Def^n :- If a sample space S has n points which are equally likely and mutually exclusive and an event A has m points then ratio $\frac{m}{n}$ is called probability of event A which is denoted by P(A)

Thus $$P(A) = \frac{n(A)}{n(S)}$$

Addition Theorem :- For events A & B of sample space S,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

This result can be extended to more events also.

De Morgan's laws

$$P(\overline{A \cup B}) = P(\bar{A} \cap \bar{B})$$
$$P(\overline{A \cap B}) = P(\bar{A} \cup \bar{B})$$

Example :- Two cards are drawn from pack of 52 cards. Find probability that 1) both of them are red

2) both of them are dimond

S: space space contains 52_{C_2} points.

1) A is event of selecting red card

$\therefore n(A) = 26_{C_2}$

$\therefore P(A) =$ probability of selecting both red cards

$$= \frac{n(A)}{n(S)} = \frac{26_{C_2}}{52_{C_2}}$$

2) B is event of selecting both dimond cards.

$\therefore n(B) = 13_{C_2}$

P(B) = Probability of selecting both cards dimond

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{13_{C_2}}{52_{C_2}}.$$

Conditional Probability

$P(B/A) \rightarrow$ probability of event B when event A is already taken place is called conditional probability similarly

$P(A/B) \rightarrow$ probability of event A when event B is already taken place.

Multiplication Theorem

$$P(A \cap B) = P(A) \cdot P(B/A)$$

OR

$$P(A \cap B) = P(B) \cdot P(A/B)$$

If two events are independant then.

$$P(A \cap B) = P(A) \cdot P(B)$$

Total probability Theorem

If $A_1, A_2 \cdots A_n$ be partitions of S and B be some event defined on S then

$$P(B) = P(B/A_1) \cdot P(A_1) + P(B/A_2) \cdot P(A_2) + \cdots + \cdots + P(B/A_n)\, P(A_n)$$

Example -: Four roads lead away from a jail. A prisoner trying to escape from jail selects a road at random $1/8$, $1/6$, $1/4$ and $9/10$ are probabilities of selecting roads A, B, C & D what is probability that prisoner will succeed in escaping from jail?

→ E is event of getting success in escaping from jail.

$P(A_1)$ = probability of selecting road A by prisoner = $1/4$

$P(A_2)$ = Probability of selecting road B by prisoner = $1/4$

Ily $P(A_3) = 1/4$ and $P(A_4) = 1/4$

$P(E/A_1)$ → Probability of getting success in escaping provided road A is selected

$\therefore P(E/A_1) = 1/8$

$P(E/A_2)$ → Probability of getting success in escaping provided road B is selected

$P(E/A_2) = 1/6$

Ily $P(E/A_3) = 1/4$

$P(E/A_4) = 9/10$

By Total probability Theorem,

$$P(E) = P(A_1) \cdot P(E/A_1) + P(A_2) \cdot P(E/A_2) + P(A_3) P(E/A_3) + P(A_4) \cdot P(E/A_4)$$

$$= (1/4)\left(\frac{1}{8}\right) + \frac{1}{4} \times \frac{1}{6} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{9}{10}$$

$$= 163/480$$

EXERCISE - I

1. The probabilities that three students A, B and C will pass the common entrance test for engineering are 4 / 9, 2 / 9 and 1 / 3 respectively. The probabilities that they will get admission in the same engineering college are 3 / 10, 1 / 2 and 4 / 5 respectively.

Find the probability that they will get admission in the same engineering college.

[**Ans. :** 23 / 45]

2. The chances that A, B and C will be the Education Minister of Government of India are in the ratio 4 : 1 : 2. The probabilities that they will introduce reservations in professional colleges for backward classes are 0·3, 0·8 and 0·5 respectively.

Find the probability that the bill for reservation will be introduced. [**Ans. :** 3 / 7]

3. In a factory an article is produced on three machines. Their respective productions are 300 units by A, 250 units by B and 450 units by C. It is found that the percentages of defective articles for A, B, C are 1, 1·2 and 2 selected at random from a days production (which are mixed).

Find the probability that the selected article is defective. [**Ans. :** 0·015]

We now state an important theorem known as Bayes' Theorem. It enables us to evaluate what may be called **reverse probabilities**. Suppose there are two boxes (I and II) which contain 2 white and 3 black balls; and 3 white and 4 black balls. If a box is chosen at random and a ball is drawn from it, what is the probability that the ball drawn is white ? We know how to calculate this probability.