Klassische Physik 1 Hausaufgaben Blatt Nr. 0

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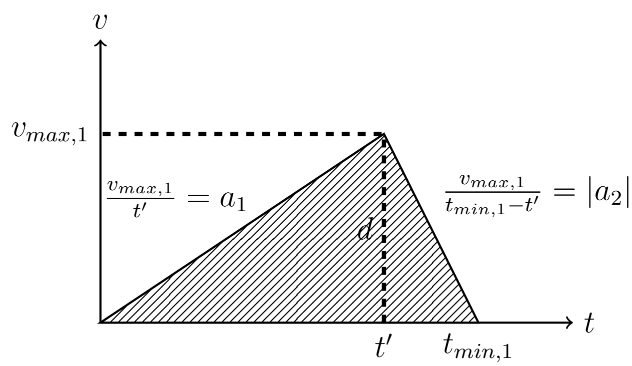
a. *Aufgabe 1.1*



(a) Man löst die Gleichungen

$$\frac{1}{2}(v_{max,1})(t_{min,1}) = d \tag{1}$$

$$v_{max,1} = a_1 t' \tag{2}$$

$$v_{max,1} = (t' - t_{min,1})a_2 \tag{3}$$

Aus (2) folgt $t' = v_{max,1}/a_1$. Wir setzen das in (3) ein. Es ergibt sich

$$v_{max,1} = \left(\frac{v_{max,1}}{a_1} - t_{min,1}\right) a_2.$$

Daraus folgt:

$$v_{max,1}\left(1 - \frac{a_2}{a_1}\right) = -t_{min,1}a_2.$$

(b) Noch einmal setzen wir das in (1) ein:

$$\frac{1}{2}\left[-t_{min,1}a_2\left(1 - \frac{a_2}{a_1}\right)^{-1}\right](t_{min,1}) = d.$$

Die Lösung ist

$$t_{min,1} = \boxed{\left[-\frac{2d}{a_2}\left(1 - \frac{a_2}{a_1}\right)\right]^{1/2}}.$$

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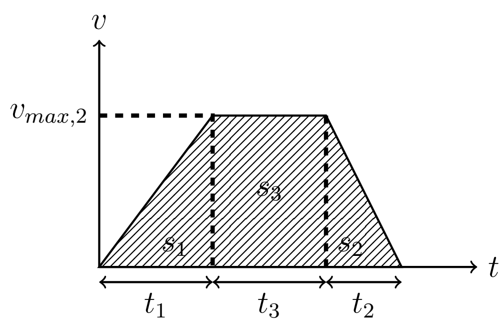
Aus (1) folgt

$$v_{max,1} = \frac{2d}{t_{mn,1}}.$$

Also

$$v_{max,1} = \boxed{\left[-\frac{1-\frac{a_2}{a_1}}{2a_2 d}\right]^{-1/2}}.$$

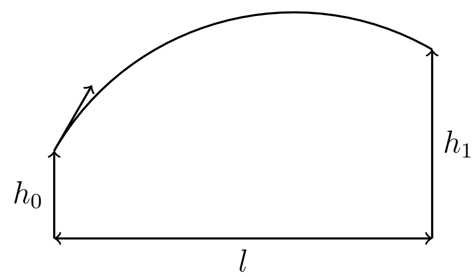


(c)

Es gilt

$$\begin{aligned}
t_1 =& \frac{v_{max,2}}{a_1} \\
t_2 =& -\frac{v_{max,2}}{a_2} \\
s_1 =& \frac{1}{2} a_1 t_1^2 = \frac{v_{max,2}^2}{2a_1} \\
s_2 =& \frac{1}{2} v_{max,2} t_2 = -\frac{v_{max,2}^2}{2a_2} \\
s_3 =& v_{max,2} t_3 = d - s_1 - s_2 \\
t_3 =& \frac{d - s_1 - s_2}{v_{max,2}} \\
=& \frac{d}{v_{max,2}} - \frac{v_{max,2}}{2a_1} + \frac{v_{max,2}}{2a_2} \\
t_{min,2} =& t_1 + t_2 + t_3 \\
=& \frac{d}{v_{max,2}} + \frac{v_{max,2}}{2a_1} - \frac{v_{max,2}}{2a_2}
\end{aligned}$$

b. Aufgabe 1.2



$$x = v_0 t \cos\theta$$
$$y = v_0 t \sin\theta - \frac{1}{2} g t^2$$
$$y = x \tan\theta - \frac{g x^2}{2 v_0^2 \cos^2\theta}$$

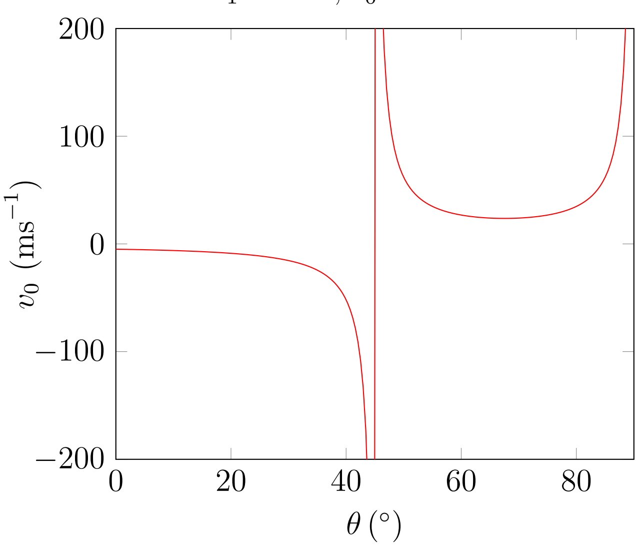
Wir brauchen $y(l) = h_1 - h_0$, oder

$$h_1 - h_0 = l \tan\theta - \frac{g l^2}{2 v_0^2 \cos^2\theta}.$$

Daraus folgt

$$v_0^2 = \frac{g l^2}{2\cos^2\theta\,(l\tan\theta - (h_1 - h_0))}.$$

$l = h_1 = 1$ m, $h_0 = 0$ m

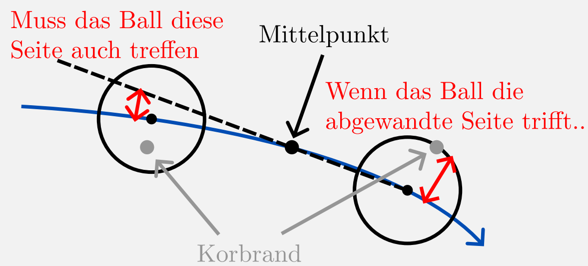


Es folgt daraus:

$$y = x\tan\theta - (l\tan\theta - (h_1 - h_0))\frac{x^2}{l^2}.$$

(b)

Bemerkung: Man muss wegen der Konvexität der Trajektorie nur das Fall betrachten, in dem der Ball nur die nähere Seite des Korbrands trifft.



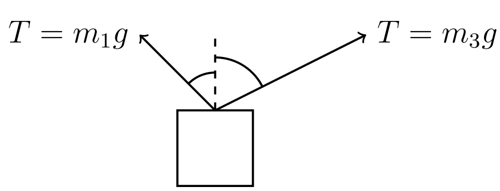
Die "linke" Seite des Korbrands hat Koordinaten bezüglich der Werfer $(l - d/2, h_1 - h_0)$. Das Ball hat Radius $\frac{U}{2\pi}$. Wir betrachten dewegen

$$d(x) := \left\| \begin{pmatrix} x \\ x\tan\theta - (l\tan\theta - (h_1 - h_0))\frac{x^2}{l^2} \end{pmatrix} - \begin{pmatrix} l - \frac{d}{2} \\ h_1 - h_0 \end{pmatrix} \right\| \leq \frac{U}{2\pi}.$$

$$\begin{aligned} d(x)^2 &= \left(x - l + \frac{d}{2}\right)^2 + \left(x\tan\theta - (l\tan\theta - (h_1 - h_0))\frac{x^2}{l^2} - h_1 + h_0\right)^2 \\ 2d(x)\frac{\mathrm{d}d(x)}{\mathrm{d}x} &= 2\left(x - l + \frac{d}{2}\right) \\ &\quad + \left(x\tan\theta - (l\tan\theta - (h_1 - h_0))\frac{x^2}{l^2}\right)\left(\tan\theta - (l\tan\theta - (h_1 - h_0))\frac{2x}{l^2}\right) \end{aligned}$$

Das ist eine kubische Gleichung, und deswegen im Allgemein nicht einfach unlösbar.

c. Aufgabe 1.3

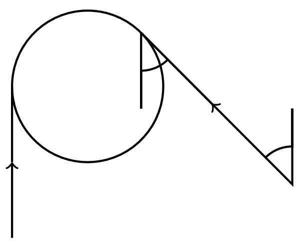


Es gilt

$$\begin{aligned} x :& m_1 g \sin\alpha = m_3 g \sin\beta \\ y :& m_1 g \cos\alpha + m_3 g \cos\beta = m_2 g \end{aligned}$$

Also

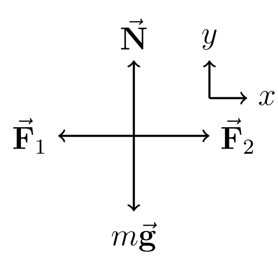
$$
\begin{aligned}
m_3 =& m_1 \frac{\sin\alpha}{\sin\beta} \\
m_1 \cos\alpha + m_1 \frac{\sin\alpha}{\sin\beta}\cos\beta =& m_2 \\
m_1 =& \frac{m_2}{\cos\alpha + \cos\beta\left(\frac{\sin\alpha}{\sin\beta}\right)} \\
=& \frac{m_2 \sin\beta}{\sin(\alpha+\beta)} \\
m_3 =& \frac{m_2 \sin\alpha}{\sin(\alpha+\beta)}
\end{aligned}
$$



$$
\begin{aligned}
\vec{\mathbf{F}} =& -\left[\begin{pmatrix} 0 \\ -m_1 g \end{pmatrix} + m_1 g \begin{pmatrix} \sin\alpha \\ -\cos\alpha \end{pmatrix}\right] \\
=& m_1 g \begin{pmatrix} -\sin\alpha \\ 1+\cos\alpha \end{pmatrix}
\end{aligned}
$$

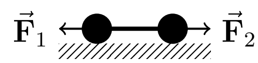
d. Aufgabe 1.4

(a)



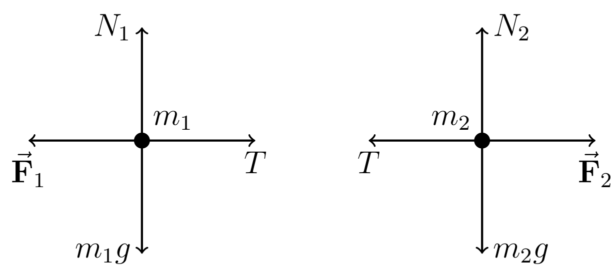
(b) $a_y = 0$ (Zwangsbedingung), $a_x = \frac{1}{3m}\left(|F_2| - |F_1|\right)$

(c)



$$a_1 = a_2 = \frac{1}{3m}\left(|\vec{\mathbf{F}}_2| - |\vec{\mathbf{F}}_1|\right).$$

(d)



(e)

$$\begin{aligned}
m_1 a =& T - F_1 \\
m_2 a =& F_2 - T \\
(m_1 + m_2) a =& T - F_1 + F_2 - T \\
=& F_2 - F_1 \\
=& 3ma \\
a =& \frac{1}{3m}\left(F_2 - F_1\right)
\end{aligned}$$