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Chapter 1

Part IV

1.1 Introduction to Sensitivity Analysis

We have to solve an uncertainty quantification problem, in which we have 5 input random parameters. We have to calculate the forces, Force 1 and Force 2, which are the forces when no current is supplied and with current respectively. In part IV we have to use global sensitivity analysis to determine which input parameters are most sensitive, which means that we have to check which input parameters have the most effect on force 1 and force 2. It is not always the case that all input parameters contribute equally to the randomness of the output variable, hence we must be able to determine which factors we need to calculate experimentally more efficiently and which factors we could have some liberty to keep a larger uncertain domain. It is seen that even if some parameters have a large domain of uncertainty, they rarely affect the uncertainty in the output. It all depends on the model. Hence the motivation behind sensitivity analysis is to get to know that our output is most sensitive to which input parameters.

In this report we are going to make comments and analyse using the notion called sobol indices. Sobol indices are a method derived from the variational approach. There are two approaches for the sensitivity analysis, one local sensitivity analysis, which concerns the question of how much change does an output y observe when a small change in x is incurred. Mathematically this can be calculated by taking the derivative of the model equation with respect to the variable x. Another approach to estimate sensitivity is the global approach (which sobol indices belong to). This approach concerns the whole domain of X and Y, rather than taking infinitesimal small perturbations of x, hence the name is a global approach. Mathematically this approach amounts to calculating the variance of the marginal expectations and expressing them in the fractional form. The indices then give the idea about how they affect the solution. If the value of first order indices are high, the outputs are highly affected by the corresponding input and vice versa. In the following sections we present the tables for first order index and total index for force 1 and force 2.

1.2 Tables of Sobol Indices

sobol indice	Sobol indices value
S1	0.11909792642898476
S2	0.07032550093477034
S3	0.7604130045426518
S4	-0.020077258474247687
S5	-0.020146597873314517
S12	0.20894103811030096
S13	0.9074064516287328
S14	0.11917744216047083
S15	0.11909792642898476
S23	0.8536952686822606
S24	0.07034909173916144
S25	0.07032550093477034
S34	0.7604671316055293
S35	0.7604130045426518
S45	-0.020077258474247687
S123	0.9999779243339719
S124	0.20897422524272283
S125	0.20894103811030096
S134	0.90746943039802
S135	0.9074064516287328
S145	0.11917744216047083
S1234	0.9999999999999951
S1235	0.9999724055078734
S1245	0.20897422524272283
S1345	0.90746943039802
S2345	0.8537092271700978
S12345	0.9999999999999951
S234	0.8537092271700978
S235	0.8536952686822606
S245	0.07034909173916144
S345	0.7604671316055293

Table 1.1: Sobol indices force 1

sobol indice	Sobol indices value
S1	0.10102543716352279
S2	0.7898697556981854
S3	0.04454593664913678
S4	0.032440470287147506
S5	0.03248428179770827
S12	0.9571780762142496
S13	0.11286424721450727
S14	0.10087717988366342
S15	0.10102543716352279
S23	0.8278587629986576
S24	0.7897323683593103
S25	0.7898697556981854
S34	0.04455495948456025
S35	0.04454593664913678
S45	0.032440470287147506
S123	1.0001267303291963
S124	0.9569373671655377
S125	0.9571780762142496
S134	0.11276818307665322
S135	0.11286424721450727
S145	0.10087717988366342
S1234	1.0
S1235	1.0001267303291963
S1245	0.9569373671655377
S1345	0.11276818307665322
S2345	0.8278366668733165
S12345	1.0
S234	0.8658773767494344
S235	0.8646116741574911
S245	0.7897323683593103
S345	0.04455495948456025

Table 1.2: sobol indices force 2

1.3 Comments and explanation of the results

In this section we discuss how we obtained the results and what tables represents.

The first question was to calculate the sobol indices for our problem. Since, we had two required outputs, therefore each one will have its own sobol indices. The first table lists the sobol indices for force 1 and the second table represents the sobol indices for force 2. Here tables have some notations - In our notation S1 means sobol index corresponding to br, S2 means sobol index corresponding to e, S3 means sobol index corresponding to ep, S4 means sobol index

sobol indice	total Sobol indices value
t1	8.942088876606398
t2	8.531944552426562
t3	14.086276876722518
t4	7.8001385796835
t5	7.799779806630109

Table 1.3: Total sobol indices force 1

sobol indice	total Sobol indices value
t1	8.68355444209466
t2	14.299079455276905
t3	7.941110973252056
t4	7.730294390260378
t5	7.731100423195353

Table 1.4: Total sobol indices force 2

corresponding to haim, S5 means sobol index corresponsing to current, S12 means sobol index corresponding to br and e and so on. As we can see from the first table, force 1 is most sensitive to ep and as we can see from the second table force 2 is most sensitive to e.

Detailed calculation details of these indices has been provided in the interactive python file.

Moving on to Table 3 and Table 4, we can see that, the inferences drawn from table 1 and table 2 above satisfy the conclusions which can be drawn from table 3 that force 1 is most sensitive to ep and force 2 is most sensitive to e. More the value of total indices more is the effect of that random variable on the output. In our notation t1 denotes the sum total indices of br and so on.

Now we come to Table 5 and Table 6, in this our task was to change the uncertain intervals of the random variables so that all the values of force 1 and force 2 are close to each other. In Table 5 we have taken the uncertainty in br to be 5 percent, the uncertainty in e to be 5 percent,the uncertainty in ep to be 30 percent,the uncertainty in haim to be 35 percent, and the uncertainty in current to be 50 percent and obtained the first order sobol indices for force

sobol indice	Sobol indices value (Force1)	Sobol indices value (Force2)
S1	0.16072158567165	0.9317808627424699
S2	-0.10489395751299303	0.9205178804594448
S3	0.682825063054439	0.9023998373155044
S4	-0.13010202231275467	0.8816201968481407
S5	-0.13016174694218338	0.8797198374665435

Table 1.5: Making first order sobol indices as same as possible for force 1

sobol indice	Sobol indices value (Force1)	Sobol indices value (Force2)
S1	-0.19641782959519966	0.7557305443157277
S2	0.22177551030571058	0.9944109613434907
S3	0.4537310426434247	0.7539049748153849
S4	-0.256423685953017	0.7517640747220844
S5	-0.25876004563501526	0.7518554265164533

Table 1.6: Making first order sobol indices as same as possible for force 2

1 close to each other. In Table 6 we have taken the uncertainty in br to be 1 percent, the uncertainty in e to be 10 percent,the uncertainty in ep to be 10 percent,the uncertainty in haim to be 10 percent, and the uncertainty in current to be 40 percent and obtained the first order sobol indices for force 2 close to each other. These values we found by first reducing the most influential factors that is ep for force 1 and e for force 2 and then performing parameter tuning and many trials.

In Table 7, our task was to reduce the new standard deviation of the output to 5 percent of actual standard deviation of the output by altering the domain of uncertainty of random inputs. We first of all decreased the region of dependence of ep and e and as the last part performed parameter tuning to obtain the results. The results can be simulated in the python files attached.

R	ratio of force 1	ratio of force 2
10	0.04192456970077849	0.05938926401497986
100	0.04628657640105828	0.036729152296065284
1000	0.05197042563027714	0.04049974006208034
10,000	0.049481132673055636	0.04106476449039983
100,000	0.049414821589951065	0.04142528321272161

Table 1.7: Standard devaition in 5 percent for both force 1 and force 2