Numerical Techniques Laboratory

Assignment 5 | Tanishq Jasoria | 16MA20047

Alternating Direction Implicit Scheme

We need to find the solution of the Partial Differential Equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

for $-1 < x \ , y < 1$

$$u(x, y, 0) = cos\frac{\pi x}{2}cos\frac{\pi y}{2}$$

$u = 0$ for $x = \pm 1$, $y = \pm 1$

Here

$$\partial x = \partial y = \frac{1}{2}$$

and

$$r = \frac{1}{6}$$

We introduce a new constant r such that

$$r = \frac{\nu}{2}\frac{\partial t}{(\partial x)^2}$$

Here the value of $\nu = 1$

$$\implies r = \frac{1}{2}\frac{\partial t}{(\partial x)^2}$$

Step 1

First we generate the Difference Equation from the implicit scheme wrt x and explicit scheme wrt y from $t_n \rightarrow t_{n+\frac{1}{2}}$

Difference Equation

$$-ru_{i-1j}^{n+\frac{1}{2}} + (1+2r)u_{ij}^{n+\frac{1}{2}} - ru_{i+1j}^{n+\frac{1}{2}} = ru_{ij-1}^{n} + (1-2r)u_{ij}^{n} + ru_{ij+1}^{n}$$

Step 2

First we generate the Difference Equation from the implicit scheme wrt y and explicit scheme wrt x from $t_{n+\frac{1}{2}} \rightarrow t_{n+1}$

Difference Equation

$$-ru_{ij-1}^{n+1} + (1+2r)u_{ij}^{n+1} - ru_{ij+1}^{n+1} = ru_{i-1j}^{n+\frac{1}{2}} + (1-2r)u_{ij}^{n+\frac{1}{2}} + ru_{i+1j}^{n+\frac{1}{2}}$$

In [1]:

```
import numpy as np
from matplotlib import pyplot as plt
from mpl_toolkits import mplot3d
from copy import copy
%matplotlib inline
plt.rcParams['figure.figsize'] = [10, 10]
```

In [2]:

```
def a(r):
    return -1*r
def b(r):
    return (1 + 2*r)
def c(r):
    return -1*r

def ThomasAlgorithm(a, b, c, d, n):
    c_dash = np.zeros(n-1)
    d_dash = np.zeros(n-1)
    c_dash[0] = c[0] / b[0]
    d_dash[0] = d[0] / b[0]
    for itr in range(1, n-1):
        c_dash[itr] = c[itr] / (b[itr] - a[itr] * c_dash[itr-1])
        d_dash[itr] = (d[itr] - a[itr]*d_dash[itr-1]) / (b[itr] - a[itr] * c_dash[i

    y = np.zeros(n-1)
    y[n-2] = d_dash[n-2]

    for itr in reversed(range(n-2)):
        y[itr] = d_dash[itr] - c_dash[itr] * y[itr+1]

    return y
```

In [3]:

```
or the ith x solve the difference equation
 Difference_Step_1(r, u_prev, space_step_x, space_step_y, u_x0, u_xn):
 u = np.zeros((int(space_step_x ), int(space_step_y )))

 for j in range(1, int(space_step_y) -1):
     A = np.array([a(r) for i in range(int(space_step_x - 2))])
     B = np.array([b(r) for i in range(int(space_step_x - 2))])
     C = np.array([c(r) for i in range(int(space_step_x - 2))])
     D = np.array([r*(u_prev[i][j-1] + u_prev[i][j+1]) + (1-2*r)*u_prev[i][j] for i
     u[1:-1, j] = ThomasAlgorithm(A, B, C, D, int(space_step_x) - 1)

 for i in range(0, int(space_step_x)):
     u[i, 0] = u_x0
     u[i, int(space_step_y) - 1] = u_xn

 return u
```

In [4]:

```
# For the jth y solve the difference equation
def Difference_Step_2(r, u_prev, space_step_x, space_step_y, u_y0, u_yn):
    u = np.zeros((int(space_step_x), int(space_step_y)))

    for i in range(1, int(space_step_x) - 1):
        A = np.array([a(r) for j in range(int(space_step_y - 2))])
        B = np.array([b(r) for j in range(int(space_step_y - 2))])
        C = np.array([c(r) for j in range(int(space_step_y - 2))])
        D = np.array([r*(u_prev[i+1][j] + u_prev[i-1][j]) + (1-2*r)*u_prev[i][j] fo
        u[i, 1:-1] = ThomasAlgorithm(A, B, C, D, int(space_step_y) - 1)

    for j in range(0, int(space_step_y)):
        u[0, j] =  u_y0
        u[int(space_step_x) - 1, j] = u_yn
    return u
```

In [5]:

```
ep_x, space_step_y, x0, y0, xn, yn):
linspace(x0, xn, space_step_x+1)
linspace(y0, yn, space_step_y+1)
i/2)*np.cos(np.pi*j/2) for i in x_coordinates] for j in y_coordinates]


, space_step_x, space_step_y, boundarypoints, boundaryconditions):
lize(space_step_x - 1, space_step_y - 1, boundarypoints[0][0], boundarypoints[1][0],


pe(u, (1, u.shape[0], u.shape[1]))

2*time_step)):
n.shape)

nce_Step_1(r, u_prev, space_step_x, space_step_y, boundaryconditions[0][0], boundary

nce_Step_2(r, u_prev, space_step_x, space_step_y, boundaryconditions[1][0], boundary
py(u)
.reshape(u, (1, u.shape[0], u.shape[1]))
np.append(Solution, u_copy, axis=0)
)
```

In [6]:

```
x0 = y0 =  -1
xn = yn = 1
u_x0 = u_xn = u_yn = u_y0 = 0
r = 1/6
t0 = 0
tn = 1
# Space Conditions
boundaryvalues = [[x0, xn], [y0, yn]]
boundaryconditions = [[u_x0, u_xn], [u_y0, u_yn]]
delta_x = delta_y = 0.2

step_x = step_y =  np.ceil((xn - x0)/delta_x) # because yn = xn and x0 = y0
# Time Conditions
delta_t = 2 * r * delta_x**2
time_step = np.ceil((tn - t0)/delta_t)

print(time_step)
```

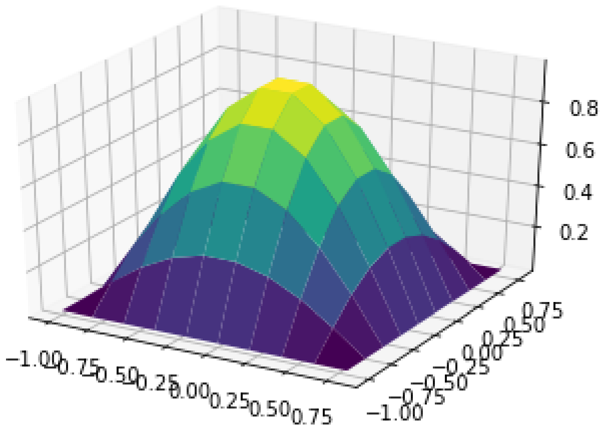
```
75.0
```

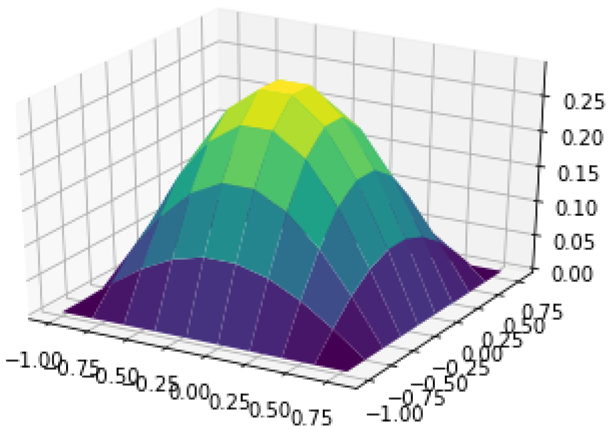
In [7]:

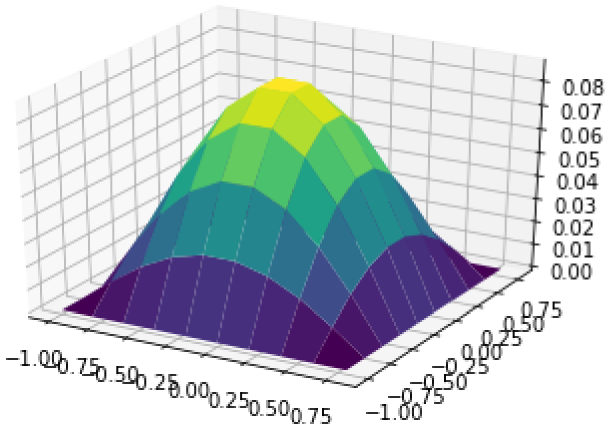
```
Solution = Solver(r, time_step, step_x, step_y, boundaryvalues, boundaryconditions)
x = np.linspace(x0, xn, step_x + 1)[:-1]
y = np.linspace(y0, yn, step_y+1)[:-1]
for i in range(0, int(time_step), int(time_step/5)):
    u = Solution[i]
    fig = plt.figure()
    ax = plt.axes(projection='3d')
    X, Y = np.meshgrid(x, y)
    ax.plot_surface(X, Y, u, cmap='viridis')
```

```
/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:2: Deprec
ationWarning: object of type <class 'numpy.float64'> cannot be safely
interpreted as an integer.

/usr/local/lib/python3.6/dist-packages/ipykernel_launcher.py:3: Deprec
ationWarning: object of type <class 'numpy.float64'> cannot be safely
interpreted as an integer.
  This is separate from the ipykernel package so we can avoid doing im
ports until
```



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0.07
0.06
0.05
0.04
0.03
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0.01
0.00
−1.00
−0.75
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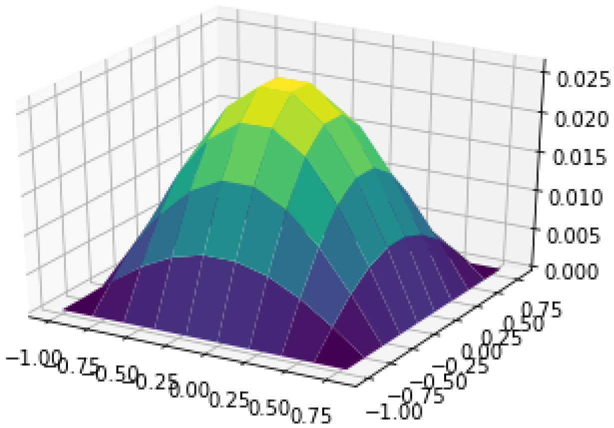
0.75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0.020
0.015
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−1.00
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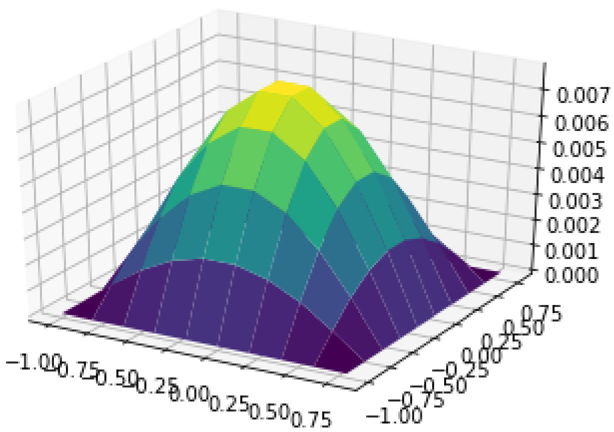
0.75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0.006
0.005
0.004
0.003
0.002
0.001
0.000
−1.00
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−0.25
0.00
0.25
0.50
0.75
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75

In [ ]:

```

```