Numerical Techniques Laboratory

Assignment 4.2 | Tanishq Jasoria | 16MA20047

Solve the ODE for the given conditions

$$y'' = 2 + y^2$$

For the conditions

$$y(0) = 0 = y(1)$$

In [1]:

```
import matplotlib.pyplot as plt
import numpy as np
%matplotlib inline
```

In [2]:

```
def ThomasAlgorithm(a, b, c, d, n):
    c_dash = np.zeros(n-1)
    d_dash = np.zeros(n-1)
    c_dash[0] = c[0]/b[0]
    d_dash[0] = d[0]/b[0]
    for itr in range(1, n-1):
        c_dash[itr] = c[itr] / (b[itr] - a[itr] * c_dash[itr-1])
        d_dash[itr] = (d[itr] - a[itr]*d_dash[itr-1]) / (b[itr] - a[itr] * c_dash[i

    y = np.zeros(n-1)
    y[n-2] = d_dash[n-2]

    for itr in reversed(range(n-2)):
        y[itr] = d_dash[itr] - c_dash[itr] * y[itr+1]

    return y
```

In [3]:

```
x0 = 0
xn = 1
y0 = 0
yn = 1
def func(x0, xn, h = 0.1):
    lst = np.arange(x0, xn, h)
    lst = np.append(lst, xn)
    return lst
```

In [4]:

```
def BVP(x0, xn, y0, yn, step, epsilon = 0.0001):
    '''Keeping the initialization y = 0.5cos(x) '''
    x = func(x0, xn, step)
    y = x - x**2
    print(y)
#     y = np.zeros(x.shape[0])
# #     y[0] = 0.5
#     y[-1] = -0.5
#     a = [1/step**2 - 2*(y[i+1] - y[i-1])/(4*step**2)for i in range(1, len(y)-1)]
#     b = [-2/step**2 + -2*y[i] + 1 for i in range(1, len(y)-1)]
#     c = [1/step**2 + 2*(y[i+1] - y[i-1]) for i in range(1, len(y) -1)]
#     d = [-(y[i]**2 - y[i] - 1 + (y[i+1] -y[i-1])**2/(4*step**2) - (y[i-1] - 2*y[i
    delta_y = np.ones(y.shape)
    while(np.amax(np.absolute(delta_y))>epsilon):
        a = [1/step**2 for i in range(1, len(y)-1)]
        b = [-2*y[i] - 2/step**2 for i in range(1, len(y)-1)]
        c = [1/step**2 for i in range(1, len(y) -1)]
        d = [2 +y[i]**2  -(y[i-1] - 2*y[i] + y[i+1])/(step**2) for i in range(1, le
        delta_y = ThomasAlgorithm(a, b, c, d, len(y)-1)
        delta_y = np.insert(delta_y, 0, 0)
        delta_y = np.append(delta_y, 0)
        print(delta_y)
        y = y + delta_y

    return y
```

In [5]:

```
y_new = BVP(x0, xn, y0, yn, step=0.02, epsilon = 0.0001)
print(y_new)
```

```
[0.     0.0196 0.0384 0.0564 0.0736 0.09   0.1056 0.1204 0.1344 0.1476
 0.16   0.1716 0.1824 0.1924 0.2016 0.21   0.2176 0.2244 0.2304 0.2356
 0.24   0.2436 0.2464 0.2484 0.2496 0.25   0.2496 0.2484 0.2464 0.2436
 0.24   0.2356 0.2304 0.2244 0.2176 0.21   0.2016 0.1924 0.1824 0.1716
 0.16   0.1476 0.1344 0.1204 0.1056 0.09   0.0736 0.0564 0.0384 0.0196
 0.    ]
[ 0.         -0.03824182 -0.07488408 -0.10992806 -0.14337572 -0.175229
66
 -0.20549297 -0.23416918 -0.26126215 -0.28677599 -0.31071497 -0.333083
49
 -0.35388595 -0.37312674 -0.39081016 -0.40694035 -0.42152127 -0.434556
62
 -0.44604985 -0.45600406 -0.46442201 -0.47130609 -0.47665828 -0.480480
15
 -0.48277281 -0.48353696 -0.48277281 -0.48048015 -0.47665828 -0.471306
09
 -0.46442201 -0.45600406 -0.44604985 -0.43455662 -0.42152127 -0.406940
35
 -0.39081016 -0.37312674 -0.35388595 -0.33308349 -0.31071497 -0.286775
99
 -0.26126215 -0.23416918 -0.20549297 -0.17522966 -0.14337572 -0.109928
06
 -0.07488408 -0.03824182  0.        ]
[ 0.         -0.00130437 -0.00260814 -0.00390958 -0.00520603 -0.006493
96
 -0.00776917 -0.00902686 -0.0102618  -0.0114684  -0.01264082 -0.013773
1
 -0.01485923 -0.01589322 -0.01686922 -0.01778158 -0.01862489 -0.019394
1
 -0.02008451 -0.02069187 -0.0212124  -0.02164285 -0.02198051 -0.022223
24
 -0.02236949 -0.02241835 -0.02236949 -0.02222324 -0.02198051 -0.021642
85
 -0.0212124  -0.02069187 -0.02008451 -0.0193941  -0.01862489 -0.017781
58
 -0.01686922 -0.01589322 -0.01485923 -0.0137731  -0.01264082 -0.011468
4
 -0.0102618  -0.00902686 -0.00776917 -0.00649396 -0.00520603 -0.003909
58
 -0.00260814 -0.00130437  0.        ]
[ 0.00000000e+00 -2.55544394e-06 -5.11016655e-06 -7.66200839e-06
 -1.02073842e-05 -1.27413067e-05 -1.52574256e-05 -1.77480863e-05
 -2.02044099e-05 -2.26163952e-05 -2.49730452e-05 -2.72625154e-05
 -2.94722839e-05 -3.15893402e-05 -3.36003898e-05 -3.54920717e-05
 -3.72511831e-05 -3.88649077e-05 -4.03210427e-05 -4.16082187e-05
 -4.27161080e-05 -4.36356170e-05 -4.43590563e-05 -4.48802872e-05
 -4.51948387e-05 -4.52999931e-05 -4.51948387e-05 -4.48802872e-05
 -4.43590563e-05 -4.36356170e-05 -4.27161080e-05 -4.16082187e-05
 -4.03210427e-05 -3.88649077e-05 -3.72511831e-05 -3.54920717e-05
 -3.36003898e-05 -3.15893402e-05 -2.94722839e-05 -2.72625154e-05
 -2.49730452e-05 -2.26163952e-05 -2.02044099e-05 -1.77480863e-05
 -1.52574256e-05 -1.27413067e-05 -1.02073842e-05 -7.66200839e-06
 -5.11016655e-06 -2.55544394e-06  0.00000000e+00]
[ 0.         -0.01994874 -0.03909733 -0.0574453  -0.07499195 -0.091736
36
 -0.1076774  -0.1228138  -0.13714416 -0.15066701 -0.16338077 -0.175283
```

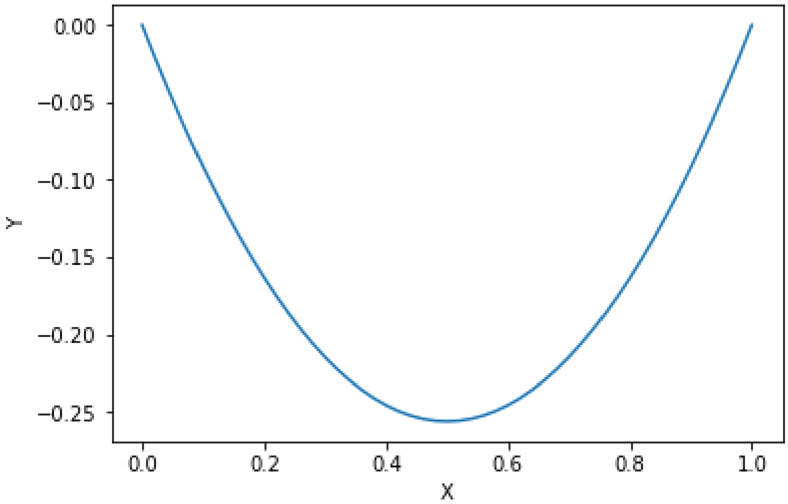
```
85
 -0.18637465 -0.19665155 -0.20611298 -0.21475742 -0.22258341 -0.229589
59
 -0.23577468 -0.24113753 -0.24567713 -0.24939258 -0.25228315 -0.254348
27
 -0.2555875  -0.25600061 -0.2555875  -0.25434827 -0.25228315 -0.249392
58
 -0.24567713 -0.24113753 -0.23577468 -0.22958959 -0.22258341 -0.214757
42
 -0.20611298 -0.19665155 -0.18637465 -0.17528385 -0.16338077 -0.150667
01
 -0.13714416 -0.1228138  -0.1076774  -0.09173636 -0.07499195 -0.057445
3
 -0.03909733 -0.01994874  0.        ]
```

In [6]:

```
x = func(x0, xn, 0.02)
plt.xlabel('X')
plt.ylabel('Y')
plt.plot(x, y_new, '-')
```

Out[6]:

```
[<matplotlib.lines.Line2D at 0x7fdf9ff95390>]
```



In [ ]: