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Problem Sheet 8
for the tutorial on June 27th, 2025
Quantum Mechanics II
Summer term 2025

Sheet handed out on June 17th, 2025; to be handed in on June 24th, 2025 until 2 pm

Exercise 8.1: Scattering phase shifts in first Born approximation [13 P.]

In first Born approximation for spherically symmetric potentials, we found for the scattering amplitude (see lecture notes)

$$f^{(1B)} = -\frac{1}{\Delta}\int_0^\infty dr\, r\, U(r)\sin(\Delta r)\,, \qquad \Delta = 2k\sin(\theta/2)\,, \tag{1}$$

for the central potential $U(r)$. Using the expansion

$$\frac{\sin(\Delta r)}{\Delta r} = \frac{\pi}{2kr}\sum_{l=0}^{\infty}(2l+1)[J_{l+1/2}(kr)]^2\, P_l(\cos\theta)\,, \tag{2}$$

show that the scattering phases introduced in HW 7.2 last week are in the first Born approximation given by

$$\delta_l^B(k) = -\frac{\pi}{2}\int_0^\infty U(r)[J_{l+1/2}(kr)]^2\, r\, dr\,. \tag{3}$$

In the equations above, $J_{l+1/2}(kr)$ denotes the Bessel functions of the first kind (these are siblings of the spherical Bessel functions $j_n(kr)$ already used in the lecture) and $P_l(\cos\theta)$ the Legendre polynomials, respectively.

Exercise 8.2: Gaussian potential [12 P.]

Show that the differential cross section in the first Born approximation for the Gaussian potential $U(r) = \frac{2m}{\hbar^2}V_0\, e^{-r^2/r_0^2}$ is given by

$$\frac{d\sigma}{d\Omega} = \frac{\pi r_0^2}{4}\left(\frac{mV_0 r_0^2}{\hbar^2}\right)^2 e^{-\Delta^2 r_0^2/2} \tag{4}$$

where $\Delta^2 = 2k^2(1-\cos(\theta)) = 4k^2\sin^2(\theta/2)$ is the modulus square of the momentum transfer $\vec{\Delta} = \vec{k} - \vec{k}'$, as introduced in the lecture.
How does $\mathrm{d}\sigma/\mathrm{d}\Omega$ change as a function of θ with increasing energy of the incident particles?
Hint: Don't hesitate to use Mathematica or similar tools to solve the complicated integral!