Digital Image Processing

Team Name	Random
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Title of Project	Multispectral Intrinsic Image Decomposition via Low Rank Constraint
Mentor TA	Surendra Gopireddy
Repo URL	https://github.com/Digital-Image-Processing-IIITH/project-random.git

Model

- The captured luminance spectrum at every point l_p is modelled as the product of Lambertian reflectance spectrum r_p and shading spectrum s_p

$$l_p = s_p . \cdot r_p$$

- Observations of Retinex model:
  - When there is significant reflectance change between two adjacent pixels p and q, the shading is typically constant. This leads to the relation

$$l_p./l_q = r_p./r_q$$

  - When the expected reflectance difference between two pixels is small, the recovered reflectance difference between the two pixels should be small

- **Assumption** : The basis of Retinex theory would continue to take effect on multispectral domain.

Independent Estimation of Reflectance and Shading

- By recognizing two adjacent pixels which have the same shading, the ratio relationship can be written as $l_p . * r_q = l_q . * r_p$, or $L_p r_q = L_q r_p$ where L_p is a diagonal matrix consisting of spectral elements in l_p
- For reflectance, the energy functions can be formulated in terms of reflectance vectors as

$$E_{sc} = \sum_{p,q \in N_{sc}} \left| w_{p,q} \left(L_p r_q - L_q r_p \right) \right|^d \qquad E_{rc} = \left| v_{p,q} \left(r_p - r_q \right) \right|^d$$

where N_{sc} and N_{rc} denote neighborhood pairs, $w_{p,q}$ and $v_{p,q}$ denote weights & d denotes error norm.

- For minimization of energy function, the minimal is found to be achieved when no other constraints are imposed on r_p i.e. when $r_p = l_p$
- Reflectance vector r_p can be written as $r_p = B_r * r'_p$ where Br is the KXJ basis matrix for representing reflectance vector. Thus

$$E_{sc} = \sum_{p,q \in N_{sc}} \left| w_{p,q} \left(L_p B_r\, r'_q - L_q B_r\, r'_p \right) \right|^d \qquad E_{rc} = \left| v_{p,q} \left(B_r r'_p - B_r r'_q \right) \right|^d$$

Independent Estimation of Reflectance and Shading

- The combined energy E = $E_{sc} + \lambda_1 E_{rc}$ i.e. $E = |W_{L,B_r}, R'|^d + \lambda_1 \, |V_{B_r}, R'|^d$
- To circumvent the ambiguity about the scaling factor, the generic constraint on the coefficient sum is expressed as MR′ = C and the original energy function is augmented to enforce this constraint

$$E_{refl} = |W_{L,B_r}, R'|^d + \lambda_1 \, |V_{B_r}, R'|^d + \lambda_2 \, |M_r R' - C|^d$$

- Similarly for shading, $E_{shad} = |W_{B_s}, S'|^d + \lambda_1 \, |V_{L,B_s}, S'|^d + \lambda_2 \, |M_s S' - C|^d$

Simultaneous Estimation of Reflectance and Shading

- For simultaneous estimation of reflectance and shading, the energy functions can directly be formulated as

$$E_{sc} = \sum_{p,q \in N_{sc}} |w_{p,q}(L_p r_q - L_q r_p)|^d + |w_{p,q}(s_p - s_q)|^d = |W_{L,B_r}, R'|^d + |W_{B_s} S'|^d$$

$$E_{rc} = \sum_{p,q \in N_{rc}} |v_{p,q}(L_p s_q - L_q s_p)|^d + |v_{p,q}(r_p - r_q)|^d = |V_{L,B_s}, S'|^d + |V_{B_r} R'|^d$$

$$E_{data} = \sum_{p} |s_p \cdot r_p - l_p|^d = |Q_{S',B_s,B_r} R' - L|^d = |Q_{R',B_r,B_s} S' - L|^d$$

- The problem is to find S′ and R′ that minimize the weighted average of three energy functions :

$$E = E_{sc} + \lambda_1 E_{rc} + 2\lambda_{data} E_{data}$$

- S′ and R′ are solved iteratively by solving shading first through each iteration.
- This solution requires initial estimation of S′ and R′.

Initiating weights

- Various methods are used to determined weights $w_{p,q}$ and $v_{p,q}$ - including pixel gradient, hue, correlation between vectors and learning.
- This implementation uses distance – normalized cosine distance, to signify the differences between spectra of pixels in one neighborhood.

$$d_{p,q \in N_{sc}} = 1 - \frac{l'_p l_q}{|l_p| \,.\, |l_q|}$$

- Shading and reflectance weights $w_{p,q}$ and $v_{p,q}$ are derived as $w_{p,q} = \frac{1}{1 + e^{\alpha(d_{p,q} - \beta)}}$ & $v_{p,q} = 1 - w_{p,q}$
- α and β are parameters of sigmoid function where $\alpha \in [1000, 10000]$ and $\beta \in [10^{-5}, 10^{-2}]$
- Implemented parameters: $\alpha = 5000$; $\beta = 3e^{-4}$

ALGORITHM

01

Assign constant-shading weights $w_{p,q}$ and constant-reflectance weights $v_{p,q}$

02

Solve an initial low rank estimate of the shading image S' using a generic constraint

03

Solve an initial low rank reflectance estimate R' using a generic constraint and the data constraint defined by the previous shading estimate

04

Solve S' using the data constraint defined by the previous reflectance estimate

05

Solve R_e using the data constraint defined by the previous shading estimate (until the solution for S' and R' converge or $\nabla E < 0.01$)

06

Reconstruct S and R to get the refined shading and reflectance

Initial Estimation

- Only horizontal and vertically adjacent pixels are considered in the neighborhood N_{sc} and N_{rc}.
- For an image with N pixels, the sizes of
  - $W_{L,Br}$, V_{Br} and R' : 4NK x NJ_r, 4NK x NJ_r and NJ_r x 1
  - W_{Bs}, $V_{L,Bs}$ and S' : 4NK x NJ_s, 4NK x NJ_s and NJ_s x 1
- With the low rank basis of reflectance, it was found that a J_r around 8 is optimum in the process of fitting reflectance spectra.
- Using L2 norm (d=2), the solution to step-2 of algorithm satisfies

$$Q_s S' = \lambda_2 M_s^T C = \left(W_{B_s}^T W_{B_s} + \lambda_1 V_{L,B_s}^T V_{L,B_s} + \lambda_2 M_s^T M_s\right) S'$$

while the solution to the step-3 of algorithm satisfies

$$Q_r R' = \lambda_{data} Q_{S'}^T L + \lambda_2 M_r^T C = \left(W_{L,B_r}^T W_{L,B_r} + \lambda_1 V_{B_r}^T V_{B_r} + \lambda_2 M_r^T M_r\right) R'$$

- Implemented weights : $\lambda_1 = 2$, $\lambda_2 = 0.01$ and $\lambda_{data} = 1$

DATASET

1

Input Images
3 input scenes

2

Masks
Masking images for 3 scenes

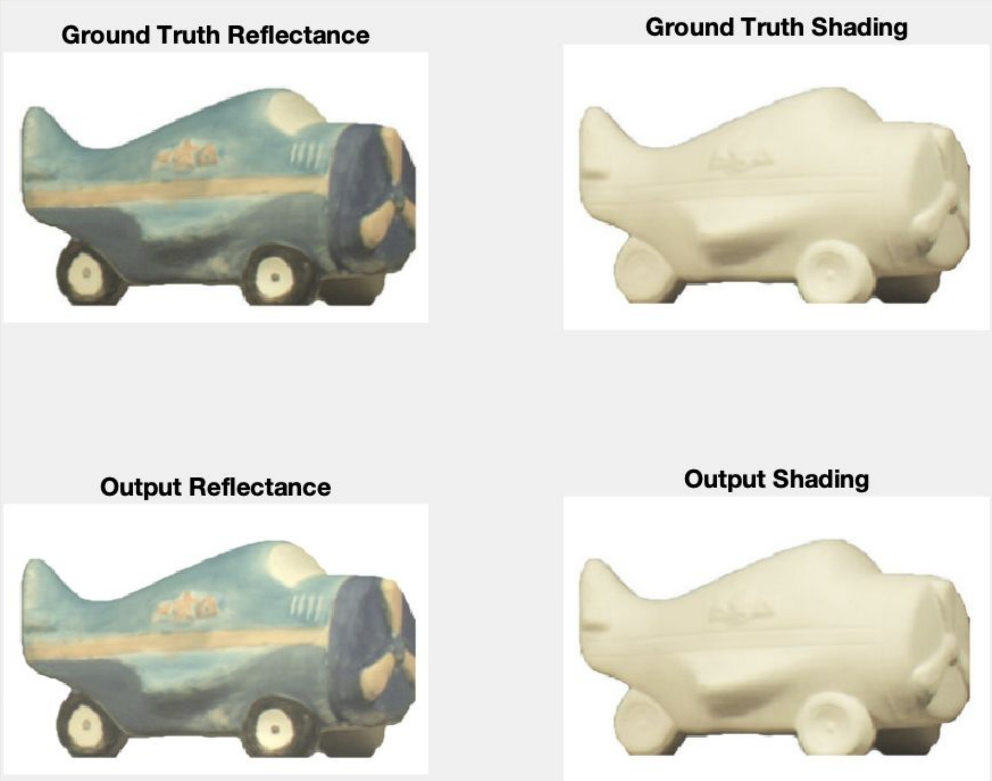
3

GT Shading
Ground truth shading for all the 3 scenes

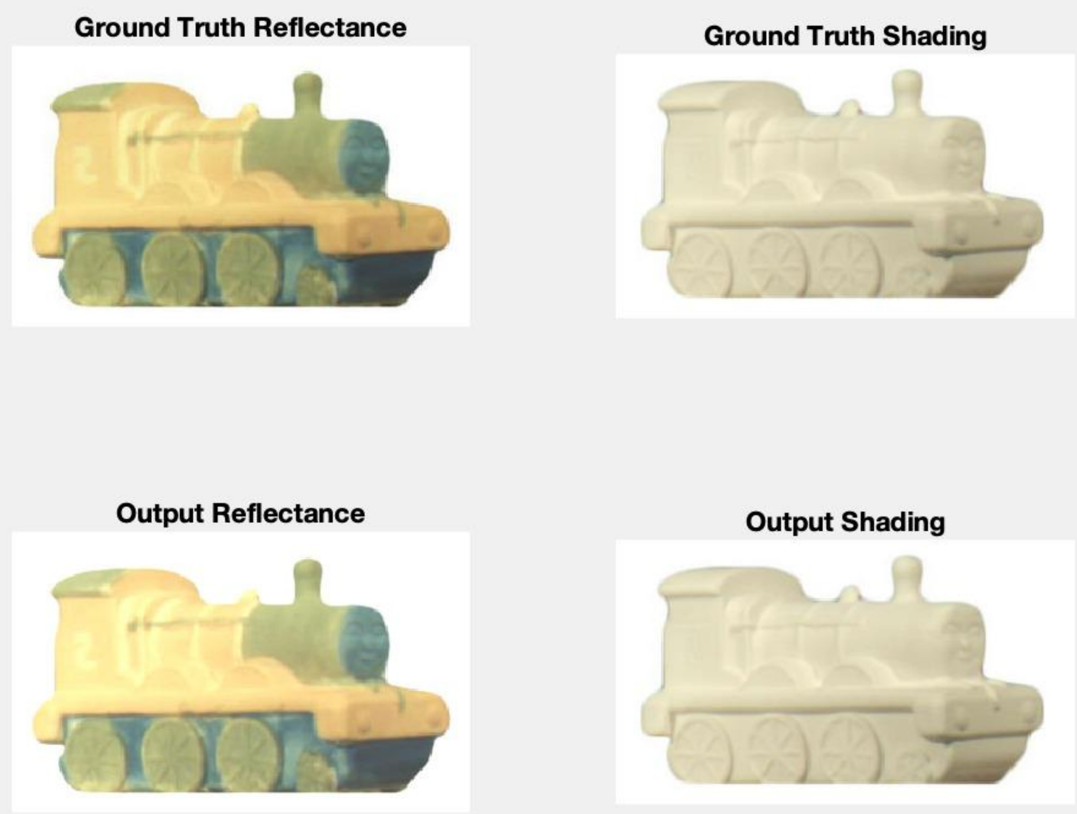
4

GT Reflectance
Ground truth reflectance for all the 3 scenes

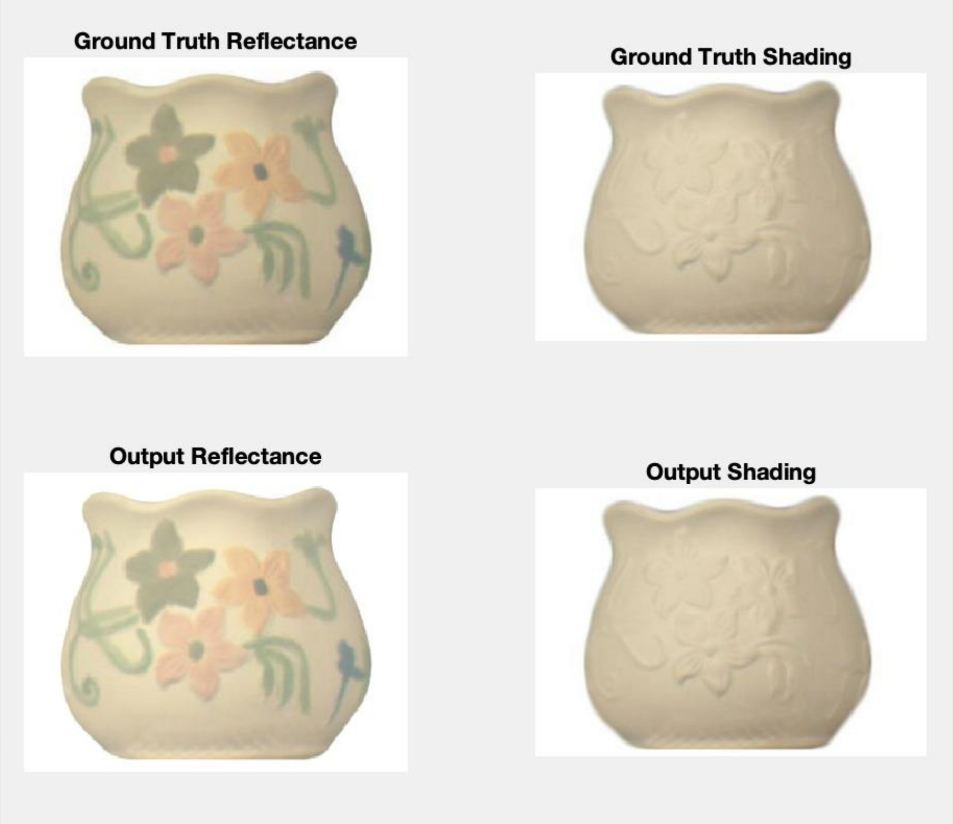
Results - Plane



Results - Train



Results - Cup



Ground Truth Reflectance
Ground Truth Shading
Output Reflectance
Output Shading

Performance - LRIID Vs SIID

Dataset	LMSE (LRIID)	LMSE (SIID)
Plane	0.015	0.024
Train	0.012	0.016
Cup	0.015	0.017

Conclusion

- The problem of the recovery of reflectance and shading from a single multispectral image captured under general spectral illumination was addressed
- Low rank based intrinsic image decomposition approach was attempted on 3 input datasets and shading and reflectance images were generated with an average LMSE of 0.014 while the average LMSE for SIID was 0.019.
- The observed limitations of this algorithm are as follows:
  - Unable to recover global structure.
  - Sensitive to parameter choice.
  - Rely heavily on initial estimation.
  - Illumination has to be known.

Work Division

- Coding and Documentation
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- Presentation and Experimentation
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