Numerical Techniques Laboratory

Assignment 7.1 | Tanishq Jasoria | 16MA20047

In [4]:

```
import math
from collections import namedtuple
from fractions import Fraction
from copy import copy
import numpy as np
from numpy.linalg import inv
from sympy import symbols
import matplotlib.pyplot as plt
import warnings
warnings.filterwarnings("ignore")
```

In [5]:

```
#some boilerplate code
Conditions = namedtuple('Conditions',('x0','xn','y0','yn','h','e'))
```

In [6]:

```

```

$$
\begin{aligned}
y'' - (y)^2 - 2 &= 0 \\
x &\in (0,1) \\
y(0) &= 0 \\
y(1) &= 0
\end{aligned}
$$

In [7]:

```
def initializer(x):
    return x*(1-x)
```

In [8]:

```
def Finite_Diff_Solver(cond, y_k, verbose=True):
    n = int(np.ceil((cond.xn - cond.x0)/cond.h))
    a, b, c, d = np.zeros((n-2)), np.zeros((n-1)), np.zeros((n-2)), np.zeros((n-1))
    #A
    b[0] = 2.0 * (-1.0/cond.h**2 + y_k[1])
    c[0] = 1.0/cond.h**2

    a[n-3] = 1.0/cond.h**2
    b[n-2] = 2.0 * (-1.0/cond.h**2 + y_k[-2])
    #d
    d[0] = 3 * y_k[1]**2 + 2
    d[n-2] = 3 * y_k[-2]**2 + 2
    for i, xi in enumerate(np.arange(cond.x0 + 2.0*cond.h, cond.xn - cond.h, cond.h
                           start=1):
        if (abs((cond.x0 + 2.0*cond.h) - (cond.xn - cond.h)) < 1e-7):
            continue
        a[i-1] = 1.0 / cond.h**2
        b[i] = 2.0 * (-1.0/cond.h**2 + y_k[i+1])
        c[i] = 1.0 / cond.h**2
        d[i] = 3 * y_k[i+1]**2 + 2

    def tridiag(a, b, c, k1=-1, k2=0, k3=1):
        return np.diag(a, k1) + np.diag(b, k2) + np.diag(c, k3)
    if verbose:
        print("A={}".format(tridiag(a,b,c)))
        print("d={}".format(d.T))

    return(a,b,c,d)
```

In [9]:

```
def Thomas_Algorithm(coeff, cond):
    a, b, c, d = coeff
    n = int(np.ceil((cond.xn - cond.x0)/cond.h))
    c1 = np.zeros((n-2))
    d1 = np.zeros((n-1))
    c1[0] = c[0]/b[0]
    d1[0] = d[0]/b[0]
    for i in range(1,n-2):
        c1[i] = c[i]/(b[i]-a[i-1]*c1[i-1])
        d1[i] = (d[i] - a[i-1] * d1[i-1])/(b[i]-a[i-1]*c1[i-1])
    d1[n-2] = (d[n-2] - a[n-3] * d1[n-3])/(b[n-2] - a[n-3]*c1[n-3])
    def backsubstitution(c,d):
        y = np.zeros_like(d)
        y[-1] = d[-1]
        cache = y[-1]
        for i in reversed(range(d.shape[0]-1)):
            y[i] = d[i] - cache * c[i]
            cache = y[i]
        return y
    fin_y = backsubstitution(c1,d1)
    #print(fin_y)
    y = np.append(fin_y, cond.yn)
    y = np.append(cond.y0, y)
    return y
```

In [10]:

```
def Newton_Solver(cond):
    n = int(np.ceil((cond.xn - cond.x0)/cond.h))
    print(n)
    y_k = np.arange(cond.x0, cond.xn + cond.h, cond.h)
    #print(y_k)
    #print(initializer(y_k))
    solutions = []
    y_k = initializer(y_k)
    tuples_coeff = Finite_Diff_Solver(cond, y_k, verbose=False)
    y_k1 = Thomas_Algorithm(tuples_coeff, cond)
    s = np.max(abs(y_k1 - y_k))
    y_k = y_k1
    while(s > cond.e):
        y_k = copy(y_k1)
        solutions.append(y_k)
        tuples_coeff = Finite_Diff_Solver(cond, y_k, verbose=False)
        y_k1 = Thomas_Algorithm(tuples_coeff, cond)
        s = np.max(abs(y_k1 - y_k))
        #print(s)

    y_k = y_k1
    solutions.append(y_k)
    return solutions
```

In [11]:

```
cond = Conditions(0.0,1.0,0.0,0.0,Fraction(1,10),0.001)
```

In [12]:

```
y = Newton_Solver(cond)
#print(len(y))
x_range = np.arange(cond.x0, cond.xn+cond.h, cond.h)
t1 = np.arange(cond.x0, cond.xn, 0.001)
fig = plt.figure(figsize=(10, 20))
ax = fig.add_subplot(311)
g = []
for i in range(len(y)):
    ax.plot(x_range, y[i])
    ax.scatter(x_range, y[i])
    g.append("iter"+str(i))
ax.legend(tuple(g))
fig.show()
```

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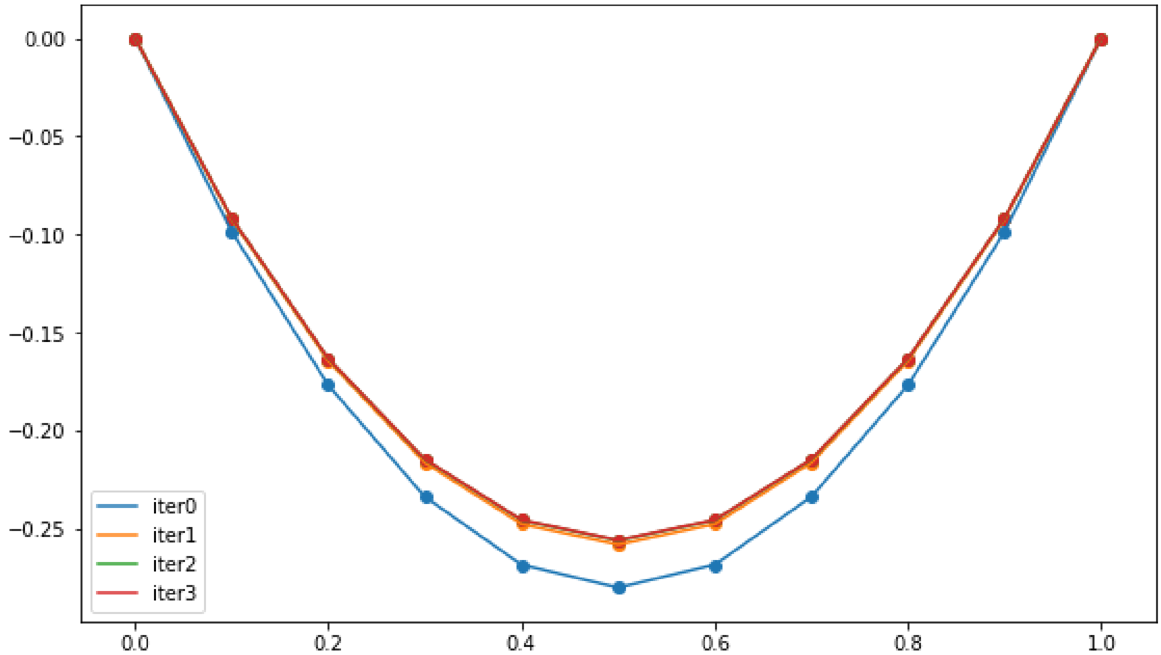

In [13]:

```
cond = Conditions(0.0,1.0,0.0,0.0,Fraction(1,5),0.001)
y = Newton_Solver(cond)
#print(len(y))
x_range = np.arange(cond.x0, cond.xn+cond.h, cond.h)
fig = plt.figure(figsize=(10,20))
ax = fig.add_subplot(311)
ax.set(title="BVP h=0.2", xlabel="x", ylabel="y")
ax.plot(x_range, y[-1])
ax.scatter(x_range, y[-1])

fig.show()
```

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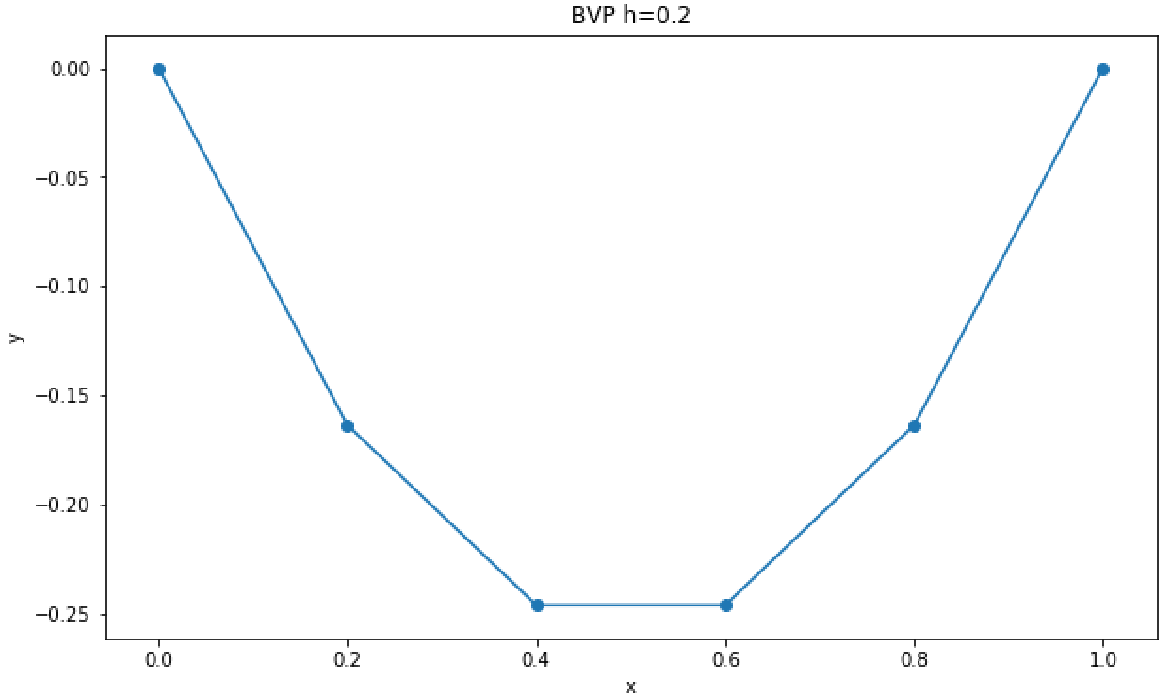
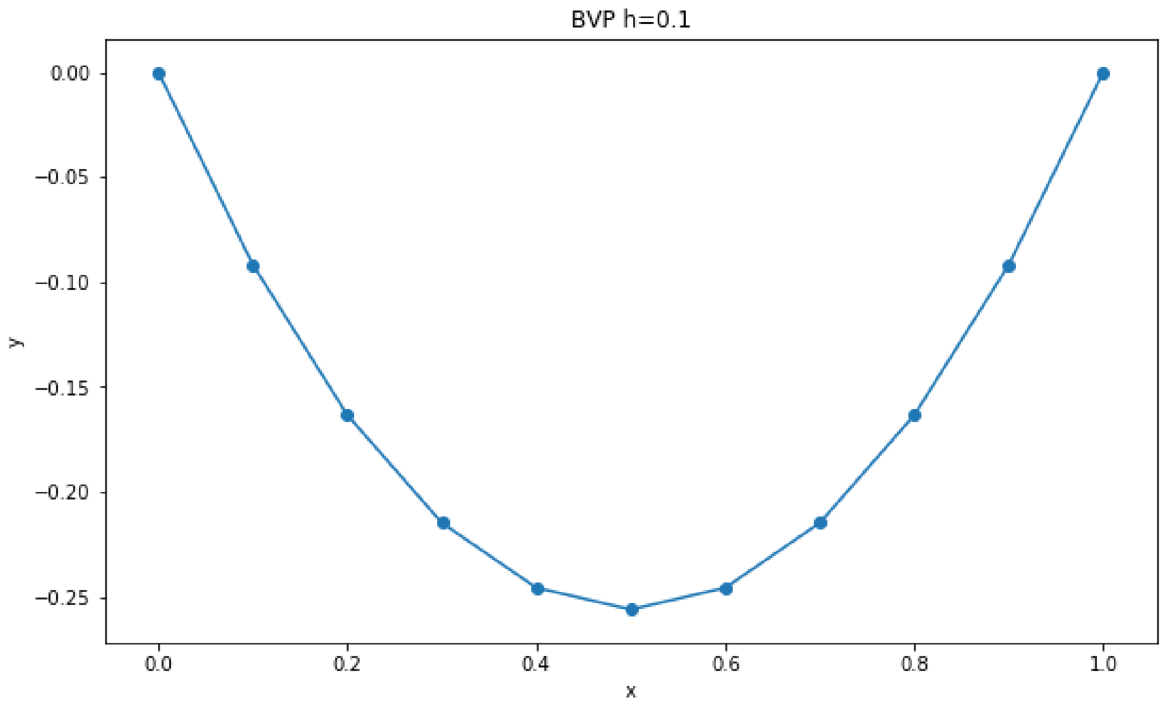


In [14]:

```
cond = Conditions(0.0,1.0,0.0,0.0,Fraction(1,10),0.001)
y = Newton_Solver(cond)
#print(len(y))
x_range = np.arange(cond.x0, cond.xn+cond.h, cond.h)
fig = plt.figure(figsize=(10,20))
ax = fig.add_subplot(311)
ax.set(title="BVP h=0.1", xlabel="x", ylabel="y")
ax.plot(x_range, y[-1])
ax.scatter(x_range, y[-1])

fig.show()
```

```
10
```



In [15]:

```
cond = Conditions(0.0,1.0,0.0,0.0,Fraction(1,100),0.001)
y = Newton_Solver(cond)
#print(len(y))
x_range = np.arange(cond.x0, cond.xn+cond.h, cond.h)
fig = plt.figure(figsize=(10,20))
ax = fig.add_subplot(311)
ax.set(title="BVP h=0.01", xlabel="x", ylabel="y")
ax.plot(x_range, y[-1])
ax.scatter(x_range, y[-1])

fig.show()
```

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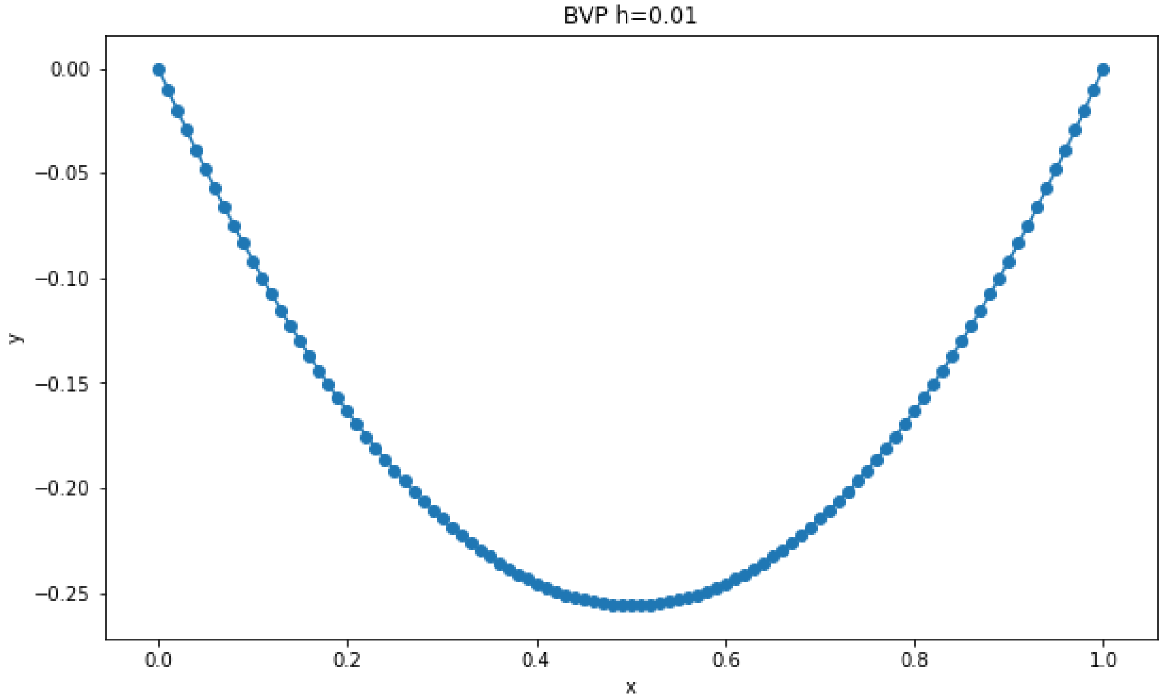

In [ ]: