ST 518: Data Analytics II

Odds and Odds Ratios

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Details about Odds and Odds Ratios

Another Example

Some General Comments

Differences Versus Odds Ratios

For the Vitamin C example, $\hat{p}_1 - \hat{p}_2 = 0.815 - 0.742 = 0.073$.

Since both of $\hat{p}_1$ and $\hat{p}_2$ are fairly large, this difference communicates a lot: The probability of getting a cold when taking Vitamin C is about 7 percentage points lower than when taking a placebo.

But what if $\hat{p}_1 = 0.10$ and $\hat{p}_2 = 0.027$?

- Then the difference in the two proportions is also 0.073
- But, there's more information here: $\hat{p}_1 \approx 4 \times \hat{p}_2$
- If we look at **odds ratios** rather than differences when the proportions are small, we'll be able to communicate that multiplicative difference.

Odds

If p is the population proportion (or probability) of a "success" (or "yes" outcome), then the **odds of a success** are:

$$\omega = \frac{p}{1-p}.$$

We often hear of $5:1$ odds, or $3:2$ odds. If we just solve

$$\frac{5}{1} = \frac{p}{1-p}$$

for p, we find that $p = 5/6$; or solving

$$\frac{3}{2} = \frac{p}{1-p}$$

for p gives $p = 3/5$.

In the Vitamin C example, The odds of getting a cold in the Vitamin C group is:

$$\hat{\omega} = \frac{0.742}{1 - 0.742} = \frac{0.742}{0.258} = 2.876.$$

That is, there are 2.876 cold cases for every 1 non-cold case; or, about 23 cold cases for every 8 non-cold cases.

It's customary to report the larger number first when reporting odds.

For example, if an event has a 0.95 chance of occurrence, we say that there are 19:1 odds *in favor* of its occurrence. Whereas if an event has a 0.05 chance of occurrence, we say that it has 19:1 odds *against* it.

Odds and Odds Ratios

Keep in mind that an odds of success corresponds to a proportion of success.

- If we want to compare two proportions of success, then we naturally compare two odds of success as well.
- That is, for the difference in two proportions we looked at $p_1 - p_2$ and talked about the sampling distribution of $\hat{p}_1 - \hat{p}_2$.
- For the *ratio of two odds*, we'll look at at $\phi = \omega_1/\omega_2$, and in the next module, we'll talk about estimating this quantity.

Interpreting Odds Ratio

Odds ratios can be interpreted as *multiplicative differences* between two odds.

For example, if

$$2 = \hat{\phi} = \frac{\hat{\omega}_1}{\hat{\omega}_2},$$

Then

$$\hat{\omega}_1 = 2\hat{\omega}_2.$$

In words, "the odds of success in population 1 is estimated to be twice the odds of success in population 2."

Salk Polio Vaccine

	Infantile Paralysis Victim?	
	Yes	No
Placebo	142	199,858
Salk Vaccine	56	199,944

- The odds of infantile paralysis in the Placebo group are

$$\frac{142}{199858} = 0.00071$$

- The odds of infantile paralysis in the Salk vaccine group are

$$\frac{56}{199944} = 0.00028$$

- And therefore the odds ratio is:

$$\frac{0.00071}{0.00028} = 2.5$$

Odds Ratios

- In the case of relatively rare events, like infantile paralysis, it's usually better to report odds ratios than to report differences in proportions.
- The odds ratio is the only parameter that can be used to compare two groups of binary outcomes from a retrospective study.
    - A retrospective study is one in which subjects are selected into a study on the basis of some outcome, and then the researchers *look back* to see if subjects with different outcomes also differ in terms of some important other variable(s).
- Odds and will crop up again when we talk about logistic regression, and that's when we'll discuss how to make inference.