}_{i-1}$ axis pointing along common perpendicular (or along normal to the plane containing both axes if they intersect)

Assign Y axes to complete right handed system

Assign frame {0} to coincide with {1} wher first joint variable is zero

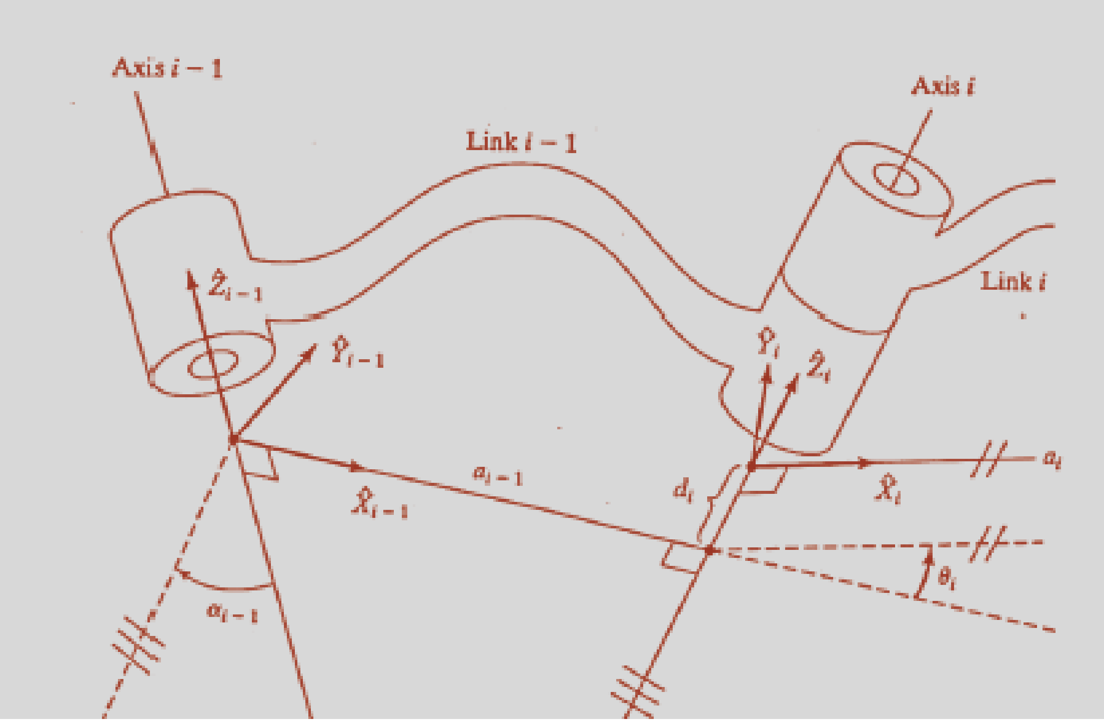
Origin of frame {N} and $\hat{X}_N$ can be chosen arbitrarily

- In general choice should be made in such a way that most linkage parameters turn out to be zero

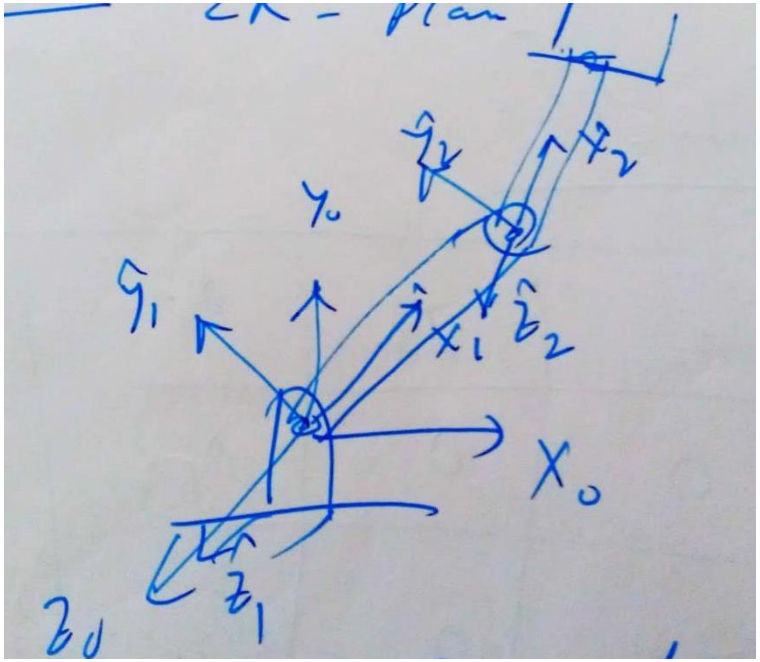


Denavit-Hartenberg notations

a_i: distance between $\hat{Z}_i$ and $\hat{Z}_{i+1}$ measured along $\hat{X}_i$
α_i: angle between $\hat{Z}_i$ and $\hat{Z}_{i+1}$measured about $\hat{X}_i$
d_i: distance between $\hat{X}_{i-1}$ and $\hat{X}_i$ measured along $\hat{Z}_i$
θ_i: angle between $\hat{X}_{i-1}$ and $\hat{X}_i$ measured about $\hat{Z}_i$

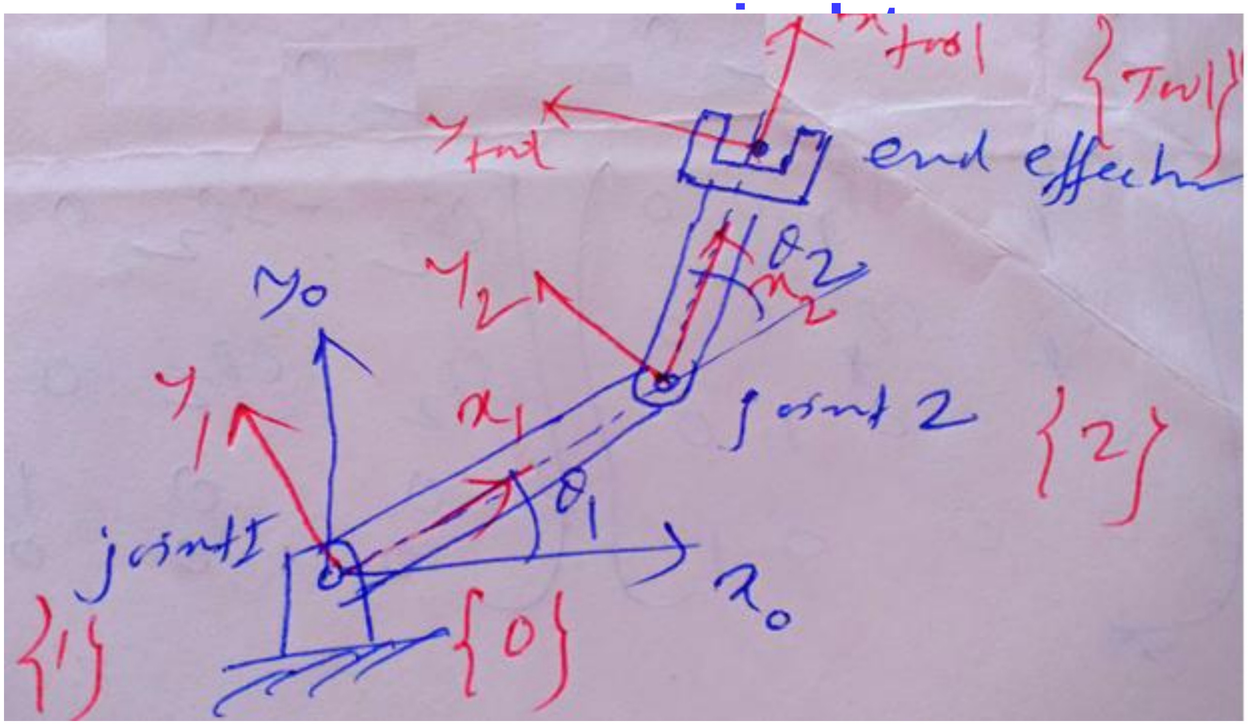


D-H parameters for 2-R manipulator



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2

Example1: Manipulator kinematics: 2R



Obtain the transformation matrix and find the position and orientation of end effector

Length of link 1 (a_1) = 6 cm;

Length of link 2 (a_2) = 9 cm;

θ_1= 60 degree; θ_2= 45 degree;

Manipulator kinematics: 2R manipulator

D-H Parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	a_1	0	θ_2

$$^{0}T_{1} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Manipulator kinematics: 2R manipulator

$$
{}^{1}T_{2} = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}T_{tool} = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Manipulator kinematics: 2R manipulator

$$ {}^{0}T_{tool} = {}^{0}T_{1}\,{}^{1}T_{2}\,{}^{2}T_{tool} $$

$$ = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_1 \\ S\theta_2 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Manipulator kinematics: 2R manipulator

Final transformation matrix

$$ {}^{0}T_{tool} = {}^{0}T_{1} \, {}^{1}T_{2} \, {}^{2}T_{tool} $$

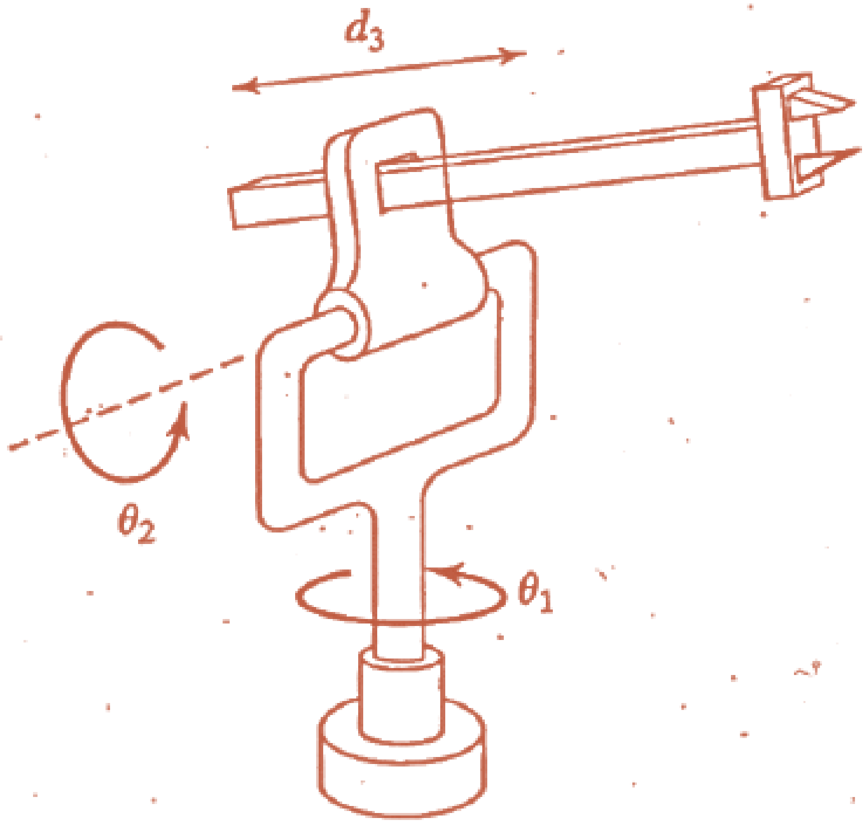
$$ \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1 c_1 + a_2 c_{12} \\ s_{12} & c_{12} & 0 & a_1 s_1 + a_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Considering the given values of D-H parameters

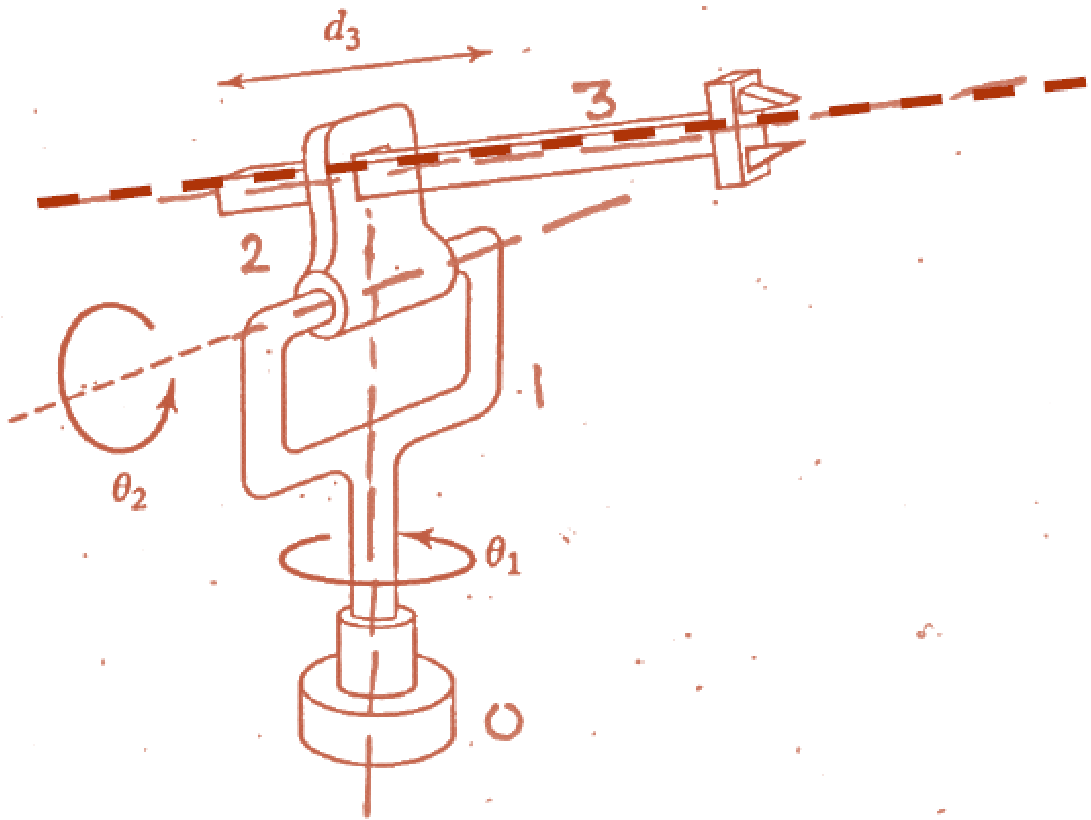
$$
{}^{0}T_{tool} = \begin{bmatrix} c(60+45^\circ) & -s(60+45^\circ) & 0 & 6c(60)+9c(60+45) \\ s(60+45) & c(60+45) & 0 & 6s(60)+9s(60+45) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} -0.2588 & -0.9659 & 0 & 0.6706 \\ 0.9659 & -0.2588 & 0 & 13.889 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

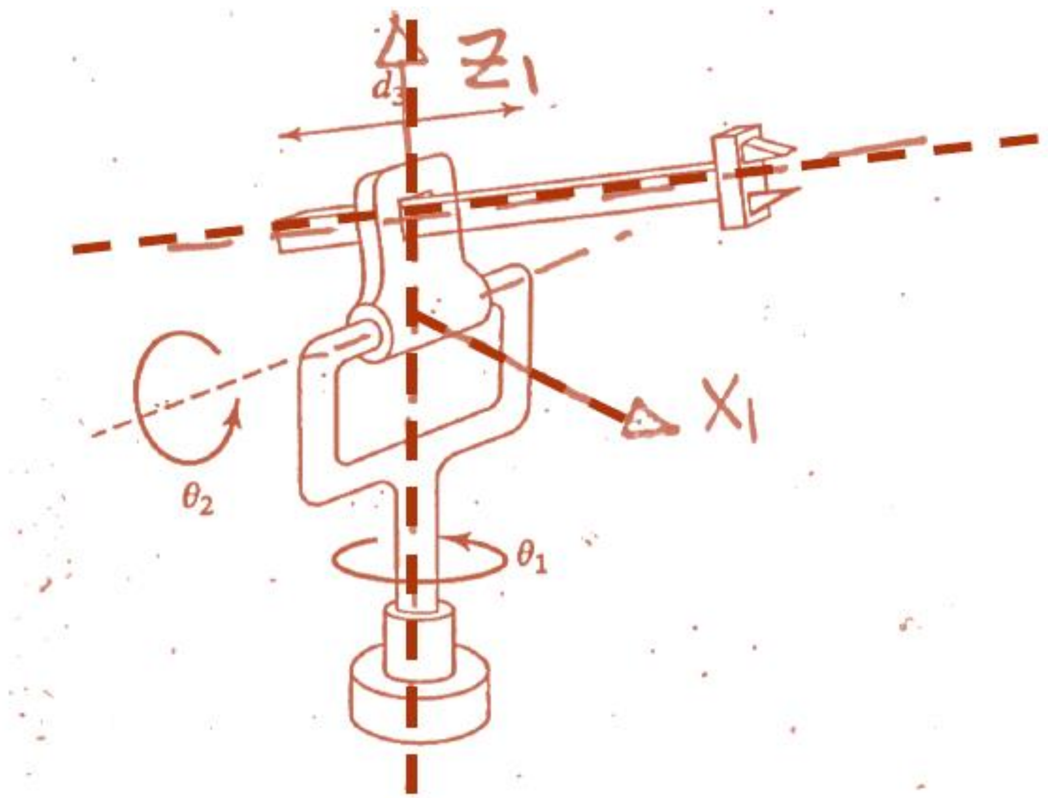
Example 2



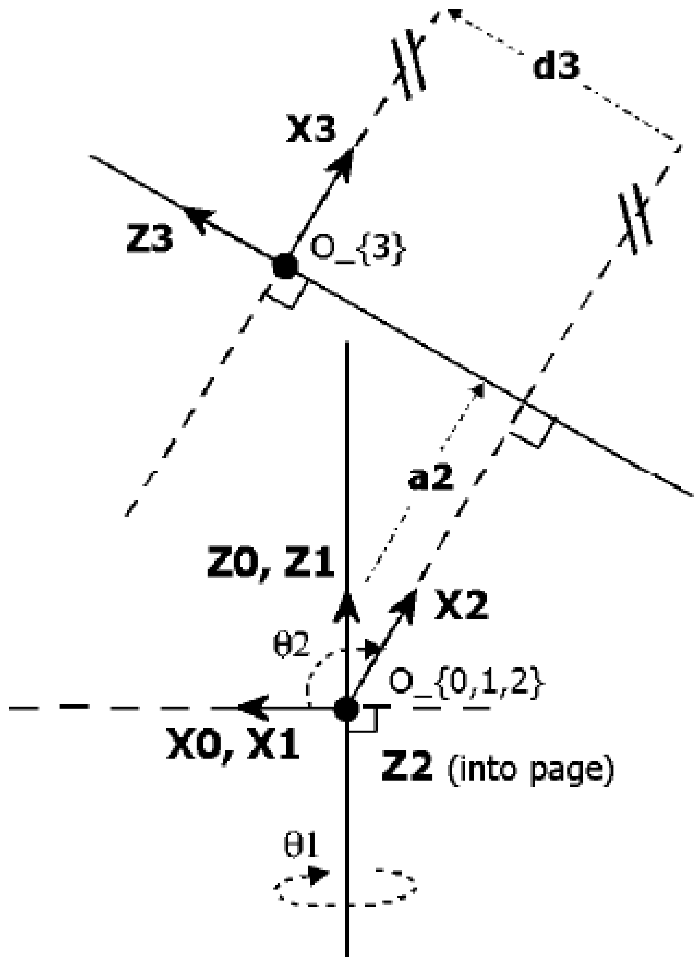
Example 2



Example 2

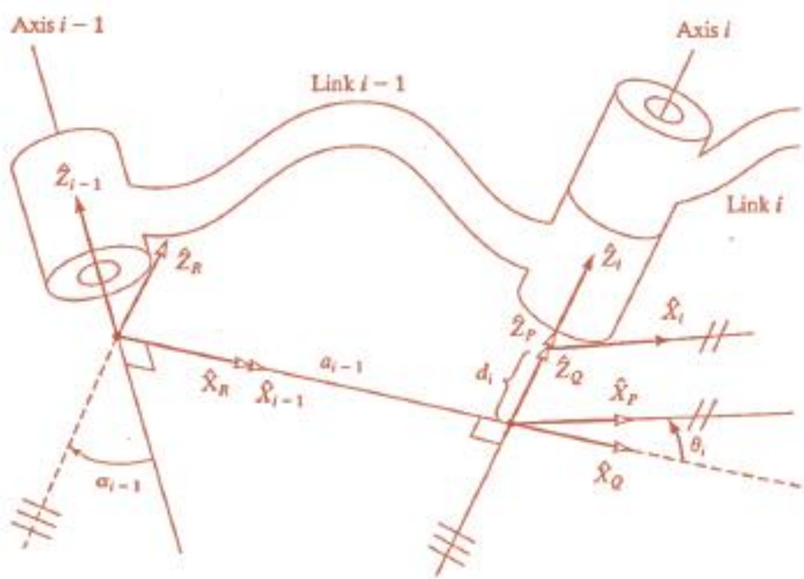


Example 1



How to find the transformation matrix for Example 2

- **Require to know Transformation of adjacent frames**



$$^{i-1}P = {}^{i-1}_{R}T \; {}^{R}_{Q}T \; {}^{Q}_{P}T \; {}^{P}_{i}T \; {}^{i}P,$$

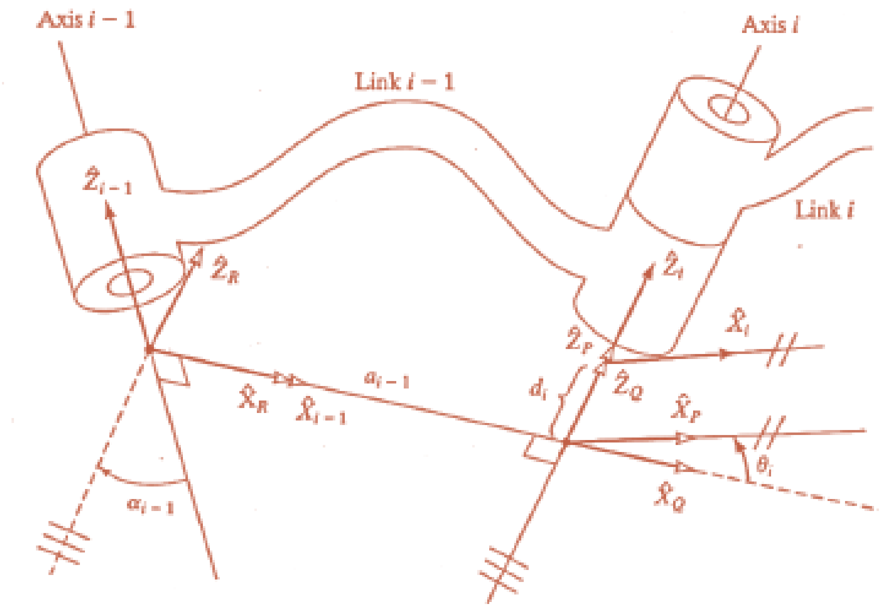
$$^{i-1}P = {}^{i-1}_{i}T \; {}^{i}P,$$

$$^{i-1}_{i}T = {}^{i-1}_{R}T \; {}^{R}_{Q}T \; {}^{Q}_{P}T \; {}^{P}_{i}T.$$

$$^{i-1}_{i}T = R_X(\alpha_{i-1}) D_X(a_{i-1}) R_Z(\theta_i) D_Z(d_i),$$

How to find the transformation matrix for Example 2

- Require to know Transformation of adjacent frames



$$D_X(a_{i-1}) = \begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_X(\alpha_{i-1}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_{i-1} & -\sin\alpha_{i-1} & 0 \\ 0 & \sin\alpha_{i-1} & \cos\alpha_{i-1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

How to find the transformation matrix for Example 2

- Require to know Transformation of adjacent frames

$$D_z(d_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D_z(d_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z(\theta_i) = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

How to find the transformation matrix for Example 2

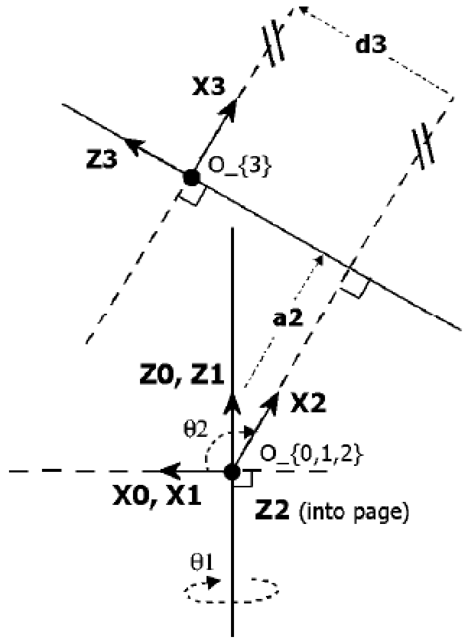
- Require to know Transformation of adjacent frames

<u>**Final transformation matrix**</u>

$$
{}^{i-1}_{\ \ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

How to find the transformation matrix for Example 2

- Denavit-Hartenberg parameters for this manipulator



i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	90^o	0	θ_2
3	a_2	90^o	d_3	0

Derive the forward kinematics for this manipulator — that is, find ${}^0_3T$.

$$
{}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^2_3T = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

How to find the transformation matrix for Example 2

- **Final transformation matrix**

$$
\begin{aligned}
{}^0_3T &= {}^0_1T\,{}^1_2T\,{}^2_3T \\
&= \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_1c_2 & s_1 & c_1s_2 & c_1c_2a_2 + c_1s_2d_3 \\ s_1c_2 & -c_1 & s_1s_2 & s_1c_2a_2 + s_1s_2d_3 \\ s_2 & 0 & -c_2 & s_2a_2 - c_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$