Ginzburg-Landau theory

Dr. Viktoriia Kornich[1] and Dr. Kristian Mæland[2]

[1]viktoriia.kornich@physik.uni-wuerzburg.de
[2] kristian.maeland@uni-wuerzburg.de

Problem 1

Show that the Ginzburg-Landau free energy of a domain wall can be written as

$$\Delta F = A\frac{u}{4}\int dx\,\left[\psi_0^4 - \psi^4(x)\right], \tag{1}$$

where $A = L^{d-1}$ is the area of the domain wall. Using this result, show that the surface tension $\sigma = \Delta F/A$ is given by

$$\sigma = \frac{\sqrt{8}}{3}\xi u \psi_0^4, \tag{2}$$

with ξ the correlation length. To get the final result, you will need to recall the functional form of the soliton, i.e. the solution to the Ginzburg-Landau equation through the domain wall.

Problem 2

Consider a two-component Dirac electron moving in one dimension through a domain wall, described by the wave equation

$$\left(-i\sigma_1\nabla_x - m(x)\sigma_3\right)\psi = E\psi, \tag{3}$$

where the mass field forms a domain wall, changing sign at the origin according to

$$m(x) = m_0 \tanh\left(\frac{x}{\sqrt{2}\xi}\right). \tag{4}$$

Asymptotically, the energy of the excitation is gapped, with an excitation spectrum $E(k) = \sqrt{k^2 + m_0^2}$. Show that the domain wall gives rise to a zero energy bound state and derive the form of its wave-function.