Aufgabe 2

a) $U = cP^2V$

$$dU = 2cPV\,dP + cP^2\,dV$$
$$= dQ - dW \quad \text{(Hauptsatz)}$$
$$= -dW \quad \text{(Adiabat)}$$
$$= -P\,dV$$

$$2cPV\,dP + (cP^2 + P)\,dV = 0$$

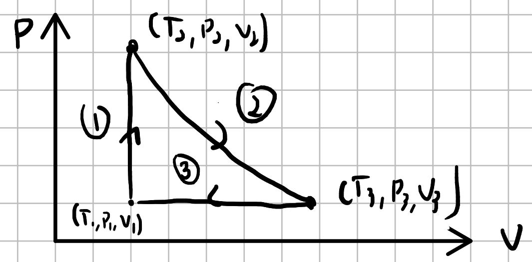
$$2c\cancel{P}V\frac{dP}{dV} = -\cancel{P}(cP+1)$$

$$2c\frac{1}{cP+1}\,dP = -\frac{1}{V}\,dV$$

$$2\ln|cP+1| = -\ln|V| + B$$

$$|cP+1|^2 = \frac{A}{|V|}$$

b)



① $W = 0$ (Isochor)

$$\Delta U = Q = C_V(T_2 - T_1)$$

②: $Q = 0$ (Adiabatisch)

$$\Delta U = -W = C_V(T_3 - T_2)$$
$$W = C_V(T_2 - T_3)$$

③: $\Delta U = C_V(T_1 - T_3)$

$$W = -P_1(V_3 - V_1)$$
$$Q = \Delta U + W$$
$$= C_V(T_1 - T_3) - P_1(V_3 - V_1)$$

Bekannt: $V_2 = V_1$, $P_3 = P_1$

$\Delta U_{ges} = 0$ (Kreisprozess)

$$Q = -W$$

$$= C_V(T_2 - T_1) + C_V(T_1 - T_3) - P_1(V_3 - V_1)$$

$$P_2 V_2^{\gamma} = P_3 V_3^{\gamma}$$

$$T_2 V_2^{\gamma-1} = T_3 V_3^{\gamma-1}$$

$$V_3 = \frac{Nk_B T_3}{P_3} = \frac{Nk_B T_3}{P_1}$$

$$T_2 V_1^{\gamma-1} = T_3^{\gamma}\left(\frac{Nk_B}{P_1}\right)^{\gamma-1}$$

$$T_3 = \left[T_2 V_1^{\gamma-1}\left(\frac{P_1}{Nk_B}\right)^{\gamma-1}\right]^{1/\gamma}$$

$$= T_2^{1/\gamma} V_1 V_1^{-1/\gamma}\left(\frac{P_1}{Nk_B}\right)\left(\frac{P_1}{Nk_B}\right)^{-1/\gamma}$$

$$= T_2^{1/\gamma} T_1 T_1^{-1/\gamma}$$

$$P_2 V_2^{\gamma} = P_3 V_3^{\gamma}$$

$$Nk_B T_2 V_2^{\gamma-1} = P_1 V_3^{\gamma}$$

$$V_3 = \left(\frac{Nk_B T_2 V_2^{\gamma-1}}{P_1}\right)^{1/\gamma}$$

$$= \left(\frac{Nk_B T_2}{P_1}\right)^{1/\gamma} V_2 V_2^{-\frac{1}{\gamma}}$$

$$Q = C_V(T_2 - T_1) + C_V(T_1 - T_3) - P_1(V_3 - V_1)$$

$$= C_V(T_2 - T_1) + C_V\left[T_1 - T_2^{1/\gamma} T_1 T_1^{-1/\gamma}\right]$$

$$-P_1\left[\left(\frac{Nk_B T_2}{P_1}\right)^{1/\gamma} V_2 V_2^{-\frac{1}{\gamma}} - V_1\right]$$

$$= C_V(T_2 - T_1) + C_V\left[T_1 - T_2^{1/\gamma} T_1 T_1^{-1/\gamma}\right]$$

$$- Nk_BT_1\left[\left(\frac{Nk_BT_2}{p_1V_2}\right)^{1/\gamma}\cancel{\frac{V_2}{V_1}}^{\,1} - 1\right]$$

$$= C_V(T_2-T_1) + C_V\left[T_1 - T_2^{1/\gamma}\,T_1\,T_1^{-1/\gamma}\right]$$

$$- Nk_BT_1\left[\left(\frac{T_2}{T_1}\right)^{1/\gamma} - 1\right]$$

$$= C_V(T_2-T_1) - (C_V + Nk_B)T_1\left[\left(\frac{T_2}{T_1}\right)^{1/\gamma} - 1\right]$$

(falsche Definition von W, wenn $U = Q - W$, ist W wie oben)

d) $$\eta = \frac{W}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Q_{out} entspricht Q in 3

Q_{in} entspricht Q in 1

$$\eta = \frac{C_V(T_2-T_1)}{(C_V+Nk_B)T_1\left[\left(\frac{T_2}{T_1}\right)^{1/\gamma}-1\right]}$$

$$\gamma = 1 + \frac{Nk_B}{C_V}$$

$$\gamma - 1 = \frac{Nk_B}{C_V}$$

$$C_V = \frac{Nk_B}{\gamma - 1}$$

$$C_V + Nk_B = \frac{\gamma Nk_B}{\gamma - 1}$$

$$\eta = \frac{\cancel{\frac{Nk_B}{\gamma-1}}(T_2-T_1)}{\gamma\cancel{\frac{Nk_B}{\gamma-1}}T_1\left[\left(\frac{T_2}{T_1}\right)^{1/\gamma}-1\right]}$$

$$= \frac{T_2 - T_1}{\gamma T_1\left[\left(\frac{T_2}{T_1}\right)^{1/\gamma}-1\right]}$$