Geometric Analysis Exam Presentation Outline

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(Dated: June 18, 2024)

I. INTRODUCTION

1. **Define** Lie groups
2. **State** example of $\mathrm{GL}(n, \mathbb{R})$ and **prove** that it is a Lie group.
3. **State** that Lie groups provide a way to move between elements (group multiplication)
4. **Prove** left translation is diffeo
5. **Prove** Lie group homos have constant rank
6. **Prove** Identity component is only connected open subgroup, all connected components are diffeo to identity component
7. **Draw** picture corresponding to previous proof
8. **State** that embedded subgroups are open and closed.

II. GROUP ACTIONS

9. **Define** what it means for a smooth function to intertwine actions
10. **Prove:** If group action on M, N is transitive and F intertwines actions, F has constant rank.
11. **Prove** orbit map $G \to M$ (fixed p) is constant rank.

III. LIE ALGEBRAS

12. State commutator properties

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13. **Define** a lie algebra
14. **Define** left invariant vector fields
15. **Prove** that left invariant vector fields are closed under the commutator
16. **Prove** that $\dim(\mathrm{Lie}(G)) = \dim(G)$ by showing that the evaluation map is an isomorphism.
17. **Deduce** as a corollary that all left invariant vector fields on a lie group are smooth.

A. Matrix Lie Group & Algebra

18. **State** that $\mathrm{GL}(n, \mathbb{R})$ is an open subset of $\mathfrak{gl}(n, \mathbb{R})$.
19. **Prove** that $\mathrm{GL}(n, \mathbb{R}) \cong T_{I_n}\mathrm{GL}(n, \mathbb{R}) \cong \mathfrak{gl}(n, \mathbb{R})$

B. Lie Algebra Homomorphisms