ELL225 PROJECT CONTROL ENGINEERING

Riderless bicycle control

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Contents

1. Abstract

The goal of this course project is to create an autopilot for a bicycle without a rider that will maintain its upright position at certain speeds. The dynamics of the bicycle can be described as a fourth order linear model, by taking the following quantities into account: i) states: roll angle, roll rate, steer angle, and steer rate; ii) input: steering torque; and iii) output: roll angle. The linear model, which is also known as a linear parameter changing model, depends on velocity and allows for the creation of linear time-invariant state space models that correspond to various fixed velocities. For this project, we have considered the following three fixed velocities: v1 = 0 meter per second (mps), v2= 3.5 mps and v3 = 5 mps.

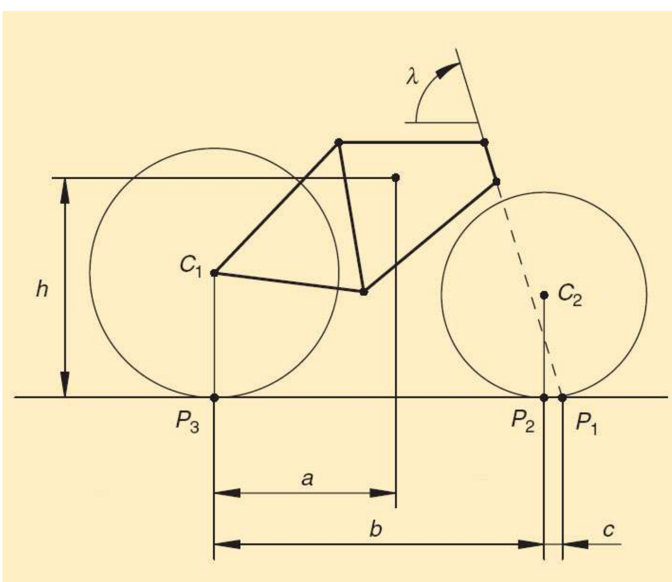
2. Introduction

Similar to the inverted pendulum, bicycles are statically unstable but can occasionally move steadily when specific conditions are met. A sophisticated bicycle model is needed because of the system's multiple degrees of freedom and complex geometry. Our opinion is that the bicycle is made up of four rigid components: two wheels, a frame, and a front fork with handlebars. Other moving components, such as the brakes, chain, and pedals, have no bearing on the dynamics. The upper body of the rider is modelled as a point mass that can move laterally with respect to the bicycle frame in order to incorporate the rider in the analysis. The handlebars can also be rotated by the rider. We make the assumption that the bicycle tyre rolls without longitudinal or lateral slippage since we do not take harsh conditions and tight turns into account. In addition, we believe that the forward velocity is constant. To sum up, we just take for granted that the bicycle moves in a horizontal plane and that the wheels are always in contact with the ground.

3. Mathematical Model of Bicycle

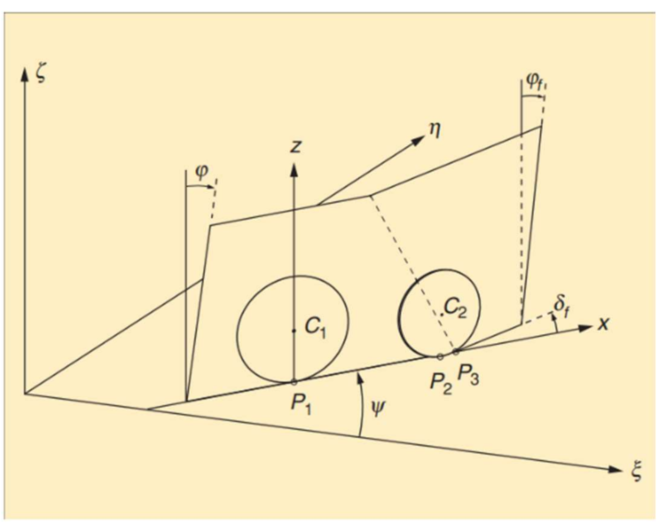
3.1 Bicycle Geometry

The parameters that define the geometry of a bicycle, such as the wheelbase (b), head angle (), and trail, are shown in the following picture (c). The front fork is angled and shaped so that the front wheel's point of contact with the ground is behind the steer axis extension. When the bike is upright and the steer angle is zero, the trail is the distance (c) between the contact point and the steer axis. While reducing steering agility, a bigger trail increases stability. The frame plane and the front fork plane of the bicycle are linked at the steer axis. The front wheel is situated in the front, while the frame and rear wheels are in the frame plane. The horizontal plane is intersected by the steer axis at point P3, whereas the points P1 and P2 indicate where the wheels make contact with the plane.



3.2 Coordinates

According to ISO 8855, the displayed figure defines the coordinates used in the system analysis. There is an inertial system with axes and origin O. The rear wheel's contact point P1 with the horizontal plane serves as the origin of the coordinate system xyz. The steer axis and the horizontal plane are intersected at point P3 on the x axis, which is aligned with the rear plane's contact line with the latter. The angle, which denotes the angle between the x-axis and the y-axis, determines the orientation of the back wheel plane. The bicycle's left side is pointed in the right direction, with the z axis vertical and the y axis perpendicular to the x axis. When leaning to the right, the roll angle of the rear frame is positive while the roll angle of the front fork plane is f. The steer angle, which is positive while turning left, is the angle at which the rear and front planes cross. The angle formed by the rear and front planes intersecting with the horizontal plane is known as the effective steer angle (f).



3.3 Equation of Motion

For analysis and design purposes, we consider the bicycle model presented, which consists of four rigid bodies, namely, the rear frame, the front fork with the handlebar, and the rear and front knife-edge wheels. The four bodies are interconnected by revolute hinges and, in the reference configuration, they are all symmetric relative to the bicycle longitudinal axis. The contact between the stiff non-slipping wheels and the flat level surface is modelled by holonomic constraints in the normal direction as well as by nonholonomic constraints in the longitudinal and lateral direction.

In spite of its simplicity, this model adequately describes the main dynamics of the bicycle as validated by the experiment. This model considers three degrees of freedom, namely, the roll angle $\phi(t)$, the steering angle $\delta(t)$, and the speed $v(t)$. The linearized equations of motion are two coupled second-order ordinary differential equations written in matrix form as

$$M\ddot{q}(t)+v(t)C_1\dot{q}(t)+(K_0+v(t)^2K_2)q(t)=f(t)$$

with

$$q(t) = \begin{bmatrix}\phi(t)\\ \delta(t)\end{bmatrix}, f(t) = \begin{bmatrix}T\phi(t)\\ T\delta(t)\end{bmatrix}$$

where $T\phi(t)$ is an exogenous roll-torque disturbance and $T\delta(t)$ is the steering torque provided by the actuator on the handlebar axis. The remaining quantities are the symmetric mass matrix M, the speed-dependent damping matrix $v(t)C_1$, and the stiffness matrix, which is the sum of a constant symmetric part K_0 and a quadratically speed-dependent part $v2K_2$. The entries of these matrices depend on the geometric parameters of the bicycle.

3.4 State Space representation in terms of v

The linearized ordinary differential equation is rewritten in state-space form choosing the roll angle $\phi(t)$, the steering angle $\delta(t)$, and their derivatives $\dot{\phi}(t)$ and $\dot{\delta}(t)$, respectively, as state variables. The control input $u(t)$ is the torque $T_\delta(t)$ applied to the handlebar axis. The measured output $y(t)$ includes all of the state variables. On the basis of the above considerations, the state-space equations are

$$\dot{x}(t) = A\big(v(t)\big)x(t) + Bu(t),$$

$$y(t) = Cx(t) + Du(t),$$

where

$$x(t) = [\phi(t)\delta(t)\phi(t)\delta(t)]_T, u(t) = T_\delta(t)$$

$$y(t) = \phi(t)$$

Using Carvallo Whipple bicycle model,

$$M\ddot{q}(t)+v(t)C_1\dot{q}(t)+(K_0+v(t)^2K_2)q(t)=f(t),$$

$$q = \begin{pmatrix}\phi\\ \delta\end{pmatrix}; T = \begin{pmatrix}0\\ T\delta\end{pmatrix}; O = \begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}; I = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},$$

$q_1 = q$; $q_2 = q'$; $q_2 = q'_1$,

$q'_2 = -M^{-1}CV\, q_2 - M^{-1}(K_0 + K_2V^2)q_1 + M^{-1}T$,

$$\begin{pmatrix}q'_1\\ q'_2\end{pmatrix} = \begin{pmatrix}O & I\\ -M^{-1}(K_0 + K_2V^2) & -M^{-1}CV\end{pmatrix}\begin{pmatrix}q_1\\ q_2\end{pmatrix} + M^{-1}\begin{pmatrix}O\\ T\end{pmatrix}$$

Comparing this equation with standard state space representation we get,

$$A = \begin{pmatrix}O & I\\ -M^{-1}(K_0 + K_2V^2) & -M^{-1}CV\end{pmatrix}$$

$$B = \begin{pmatrix}O\\ M^{-1}\begin{pmatrix}0\\ 1\end{pmatrix}\end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 \end{pmatrix}$$

The entries of the matrices A(v(t)), B, C, and D depend on the geometry as well as the physical parameters of the bicycle. In particular, the numerical values of the matrices A(v(t)), B, C, and D for the prototype shown are

$$A(v) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 13.67 & 0.225 - 1.319\mathrm{v}^2(\mathrm{t}) & -0.164\mathbf{v}(\mathbf{t}) & -0.552\mathbf{v}(\mathbf{t}) \\ 4.857 & 10.81 - 1.125\mathrm{v}^2(\mathrm{t}) & 3.621\mathrm{v}(\mathrm{t}) & -2.388\mathbf{v}(\mathbf{t}) \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ -0.339 \\ 7.457 \end{bmatrix}$$

4. Project Tasks

4.1 Case 1: v = 0 m/s

4.1.1: State Space Representation

$$\begin{pmatrix} \phi' \\ \delta' \\ \phi'' \\ \delta'' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 13.67 & 0.225 & 0 & 0 \\ 4.857 & 10.81 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi \\ \delta \\ \phi' \\ \delta' \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -0.339 \\ 7.457 \end{pmatrix} T_\delta$$

$$y = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi \\ \delta \\ \phi' \\ \delta' \end{pmatrix}$$

4.1.2: Transfer Function Representation

$$G(s) = \frac{-0.34s^2 + 5.34}{s^4 - 24.48s^2 + 146.68}$$

4.1.3: Poles

$p_1 = -3.7432; p_2 = -3.2355; \; p_3 = 3.2355; p_4 = 3.743$

4.1.4: Zeroes

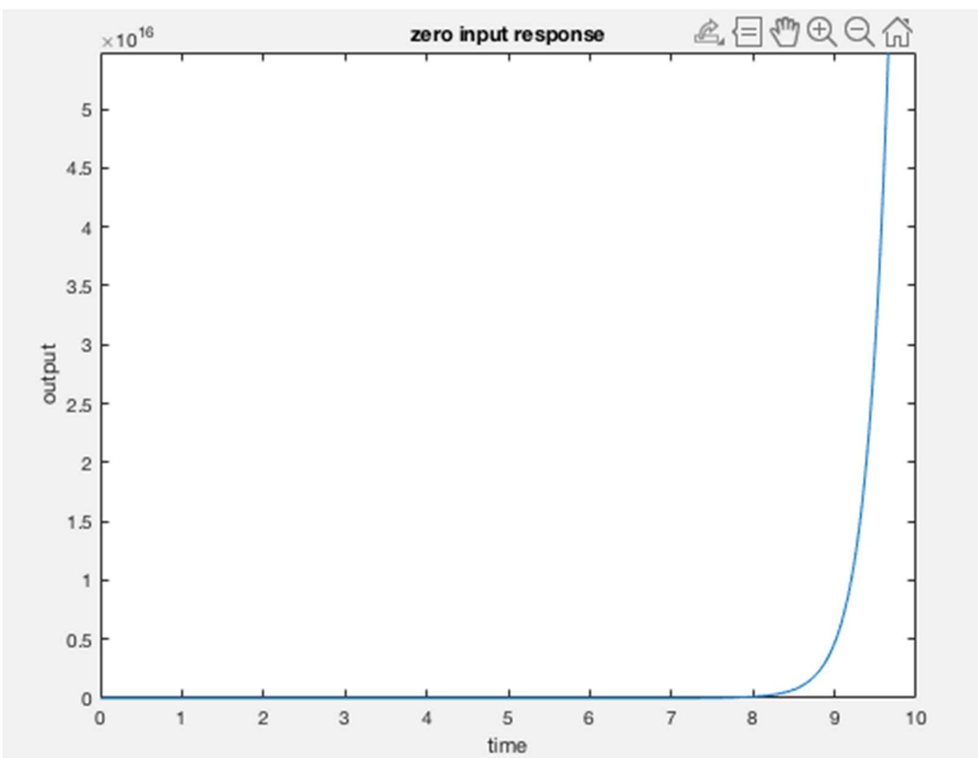
$$z_1 = -3.9631; z_2 = 3.9631$$

4.1.5: Eigenvalues of Matrix A

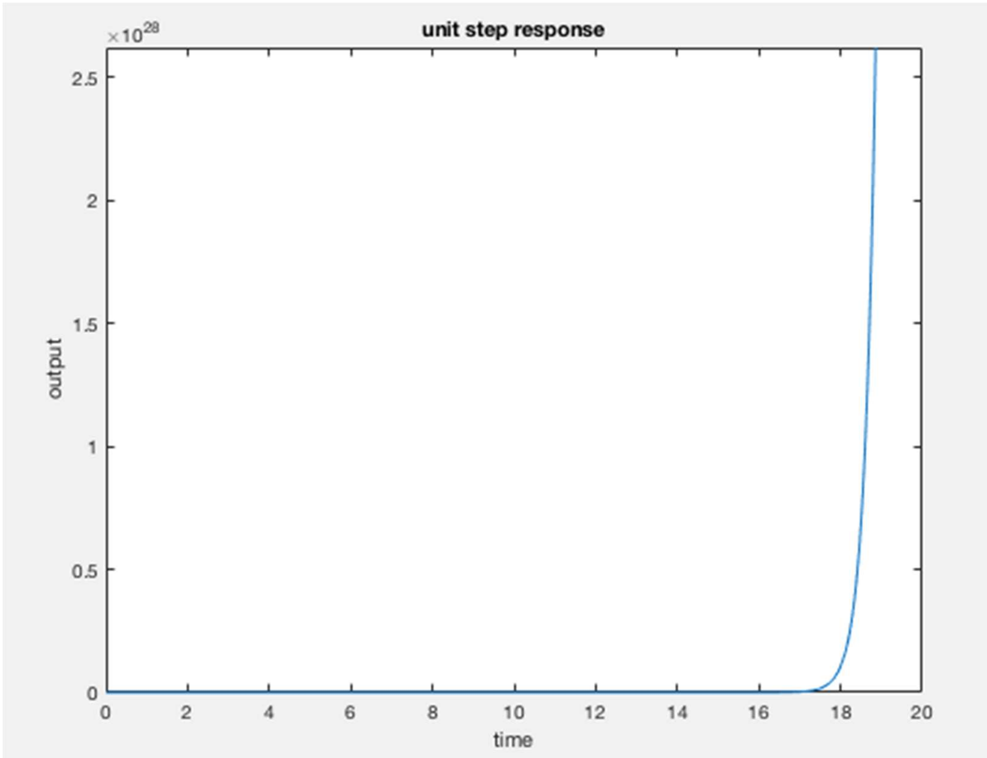
- \- 3.7432, - 3.2355, 3.2355, 3.7432

4.1.6: Zero Input Response with Non-zero Initial States

$$x_0 = \begin{pmatrix} 21 \\ 15 \\ 7 \\ 13 \end{pmatrix}$$



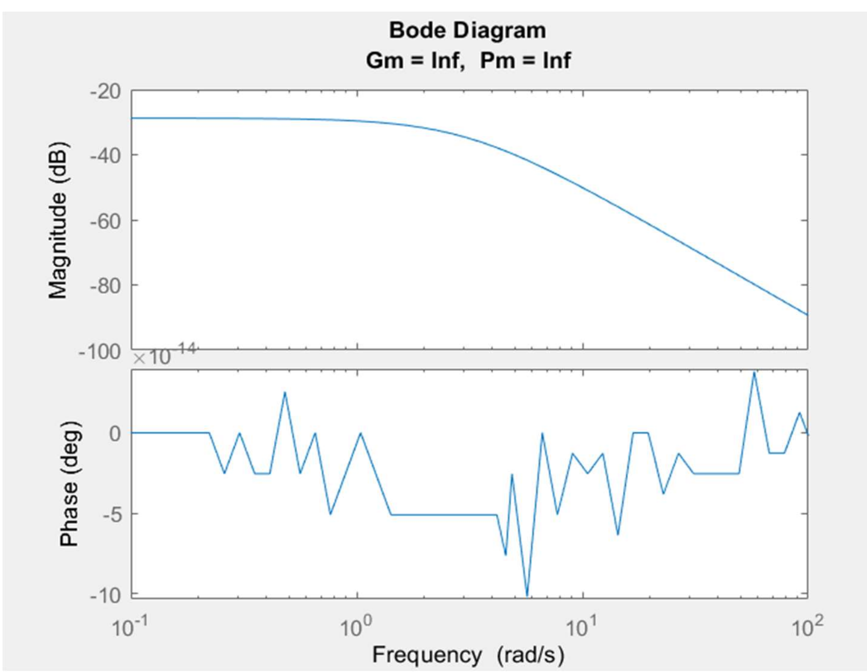
4.1.7: Unit Step Input Response



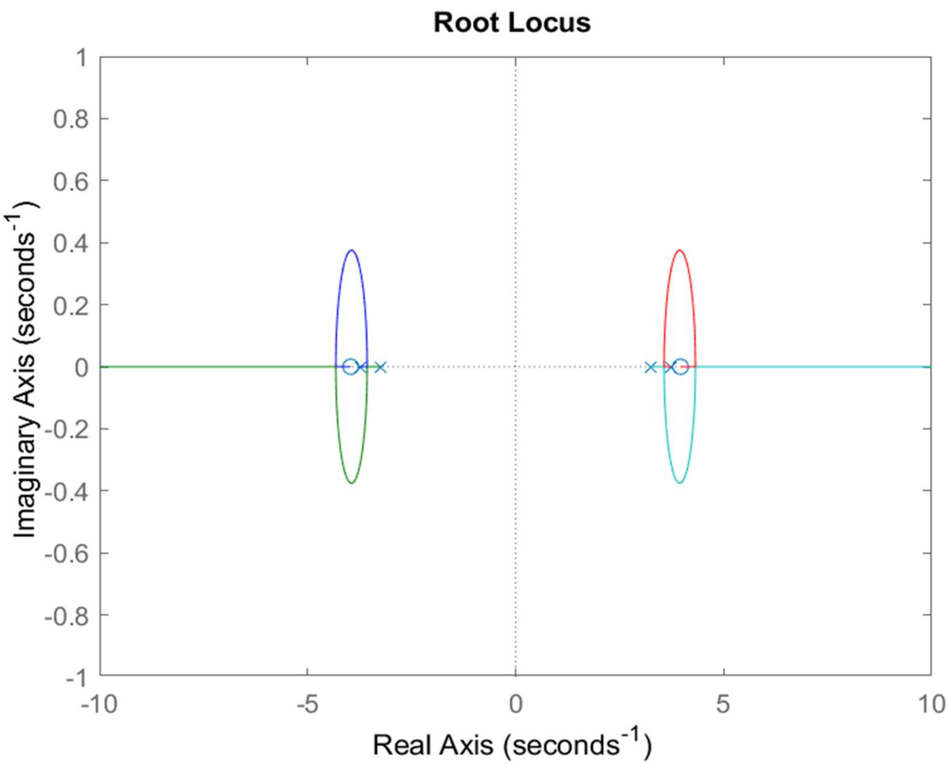
4.1.8: Stability

The system is unstable at this velocity because 2 of the poles lie in the right half plane.

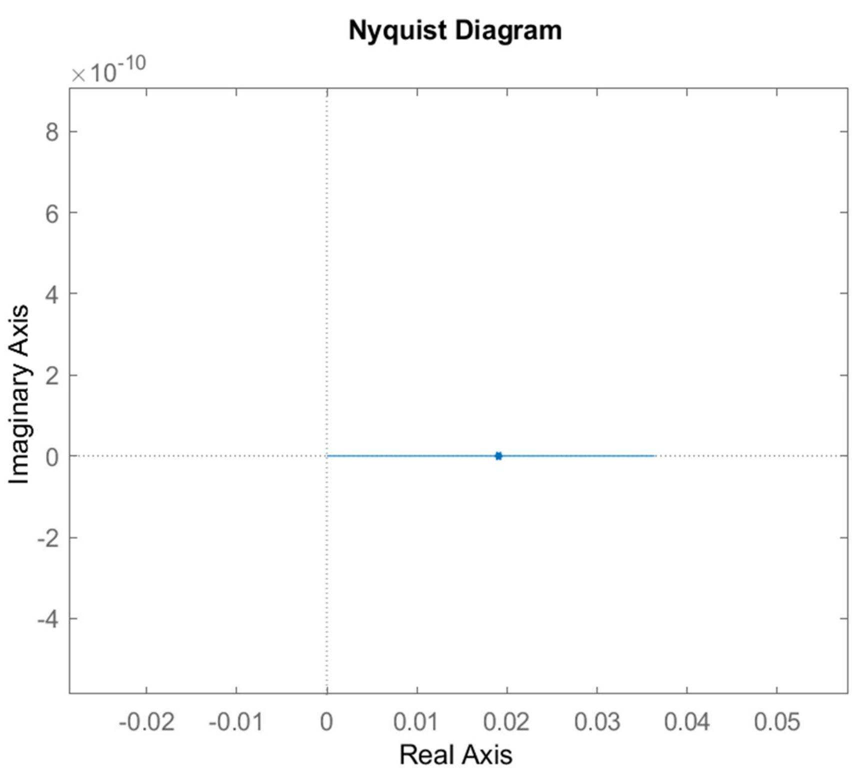
4.1.9: Bode Plot



4.1.10: Root Locus



4.1.11: Nyquist Plot



At V = 0 Closed loop Unstable

4.1.12: Gain Margin and Phase Margin

Gain Margin = infinity

Phase Margin = infinity

4.2 Case 2: v = 3.5 m/s

4.2.1: State Space Representation

$$\begin{pmatrix} \phi' \\ \delta' \\ \phi'' \\ \delta'' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 13.67 & -15.93 & -0.574 & -1.932 \\ 4.857 & -2.971 & 12.673 & -8.358 \end{pmatrix} \begin{pmatrix} \phi \\ \delta \\ \phi' \\ \delta' \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -0.339 \\ 7.457 \end{pmatrix} T_\delta$$

$$y = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi \\ \delta \\ \phi' \\ \delta' \end{pmatrix}$$

4.2.2: Transfer Function Representation

$$G(s) = \frac{-0.34s^2 - 17.249s - 119.796}{s^4 + 8.932s^3 + 18.583s^2 + 98.716s + 36.758}$$

4.2.3: Poles

$p_1 = -0.39; p_2 = -8.07;\ p_3 = -0.23 + 3.38i; p_4 = -0.23 - 3.38\mathrm{i}$

4.2.4: Zeroes

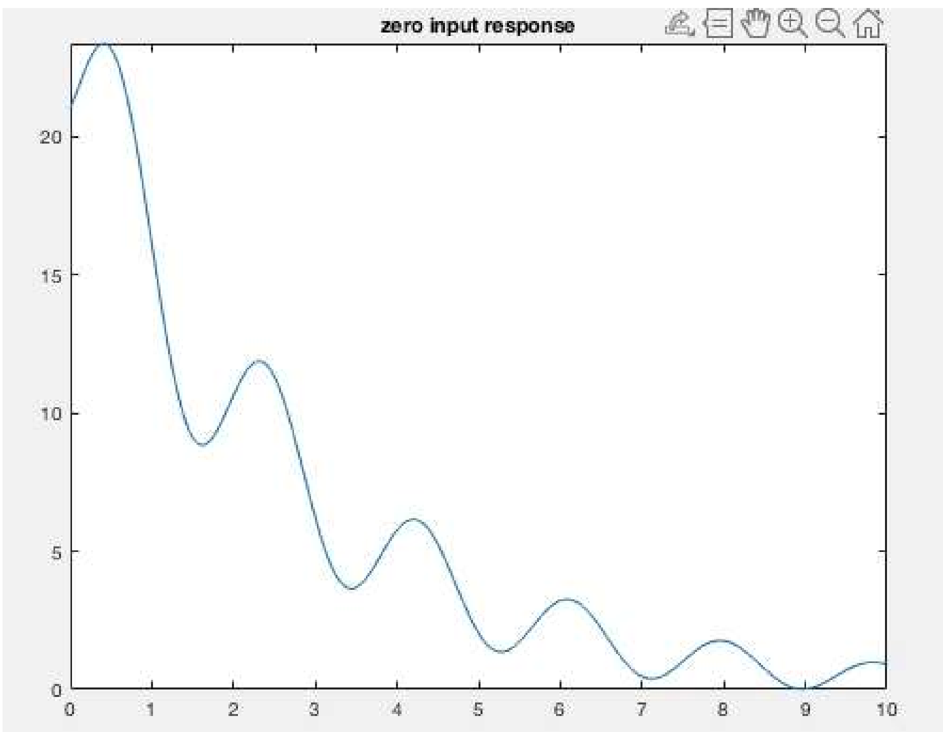
$$z_1 = -8.305; z_2 = -42.550$$

4.2.5: Eigenvalues of Matrix A

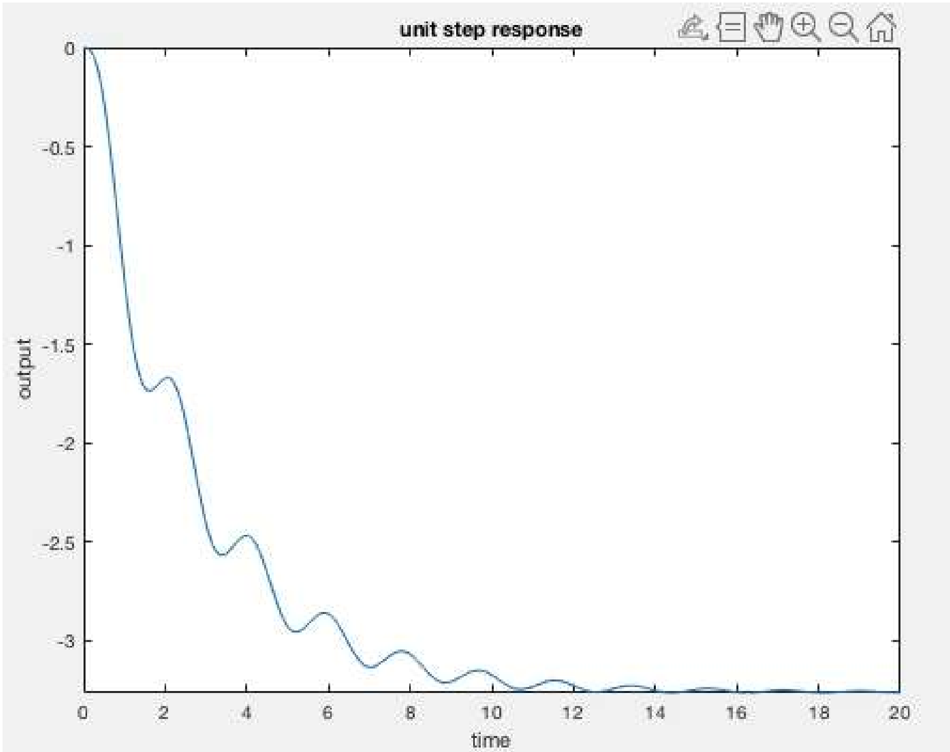
- -0.2301+ 3.3809j, -0.2301 – 3.3809j, -0.3695, -8.073

4.2.6: Zero Input Response with Non-zero Initial States

$$x_0 = \begin{pmatrix} 21 \\ 15 \\ 7 \\ 13 \end{pmatrix}$$



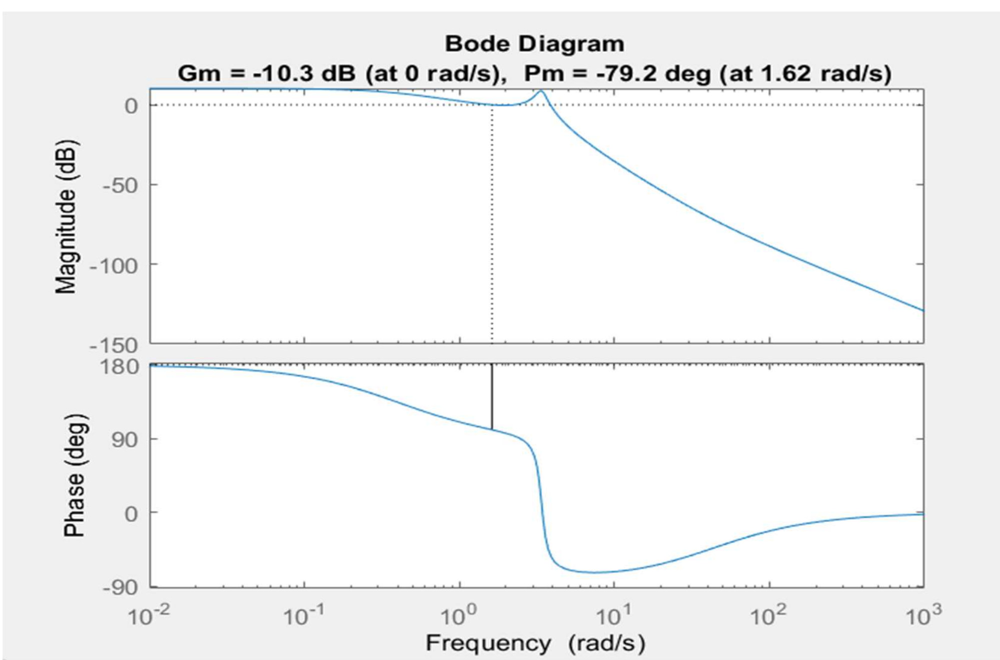
4.2.7: Unit Step Input Response



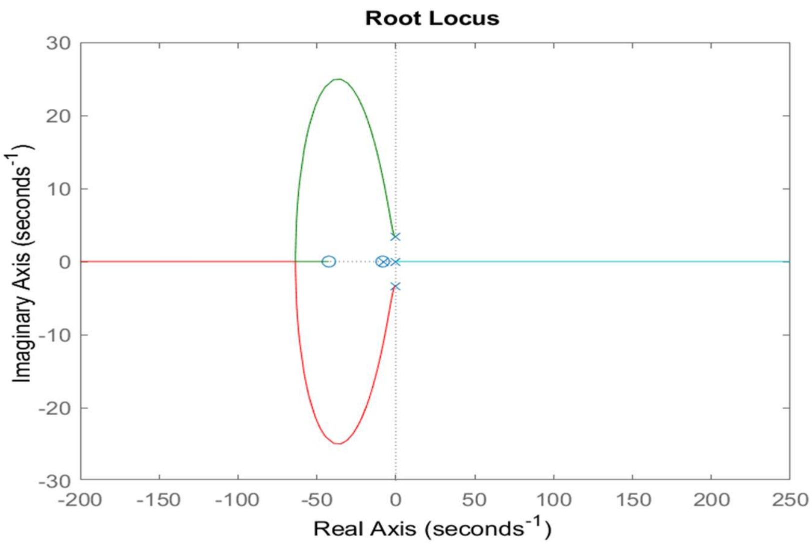
4.2.8: Stability

The system is stable at this velocity because none of the poles lie in the right half plane.

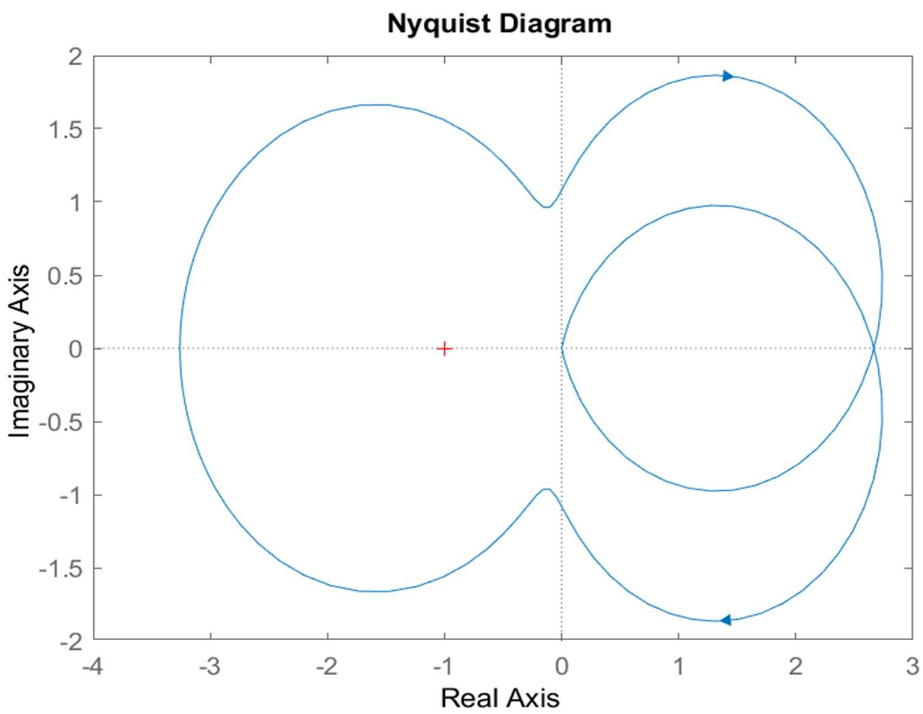
4.2.9: Bode Plot



4.2.10: Root Locus



4.2.11: Nyquist Plot



At V = 3.5 closed loop unstable

4.2.12: Gain Margin and Phase Margin

Gain Margin = -10.3 dB

Phase Margin = -79.2 deg

4.3 Case 3: v = 5 m/s

4.3.1: State Space Representation

$$\begin{pmatrix} \phi' \\ \delta' \\ \phi'' \\ \delta'' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 13.67 & -32.75 & -0.82 & -2.76 \\ 4.857 & -17.315 & 18.105 & -11.94 \end{pmatrix} \begin{pmatrix} \phi \\ \delta \\ \phi' \\ \delta' \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -0.339 \\ 7.457 \end{pmatrix} T_\delta$$

$$y = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi \\ \delta \\ \phi' \\ \delta' \end{pmatrix}$$

4.3.2: Transfer Function Representation

$$G(s) = \frac{-0.34s^2 - 24.63s - 250.86}{s^4 + 12.76s^3 + 63.40s^2 + 457.32s - 77.62}$$

4.3.3: Poles

- $p_1 =$ -1.0330 + 6.4842j; $p_2 =$ − 1.0330 − 6.4842j; $p_3 =$-10.8598 ; $p_4 =$ 0.1658

4.3.4: Zeroes

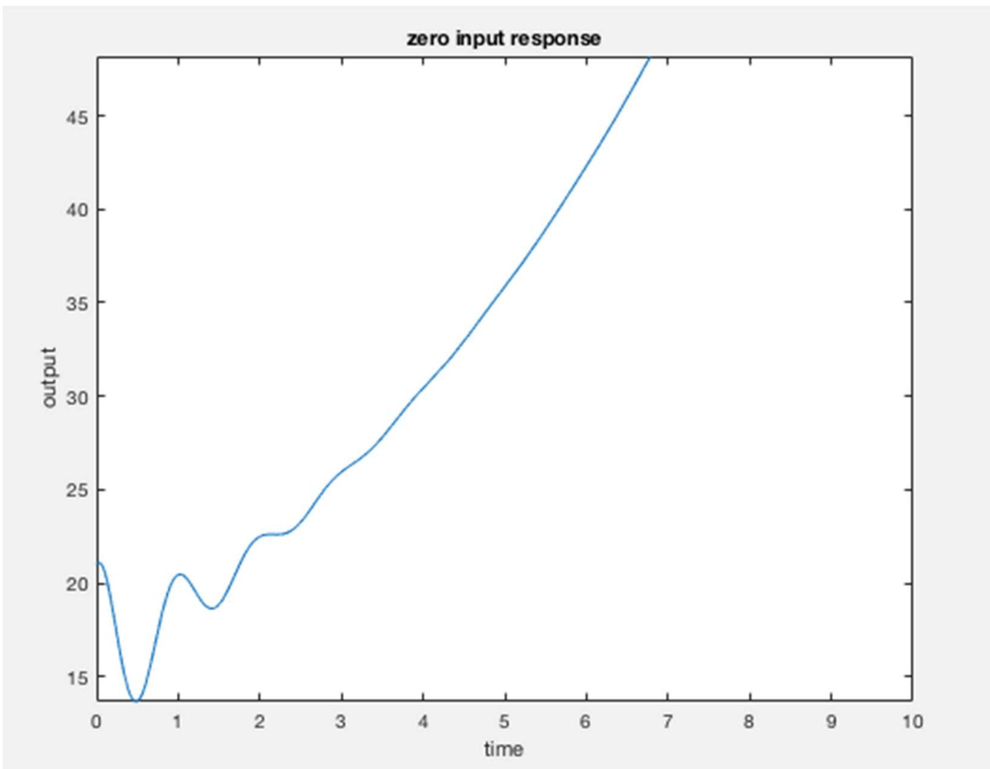
- $z_1 =$- 12.204; $z_2 =$ - 60.448

4.3.5: Eigenvalues of Matrix A

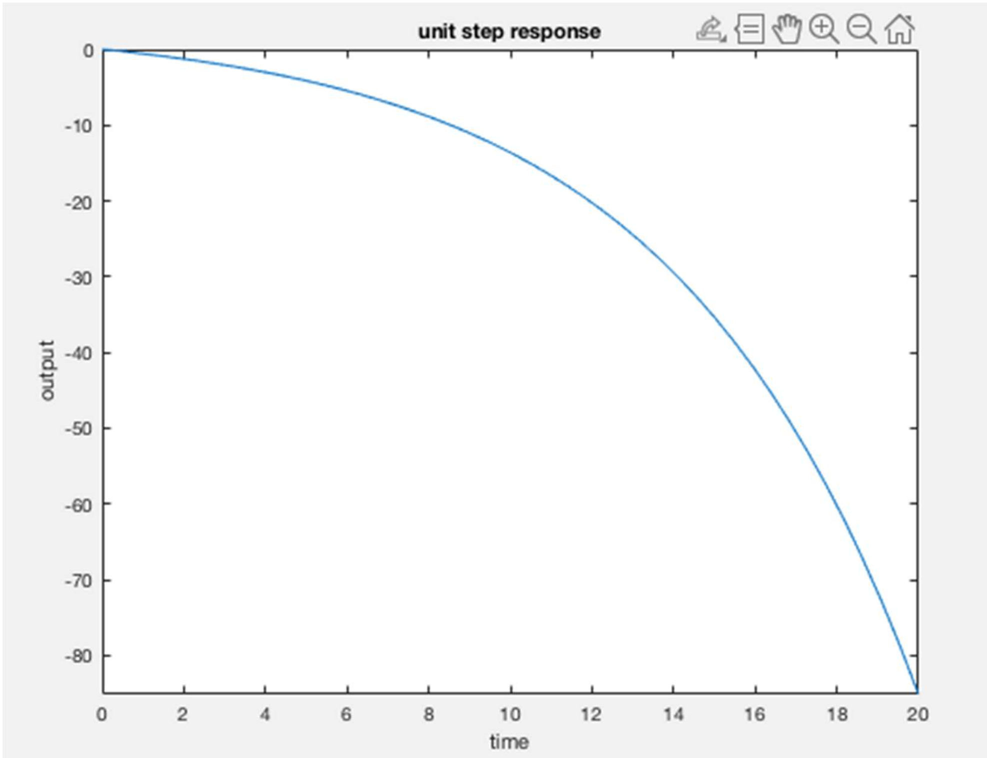
- - 1.0330 + 6.4842j, - 1.0330 - 6.4842j, - 10.8598, 0.1658

4.3.6: Zero Input Response with Non-zero Initial States

$$x_0 = \begin{pmatrix} 21 \\ 15 \\ 7 \\ 13 \end{pmatrix}$$



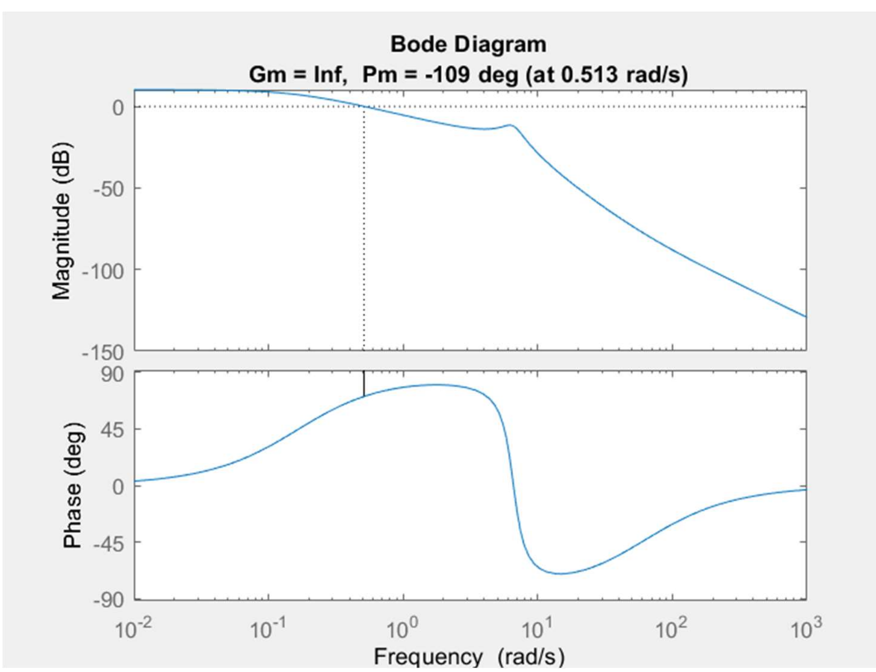
4.3.7: Unit Step Input Response



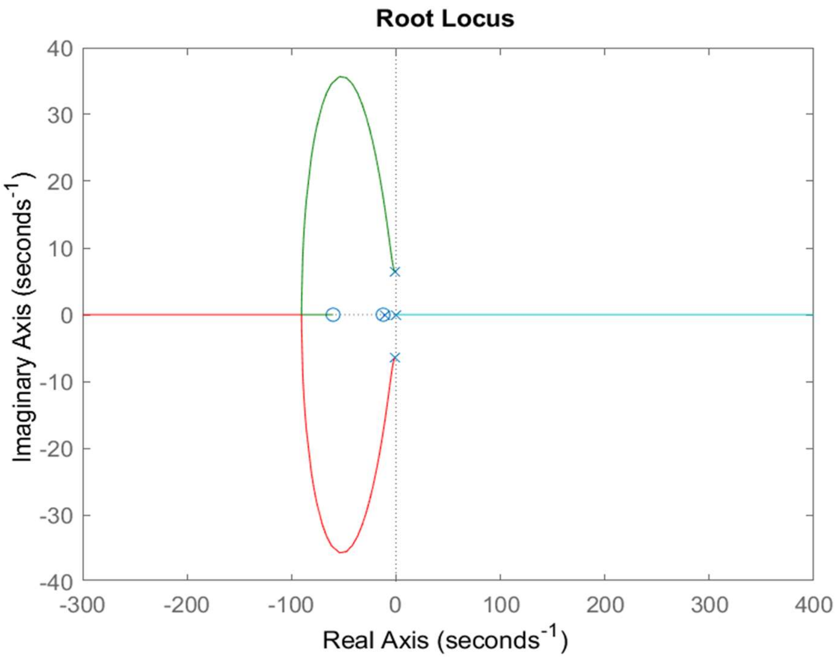
4.3.8: Stability

The system is unstable at this velocity because 1 of the poles lie in the right half plane.

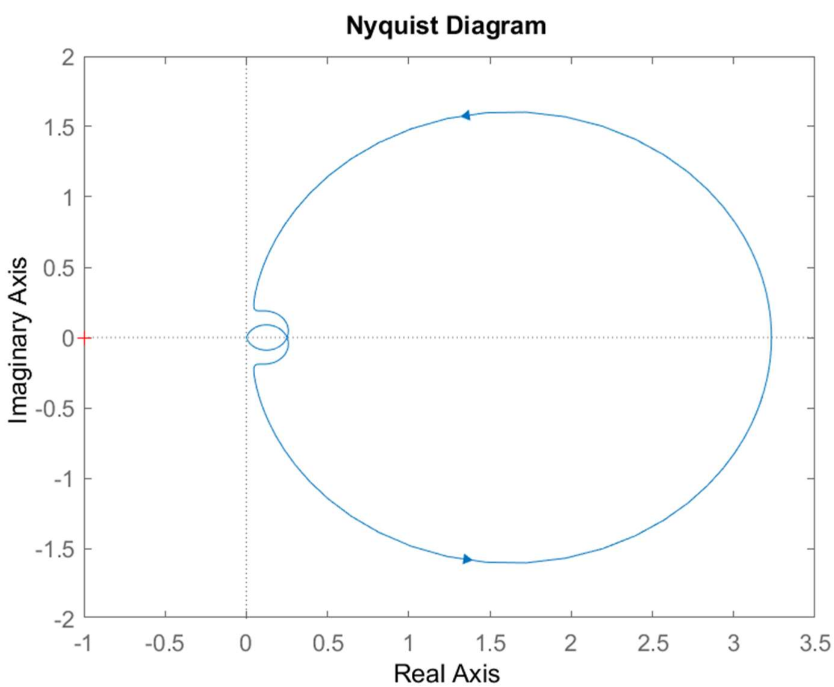
4.3.9: Bode Plot



4.3.10: Root Locus



4.3.11: Nyquist Plot



At V = 5 closed loop unstable

4.3.12: Gain Margin and Phase Margin

Gain Margin = infinity

Phase Margin = -109 deg

5. Controller Design

5.1 Case 1: v = 0

Achieving stability in a system with zero forward velocity can only be done through pole cancellations, but this method is generally considered unreliable. The problem lies in the fact that even if an extra zero is introduced to cancel out the pole, it is unlikely to cancel it out completely, leading to a portion of the root locus getting stuck in the right-half plane and resulting in an unstable closed-loop response. As a result, it is not possible to design a PID Controller or Lag-Lead Controller that can both minimize steady-state error and achieve the desired transient conditions.

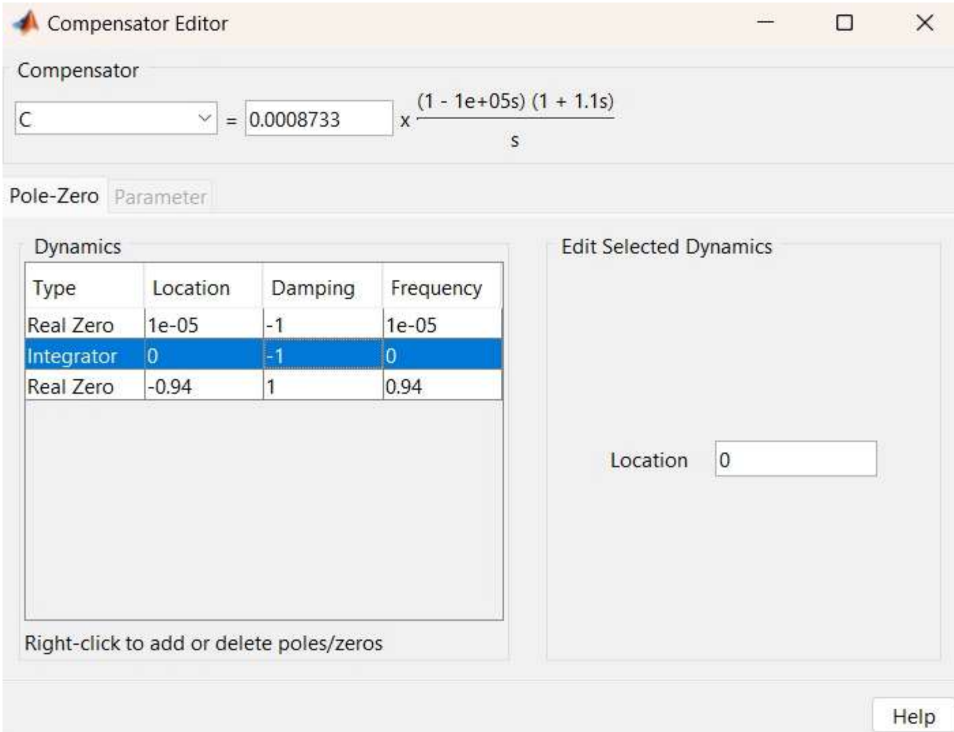
5.2 Case: v = 3.5

5.2.1 Design Specifications

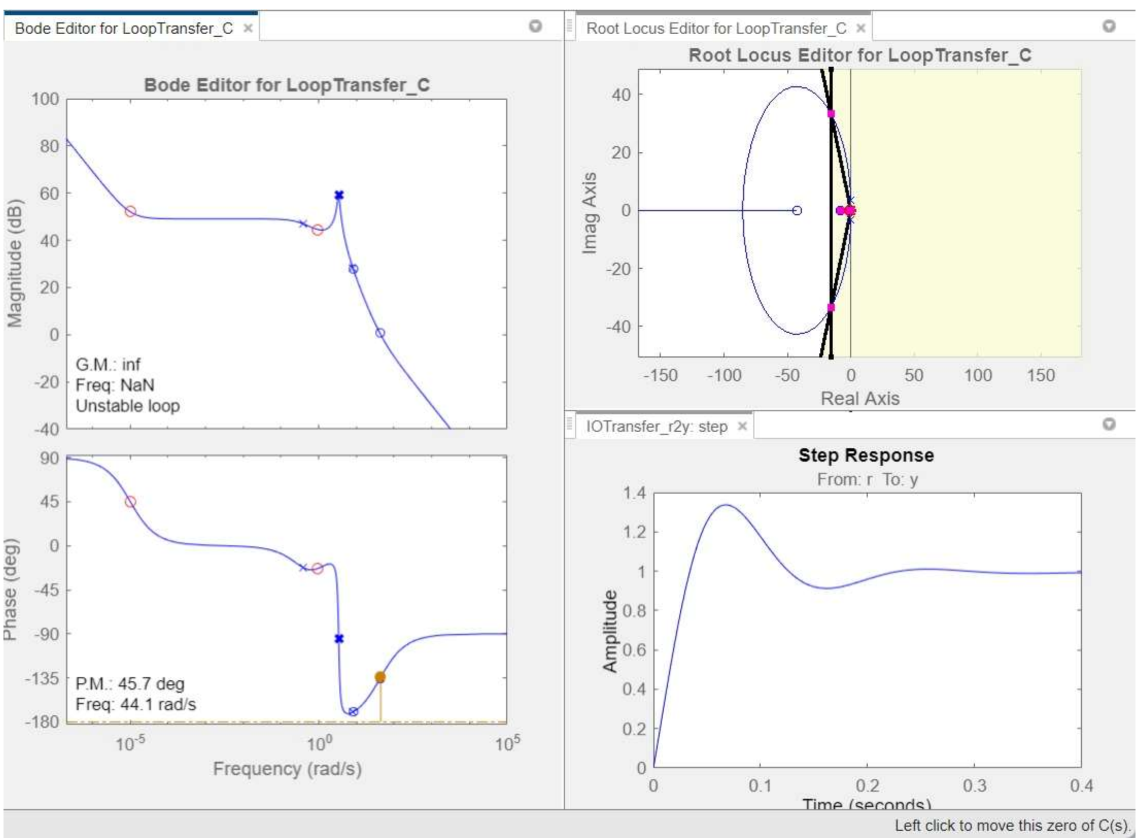
Damping ratio = 0.4252

Settling time = 0.25

5.2.2 Compensator Design



5.2.3 Updated Root Locus and Step Response



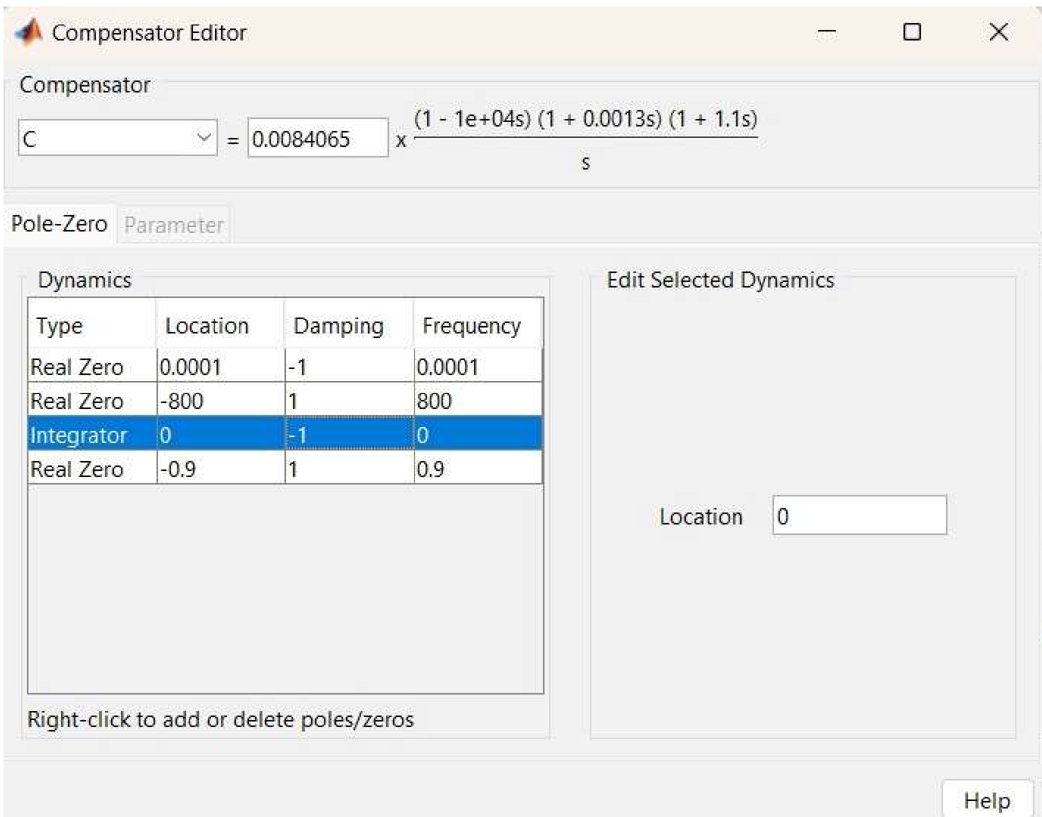
5.3 Case: v = 5

5.2.1 Design Specifications

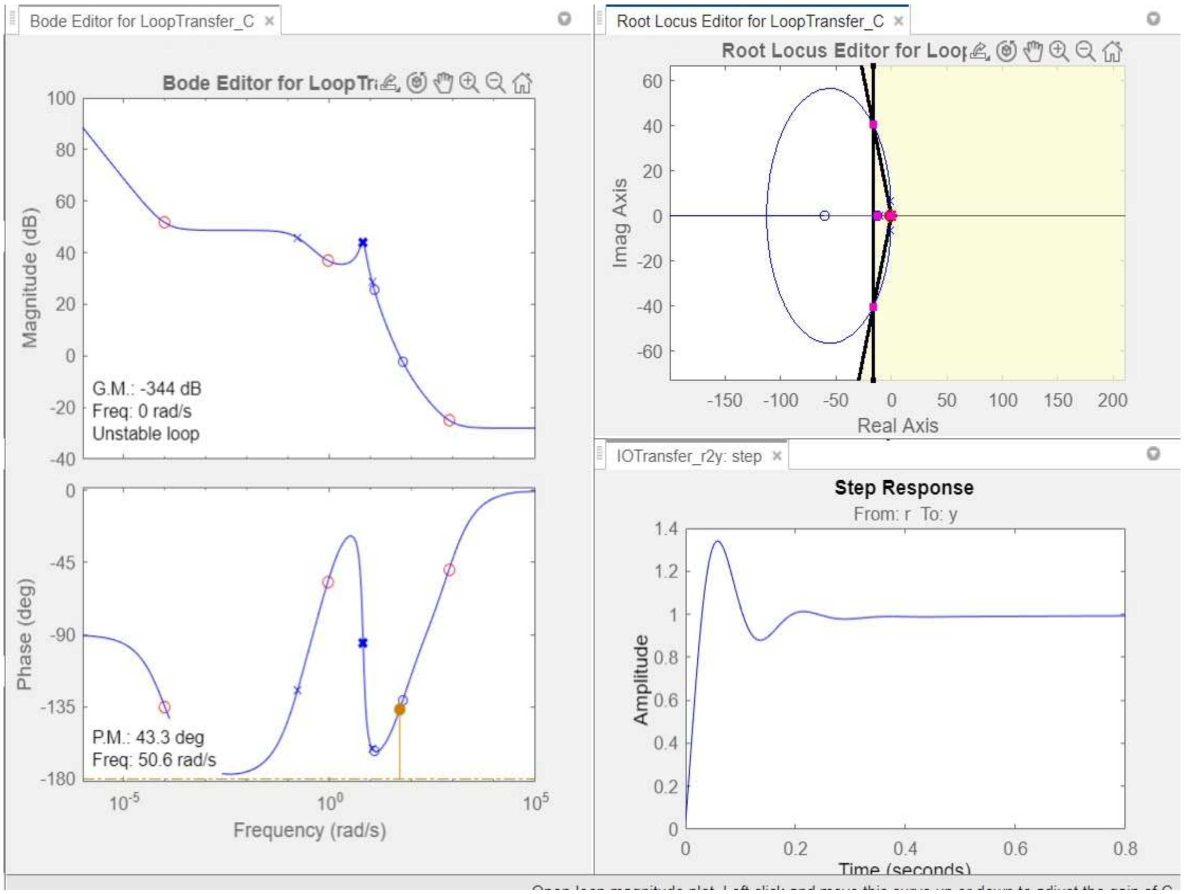
Damping ratio = 0.38

Settling time = 0.23

5.2.2 Compensator Design



5.2.3 Updated Root Locus and Step Response



6. Concluding Remarks

- We obtained the state space representation of Carvallo Whipple bicycle model.
- We obtained transfer function, poles, zeroes and eigen values using MATLAB and plotted the zero input non-zero initial state, and unit step responses for v=0, 3.5 and 5 mps speeds.
- We also observe that eigen values of A are equal to the poles of the transfer function.
- By observing the poles of the transfer function in each case, it can be concluded that the bicycle is unstable for v=0 and v=5 mps. The bicycle is stable for v=3.5mps
- Using MATLAB, Nyquist plots, Bode plots and Root Locus were drawn for each case. Nyquist plots and Root Locus are for closed loop systems while Bode plots are for open loop systems. Such plots help in commenting on the stability of the system.
- It shows that the close loop system is unstable for all the 3 velocities.
- The **sisotool** in MATLAB was used to design controllers for each case considering unity negative feedback. It was found that

designing a controller for v1 = 0m/s case is not possible (excluding pole cancellation).

7. References

- Åström, K. J., Klein, R. E., Lennartsson, A. (2005), "Bicycle Dynamics and Control, "Control Systems Magazine 25(4), 26-47. https://doi.org/10.1109/MCS.2005.1499389
- L. Schwab J. P. Meijaard (2013): "A review on bicycle dynamics and rider control, Vehicle System Dynamics:" International Journal of Vehicle Mechanics and Mobility, 51:7,1059-1090