

Topological Field Theory WS 2025

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PROBLEM SET 5

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1. Dispersion relation of magnons in ferromagnetic spin chains

In the lectures, we derived the effective action for a ferromagnetic spin chain with Hamiltonian

$$H = -J_F \sum_i \mathbf{S}_i \mathbf{S}_{i+1} \quad (1)$$

and obtained

$$S[\hat{\mathbf{n}}] = \frac{1}{a} \int d\tau dx \left\{ \frac{J_F s^2 a^2}{2} (\partial_x \hat{\mathbf{n}})^2 + i s \mathcal{L}_{WZ}(\hat{\mathbf{n}}, \partial_\tau \hat{\mathbf{n}}) \right\}, \quad (2)$$

with

$$\mathcal{L}_{WZ}(\hat{\mathbf{n}}, \partial_\tau \hat{\mathbf{n}}) = (1 - \cos \theta) \partial_\tau \phi. \quad (3)$$

- (a) Write the Lagrangian density for the action in (2) and (3) in terms of θ , ϕ , as well as their spacial (∂_x) and temporal (∂_τ) derivatives.
- (b) Obtain the equation of motion and show that the dispersion for spin wave excitations (magnons) is quadratic, *i.e.*, $\omega \propto q^2$.