

Machine Learning

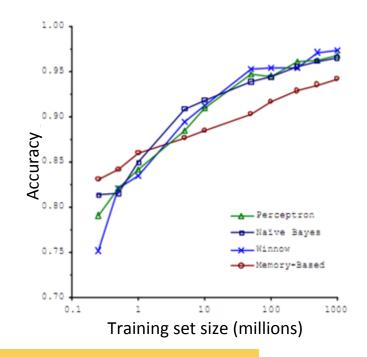
# Large scale machine learning

Learning with large datasets

## Machine learning and data

Classify between confusable words. E.g., {to, two, too}, {then, than}.

For breakfast I ate two eggs.



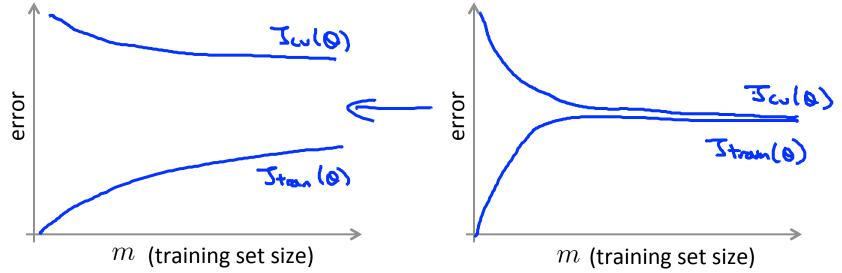
"It's not who has the best algorithm that wins.

It's who has the most data."

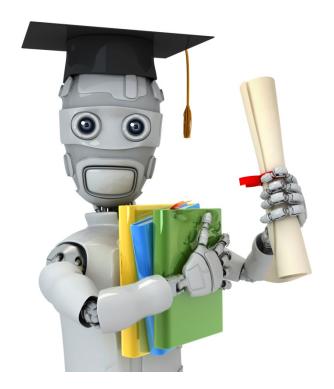
[Figure from Banko and Brill, 2001] Andrew Ng

#### **Learning with large datasets**

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$



Andrew Ng



Machine Learning

# Large scale machine learning

Stochastic gradient descent

### Linear regression with gradient descent

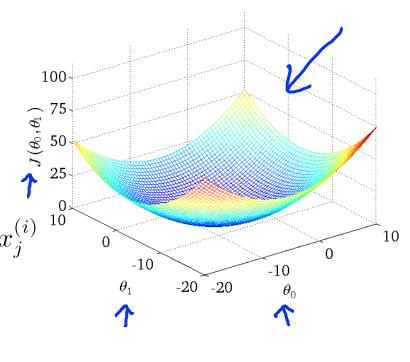
$$h_{\theta}(x) = \sum_{j=0}^{n} \theta_j x_j$$

$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

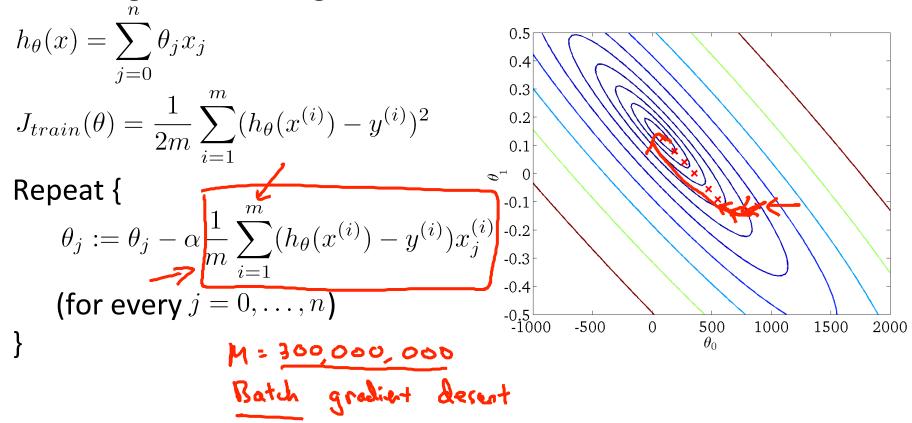
$$Repeat \{$$

### Repeat {

$$\Rightarrow \theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$
(for every  $j = 0, \dots, n$ )



### Linear regression with gradient descent



#### **Batch gradient descent**

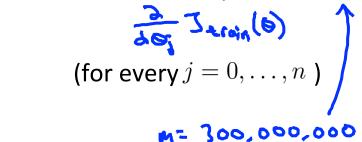
# Stochastic gradient descent

$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)})^{2} > \underbrace{cost(\theta, (x^{(i)}, y^{(i)}))}_{1 \quad \underline{m}} = \underbrace{\frac{1}{2} (h_{\theta}(x^{(i)}) - y^{(i)})^{2}}_{1 \quad \underline{m}}$$

$$cost(\theta, (x^{(i)}, y^{(i)})) = \frac{1}{2}(h_{\theta}(x^{(i)}) - y^{(i)})$$

$$J_{train}(\theta) = \frac{1}{m} \sum_{i=1}^{m} cost(\theta, (x^{(i)}, y^{(i)}))$$

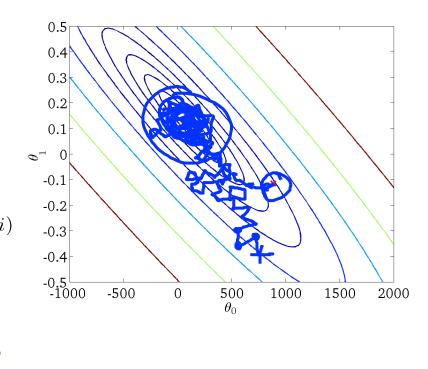
Repeat {
$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (\underline{h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)}$$



#### Stochastic gradient descent

1. Randomly shuffle (reorder) training examples

→ 2. Repeat { 1-10×  $\begin{array}{c} \text{for } i := 1, \dots, m \, \{ & \begin{array}{c} -0.1 \\ -0.2 \\ \end{array} \\ \Rightarrow \theta_j := \theta_j - \alpha (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)} & \begin{array}{c} -0.3 \\ -0.3 \\ \end{array} \\ \text{(for } j = 0, \dots, n & \begin{array}{c} -0.5 \\ -0.3 \\ -0.4 \end{array} \end{array} \\ \end{array}$ -> m = 300,000,000





Machine Learning

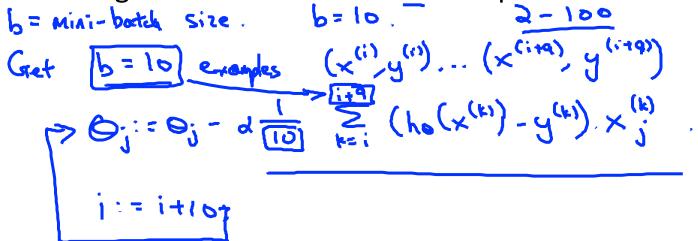
# Large scale machine learning

Mini-batch gradient descent

#### Mini-batch gradient descent

- $\rightarrow$  Batch gradient descent: Use <u>all</u> examples in each iteration
- Stochastic gradient descent: Use 1 example in each iteration

Mini-batch gradient descent: Use b examples in each iteration



# Mini-batch gradient descent

Say 
$$b = 10, m = 1000$$
.

$$\rightarrow$$
 for  $i = 1, 11, 21, 31, \dots, 991 {$ 

$$\theta_j := \theta_j - \alpha \frac{1}{10} \sum_{k=i}^{i+9} (h_{\theta}(x^{(k)}) - y^{(k)}) x_j^{(k)}$$

(for every 
$$j = 0, \ldots, n$$
)



Machine Learning

# Large scale machine learning

Stochastic gradient descent convergence

#### **Checking for convergence**

- Batch gradient descent:
  - $\rightarrow$  Plot  $J_{train}(\theta)$  as a function of the number of iterations of
  - gradient descent.  $J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) y^{(i)})^2$

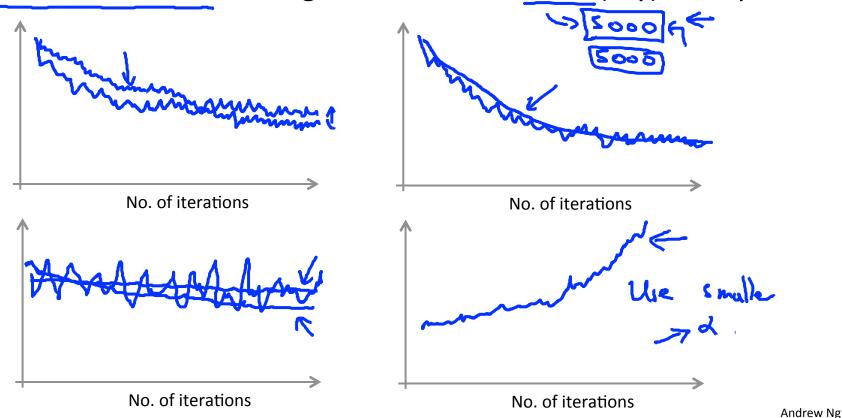
 $\gg (\chi^{(i)}, y^{(i)})$  ,  $(\chi^{(in)}, y^{(in)})$ 

- Stochastic gradient descent:

  - $\Rightarrow cost(\theta, (x^{(i)}, y^{(i)})) = \frac{1}{2}(h_{\theta}(x^{(i)}) y^{(i)})^2$   $\Rightarrow \text{During learning, compute } cost(\theta, (x^{(i)}, y^{(i)})) \text{ before updating } \theta$ using  $(x^{(i)}, y^{(i)})$ .
  - $\rightarrow$  Every 1000 iterations (say), plot  $cost(\theta, (x^{(i)}, y^{(i)}))$  averaged over the last 1000 examples processed by algorithm.

#### **Checking for convergence**

Plot  $cost(\theta, (x^{(i)}, y^{(i)}))$ , averaged over the last 1000 (say) examples

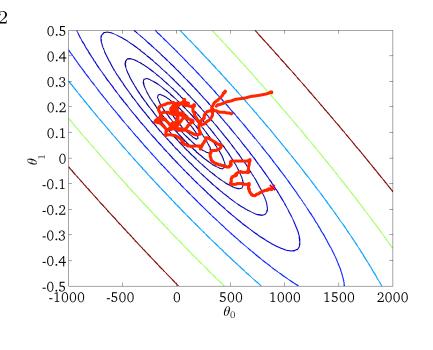


### Stochastic gradient descent

$$cost(\theta, (x^{(i)}, y^{(i)})) = \frac{1}{2} (h_{\theta}(x^{(i)}) - y^{(i)})^{2}$$
$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^{m} cost(\theta, (x^{(i)}, y^{(i)}))$$

- Randomly shuffle dataset.

```
Repeat {
   for i = 1, ..., m {
\theta_j := \theta_j - \alpha(h_{\theta}(x^{(i)}) - y^{(i)})x_j^{(i)}
                     (for i = 0, ..., n)
```

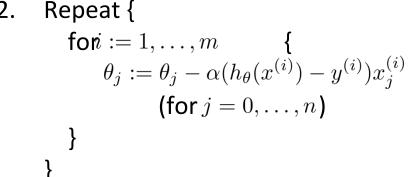


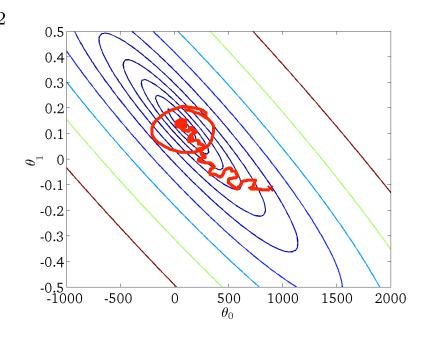
Learning rate  $\alpha$  is typically held constant. Can slowly decrease  $\alpha$ over time if we want  $\theta$  to converge. (E.g.  $\alpha = \frac{\text{const}}{\text{iteration Number}}$ 

### Stochastic gradient descent

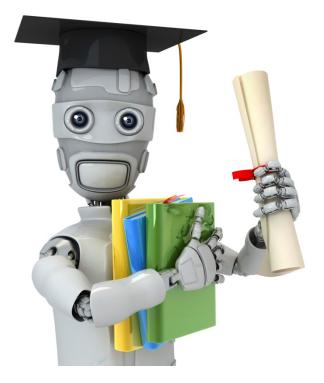
$$cost(\theta, (x^{(i)}, y^{(i)})) = \frac{1}{2} (h_{\theta}(x^{(i)}) - y^{(i)})^{2}$$
$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^{m} cost(\theta, (x^{(i)}, y^{(i)}))$$

- Randomly shuffle dataset.





Learning rate  $\alpha$  is typically held constant. Can slowly decrease  $\alpha$ over time if we want  $\theta$  to converge. (E.g.  $\alpha = \frac{\text{const1}}{\text{iterationNumber} + \text{const2}}$ 



#### Machine Learning

# Large scale machine learning

# Online learning

#### **Online learning**

Shipping service website where user comes, specifies origin and destination, you offer to ship their package for some asking price, and users sometimes choose to use your shipping service (y = 1), sometimes not (y = 0).

Features x capture properties of user, of origin/destination and asking price. We want to learn  $p(y=1|x;\theta)$  to optimize price.

Repeat forever 
$$\mathcal{E}$$
 price logistic regression

Get  $(x,y)$  corresponding to user.

Update  $0$  using  $(x,y)$ :  $(x,y)$ :

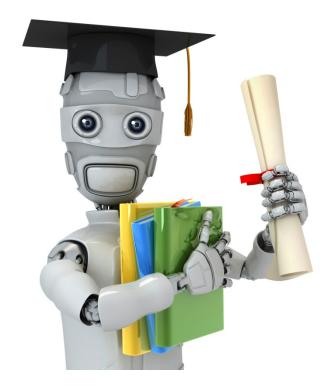
 $\Rightarrow 0_j := 0_j - \alpha (h_0(x) - y) \cdot x_j$   $(j=0,...,n)$ 
 $\Rightarrow Can adopt to charging user preference.$ 

#### Other online learning example:

Product search (learning to search)

User searches for "Android phone 1080p camera" <-- Have 100 phones in store. Will return 10 results.

- $\Rightarrow x = \text{features of phone}$ , how many words in user query match name of phone, how many words in query match description of phone, etc.  $(x,y) \leftarrow$
- $\Rightarrow y = 1$  if user clicks on link. y = 0 otherwise.
- $\Rightarrow$  Learn  $p(y=1|x;\theta)$ .  $\leftarrow$  predicted CTR
- Use to show user the 10 phones they're most likely to click on. Other examples: Choosing special offers to show user; customized selection of news articles; product recommendation; ...



Machine Learning

# Large scale machine learning

Map-reduce and data parallelism

#### Map-reduce

Batch gradient descent:

$$\text{m: } \theta_j := \theta_j - \alpha \frac{1}{400} \sum_{i=1}^{400} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)} \longleftarrow$$

m = 400,000,000

Machine 1: Use 
$$(x^{(1)}, y^{(1)}), \dots, (x^{(100)}, y^{(100)})$$
.

Hence  $(x^{(1)}, y^{(1)}), \dots, (x^{(100)}, y^{(100)})$ .

Machine 2: Use  $(x^{(101)}, y^{(101)}), \dots, (x^{(200)}, y^{(200)})$ .

$$temp_j^{(2)} = \sum_{i=101}^{200} (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_j^{(i)}$$

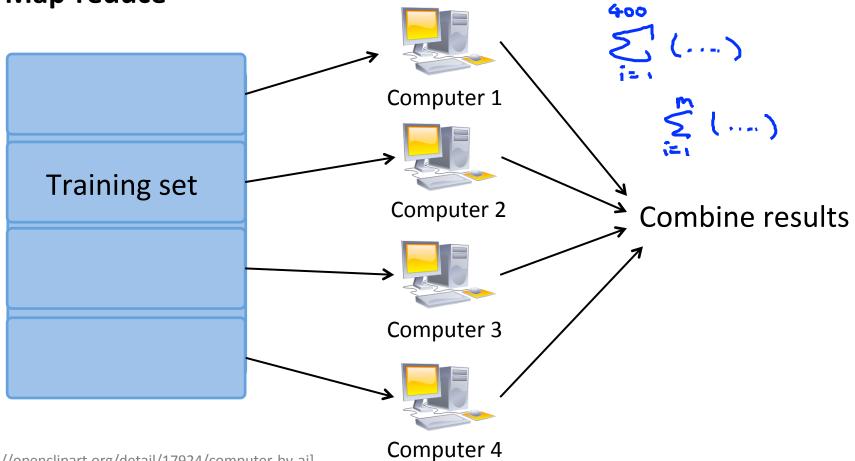
Machine 3: Use  $(x^{(201)}, y^{(201)}), \dots, (x^{(300)}, y^{(300)})$ .

$$temp_j^{(3)} = \sum_{i=201}^{300} (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_j^{(i)}$$

Machine 4: Use  $(x^{(301)}, y^{(301)}), \dots, (x^{(400)}, y^{(400)})$ .

$$temp_j^{(4)} = \sum_{i=301}^{400} (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_j^{(i)}$$

### Map-reduce



#### Map-reduce and summation over the training set

Many learning algorithms can be expressed as computing sums of functions over the training set.

E.g. for advanced optimization, with logistic regression, need:

$$J_{train}(\theta) = -\frac{1}{m} \sum_{i=1}^{m} y^{(i)} \log h_{\theta}(x^{(i)}) - (1 - y^{(i)}) \log(1 - h_{\theta}(x^{(i)}))$$

$$\Rightarrow \frac{\partial}{\partial \theta_{j}} J_{train}(\theta) = \frac{1}{m} \sum_{i=1}^{m} (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_{j}^{(i)}$$

$$+ \sum_{i=1}^{m} h_{\theta}(x^{(i)}) - y^{(i)} \cdot x_{j}^{(i)}$$

