Quantum field theory in the solid state, Exercise sheet 10

Corrections: July 14$^{\text{th}}$

1. U(1) spin-liquid on the triangular lattice. As we saw in class, the mean field theory of the Heisenberg model on the triangular lattice boils down to solving the free fermion problem:

$$\hat{H} = |\chi| \sum_{\langle i,j \rangle} \left(\hat{f}_i^\dagger e^{i \int_i^j \boldsymbol{a}(\boldsymbol{l}) \cdot \boldsymbol{dl}} \hat{f}_j + \text{H.c.} \right) - \lambda \sum_i \hat{f}_i^\dagger \hat{f}_i \tag{1}$$

(a) Consider the triangular lattice as depicted in the figure. Compute the dispersion relation

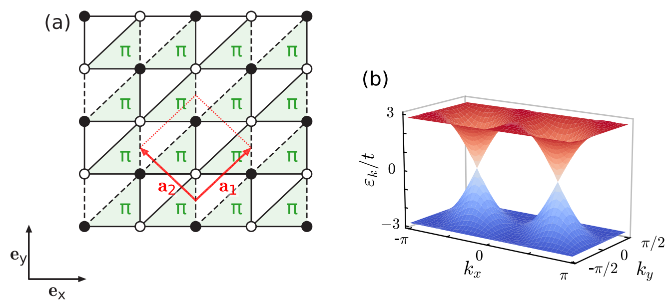

FIG. 1: Triangular lattice, and flux pattern. Taken from: Yuichi Otsuka, Kazuhiro Seki, Sandro Sorella, and Seiji Yunoki, Phys. Rev. B 98 (2018), 035126.

for the zero flux $\boldsymbol{a} = 0$ and π-flux state shown in the figure.

(b) Tune the value of λ so as to ensure half-filling, and show that for a small lattice the ground state of the π-flux state is smaller than that of the zero flux one.

(c) For the π-flux state, expand the dispersion around the Dirac points and show that you obtain a Dirac Hamiltonian. Show that the single fermion density of states has the form $N(\omega) \simeq |\omega|$.

(d) Compute the dynamical spin-structure factor, $\chi(\boldsymbol{q}, \omega)$ as shown in class, and write a code to plot it. Here is a hint: For the π-flux, the unit cell has 2 orbitals that we denote by α. The dynamical spin-structure factor will then be a 2×2 matrix $\chi_{\alpha,\beta}(\boldsymbol{Q}, \omega)$ where Q is in the first Brillouin zone. The quantity you should plot reads:

$$\chi(\boldsymbol{q}, \omega) = \sum_{\alpha,\beta} \chi_{\alpha,\beta}(\boldsymbol{q}, \omega) e^{i\boldsymbol{q} \cdot (\boldsymbol{R}_\alpha - \boldsymbol{R}_\beta)} \tag{2}$$

where $\boldsymbol{R}_\alpha$ and $\boldsymbol{R}_\beta$ are the positions of the two orbitals in the unit cell. This corresponds to the dynamical spin structure factor in the extended zone scheme.

2. Kondo effect in Dirac systems

In class, we solved the mean field equations for the Kondo model for a band with a flat density of states and in the wide band limit. The result was that for any value of the Kondo coupling, J_K, we obtained a finite value of the mean-field order parameter. Thereby a spin 1/2-impurity in a metallic environment is always screened in the limit of vanishingly small temperatures.

Consider the density of states of a Dirac system:

$$N(\omega) = \begin{cases} \alpha|\omega| & \text{for } |\omega| < W/2 \\ 0 & \text{for } |\omega| \geq W/2 \end{cases} \tag{3}$$

and show that in the wide band limit the Kondo effect is absent in the limit of small values of J_K, but that it is present in the large J_K limit. Note that in the wide band limit, you can omit the real part of the hybridization function, $\Delta(\omega)$.