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Chapter 1

Part III

We have to solve an uncertainty quantification problem, in which we have 5 input random parameters. We have to calculate the forces, Force 1 and Force 2, which are the forces when no current is supplied and with current respectively. In part III we have to improve the convergence rates as obtained in the Crude Monte Carlo algorithm. Crude Monte Carlo algorithm is an exceptionally good method but at the same time, it is very slow to converge. The method converges at the rate proportional to $1/sqrt(R)$, which is not acceptable in todays computing industry where a lot of focus is laid upon the computing time of the scheme. In this report we simulate three methods which are very popular in increasing the convergence rate of the Crude Monte Carlo algorithms. The algorithms namely, Stratified sampling, Importance sampling and Antithetic variates are based on the general idea of Crude MC, which have some changes in the algorithm to focus on important samples or regions. To recall, we have to calculate the outputs taking 1 random variable at a time and keeping the rest 4 to be their nominal values (for the RV e and ep as they were the most influential parameters). We also present the tables for 1 sample with their mean and calculate the convergence factor for. Finally we also show the convergence comparison of all the methods.

1.1 Part III - Improving the Crude Monte Carlo algorithm

Three methods will be discussed viz,

- Stratified Sampling

- Antithetic Variates
- Improvement Sampling

1.2 Stratified Sampling

In this section we start with the algorithm of Stratified Sampling which is used to calculate the output parameters. Then we present the tables of obtained results and histograms of Force 1 and Force 2. At the end of report, we give some comments about convergence.

1.2.1 Algorithm

Stratified sampling is the method used to calculate the output for random input parameters. The algorithm of the method is as shown: Here j is the number of random input parameters and R is the number of samples generated for each random input parameter. The uncertain interval of the random variable X is divided into a number of sub-intervals. For each sub-interval we assign the number of samples to be generated. The sum of number of samples on all the sub-intervals should be equal to R. Then for each subinterval we input its samples in the model to generate local outputs. Then we combine all the local outputs to form a global kind of output. Final post processing of the output consist of the step of calculating its mean and variance etc.

start
for $param \leftarrow 1$ **to** j **by** 1 **do**
  Divide the uncertain interval of each RV in sub intervals.
  Generate a sample from required distribution in subintervals
end
Calculate the random samples in the model to obtain local output y
Combine local outputs to obtain the global output y Calculation of mean and variance of y for post-processing
end

The benefit of using stratified sampling lies in the fact that any number of samples could be considered in any of the sub intervals as long as sum of samples from each subinterval is equal to R. This allows the user to define more samples in the region where more changes are likely to be obtained. Generally, the user defines more samples in the region where the PDF of the function changes more and less samples in the region where the PDF is constant. As, in our case we will be considering only the uniform distribution, hence we assume equal number of samples in each sub interval. because the pdf is constant. Next, we show some results obtained from the simulation.

1.2.2 Results

R	S1	conv
100	147.21149578278902	0.8099691258236184
1000	147.48602419540505	0.24943306661937657
10000	147.42679786858213	0.07930414644546487
100000	147.43719651349423	0.025167233386197457
1000000	147.43793060053372	0.007963322764195703

Table 1.1: e as random force 1

R	S1	conv
100	1.1749625255919918	0.11402323352786026
1000	1.1511526564465433	0.0332463450939305
10000	1.1552471743980166	0.0106863324768872
100000	1.151615188607741	0.003409215174427287
1000000	1.150840092375203	0.0010783374359141771

Table 1.2: e as random force 2

R	S1	conv
100	150.70770506954622	2.27954838343086
1000	150.41461857470574	0.6831364890001358
10000	150.0562345702291	0.21494588226253358
100000	150.07728717365248	0.06785220764564463

Table 1.3: ep as random force 1

1.3 Antithetic Variates

In this section we start with the algorithm of Antithetic Variates which is used to calculate the output parameters. Then we present the tables of obtained results and histograms of Force 1 and Force 2.

1.3.1 Algorithm

Antithetic Variates is the method used to calculate the output for random input parameters. The algorithm of the method is as shown: Here j is the number of random input parameters and R is the number of samples generated for each random input parameter. The logic behind

R	S1	conv
100	0.00016125405661495786	2.4456916659651948e-06
1000	0.0001609941882362235	7.329264120947448e-07
10000	0.0001609929778117643	2.3061206189088062e-07
100000	0.00016101556481939867	7.279756813344523e-08

Table 1.4: ep as random force 2

R	S1	conv
100	149.8250199690889	0.7981275415241923
1000	149.27216633773136	0.24781412791309865
10000	149.2981367668484	0.0779025827165367
100000	149.2430596503005	0.02488702133165043

Table 1.5: all as random force 1

antithetic variates is very obvious and pleasing. It says, if X is a random variable uniformly generated in $[0, 1]$ then $1 - X$ is also a random variable in the same interval. We can then find the outputs for both these samples and take the average of the resulting outputs to find the final output.

start
for $param \leftarrow 1$ **to** j **by** 1 **do**
  Generate a sample from required distribution in subintervals
  Name the sample to be X
  Calculate 1-X
end
Calculate the random samples X in the model to obtain output Y
Calculate the random samples 1-X in the model to obtain output Y prime
Take the average of Y and Y prime.
Calculation of mean and variance of final output for post-processing
end

1.3.2 Results

In this section we present tables and histograms for force 1 and force 2 using antithetic variates.

1.4 Importance Sampling

In this section we start with the algorithm of Importance sampling which is used to calculate the output parameters. Then we present the tables of obtained results and histograms of Force 1 and Force 2.

R	S1	conv
100	0.5979740160551965	0.07055615305585015
1000	0.6509881508609359	0.023622631942853148
10000	0.6219973503336643	0.007122233353644568
100000	0.6243905449492281	0.0022290894240190695

Table 1.6: all as random force 2

R	S1	conv
100	147.3833606857812	0.8296135761491448
1000	147.4422407130798	0.2505858093158745
10000	147.437151419219	0.07954409830263023
100000	147.44001069415438	0.025238388406292143

Table 1.7: e as random force 1

1.4.1 Algorithm

Importance sampling is not actually a method to calculate the mean and variance for the output, actually the method importance sampling is used to calculate the hit and miss probabilities for nuclear reactor problems. But, nonetheless, we can use it as a tool to find the mean as well. The only problem lies in the fact is the requirement of the knowledge of a function g(x) which is not easy to find. g(x) is defined as the the minimum of a functional whose basic aim is to reduce the variance of the output.

start
for *param* ← 1 **to** *j* **by** 1 **do**
  Generate a sample from required distribution in subintervals
  Find the probability density function f of X
  Calculate the function g
end
Calculation of the product Mf/g to obtain the pdf of inverse mapping
Calculate the inverse mapping
Calculation of mean and variance of y for post-processing
end

1.4.2 Results

1.5 Final Comments

From the tables which below clearly shows that stratified shows the best convergence result as compare to other methods such as importance sampling, Monte Carlo and antithetic sampling for all the cases i.e, e, ep and all random parameters as random.

R	S1	conv
100	1.1696665504225274	0.11749291155634782
1000	1.0721129410473544	0.034256736805207286
10000	1.0828320775044449	0.010818516644486784
100000	1.0905962277703927	0.0034192761833354447

Table 1.8: e as random force 2

R	S1	conv
100	150.21177646499495	2.220098931533893
1000	150.1178290582321	0.6813941231367602
10000	150.06202545264924	0.2155718638183268
100000	150.06954514286667	0.06798118331603682

Table 1.9: ep as random force 1

R	S1	conv
100	0.00016070245122570164	2.381909282530199e-06
1000	0.0001609214135976004	7.310570551194362e-07
10000	0.00016099919075858156	2.312836676726132e-07
100000	0.00016100814791959321	7.2935944104968e-08

Table 1.10: ep as random force 2

R	S1	conv
100	150.02414066704077	2.596593266955689
1000	149.83599977724487	0.7559168609706008
10000	149.86844897018784	0.24380129158800973
100000	149.82104094758148	0.07695816820686315

Table 1.11: all as random force 1

R	S1	conv
100	1.8537119480761233	0.219925523881926
1000	1.71718366588981	0.070427484211383
10000	1.7275435004271629	0.022286845683252562
100000	1.7316591275340878	0.00703602354628807

Table 1.12: all as random force 2

R	S1	conv
100	146.36597	0.82118
1000	147.33725	0.25223
10000	146.62619	0.07978
100000	146.72767	0.02524

Table 1.13: e as random force 1

R	S1	conv
100	1.2874	0.11782
1000	1.11809	0.03552
10000	1.18622	0.01108
100000	1.182008	0.0035

Table 1.14: e as random force 2

R	S1	conv
100	154.67204	2.07235
1000	155.82625	0.69055
10000	155.58076	0.2178
100000	155.45142	0.06901

Table 1.15: ep as random force 1

R	S1	conv
100	0.000165	2.20e-06
1000	0.000167	7.40e-07
10000	0.000166	2.34e-07
100000	0.000166	7.40e-08

Table 1.16: ep as random force 2

R	S1	conv
100	153.84013	0.51245
1000	153.89638	0.16214
10000	153.59309	0.05119
100000	153.56071	0.01622

Table 1.17: all as random force 1

R	S1	conv
100	0.61066	0.05924
1000	0.57627	0.01831
10000	0.57571	0.00567
100000	0.5775	0.00179

Table 1.18: all as random force 2

methods	mean	conv
Monte Carlo	147.47139	0.07969
Stratified	147.43719651349423	0.025167233386197457
Anthetic	147.437151419219	0.07954409830263023
Improve Sampling	146.62619	0.07978

Table 1.19: e random force 1

methods	mean	conv
Monte Carlo	1.15112	0.01069
Stratified	1.151615188607741	0.003409215174427287
Anthetic	1.0828320775044449	0.010818516644486784
Improve Sampling	1.18622	0.01108

Table 1.20: e random force 2

methods	mean	conv
Monte Carlo	150.50391	0.21620
Stratified	150.0562345702291	0.21494588226253358
Anthetic	150.06202545264924	0.2155718638183268
Importance Sampling	155.58076	0.2178

Table 1.21: ep random force 1

methods	mean	conv
Monte Carlo	0.00016	2.3e-07
Stratified	0.0001609929778117643	2.3061206189088062e-07
Anthetic	0.00016099919075858156	2.312836676726132e-07
Improve Sampling	0.000166	2.34e-07

Table 1.22: ep random force 2

methods	mean	conv
Monte Carlo	149.52385	0.24487
Stratified	149.2981367668484	0.0779025827165367
Anthetic	149.82104094758148	0.07695816820686315
Improve Sampling	153.59309	0.05119

Table 1.23: all random force 1

methods	mean	conv
Monte Carlo	1.77449	0.02177
Stratified	0.6219973503336643	0.007122233353644568
Anthetic	1.7275435004271629	0.022286845683252562
Improve Sampling	0.57571	0.00567

Table 1.24: all random force 2