TUTORIAL 4

Ans 1) $T(n) = 3T(n/2) + n^2$

$a = 3, \; b = 2, \quad f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

Comparing $n^{\log_2 3}$ and n^2

$n^{\log_2 3} < n^2$ (Case 3)

∴ according to Master's Theorem

$T(n) = \Theta(n^2)$

Ans 2) $T(n) = 4T(n/2) + n^2$

$a = 4, \; b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2 = f(n)$ (case 2)

∴ According to Master's Theorem

$T(n) = \Theta(n^2 \log n)$

Ans 3) $T(n) = T(n/2) + 2^n$

$a = 1, \; b = 2$

$n^{\log_2 1} = n^0 = 1$

$1 < 2^n$ (case 3)

∴ According to Master's Theorem $T(n) = \Theta(2^n)$

Ans 4) $T(n) = 2^n T(n/2) + n^n$

∴ Master's Theorem is not applicable as a is function of n

Ans 5) $T(n) = 16T(n/4) + n$

$a = 16, b = 4, f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 > f(n)$ (Case 1)

$T(n) = \theta(n^2)$

Ans 6) $T(n) = 2T(n/2) + n\log n$

$a = 2, b = 2, f(n) = n\log n$

$n^{\log_b a} = n^{\log_2 2} = n$

Now $f(n) > n$

∴ According to master's Theorem

$T(n) = \theta(n\log n)$

Ans 7) $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

$a = 2, b = 2, f(n) = \frac{n}{\log n}$

$n^{\log_b a} = n^{\log_2 2} = n$

$n > f(n)$

∴ According to master's theorem $T(n) = \theta(n)$

Ans 8) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a = 2$, $b = 4$, $f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$n^{0.5} < f(n)$

∴ According to master's theorem $T(n) = \theta(n^{0.5})$

Ans 9) $T(n) = 0.5\,T(n/2) + \frac{1}{n}$

∴ Master's Theorem not applicable as $a < 1$

Ans 10) $T(n) = 16\,T\left(\frac{n}{4}\right) + n!$

$a = 16$, $b = 4$, $f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 < n!$

∴ According to master's theorem, $T(n) = \theta(n!)$

Ans 11) $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

$a = 4$, $b = 2$, $f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > f(n)$

∴ According to master's theorem, $T(n) = \theta(n^2)$

Ans 12) $T(n) = \text{sqrt}(n) + (n/2) + \log n$

∴ Master's theorem not applicable as a is not a constant

Ans 13) $T(n) = 3T(n/2) + n$

$a = 3$, $b = 2$, $f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > f(n)$

∴ According to Master's Theorem, $T(n) = O(n^{\log_2 3})$

Ans 14) $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3$, $b = 3$, $f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

∴ According to Master's Theorem, $T(n) = \theta(n)$

Ans 15) $T(n) = 4T(n/2) + cn$

$a = 4$, $b = 2$, $f(n) = c * n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > c * n$

∴ According to Master's Theorem, $T(n) = \theta(n^2)$

Ans 16) $T(n) = 3T(n/4) + n\log n$

$a = 3$, $b = 4$, $f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n\log n$

∴ According to Master's Theorem, $T(n) = \theta(n\log n)$

Ans 17) $T(n) = 3T(n/3) + n/2$

$a = 3$, $b = 3$, $f(n) = n/2$

$n^{\log_b a} = n^{\log_3 3} = n$ $\quad \theta(n) = \theta(n/2)$

∴ According to Master's Theorem

$T(n) = \theta(n\log n)$

Ans 18) $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, b = 3, f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{1.63} < n^2 \log n$

∴ According to Master's theorem

$T(n) = \theta(n^2 \log n)$

Ans 19) $T(n) = 4T(n/2) + n/\log n$

$a = 4, b = 2, f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > n/\log n$

∴ According to Master's theorem

$T(n) = \theta(n^2)$

Ans 20) $T(n) = 64T(n/8) - n^2 \log n$

Master's theorem not applicable as $f(n)$ is not increasing function

Ans 21) $T(n) = 7T(n/3) + n^2$

$a = 7, b = 3, f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

$n^{1.7} < n^2$

∴ According to Master's Theorem, $T(n) = \theta(n^2)$

Ans 22) $T(n) = T(n/2) + n(2 - \cos n)$

Master's Theorem not applicable since regularity condition is isolated in case 3.