



Department of Mathematics

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M2CHPS

UQ Project Part I and II

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Chapter 1

Part I

We have to solve an uncertainty quantification problem, in which we have 5 input random parameters. We have to calculate the forces, Force 1 and Force 2, which are the forces when no current is supplied and with current respectively. In part 1.1 we have to calculate the outputs taking 1 random variable at a time and keeping the rest 4 to be their nominal values. We present the tables for 3 samples with their mean and calculate the convergence factor for first sample. Histogram plots for 10,000 samples are also shown for each calculation. In part I.II we take all 5 input parameters to be random simultaneously. Both Gaussian and uniform distribution is considered for the case of Part I.

1.1 Part I.I

In this section we start with the algorithm of Crude Monte Carlo algorithm which is used to calculate the output parameters. Then we present the tables of obtained results and histograms of Force 1 and Force 2. At last, we end the section with answer to all the questions posed in the project.

1.1.1 Algorithm

Crude Monte Carlo is the method used to calculate the output for random input parameters. The algorithm of the method is as shown: Here j is the number of random input parameters and R is the number of samples generated for each random input parameter.

Crude Monte Carlo algorithm can be summarized in three major steps, which is generating

```

start
for  $param \leftarrow 1$  to  $j$  by 1 do
  for  $samples \leftarrow 1$  to  $R$  by 1 do
    | Generate a sample from required distribution
  end
end
end
Calculate the random samples in the model to obtain output  $y$ 
Calculation of mean and variance of  $y$  for post-processing
end

```

random samples from the required distribution for the uncertain input parameters. Then calculating the output by using the physical model which is provided. Then for the post processing of the solution or the analysis of the solution, its mean, standard deviation etc. could be studied.

1.1.2 Results

In this section we present the tables and histogram plots for the analysis of the method. Three Samples are taken for each case. Each table includes a caption, which means only that parameter is taken to be random in that case and rest 4 have their nominal values. The caption also entails if the distribution taken is normal or uniform. Histogram plots are shown for $R=10000$.

| R | S1 | S2 | S3 | sigma/R |
|-----------|-----------|-----------|-----------|---------|
| 10 | 145.47228 | 151.23681 | 147.55152 | 2.17419 |
| 100 | 146.73397 | 148.59867 | 147.01965 | 0.74390 |
| 1000 | 147.89310 | 147.68017 | 147.43228 | 0.26031 |
| 10,000 | 147.47139 | 147.49201 | 147.47244 | 0.07969 |
| 1,00,000 | 147.45406 | 147.44603 | 147.44131 | 0.02517 |
| 10,00,000 | 147.42910 | 147.43368 | 147.43796 | 0.00756 |
| 50,00,000 | 147.4316 | 147.43480 | 147.43517 | 0.00356 |

Table 1.1: RV - e; Uniform Distribution; Force 1

| R | S1 | S2 | S3 | sigma/R |
|-----------|----------|----------|---------|---------|
| 10 | 0.96417 | 1.91396 | 1.27325 | 0.27242 |
| 100 | 1.043177 | 1.00587 | 1.11226 | 0.09865 |
| 1000 | 1.22213 | 1.14342 | 1.10148 | 0.03429 |
| 10,000 | 1.15112 | 1.16686 | 1.15573 | 0.01069 |
| 1,00,000 | 1.14974 | 1.15100 | 1.15583 | 0.00341 |
| 10,00,000 | 1.15180 | 1.149322 | 1.15175 | 0.00107 |
| 50,00,000 | 1.15120 | 1.5156 | 1.15143 | 0.00048 |

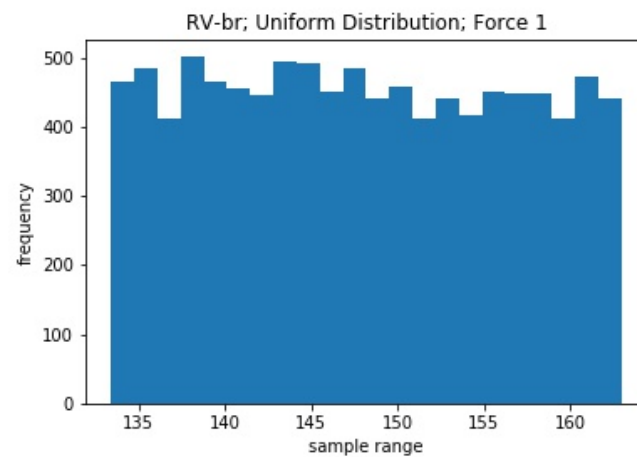
Table 1.2: RV - e; Uniform Distribution; Force 2

| R | S1 | S2 | S3 | sigma/R |
|-----------|-----------|-----------|------------|---------|
| 10 | 145.18426 | 145.55145 | 147.33387 | 2.73762 |
| 100 | 147.60098 | 147.19140 | 147.77953 | 0.85740 |
| 1000 | 147.66448 | 147.60475 | 148.126120 | 0.27413 |
| 10,000 | 148.05517 | 148.0153 | 147.91236 | 0.08573 |
| 1,00,000 | 147.94739 | 147.94933 | 147.91914 | 0.02698 |
| 10,00,000 | 147.93912 | 147.9414 | 147.92255 | 0.00853 |
| 50,00,000 | 147.93155 | 147.92565 | 147.92397 | 0.00381 |

Table 1.3: RV - br; Uniform Distribution; Force 1

| R | S1 | S2 | S3 | sigma/R |
|-----------|---------|---------|----------|---------|
| 10 | 0.14256 | 0.14916 | 0.17814 | 0.22401 |
| 100 | 0.13515 | 0.10507 | 0.12475 | 0.01150 |
| 1000 | 0.11794 | 0.12011 | 0.012696 | 0.00340 |
| 10,000 | 0.12457 | 0.12436 | 0.00110 | 0.12263 |
| 1,00,000 | 0.12288 | 0.12327 | 0.12336 | 0.00034 |
| 10,00,000 | 0.12326 | 0.12325 | 0.12333 | 0.00011 |
| 50,00,000 | 0.12334 | 0.12343 | 0.12337 | 4.9e-5 |

Table 1.4: RV - br; Uniform Distribution; Force 2



Uniform Distribution; Force 1.jpg

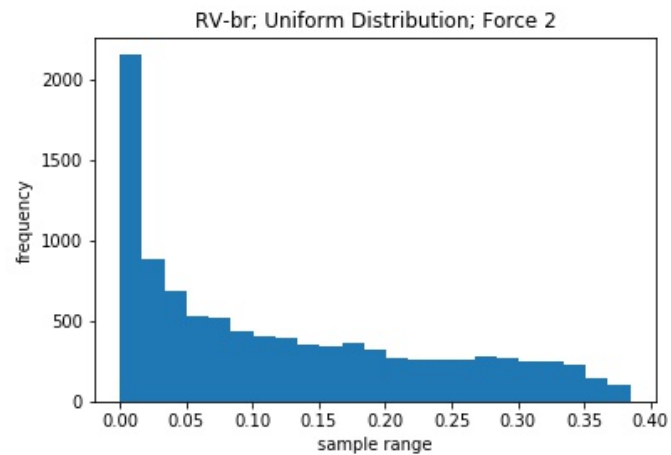
Figure 1.1: RV-br; Uniform Distribution; Force 1

| R | S1 | S2 | S3 | sigma/R |
|-----------|-----------|-----------|-----------|---------|
| 10 | 151.51356 | 133.01096 | 141.68261 | 6.4e-06 |
| 100 | 149.97327 | 149.06359 | 153.85526 | 2.3e-06 |
| 1000 | 149.87101 | 150.81055 | 150.74356 | 7.3e-07 |
| 10,000 | 150.50391 | 149.84744 | 149.47618 | 0.21620 |
| 1,00,000 | 150.04085 | 150.02993 | 150.13318 | 0.06780 |
| 10,00,000 | 150.04450 | 150.03806 | 150.06047 | 0.02146 |
| 50,00,000 | 150.09024 | 150.07137 | 150.06776 | 0.00960 |

Table 1.5: RV - ep; Uniform Distribution; Force 1

| R | S1 | S2 | S3 | sigma/R |
|-----------|---------|---------|---------|----------|
| 10 | 0.00016 | 0.00014 | 0.00015 | 6.4e-06 |
| 100 | 0.00016 | 0.00015 | 0.00016 | 2.3e-06 |
| 1000 | 0.00016 | 0.00016 | 0.00016 | 7.3e-07 |
| 10,000 | 0.00016 | 0.00016 | 0.00016 | 2.3e-07 |
| 1,00,000 | 0.00016 | 0.00016 | 0.00016 | 7.2e-08 |
| 10,00,000 | 0.00016 | 0.00016 | 0.00016 | 2.3e-08 |
| 50,00,000 | 0.00016 | 0.00016 | 0.00016 | 1.03e-08 |

Table 1.6: RV - ep; Uniform Distribution; Force 2



Uniform Distribution; Force 2.jpg

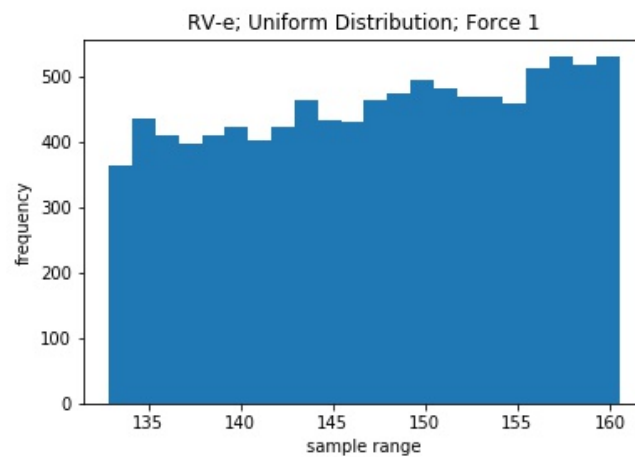
Figure 1.2: RV-br; Uniform Distribution; Force 2

| R | S1 | S2 | S3 | sigma/R |
|-----------|-----------|-----------|-----------|---------|
| 10 | 147.79844 | 147.80413 | 147.97000 | 0.13644 |
| 100 | 147.79795 | 147.79928 | 147.80432 | 0.00514 |
| 1000 | 147.80081 | 147.80129 | 147.80479 | 0.00181 |
| 10,000 | 147.80235 | 147.80225 | 147.80179 | 0.00057 |
| 1,00,000 | 147.80270 | 147.80305 | 147.80264 | 0.00018 |
| 10,00,000 | 147.80266 | 147.80276 | 147.80256 | 5.7e-05 |
| 50,00,000 | 147.80277 | 147.80280 | 147.80281 | 2.5e-05 |

Table 1.7: RV - haim; Uniform Distribution; Force 1

| R | S1 | S2 | S3 | sigma/R |
|-----------|---------|---------|---------|---------|
| 10 | 0.00021 | 0.00024 | 0.00038 | 4.2e-05 |
| 100 | 0.00024 | 0.00024 | 0.00022 | 2.0e-05 |
| 1000 | 0.00023 | 0.00022 | 0.00021 | 6.5e-06 |
| 10,000 | 0.00022 | 0.00022 | 0.00022 | 2.0e-06 |
| 1,00,000 | 0.00022 | 0.00022 | 0.00022 | 6.5e-07 |
| 10,00,000 | 0.00022 | 0.00022 | 0.00022 | 2.0e-07 |
| 50,00,000 | 0.00022 | 0.00022 | 0.00022 | 9.2e-08 |

Table 1.8: RV - haim; Uniform Distribution; Force 2



Uniform Distribution; Force 1.jpg

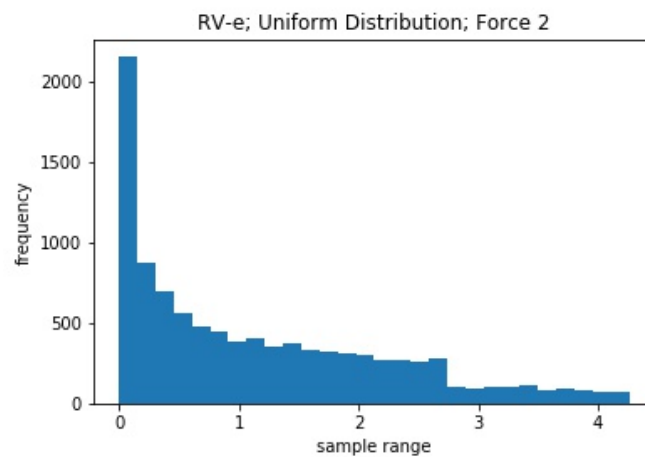
Figure 1.3: RV-e; Uniform Distribution; Force 1

| R | S1 | S2 | S3 | sigma/R |
|-----------|---------|---------|---------|---------|
| 10 | 064506 | 0.43102 | 0.33597 | 0.31095 |
| 100 | 0.49882 | 0.52190 | 0.50459 | 0.04512 |
| 1000 | 0.49753 | 0.57332 | 0.50359 | 0.01397 |
| 10000 | 0.49069 | 0.49379 | 0.49120 | 0.0044 |
| 1,00,000 | 0.49377 | 0.49452 | 0.49457 | 0.00140 |
| 10,00,000 | 0.49398 | 0.49371 | 0.49377 | 0.00044 |
| 50,00,000 | 0.49420 | 0.49372 | 0.49403 | 0.00019 |

Table 1.9: RV - current; Uniform Distribution; Force 2

| R | S1 | S2 | S3 | sigma/R |
|-----------|-----------|-----------|-----------|---------|
| 10 | 147.77827 | 147.72617 | 147.87282 | 0.09817 |
| 100 | 147.79063 | 147.81671 | 147.81825 | 0.03101 |
| 1000 | 147.80976 | 147.80493 | 147.83023 | 0.00883 |
| 10,000 | 147.80691 | 147.80495 | 147.80501 | 0.00296 |
| 1,00,000 | 147.80531 | 147.80487 | 147.80422 | 0.00093 |
| 10,00,000 | 147.80425 | 147.8043 | 147.80453 | 0.00029 |
| 50,00,000 | 147.80451 | 147.80467 | 147.80459 | 0.00013 |

Table 1.10: RV - br; Normal Distribution; Force 1



Uniform Distribution; Force 2.jpg

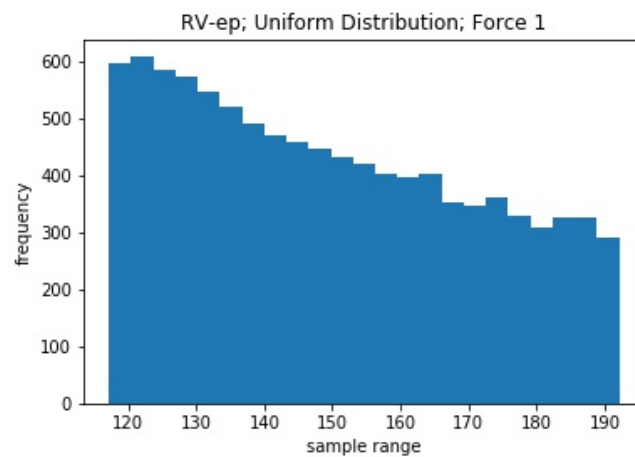
Figure 1.4: RV-e; Uniform Distribution; Force 2

| R | S1 | S2 | S3 | sigma/R |
|---------|---------|---------|---------|----------|
| 10 | 0.00035 | 0.00039 | 0.00037 | 0.00011 |
| 100 | 0.00033 | 0.00029 | 0.00027 | 3.90e-05 |
| 1000 | 0.00028 | 0.00029 | 0.00027 | 1.08e-05 |
| 10000 | 0.0003 | 0.0003 | 0.0003 | 3.60e-06 |
| 100000 | 0.0003 | 0.0003 | 0.0003 | 1.10e-06 |
| 1000000 | 0.0003 | 0.0003 | 0.0003 | 3.70e-07 |
| 5000000 | 0.0003 | 0.0003 | 0.0003 | 1.60e-07 |

Table 1.11: RV - br; Normal Distribution; Force 2

| R | S1 | S2 | S3 | sigma/R |
|---------|-----------|-----------|-----------|----------|
| 10 | 147.80432 | 147.80442 | 147.80456 | 9.80e-05 |
| 100 | 147.80432 | 147.80432 | 147.80432 | 3.10e-05 |
| 1000 | 147.80431 | 147.80442 | 147.80487 | 1.00e-05 |
| 10000 | 147.80442 | 147.80441 | 147.80443 | 3.40e-06 |
| 100000 | 147.80442 | 147.80442 | 147.80442 | 1.00e-06 |
| 1000000 | 147.80442 | 147.80442 | 147.80442 | 3.40e-07 |
| 5000000 | 147.80442 | 147.80442 | 147.80442 | 1.50e-07 |

Table 1.12: RV - e; Normal Distribution; Force 1



Uniform Distribution; Force 1.jpg

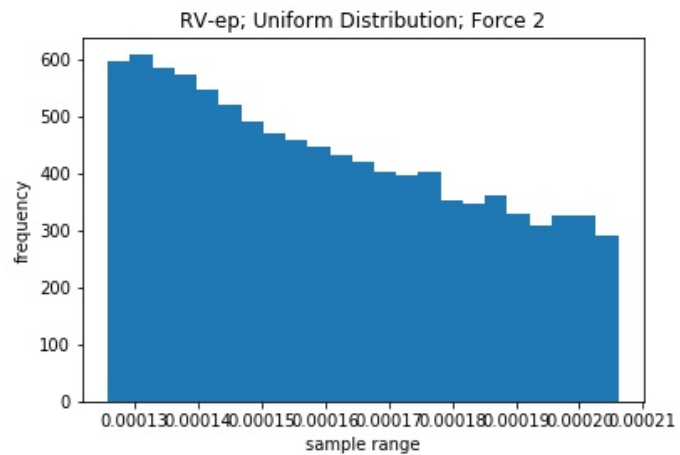
Figure 1.5: RV-ep; Uniform Distribution; Force 1

| R | S1 | S2 | S3 | sigma/R |
|---------|---------|---------|---------|----------|
| 10 | 0.00015 | 0.00015 | 0.00015 | 3.30e-07 |
| 100 | 0.00015 | 0.00015 | 0.00015 | 1.00e-07 |
| 1000 | 0.00015 | 0.00015 | 0.00015 | 3.50e-08 |
| 10000 | 0.00015 | 0.00015 | 0.00015 | 1.10e-08 |
| 100000 | 0.00015 | 0.00015 | 0.00015 | 3.60e-09 |
| 1000000 | 0.00015 | 0.00015 | 0.00015 | 1.10e-09 |
| 5000000 | 0.00015 | 0.00015 | 0.00015 | 5.17e-10 |

Table 1.13: RV - e; Normal Distribution; Force 2

| R | S1 | S2 | S3 | sigma/R |
|---------|-----------|-----------|-----------|----------|
| 10 | 147.80446 | 147.80451 | 147.80467 | 6.20e-05 |
| 100 | 147.80441 | 147.8044 | 147.80443 | 2.60e-05 |
| 1000 | 147.80442 | 147.80442 | 147.80442 | 7.60e-06 |
| 10000 | 147.80442 | 147.80442 | 147.80442 | 2.40e-06 |
| 100000 | 147.80442 | 147.80442 | 147.80442 | 7.60e-07 |
| 1000000 | 147.80442 | 147.80442 | 147.80442 | 2.40e-07 |
| 5000000 | 147.80442 | 147.80442 | 147.80442 | 1.00e-07 |

Table 1.14: RV - ep; Normal Distribution; Force 1



Uniform Distribution; Force 2.jpg

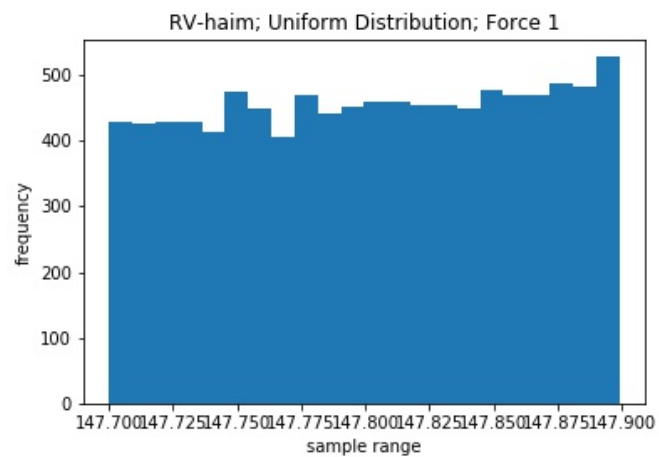
Figure 1.6: RV-ep; Uniform Distribution; Force 2

| R | S1 | S2 | S3 | sigma/R |
|---------|---------|---------|---------|----------|
| 10 | 0.00015 | 0.00015 | 0.00015 | 6.70e-11 |
| 100 | 0.00015 | 0.00015 | 0.00015 | 2.80e-11 |
| 1000 | 0.00015 | 0.00015 | 0.00015 | 8.20e-12 |
| 10000 | 0.00015 | 0.00015 | 0.00015 | 2.60e-12 |
| 100000 | 0.00015 | 0.00015 | 0.00015 | 8.19e-13 |
| 1000000 | 0.00015 | 0.00015 | 0.00015 | 2.59e-13 |
| 5000000 | 0.00015 | 0.00015 | 0.00015 | 1.16e-13 |

Table 1.15: RV - ep; Normal Distribution; Force 2

| R | S1 | S2 | S3 | sigma/R |
|---------|-----------|-----------|-----------|----------|
| 10 | 147.80446 | 147.80451 | 147.80467 | 5.16e-05 |
| 100 | 147.80441 | 147.8044 | 147.80443 | 1.70e-06 |
| 1000 | 147.80442 | 147.80442 | 147.80442 | 5.16e-07 |
| 10000 | 147.80442 | 147.80442 | 147.80442 | 1.60e-07 |
| 100000 | 147.80442 | 147.80442 | 147.80442 | 5.20e-08 |
| 1000000 | 147.80442 | 147.80442 | 147.80442 | 1.60e-08 |
| 5000000 | 147.80442 | 147.80442 | 147.80442 | 7.30e-09 |

Table 1.16: RV - haim; Normal Distribution; Force 1



Uniform Distribution; Force 1.jpg

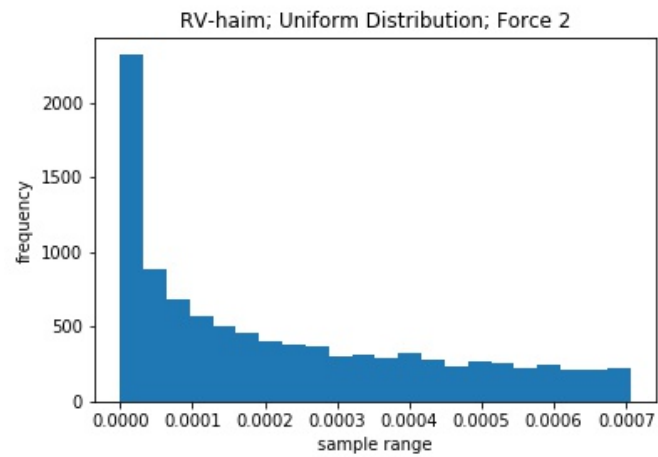
Figure 1.7: RV-haim; Uniform Distribution; Force 1

| R | S1 | S2 | S3 | sigma/R |
|---------|---------|---------|---------|----------|
| 10 | 0.00015 | 0.00015 | 0.00015 | 1.70e-08 |
| 100 | 0.00015 | 0.00015 | 0.00015 | 5.80e-09 |
| 1000 | 0.00015 | 0.00015 | 0.00015 | 1.70e-09 |
| 10000 | 0.00015 | 0.00015 | 0.00015 | 5.50e-10 |
| 100000 | 0.00015 | 0.00015 | 0.00015 | 1.70e-10 |
| 1000000 | 0.00015 | 0.00015 | 0.00015 | 5.50e-11 |
| 5000000 | 0.00015 | 0.00015 | 0.00015 | 2.40e-11 |

Table 1.17: RV - haim; Normal Distribution; Force 2

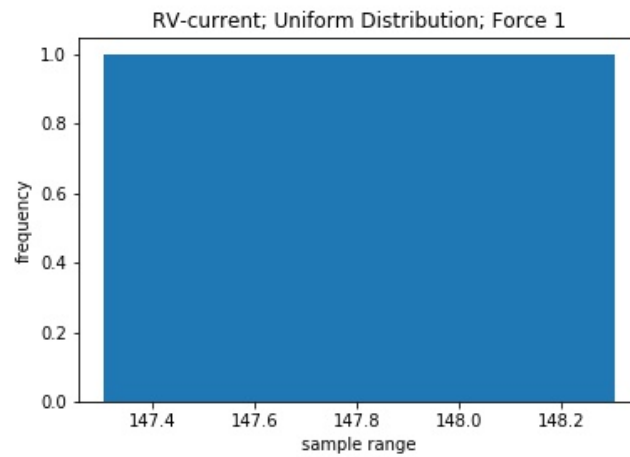
| R | S1 | S2 | S3 | sigma/R |
|---------|---------|---------|---------|----------|
| 10 | 0.00015 | 0.00015 | 0.00015 | 1.70e-07 |
| 100 | 0.00015 | 0.00015 | 0.00015 | 3.90e-08 |
| 1000 | 0.00015 | 0.00015 | 0.00015 | 1.20e-08 |
| 10000 | 0.00015 | 0.00015 | 0.00015 | 3.80e-09 |
| 100000 | 0.00015 | 0.00015 | 0.00015 | 1.20e-09 |
| 1000000 | 0.00015 | 0.00015 | 0.00015 | 3.90e-10 |
| 5000000 | 0.00015 | 0.00015 | 0.00015 | 1.70e-10 |

Table 1.18: RV - current; Normal Distribution; Force 2



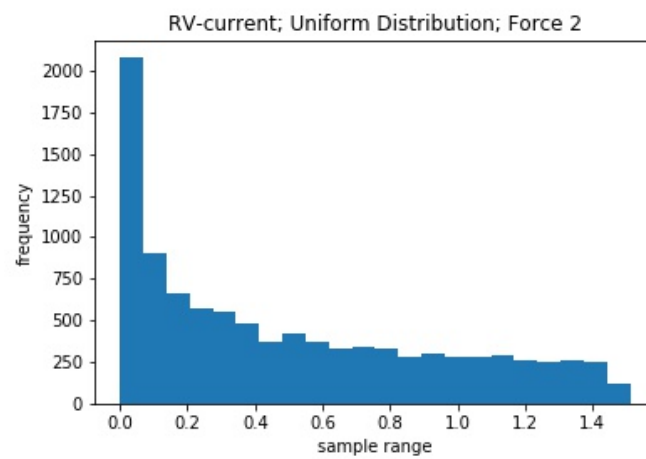
Uniform Distribution; Force 2.jpg

Figure 1.8: RV-haim; Uniform Distribution; Force 2



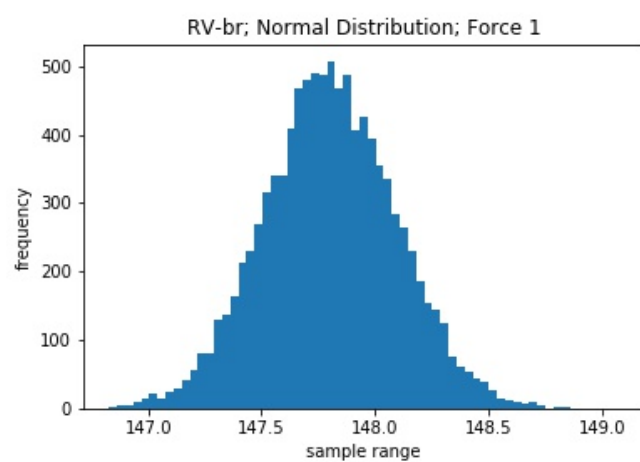
Uniform Distribution; Force 1.jpg

Figure 1.9: RV-current; Uniform Distribution; Force 1



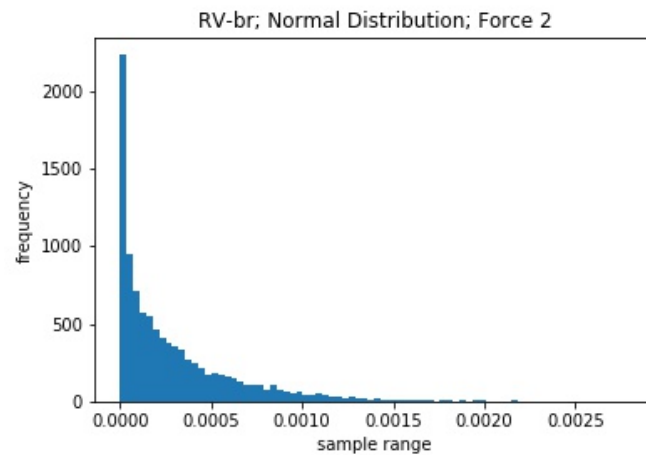
Uniform Distribution; Force 2.jpg

Figure 1.10: RV-current; Uniform Distribution; Force 2



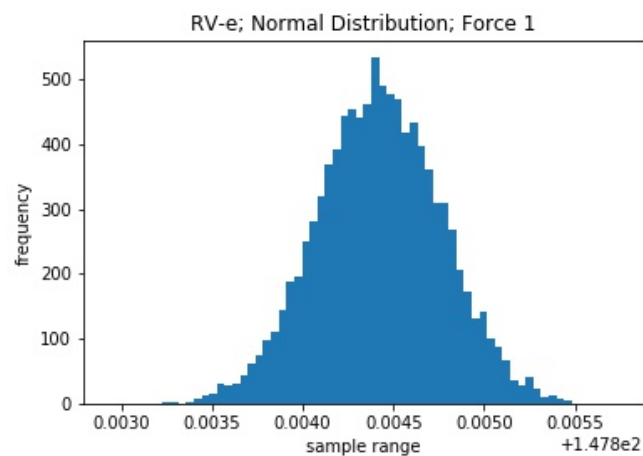
Normal Distribution; Force 1.jpg

Figure 1.11: RV-br; Normal Distribution; Force 1



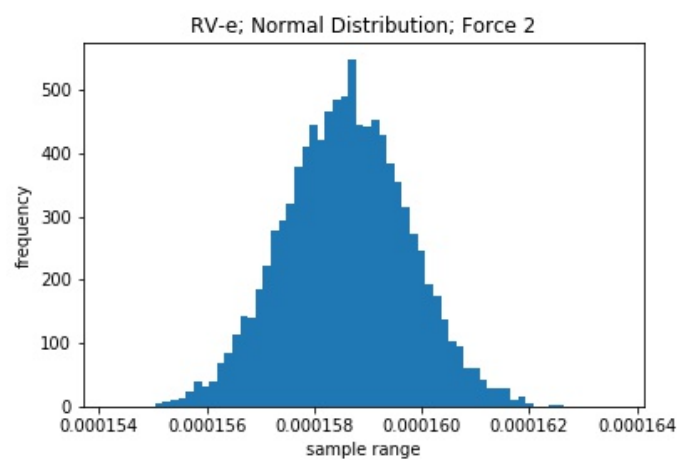
Normal Distribution; Force 2.jpg

Figure 1.12: RV-br; Normal Distribution; Force 2



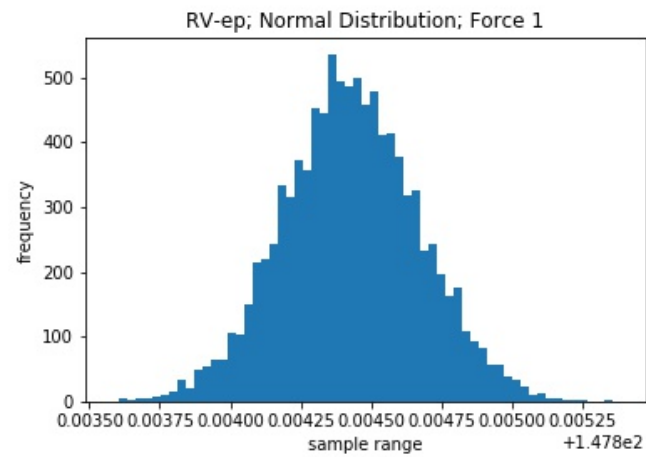
Normal Distribution; Force 1.jpg

Figure 1.13: RV-e; Normal Distribution; Force 1



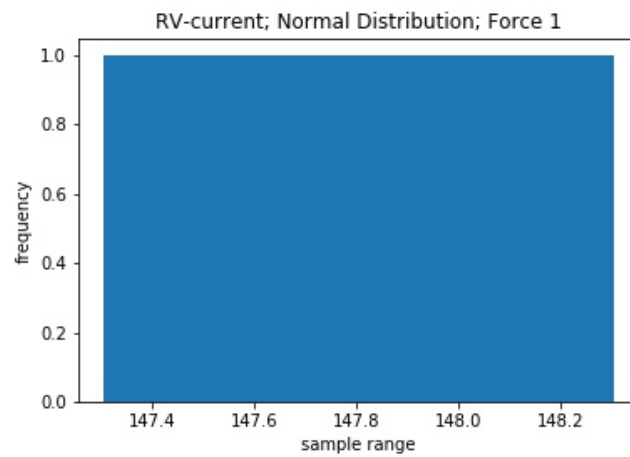
Normal Distribution; Force 2.jpg

Figure 1.14: RV-e; Normal Distribution; Force 2



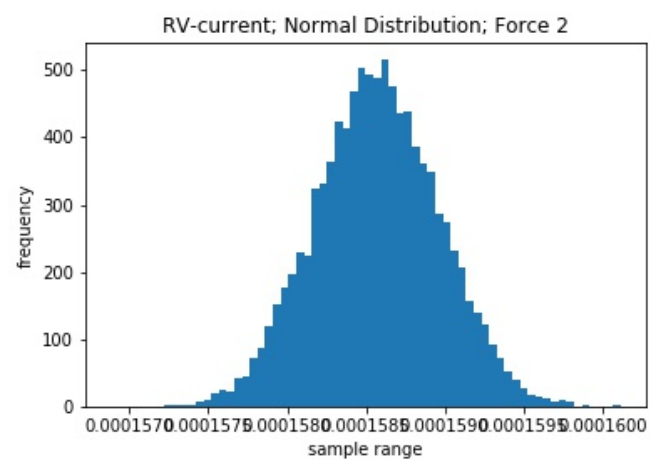
Normal Distribution; Force 1.jpg

Figure 1.15: RV-ep; Normal Distribution; Force 1



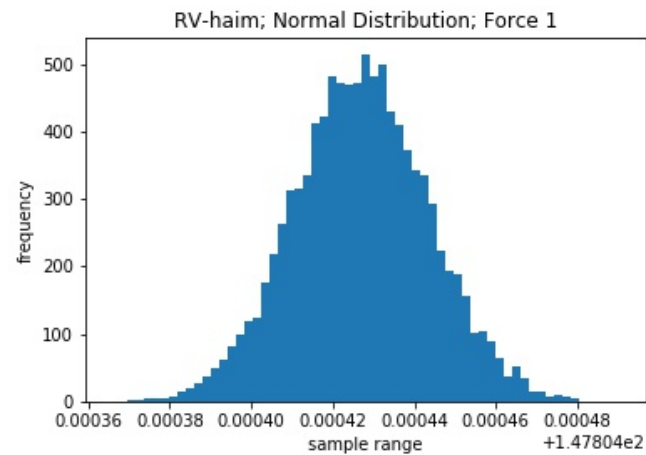
Normal Distribution; Force 1.jpg

Figure 1.16: RV-current; Normal Distribution; Force 1



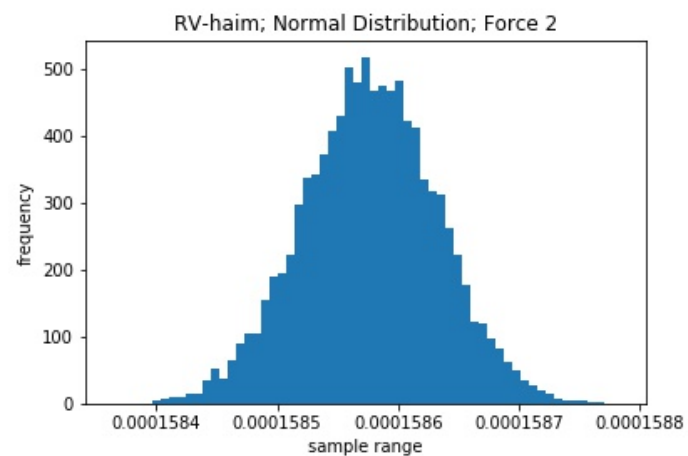
Normal Distribution; Force 2.jpg

Figure 1.17: RV-current; Normal Distribution; Force 2



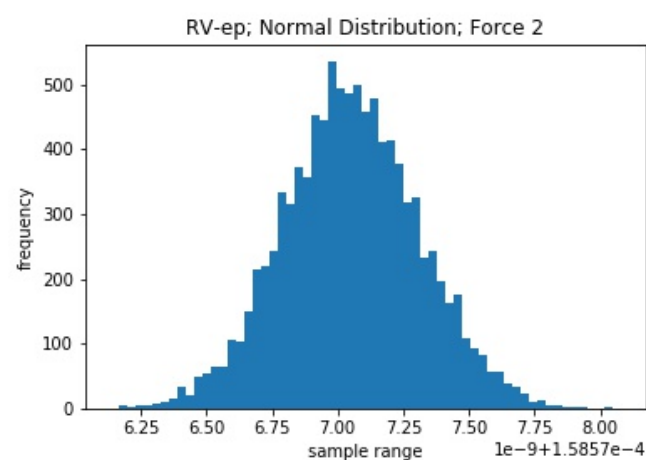
Normal Distribution; Force 1.jpg

Figure 1.18: RV-haim; Normal Distribution; Force 1



Normal Distribution; Force 2.jpg

Figure 1.19: RV-haim; Normal Distribution; Force 2



Normal Distribution; Force 2.jpg

Figure 1.20: RV-ep; Normal Distribution; Force 2

1.1.3 Is there an Influence on the PDF

Yes, there is an influence of the PDF choosen on the outputs. The influence is that normal distribution achieves the same answer on very small number of samples, which uniforml distribution achieves for a very large number of samples. This also confirms the central limit theorem.

1.1.4 Which value seems to be most influent on force 1 and force 2

The RV ep seems most influent on force 1 and the RV e seems most influent on force 2. We reached on this inference by observing the difference between the saturation mean values obtained by considering each RV alone and the actual values of force 1 and force 2 of no randomness would have been there.

1.2 Part I.II

In this part we show the results obtained when all the input parameters are taken to be random.

We show 4 tables showing the results and corresponding histogram plots of the forces.

| R | S1 | S2 | S3 | sigma/R |
|---------|-----------|-----------|-----------|---------|
| 10 | 151.99065 | 160.32926 | 140.91721 | 7.36256 |
| 100 | 150.35774 | 151.69125 | 150.83038 | 2.43098 |
| 1000 | 150.70704 | 149.86041 | 150.0672 | 0.75865 |
| 10000 | 149.52385 | 149.79439 | 150.03615 | 0.24487 |
| 100000 | 149.92523 | 149.83358 | 149.93442 | 0.07736 |
| 1000000 | 149.80654 | 149.84531 | 149.81789 | 0.02443 |
| 5000000 | 149.83306 | 149.82605 | 149.83545 | 0.01093 |

Table 1.19: RV - All 5; Uniform Distribution; Force 1

| R | S1 | S2 | S3 | sigma/R |
|---------|---------|---------|---------|---------|
| 10 | 1.0232 | 1.27794 | 1.57816 | 0.39844 |
| 100 | 1.82531 | 1.89108 | 1.65273 | 0.21315 |
| 1000 | 1.78409 | 1.97474 | 1.72091 | 0.06984 |
| 10000 | 1.77449 | 1.80182 | 1.80257 | 0.02177 |
| 100000 | 1.80677 | 1.79424 | 1.80913 | 0.00702 |
| 1000000 | 1.80056 | 1.80678 | 1.80138 | 0.00221 |
| 5000000 | 1.80093 | 1.80228 | 1.80312 | 0.00099 |

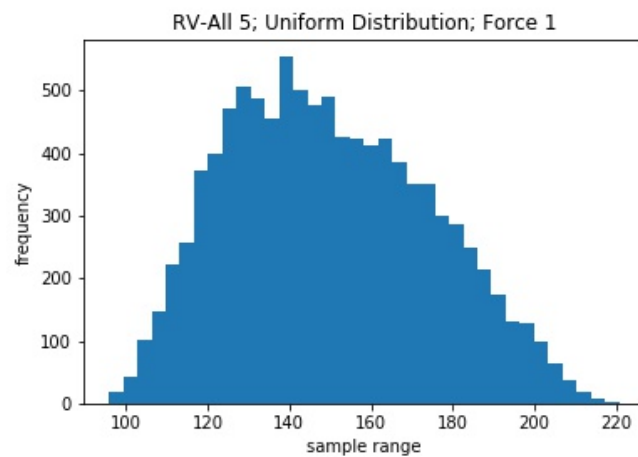
Table 1.20: RV - All 5; Uniform Distribution; Force 2

| R | S1 | S2 | S3 | sigma/R |
|---------|-----------|-----------|-----------|---------|
| 10 | 147.71888 | 147.72987 | 147.83896 | 0.07656 |
| 100 | 147.73095 | 147.85299 | 147.82279 | 0.0285 |
| 1000 | 147.79485 | 147.80893 | 147.8025 | 0.00952 |
| 10000 | 147.80043 | 147.80412 | 147.80718 | 0.003 |
| 100000 | 147.80289 | 147.8065 | 147.80638 | 0.00093 |
| 1000000 | 147.80473 | 147.80499 | 147.80469 | 0.00029 |
| 5000000 | 147.8043 | 147.80453 | 147.80476 | 0.00013 |

Table 1.21: RV - All 5; Normal Distribution; Force 1

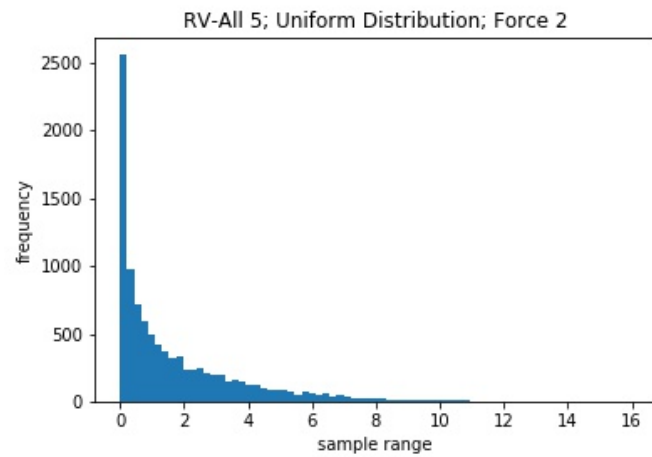
| R | S1 | S2 | S3 | sigma/R |
|---------|---------|---------|---------|----------|
| 10 | 0.00035 | 0.0002 | 0.00027 | 0.00013 |
| 100 | 0.00038 | 0.00026 | 0.00028 | 3.90E-05 |
| 1000 | 0.00032 | 0.00029 | 0.0003 | 1.20E-05 |
| 10000 | 0.00031 | 0.0003 | 0.0003 | 3.80E-06 |
| 100000 | 0.0003 | 0.0003 | 0.0003 | 1.10E-06 |
| 1000000 | 0.0003 | 0.0003 | 0.0003 | 3.70E-07 |
| 5000000 | 0.0003 | 0.0003 | 0.0003 | 1.60E-07 |

Table 1.22: RV - All 5; Normal Distribution; Force 2



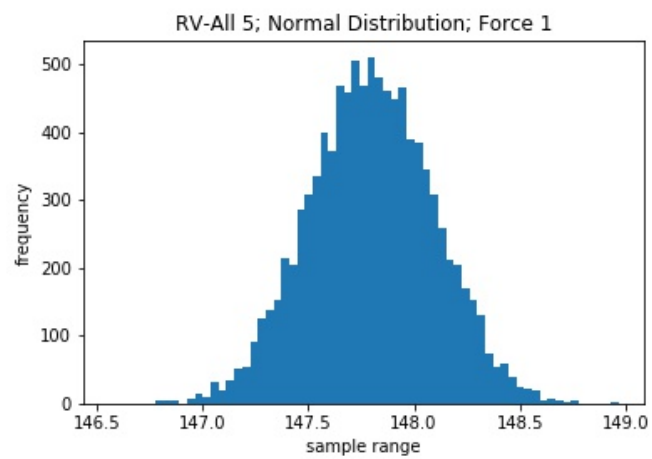
5; Uniform Distribution; Force 1.jpg

Figure 1.21: RV-All 5; Uniform Distribution; Force 1



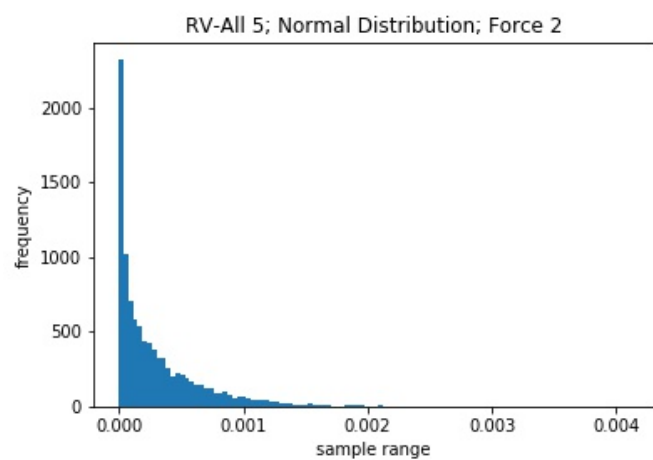
5; Uniform Distribution; Force 2.jpg

Figure 1.22: RV-All 5; Uniform Distribution; Force 2



5; Normal Distribution; Force 1.jpg

Figure 1.23: RV-All 5; Normal Distribution; Force 1



5; Normal Distribution; Force 2.jpg

Figure 1.24: RV-All 5; Normal Distribution; Force 2

1.2.1 Comparison with case 1

The comparison with case 1 clearly shows that the uncertainty in the final solution is lot more in case II than in case I. The values obtained in case I are very close to the deterministic case as compared to the case II.

1.2.2 Comment on convergence speed when increasing RV from 1 to 5

The convergence speed has no effect because of the increase in number of random variables. This is a very special property of the Crude Monte Carlo method, that its convergence is not affected whether we are considering thousands of RV or only one RV. However, the convergence is pretty slow. Due to which interest to learn other methods inspired from Monte Carlo are studied.

Chapter 2

Part 2

In this part we have to calculate the probabilities that F1 is lower than 120; F2 is greater than 10 and both.

We will consider three cases. One taking only ep as RV and taking nominal values of other 4 RVs. ep is taken because it is most influential on force 1 in part 1. Table 2.1 to 2.3 shows the results for this case with three samples.

Secondly, we take only e as RV and other 4 as nominal values. we have taken e because it is most influential for force 2 from previous part. This is illustrated in table 2.4 to 2.6.

Lastly, we consider all input random variables (5), Last three tables have answer to this case.

Algorithm to calculate probability is as follows:

```
start
N = 0
for  $i \leftarrow 1$  to  $R$  by 1 do
  if Force condition satisfied then
    | N = N + 1;
  end
end
prob = N/R
end
```


| R | S1 | S2 | S3 | $\sqrt{(p)(1-p)/R}$ |
|---------|---------|---------|---------|---------------------|
| 10 | 0 | 0 | 0 | 0 |
| 100 | 0.09 | 0.09 | 0.07 | 0.02861 |
| 1000 | 0.058 | 0.062 | 0.061 | 0.00739 |
| 10000 | 0.0526 | 0.0524 | 0.0541 | 0.00223 |
| 100000 | 0.05474 | 0.05367 | 0.05282 | 0.00071 |
| 1000000 | 0.05314 | 0.05347 | 0.05321 | 0.00022 |
| 5000000 | 0.05326 | 0.05324 | 0.05323 | 0.0001 |

Table 2.1: ep as random force 1

| R | S1 | S2 | S3 | $\sqrt{(p)(1-p)/R}$ |
|---------|----|----|----|---------------------|
| 10 | 0 | 0 | 0 | 0 |
| 100 | 0 | 0 | 0 | 0 |
| 1000 | 0 | 0 | 0 | 0 |
| 10000 | 0 | 0 | 0 | 0 |
| 100000 | 0 | 0 | 0 | 0 |
| 1000000 | 0 | 0 | 0 | 0 |
| 5000000 | 0 | 0 | 0 | 0 |

Table 2.2: ep as random force 2

| R | S1 | S2 | S3 | $\sqrt{(p)(1-p)/R}$ |
|---------|---------|---------|---------|---------------------|
| 10 | 0 | 0 | 0 | 0 |
| 100 | 0.09 | 0.09 | 0.07 | 0.02861 |
| 1000 | 0.058 | 0.062 | 0.061 | 0.00739 |
| 10000 | 0.0526 | 0.0524 | 0.0541 | 0.00223 |
| 100000 | 0.05474 | 0.05367 | 0.05282 | 0.00071 |
| 1000000 | 0.05314 | 0.05347 | 0.05321 | 0.00022 |
| 5000000 | 0.05326 | 0.05324 | 0.05323 | 0.0001 |

Table 2.3: ep as random force 1 or force2

| R | S1 | S2 | S3 | $\sqrt{(p)(1-p)/R}$ |
|---------|----|----|----|---------------------|
| 10 | 0 | 0 | 0 | 0 |
| 100 | 0 | 0 | 0 | 0 |
| 1000 | 0 | 0 | 0 | 0 |
| 10000 | 0 | 0 | 0 | 0 |
| 100000 | 0 | 0 | 0 | 0 |
| 1000000 | 0 | 0 | 0 | 0 |
| 5000000 | 0 | 0 | 0 | 0 |

Table 2.4: e as random force 1

| R | S1 | S2 | S3 | $\sqrt{(p)(1-p)/R}$ |
|---------|----|----|----|---------------------|
| 10 | 0 | 0 | 0 | 0 |
| 100 | 0 | 0 | 0 | 0 |
| 1000 | 0 | 0 | 0 | 0 |
| 10000 | 0 | 0 | 0 | 0 |
| 100000 | 0 | 0 | 0 | 0 |
| 1000000 | 0 | 0 | 0 | 0 |
| 5000000 | 0 | 0 | 0 | 0 |

Table 2.5: e as random force 2

| R | S1 | S2 | S3 | $\sqrt{(p)(1-p)/R}$ |
|---------|----|----|----|---------------------|
| 10 | 0 | 0 | 0 | 0 |
| 100 | 0 | 0 | 0 | 0 |
| 1000 | 0 | 0 | 0 | 0 |
| 10000 | 0 | 0 | 0 | 0 |
| 100000 | 0 | 0 | 0 | 0 |
| 1000000 | 0 | 0 | 0 | 0 |
| 5000000 | 0 | 0 | 0 | 0 |

Table 2.6: e as random force 1 or force 2

| R | S1 | S2 | S3 | $\sqrt{(p)(1-p)/R}$ |
|---------|---------|---------|---------|---------------------|
| 10 | 0.5 | 0.1 | 0 | 0.15811 |
| 100 | 0.11 | 0.13 | 0.09 | 0.03128 |
| 1000 | 0.123 | 0.115 | 0.108 | 0.01038 |
| 10000 | 0.103 | 0.105 | 0.1065 | 0.00303 |
| 100000 | 0.11271 | 0.1104 | 0.1117 | 0.001 |
| 1000000 | 0.11109 | 0.11065 | 0.11116 | 0.00031 |
| 5000000 | 0.11115 | 0.11111 | 0.11111 | 0.00014 |

Table 2.7: all parameters are random force1

| R | S1 | S2 | S3 | $\sqrt{(p)(1-p)/R}$ |
|---------|---------|---------|---------|---------------------|
| 10 | 0.1 | 0 | 0 | 0.09486 |
| 100 | 0.02 | 0 | 0.01 | 0.014 |
| 1000 | 0.005 | 0.008 | 0.005 | 0.00223 |
| 10000 | 0.0091 | 0.0105 | 0.0093 | 0.00094 |
| 100000 | 0.00956 | 0.00929 | 0.0096 | 0.0003 |
| 1000000 | 0.00929 | 0.00922 | 0.00942 | 9.50E-05 |
| 5000000 | 0.00918 | 0.0092 | 0.00923 | 4.20E-05 |

Table 2.8: all parameters are random force2

| R | S1 | S2 | S3 | $\sqrt{(p)(1-p)/R}$ |
|---------|---------|---------|---------|---------------------|
| 10 | 0.5 | 0.1 | 0 | 0.15811 |
| 100 | 0.12 | 0.13 | 0.1 | 0.03249 |
| 1000 | 0.128 | 0.121 | 0.112 | 0.01564 |
| 10000 | 0.1112 | 0.1145 | 0.115 | 0.00314 |
| 100000 | 0.12107 | 0.11875 | 0.12018 | 0.00103 |
| 1000000 | 0.11937 | 0.11876 | 0.11948 | 0.00102 |
| 5000000 | 0.11955 | 0.1193 | 0.11931 | 0.00014 |

Table 2.9: all parameters are random force1 or force 2