Problem Set 4

Statistical Methods In Engineering And Science

Due Date: 10:00 PM, October 27, 2023
Last Update: October 21, 2023

Prof. Alexander Giessing
Spring Quarter, 2023

Study Group: _____ _____ _____

Please upload your solution in a single pdf file on Canvas. Include all calculations, R-code, and figures (if applicable). All data sets are available on Canvas https://canvas.uw.edu/courses/1635461.

Question 1. Despite warnings of your statistics professor, you decide to gamble every month in two independent lotteries. Your strategy is to stop playing as soon as you win in at least one of the two lotteries. Suppose that every time you play in these two lotteries, the probabilities of winning are p_1 and p_2, respectively. Let T be the number of times you play until winning in at least one of the two lotteries.

(a) What is the distribution of T and what is/ are its parameter(s)?

(b) What is the expected number of times you need to play until you win at least once?

Question 2. Twelve headsets have been returned to a seller because of an audible, high-pitched oscillating noise. Seven of these headsets have defective amplifiers which cost \$75 to replace, and five of them have loose contacts which cost only \$30 to repair. Suppose that the headsets are examined in random order and let X be the number of headsets with defective amplifiers among the first six headsets that are examined.

(a) Calculate $P(X = 4)$ and $P(X \leq 4)$.

(b) What is the expected cost of repairing these six headsets? What is the associated standard deviation?

Question 3. Let X be a continuous random variable with probability density function

$$f_X(x) = \begin{cases} \frac{3}{4}x(2-x) & \text{for } 0 \leq x \leq 2, \\ 0 & o/w. \end{cases}$$

(a) Determine the distribution function F_X.

(b) Compute $\mathrm{E}[X]$ and $\mathrm{Var}(X)$.

(c) Let $Y = \sqrt{X}$. Derive the distribution function F_Y.

(d) Find the probability density of Y.

(e) Compute $\mathrm{E}[Y]$ and $\mathrm{Var}(Y)$.

Question 4. Compute expected value $\mathrm{E}[X]$ and variance $\mathrm{Var}(X)$ of the following random variables.

(a) $X = 17 \times (Y - 3)$, where $Y \sim N(3, 4^2)$;

(b) $X = \sin(Y)$, where $Y \sim Ber(p)$;

(c) $X \sim Pois(\lambda)$;

(d) $X = U^2$, where $U \sim Unif(-2, 5)$;

(e) $X \sim Exp(\lambda)$.

Question 5. Vehicle speed in the State Route 99 Tunnel in Downtown Seattle follows a normal distribution.

(a) If the mean and standard deviations are 50 m/h and 10 m/h, respectively, what percent of vehicles have a speed between 45 m/h and 65 m/h?

(b) If the fastest 3% of all vehicles are to be ticketed, what is the minimum speed to be ticketed?

(c) If 5% of all vehicles travel less than 40 m/h and 10% travel more than 70 m/h, what are the mean and standard deviation of the vehicle speed?

Question 6. The data sets `APPL.csv` and `JNJ.csv` contain the adjusted closing prices of Apple Inc and Johnson & Johnson from Jan. 1, 2000 to September 8, 2016. Use R to answer the following questions.

(a) Do the log returns of Apple Inc, and Johnson & Johnson follow a normal distribution?

(b) Compare the tails of the log returns of Apple Inc and Johnson & Johnson with a t-distribution with 4 degrees of freedom.

(c) Compare the distributions of the log returns of Johnson & Johnson during the 2008 financial crisis (index: 2063:1812, from 7/1/08 – 6/30/09) with those two years after the financial crisis (index: 1306:1, from 7/1/11 – 9/8/16) via side-by-side boxplots, side-by-side histograms, and QQ-plots.

(d) What is the appropriate degree of freedom of the t-distribution for modeling the log returns of the Apple Inc stock two years after the financial crisis (index: 1306:1, from 7/1/11 – 9/8/16)? Provide a QQ-plot and a histogram with overlayed density of the best fitting t-distribution.