Numerical Techniques Laboratory

Assignment Crank Nicholson scheme | Tanishq Jasoria | 16MA20047

Crank Nicholson scheme

We want to solve the parabolic PDE

$$u_t = \alpha u_{xx}$$

using the Crank Nicholson Technique

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \frac{\alpha}{2(\Delta x)^2}\left((u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}) + (u_{i+1}^n - 2u_i^n + u_{i-1}^n)\right)$$

and letting $r = \dfrac{\alpha \Delta t}{2(\Delta x)^2}$

And this ultimately results to a finite difference equation given below

$$-ru_{i+1}^{n+1} + (1+2r)u_i^{n+1} - ru_{i-1}^{n+1} = ru_{i+1}^n + (1-2r)u_i^n + ru_{i-1}^n$$

In [1]:

```
import matplotlib.pyplot as plt
import numpy as np
%matplotlib inline
plt.rcParams['figure.figsize'] = [10, 15]
```

In [2]:

```
def a(r):
    return -1*r
def b(r):
    return (1 + 2*r)
def c(r):
    return -1*r

def ThomasAlgorithm(a, b, c, d, n):
    c_dash = np.zeros(n-1)
    d_dash = np.zeros(n-1)
    c_dash[0] = c[0] / b[0]
    d_dash[0] = d[0] / b[0]
    for itr in range(1, n-1):
        c_dash[itr] = c[itr] / (b[itr] - a[itr] * c_dash[itr-1])
        d_dash[itr] = (d[itr] - a[itr]*d_dash[itr-1]) / (b[itr] - a[itr] * c_dash[i

    y = np.zeros(n-1)
    y[n-2] = d_dash[n-2]

    for itr in reversed(range(n-2)):
        y[itr] = d_dash[itr] - c_dash[itr] * y[itr+1]

    return y
```

In [3]:

```
def Solver(r, y_prev, step, y_x0, y_xn):
  A = np.array([a(r) for i in range(int(step - 1))])
  B = np.array([b(r) for i in range(int(step - 1))])
  C = np.array([c(r) for i in range(int(step - 1))])
  D = np.array([r*(y_prev[i+1] + y_prev[i-1]) + (1-2*r)*y_prev[i] for i in range(1,
  y = ThomasAlgorithm(A, B, C, D, int(step))
  y = np.append(y, y_xn)
  y = np.insert(y, 0, y_x0)
  return y
```

Here $\alpha = 1$

In [4]:

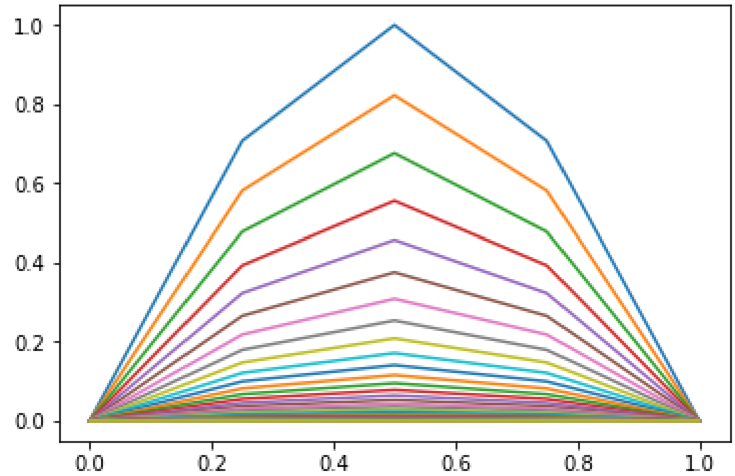
```
x0 = 0
xn = 1
t0 = 0
tn = 1
y_x0 = 0
y_xn = 0
delta_x = 0.25
r = 1/6
delta_t = 2 * r * delta_x**2
step = np.ceil((xn - x0)/delta_x)
time_step = np.ceil((tn - t0)/delta_t)
def f(x0, xn, delta_x, i):
  temp = x0 + delta_x*i
  return temp
```

In [5]:

```
count = 0
y_init = np.array([np.sin(np.pi*(f(x0, xn, delta_x, i))) for i in range(int(step+1)
y = y_init
# print("Initial Solution Iteration")
Solution = np.array([[]])
Solution = np.append(Solution, np.reshape(y, (1, y.shape[0])), axis = 1)
# print(Solution)
while(count < time_step):
  temp = Solver(r, y, step, y_x0, y_xn)
  count = count + 1
  Solution = np.append(Solution, np.reshape(temp, (1, temp.shape[0])), axis=0)
  y = temp
#   print(Solution[count])
# print(Solution.shape)
```

In [6]:

```
def func(x0, xn, delta_x, step):
  temp = np.array([f(x0, xn, delta_x, i) for i in range(int(step + 1))])
  return temp
x = func(x0, xn, delta_x, step)
for i in range(Solution.shape[0]):
  plt.plot(x, Solution[i])
plt.show()
plt.savefig("Plot.png")
```




```
<Figure size 432x288 with 0 Axes>
```