Quantum field theory in the solid state, Exercise sheet 2

Corrections: Monday 12^{th} of May

The periodic Anderson and Kondo lattice models: symmetries and strong coupling limit.

The periodic Anderson model (PAM) describes the physics of a wide band that hybridizes with a narrow band of nearly localized electrons. To define the model we will consider a hyper-cubic lattice of linear length L in d dimensions with 2 orbitals per unit cell, one extended, the c-orbital, and one localized, the f-orbital. Since the f-orbital is localized the Coulomb repulsion will have to be taken into account. The periodic SU(N) generalization of Anderson model describes this situation and and reads:

$$\hat{H}_{\mathrm{PAM}} = -t \sum_{\langle i,j \rangle} \sum_{\sigma=1}^{N} \left(\hat{c}^{\dagger}_{i,\sigma} \hat{c}_{j,\sigma} + \mathrm{H.c.} \right) + V \sum_{i} \sum_{\sigma=1}^{N} \left(\hat{c}^{\dagger}_{i,\sigma} \hat{f}_{i,\sigma} + \mathrm{H.c.} \right) + \frac{U_f}{N} \sum_{i} \left(\hat{n}^f_i - \frac{N}{2} \right)^2 . \quad (1)$$

Here $\hat{n}^f_i = \sum_{\sigma=1}^{N} \hat{f}^{\dagger}_{i,\sigma} \hat{f}_{i,\sigma}$.

1. Symmetries

(a) Show that the PAM enjoys the same global symmetries, particle number, total spin, total momentum, as the Hubbard model that we discussed in class.

(b) Let $\hat{c}^{\dagger}_{\boldsymbol{k},\sigma} = \frac{1}{\sqrt{L^d}} \sum_{\boldsymbol{i}} e^{i\boldsymbol{k}\cdot\boldsymbol{i}} \hat{c}^{\dagger}_{\boldsymbol{i},\sigma}$ and an equivalent form holds for the f-operator. For the $SU(2)$ case, under which conditions does

$$\langle \hat{c}^{\dagger}_{\boldsymbol{k}_1,\sigma_1} \hat{c}_{\boldsymbol{k}_2,\sigma_2} \hat{c}^{\dagger}_{\boldsymbol{k}_3,\sigma_3} \hat{f}_{\boldsymbol{k}_4,\sigma_4} \rangle \quad (2)$$

not vanish?, Here, for an operator $\hat{O}$, $\langle \hat{O} \rangle = \frac{\mathrm{Tr}\left(e^{-\beta \hat{H}_{PAM}} \hat{O}\right)}{\mathrm{Tr}\left(e^{-\beta \hat{H}_{PAM}}\right)}$ and β is the inverse temperature.

2. Non-interacting limit

Consider the specific case of $U_f = 0$, and d=1. Can you diagonalize the Hamiltonian and plot the band structure.

3. Strong coupling limit

(a) Consider a single site periodic Anderson model in the Hilbert with a total of two particles for the SU(2) case. Use symmetries to diagonalize the Hamiltonian.

(b) Analyze the strong coupling limit, $U_f >> t$ and show that the low energy spectrum is the same as that of the single site Kondo lattice model:

$$\hat{H}_{KLM} = -t \sum_{\langle i,j \rangle} \left(\hat{\boldsymbol{c}}_i^\dagger \hat{\boldsymbol{c}}_j + \text{H.c.} \right) + \frac{2J_K}{N} \sum_i \boldsymbol{c}_i^\dagger \frac{\boldsymbol{\sigma}}{2} \boldsymbol{c}_i \cdot \hat{\boldsymbol{S}}_i \tag{3}$$

with $\hat{\boldsymbol{S}}_i = \frac{1}{2} \hat{\boldsymbol{f}}_i^\dagger \boldsymbol{\sigma} \hat{\boldsymbol{f}}_i$ and

$$\boldsymbol{\sigma} = \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right). \tag{4}$$

Here we have used a spinor notation $\hat{\boldsymbol{c}}_i^\dagger = \left(\hat{c}_{i,\uparrow}^\dagger, \hat{c}_{i,\downarrow}^\dagger \right)$ and $\hat{\boldsymbol{f}}_i^\dagger = \left(\hat{f}_{i,\uparrow}^\dagger, \hat{f}_{i,\downarrow}^\dagger \right)$. The Kondo lattice model is defined in the Hilbert space where $\hat{\boldsymbol{f}}_i^\dagger \hat{\boldsymbol{f}}_i = 1$.