Prof. Dr. Adriana Pálffy-Buß
Fabian Richter
Julius-Maximilians-Universität Würzburg
Institute for Theoretical Physics and Astrophysics

Problem Sheet 11
for the tutorial on July 18th, 2025
Quantum Mechanics II
Summer term 2025

Sheet handed out on July 8th, 2025; to be handed in on July 15th, 2025 until 2 pm

---

Exercise 11.1: Spin [7+5 P.]

a) Consider two spin-1/2 particles. Show explicitly that the total spin is $S = 0$ for the singlet-state

$$|\chi_1\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \tag{1}$$

and $S = 1$ for each of the following triplet states

$$\begin{aligned} |\chi_2\rangle &= |\uparrow\uparrow\rangle \ , \\ |\chi_3\rangle &= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right) \ , \\ |\chi_4\rangle &= |\downarrow\downarrow\rangle \ . \end{aligned} \tag{2}$$

b) Show that

$$\hat{P}_1 = \frac{3}{4} + \frac{1}{\hbar^2} \hat{\boldsymbol{S}}_1 \cdot \hat{\boldsymbol{S}}_2 \tag{3}$$

projects onto the triplet subspace.

Exercise 11.2: Perturbative approach for Helium's first autoionizing state [13 P.]

Let us consider the Auger decay of the autoionizing state $2s^2$ of Helium. Hereby one electron is emitted in the continuum while simultaneously the second one decays to the $1s$ ground state of the resulting He^+ ion. Using energy conservation arguments, find the continuum electron energy in eV. You may use the Schrödinger energy solution known for the H-like helium ion and apply first order perturbation theory like in the lecture to determine the energy of the autoionizing state. Thereby you may employ the non-relativistic radial wave functions for the 2s electrons given by

$$\Psi_{200}(r) = \sqrt{\frac{Z^3}{32\pi a_0^3}} \left(-\frac{Zr}{a_0} + 2 \right) \mathrm{e}^{-Zr/(2a_0)} \, . \tag{4}$$