Instructions:

1. Answer **ALL** questions.
2. All relevant workings must be shown.
3. Turn in to the respective tutorial google classroom by **9 April 2021, 6pm**.

Question 1

Let $A = \{2, 4, 8, 10, 16\}$ and R be a relation on A defined by xRy if and only if $4 \mid (x - y)$.

(a) Write R as set of ordered pairs. (2 marks)
(b) Explain how R is determined as an equivalence relation. Then, determine A/R. (3 marks)

Question 2

Let $B = \{1, 2, 3, 4, 5\}$. A relation on B is given by $S = \{(1,2), (1,3), (2,2), (3,1), (3,2), (3,5), (4,4), (5,1)\}$. Find,

(a) the reflexive closure of S; (2 marks)
(b) the symmetric closure of S; (2 marks)
(c) the matrix of transitive closure of S by using Warshall's algorithm. (7 marks)

Question 3

Let $A = \{1, 2, 3, 4, 5\}$. Two relations on A are given by $R = \{(1,1), (1,2), (2,1), (2,5), (3,1), (3,5), (4,4), (5,1)\}$ and $S = \{(1,2), (1,3), (1,5), (2,1), (3,2), (3,3), (5,2)\}$. Find the **matrix** of each of the followings.

(a) $(R \cap S)^{-1}$ (b) $R^{-1} \cup S^{-1}$ (c) $S \circ \overline{R}$ (d) $(S^{-1} \circ \overline{R})^{-1}$ (10 marks)

Question 4

Let ρ_1 and ρ_2 be two permutations on set $A = \{1, 2, 3, 4, 5, 6, 7\}$ where

$$\rho_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 5 & 4 & 6 & 3 & 7 \end{pmatrix} \text{ and } \rho_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 5 & 1 & 4 & 6 & 7 \end{pmatrix}.$$

(a) Determine whether ρ_1 and ρ_2 are odd or even permutations. (4 marks)
(b) Find
(i) $\rho_1 \circ \rho_2$ (ii) $(\rho_1 \circ \rho_2)^2$ (iii) $\rho_2^{-1} \circ \rho_1$ (7 marks)

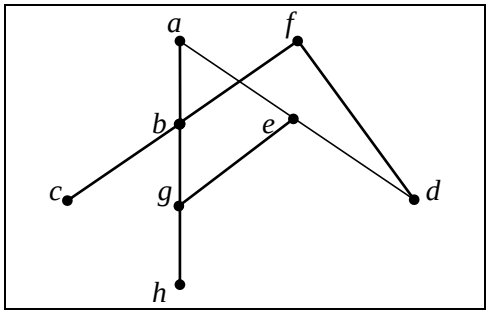
Question 5

Let set $A = \{2, 4, 6, 8, 12, 24\}$ and a relation R is defined on A by xRy if and only if x is a factor of y.

(a) Find R and explain how R is determined as a partial order on A. (3 marks)
(b) Draw the Hasse diagram of R. (3 marks)
(c) Is R linearly ordered? (1 mark)

Question 6

The Hasse diagram of a poset P is shown below. Find, if exist(s):



a) The minimal element(s) and the maximal element(s) of P. (2 marks)
b) The lower bound(s) and the greatest lower bound of $\{g, e\}$. (2 marks)
c) The upper bound(s) and the least upper bound of $\{c, g\}$. (2 marks)

END