Intro to ML - Lecture Notes

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1 MLE for Tossing Coin Problem

Each outcome of this experiment can be thought as a sample of bernoulli distribution. Where θ is a parameter that controls the distribution.

$$p\big(y_n|\theta\big) = \theta^{y_n}(1-\theta)^{1-y_n} \tag{1}$$

Since, each outcome is IID. So, overall likelihood distribution is given by the formula below.

$$p\big(y|\theta\big) = \prod_{n=1}^{N} \theta^{y_n}(1-\theta)^{1-y_n}$$

1.1 Maximum Likelihood Estimation

The MLE solution is given by the objective function written below.

$$\theta_{MLE} = \underset{\theta}{\operatorname{argmax}} \Big\{ p\big(y|\theta\big) \Big\}$$

$$\therefore \theta_{MLE} = \underset{\theta}{\operatorname{argmax}} \Bigg\{ \prod_{n=1}^{N} \theta^{y_n}(1-\theta)^{1-y_n} \Bigg\} \tag{2}$$

Logarithm function is monotonic. So, we can write above objective as maximizing log-likelihood as given below.

$$\theta_{MLE} = \underset{\theta}{\operatorname{argmax}} \Big\{ \log \Big[p\big(y|\theta\big) \Big] \Big\}$$

$$\therefore \theta_{MLE} = \underset{\theta}{\operatorname{argmax}} \Bigg\{ \sum_{n=1}^{N} \Big[\theta \log y_n + (1-\theta)\log(1-y_n) \Big] \Bigg\} \tag{3}$$

Closed form solution for MLE is given by:

$$\boxed{\boxed{\theta_{MLE} = \frac{\sum_{n=1}^{N} I[y_n = 1]}{N}}} \tag{4}$$