

Recursion.

The Simple Engineer.



ATM Analogy.

How many people are in front of me?



ATM Analogy.



ATM Analogy.



ATM Analogy.



ATM Analogy.



ATM Analogy.

```
 1 public int getMyPositionInLine(Person person) {  
 2     if (person.nextInLine == null) {  
 3         return 1;  
 4     }  
 5  
 6     return 1 + getMyPositionInLine(person.nextInLine);  
 7 }
```



What.

```
 1 public void recursion(int someValue) {  
 2     if (someValue == 10){  
 3         return;  
 4     }  
 5     return recursion(someValue + 1);  
 6 }
```

Base case.

Recursive Call.



Why & Why Not.

Pros	Cons
Bridges the gap between elegance and complexity	Slowness due to CPU overhead
Reduces the need for complex loops and auxiliary data structures	Can lead to out of memory errors / stack overflow exceptions
Can reduce time complexity easily with memoization	Can be unnecessarily complex if poorly constructed
Works really well with recursive structures like trees and graphs	



SUBSCRIBE

Call Stack.

How Memory Works.



Call Stack.



Check Email



Call Stack.



Attend Meeting



Check Email



Call Stack.



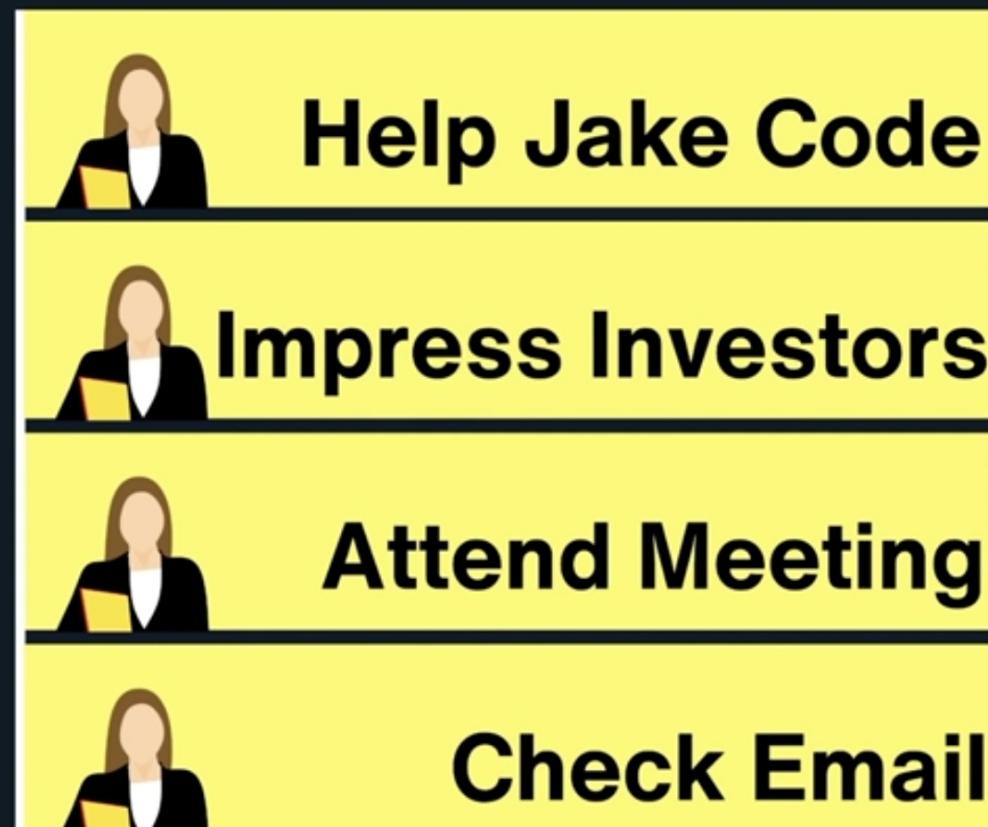
Impress Investors

Attend Meeting

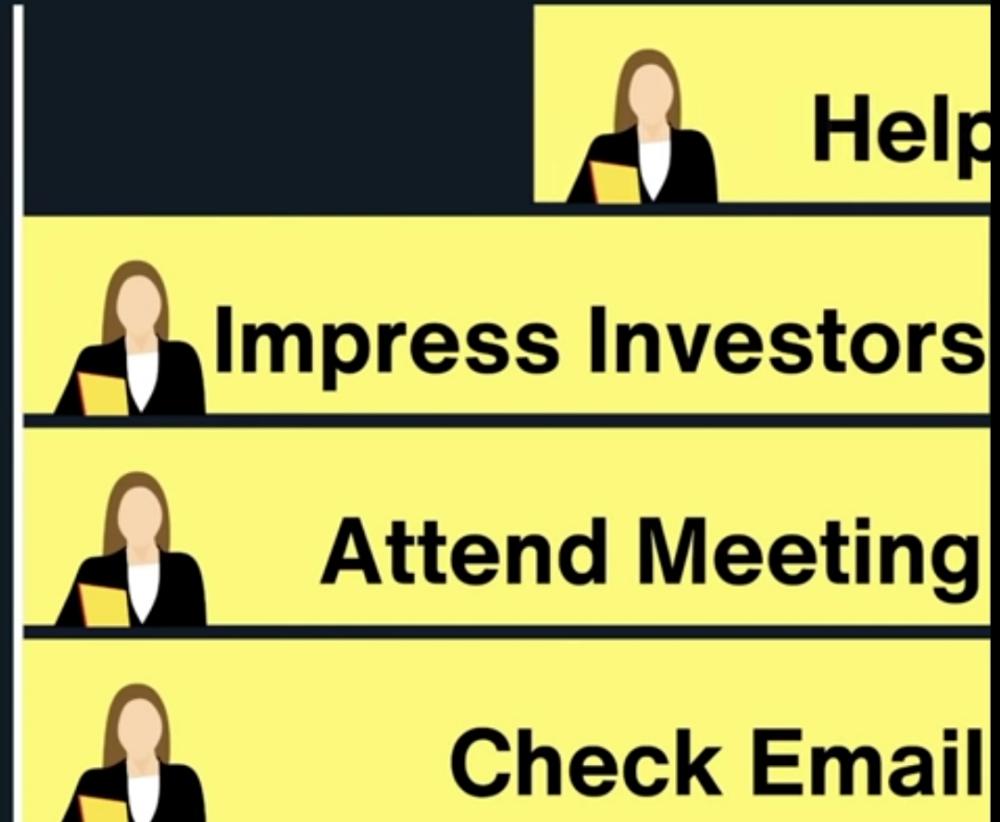
Check Email



Call Stack.



Call Stack.



Call Stack.



Impress In



Attend Meeting



Check Email



Call Stack.



Check Email



Call Stack.



Ch



Call Stack.



Call Stack.

```
1 // Expected output = "hello my friends."
2 public String A() {
3     return "hello " + B();
4 }
5
6 public String B() {
7     return "my " + C();
8 }
9
10 public String C() {
11     return "friends.";
12 }
```

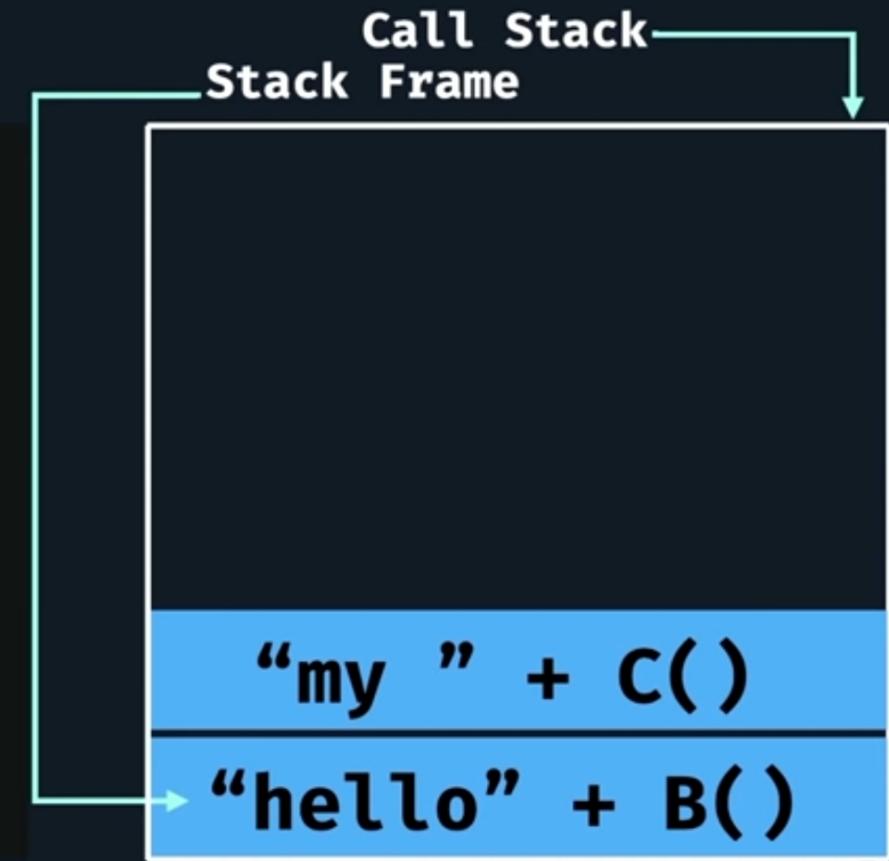
Call Stack
Stack Frame

“hello” + B()



Call Stack.

```
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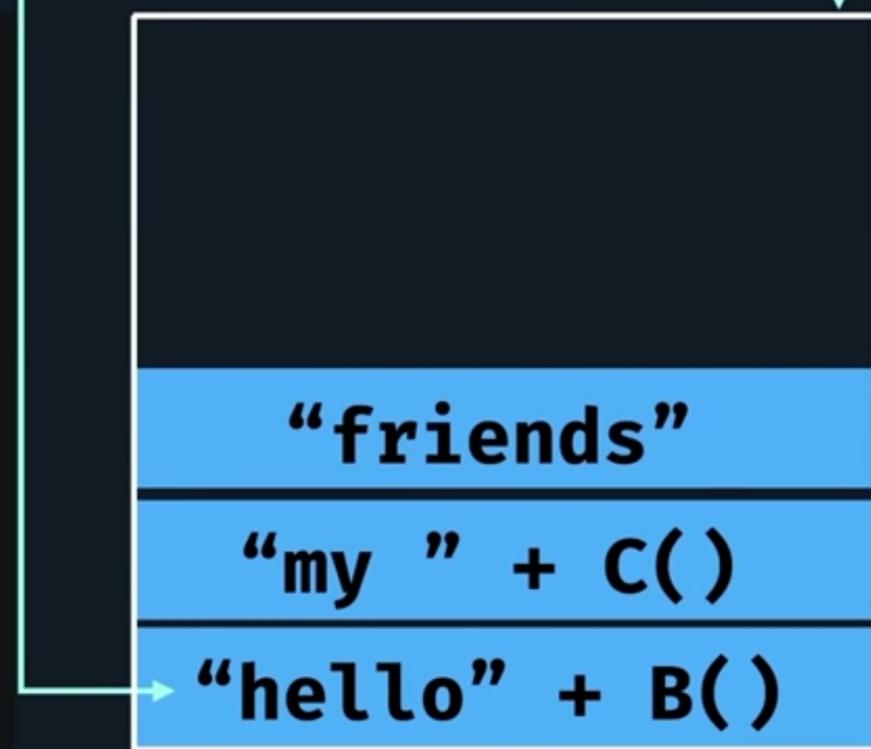


SUBSCRIBE

Call Stack.

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```

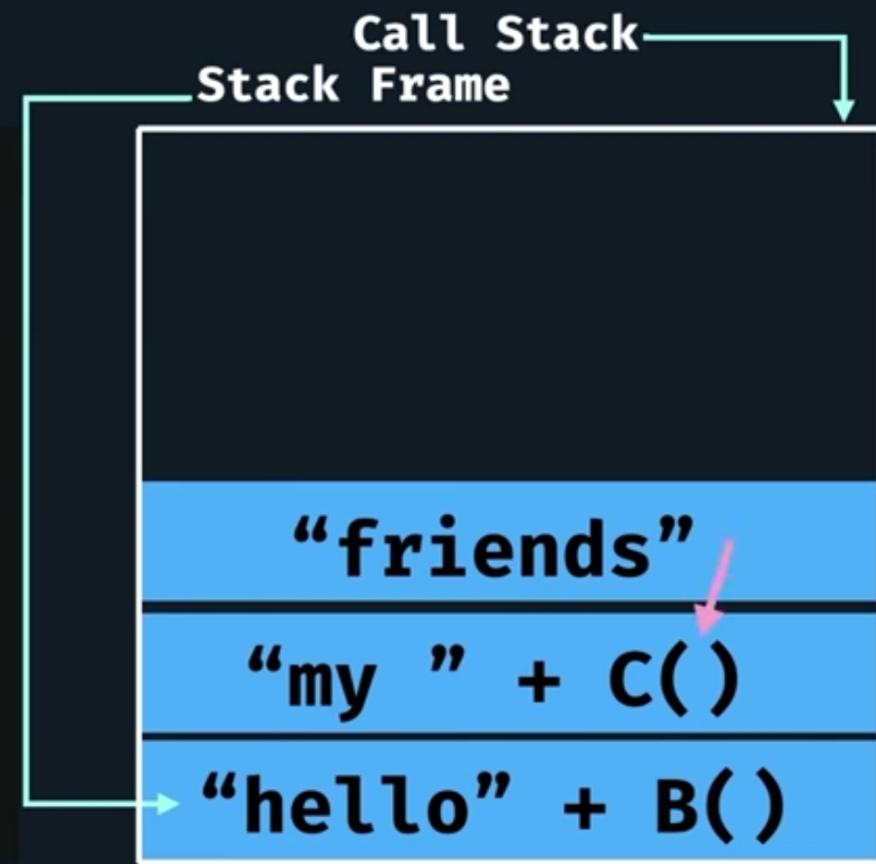
Call Stack
Stack Frame



SUBSCRIBE

Call Stack.

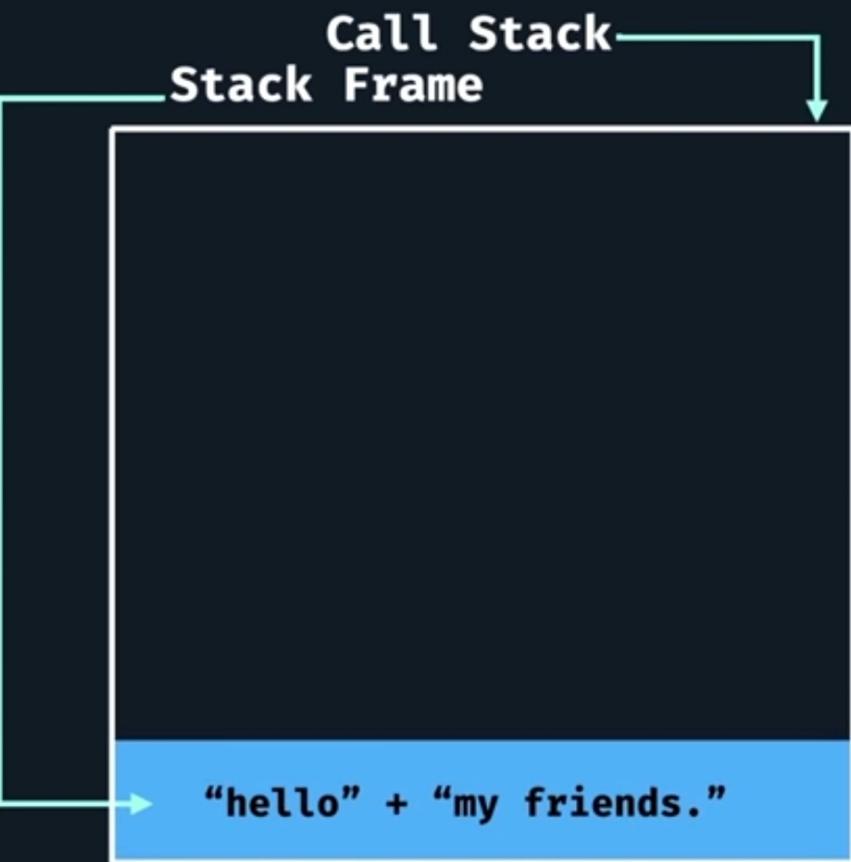
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SUBSCRIBE

Call Stack.

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Call Stack.

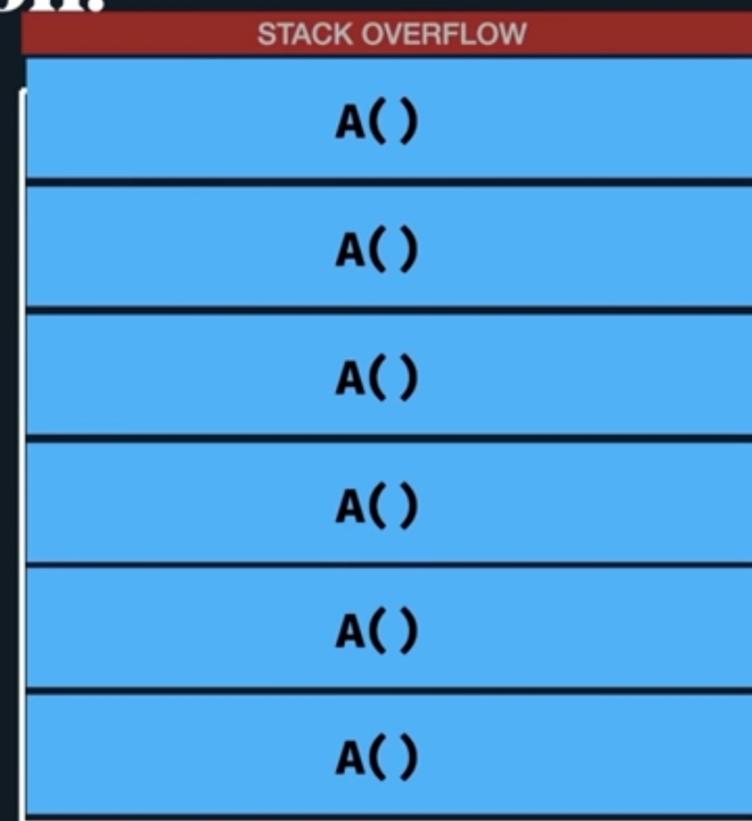
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4 }  
5  
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7     return "my " + C();  
8 }  
9  
10 public String C() {  
11     return "friends.";  
12 }
```



Call Stack With Recursion.



```
1 public String A() {  
2     return A();  
3 }
```



SUBSCRIBE

String Reversal.



```
1 public String reverseString(String input) {  
2     // What is the base case?  
3     // What is the smallest amount of work I can do in each iteration?  
4 }
```

When can I no longer continue?

One letter.

Empty String.



String Reversal.



```
1 public String reverseString(String input) {  
2     if (input.equals("")) {  
3         return "";  
4     }  
5     // What is the smallest amount of work I can do in each iteration?  
6 }
```

What's the smallest unit of work I can do to contribute to the goal?

Pick a single character and continue.



String Reversal.



```
1 public String reverseString(String input) {  
2     if (input.equals("")) {  
3         return "";  
4     }  
5     // What is the smallest amount of work I can do in each iteration?  
6     return reverseString(input.substring(1)) + input.charAt(0);  
7 }
```



String Reversal.

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 1 public String reverseString(String input) {  
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 7 }
```

The diagram illustrates the recursive calls for the string "hello". A green arrow points from the recursive call in line 6 of the code to the first stack frame. The stack frames are:

- reverseString("") + "o"
- reverseString("o") + "l"
- reverseString("lo") + "l"
- reverseString("llo") + "e"
- reverseString("ello") + "h"

Below the stack frames, the text "input: 'hello'" is displayed.



String Reversal.

```
 1 public String reverseString(String input) {  
 2     if (input.equals("")) {  
 3         return "";  
 4     }  
 5     // What is the smallest amount of work I can do in each iteration?  
 6     return reverseString(input.substring(1)) + input.charAt(0);  
 7 }
```

The diagram illustrates the recursive call stack for the `reverseString` function. It shows five vertical teal-colored boxes representing the state of the function at different points in its execution:

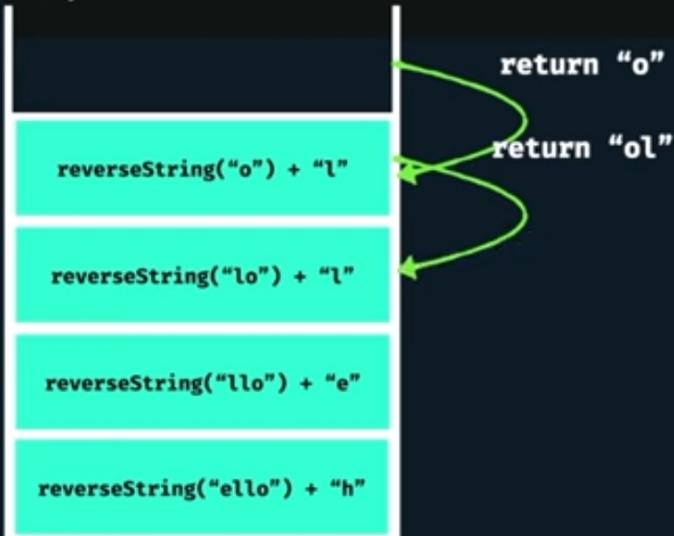
- Top box: `reverseString("") + "o"`
- Second box: `reverseString("o") + "l"`
- Third box: `reverseString("lo") + "l"`
- Fourth box: `reverseString("llo") + "e"`
- Bottom box: `reverseString("ello") + "h"`

A green arrow points from the text "return "o"" to the top box. Another green arrow points from the bottom of the second box up to the top of the first box, indicating the flow of the recursive call.



String Reversal.

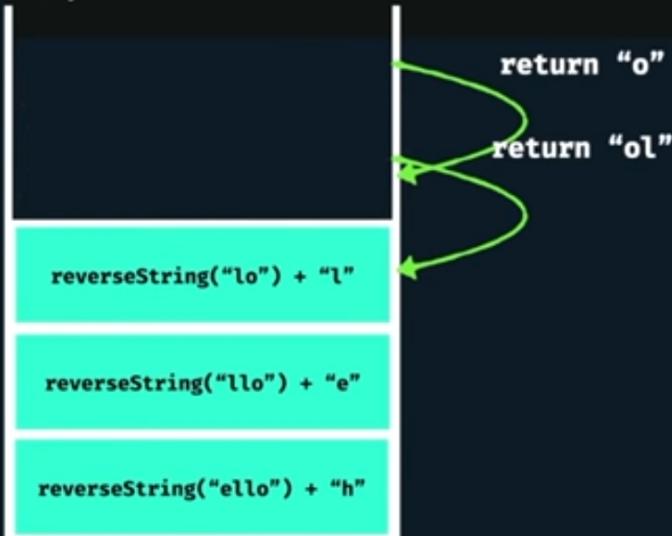
```
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 4     }  
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 7 }
```



SUBSCRIBE

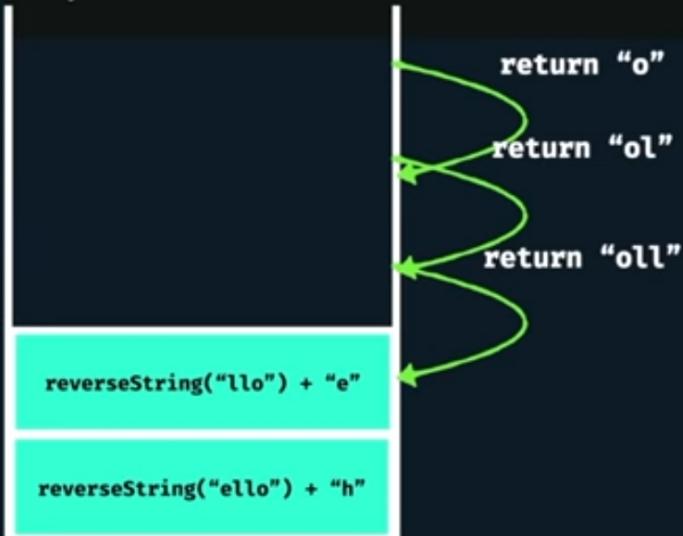
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String Reversal.

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7 }
```

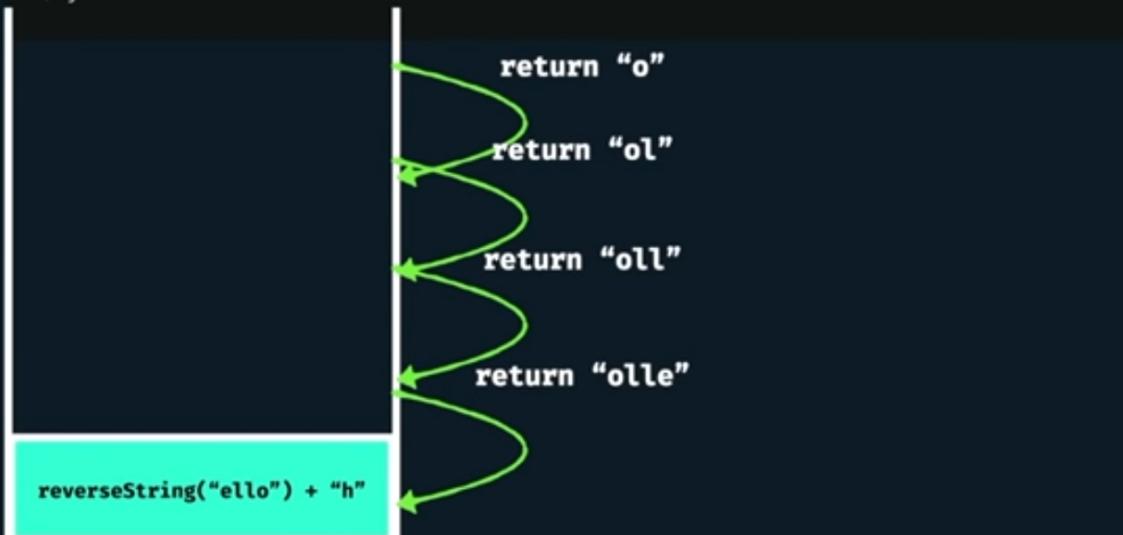


SUBSCRIBE

String Reversal.



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SUBSCRIBE

String Reversal.

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4     }  
5     // What is the smallest amount of work I can do in each iteration?  
6     return reverseString(input.substring(1)) + input.charAt(0);  
7 }
```

Shrinks the problem space.

Smallest unit of work to contribute.



SUBSCRIBE

Palindrome.

```
1 public static boolean isPalindrome(String input) {  
2     // Define the base-case / stopping condition  
3     if (input.length() == 0 || input.length() == 1) {  
4         return true;  
5     }  
6  
7     // Do some work to shrink the problem space  
8     if (input.charAt(0) == input.charAt(input.length() - 1)) {  
9         return isPalindrome(input.substring(1, input.length() - 1));  
10    }  
11  
12    // Additional base-case to handle non-palindromes  
13    return false;  
14 }
```

isPalindrome("racecar")

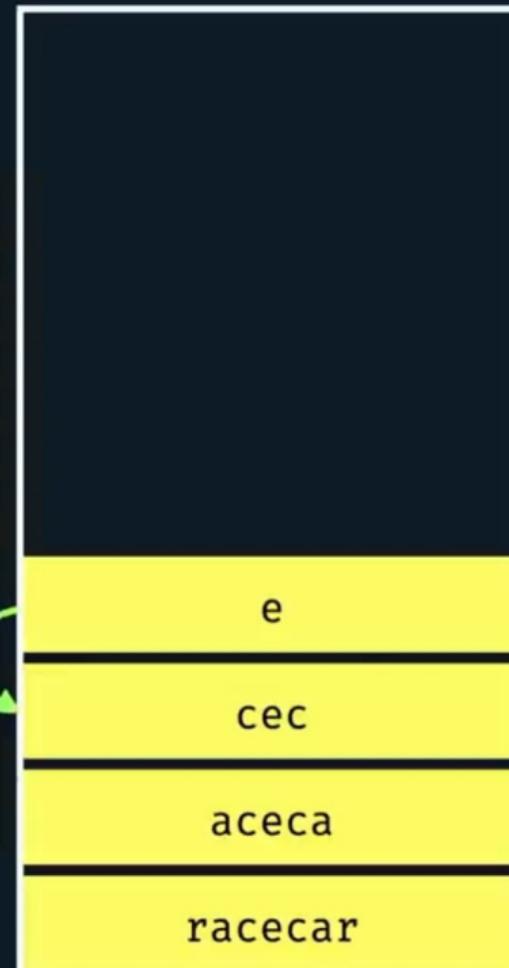


Palindrome.

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5     }  
6  
7     // Do some work to shrink the problem space  
8     if (input.charAt(0) == input.charAt(input.length() - 1)) {  
9         return isPalindrome(input.substring(1, input.length() - 1));  
10    }  
11  
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14 }
```

isPalindrome("racecar")

true

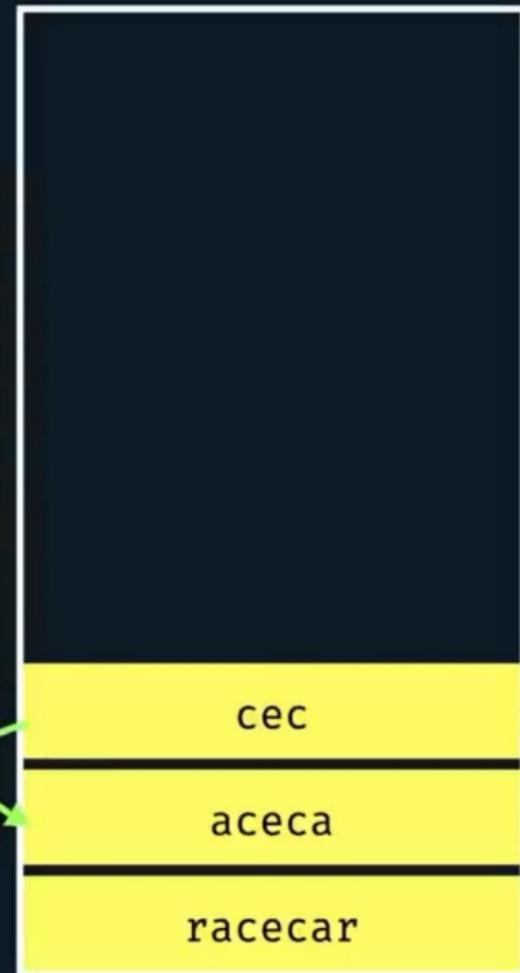


Palindrome.

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isPalindrome("racecar")

true



Palindrome.

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14 }
```

isPalindrome("racecar")

true

racecar

Decimal To Binary.

Decimal To Binary.

233 // 2 = 116 rem = 1

116 // 2 = 58 rem = 0

58 // 2 = 29 rem = 0

29 // 2 = 14 rem = 1

14 // 2 = 7 rem = 0

7 // 2 = 3 rem = 1

3 // 2 = 1 rem = 1 → 1 // 2 = 0

rem = 1

Decimal To Binary.

$$233 \text{ // } 2 = 116$$

$$116 \text{ // } 2 = 58$$

$$58 \text{ // } 2 = 29$$

$$29 \text{ // } 2 = 14$$

$$14 \text{ // } 2 = 7$$

$$7 \text{ // } 2 = 3$$

$$3 \text{ // } 2 = 1$$

```
rem = 1  
rem = 1  
rem = 1  
rem = 0  
rem = 1  
rem = 0  
rem = 0  
rem = 1
```

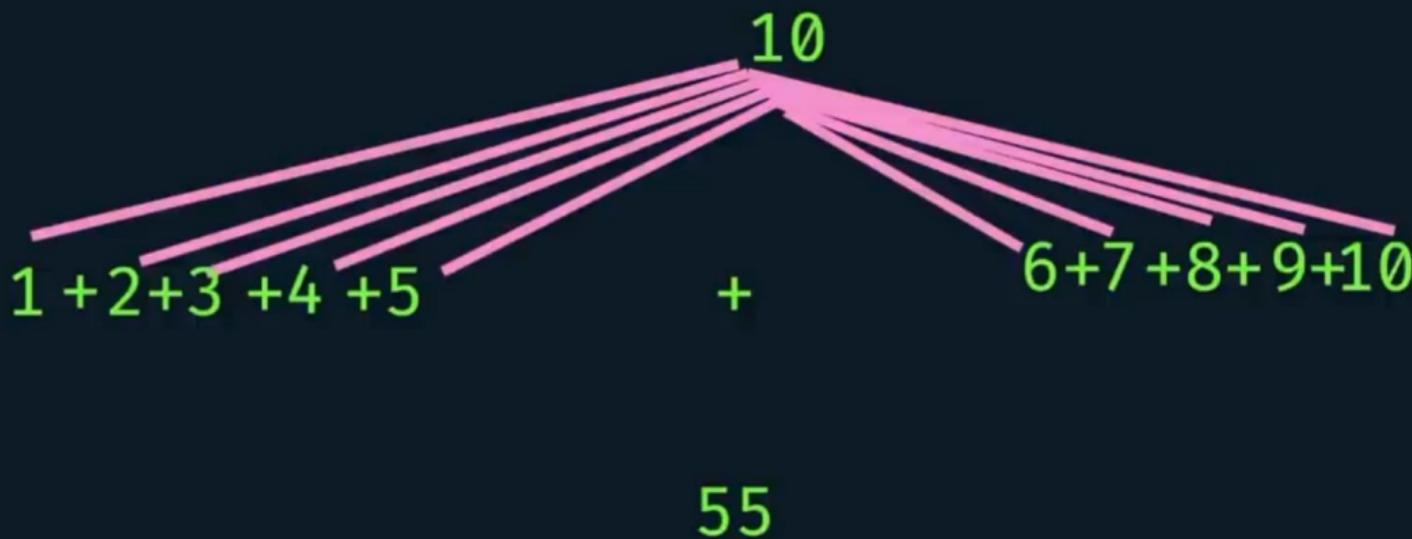
$$\rightarrow 1 \text{ // } 2 = 0$$

Decimal To Binary.



```
1 public static String findBinary(int decimal, String result) {  
2     if (decimal == 0)  
3         return result;  
4  
5     result = decimal % 2 + result;  
6     return findBinary(decimal / 2, result);  
7 }
```

Sum of Natural Numbers.



Sum of Natural Numbers.



```
1  public static int recursiveSummation(int inputNumber) {  
2      if (inputNumber <= 1)  
3          return inputNumber;  
4      return inputNumber + recursiveSummation(inputNumber - 1);  
5  }
```

Divide & Conquer.



- 1 (1) Divide problem into several smaller subproblems
- 2 Normally, the subproblems are similar to the original
- 3
- 4 (2) Conquer the subproblems by solving them recursively
- 5 Base case: solve small enough problems by brute force
- 6
- 7 (3) Combine the solutions to get a solution to the subproblems
- 8 And finally a solution to the orginal problem
- 9
- 10 (4) Divide and Conquer algorithms are normally recursive

Binary Search.

```
● ● ●  
1 public static int binarySearch(int[] A, int left, int right, int x) {  
2     if (left > right) {  
3         return -1;  
4     }  
5     int mid = (left + right) / 2;  
6  
7     if (x == A[mid]) {  
8         return mid;  
9     }  
10    if (x < A[mid]) {  
11        return binarySearch(A, left, mid - 1, x);  
12    }  
13    return binarySearch(A, mid + 1, right, x);  
14 }  
15 }
```

-1	0	1	2	3	4	7	9	10	20
----	---	---	---	---	---	---	---	----	----

Binary Search.

find 10.

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```

-1	0	1	2	3	4	7	9	10	20
----	---	---	---	---	---	---	---	----	----

↑ this array is "A"

binarySearch(A, 0, 9, 10)

Binary Search.

find 10.

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Binary Search.

find 10.

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```

-1 | 0 | 1 | 2 | 3 | 4 | 7 | 9 | 10 | 20

binarySearch(A, 5, 9, 10)

binarySearch(A, 0, 9, 10)

Binary Search.

find 10.

```
1 public static int binarySearch(int[] A, int left, int right, int x) {  
2     if (left > right) {  
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4     }  
5     int mid = (left + right) / 2;  
6  
7     if (x == A[mid]) {  
8         return mid;  
9     }  
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13    }  
14  
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```

-1 | 0 | 1 | 2 | 3 | 4 | 7 | 9 | 10 | 20



binarySearch(A, 8, 9, 10)

binarySearch(A, 5, 9, 10)

binarySearch(A, 0, 9, 10)

Binary Search.

find 10.

```
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9     }  
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12    }  
13  
14    return binarySearch(A, mid + 1, right, x);  
15}  
16}
```

return 8.



binarySearch(A, 8, 9, 10)

binarySearch(A, 5, 9, 10)

binarySearch(A, 0, 9, 10)

Binary Search.

find 10.

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```

return 8.



binarySearch(A, 5, 9, 10)

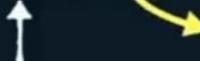
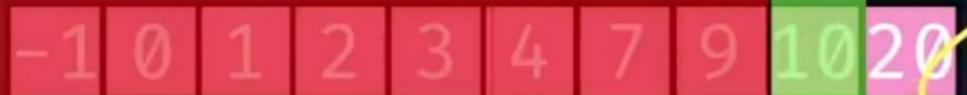
binarySearch(A, 0, 9, 10)

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binarySearch(A, 0, 9, 10)

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```

-1	0	1	2	3	4	7	9	10	20
----	---	---	---	---	---	---	---	----	----

return 8.
↑

Fibonacci (Non-Optimized).



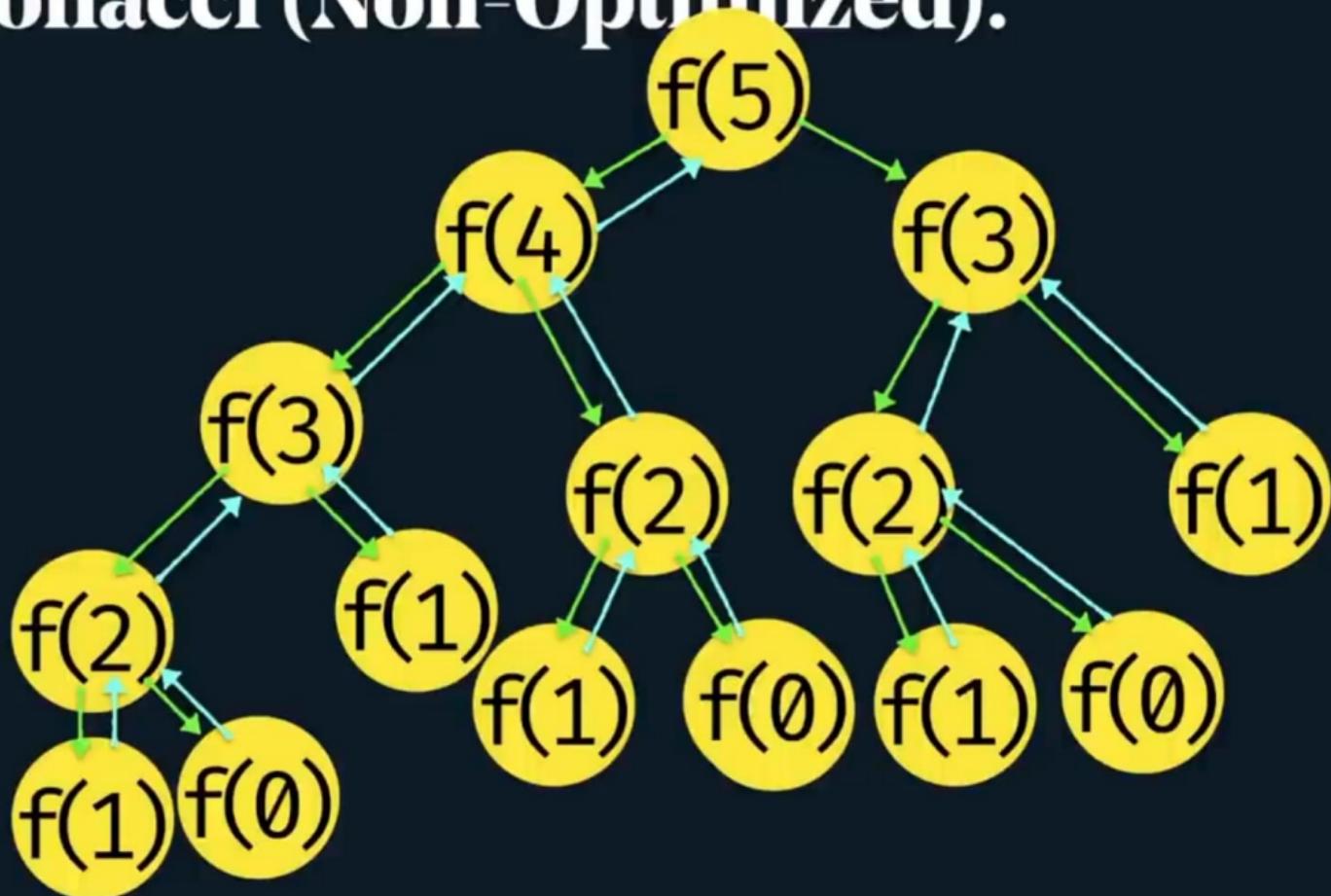
```
1 public static long fib(long n) {  
2     if ((n == 0) || (n == 1))  
3         return n;  
4     else  
5         return fib(n - 1) + fib(n - 2);  
6 }
```

$$F_n = F_{n-1} + F_{n-2}$$

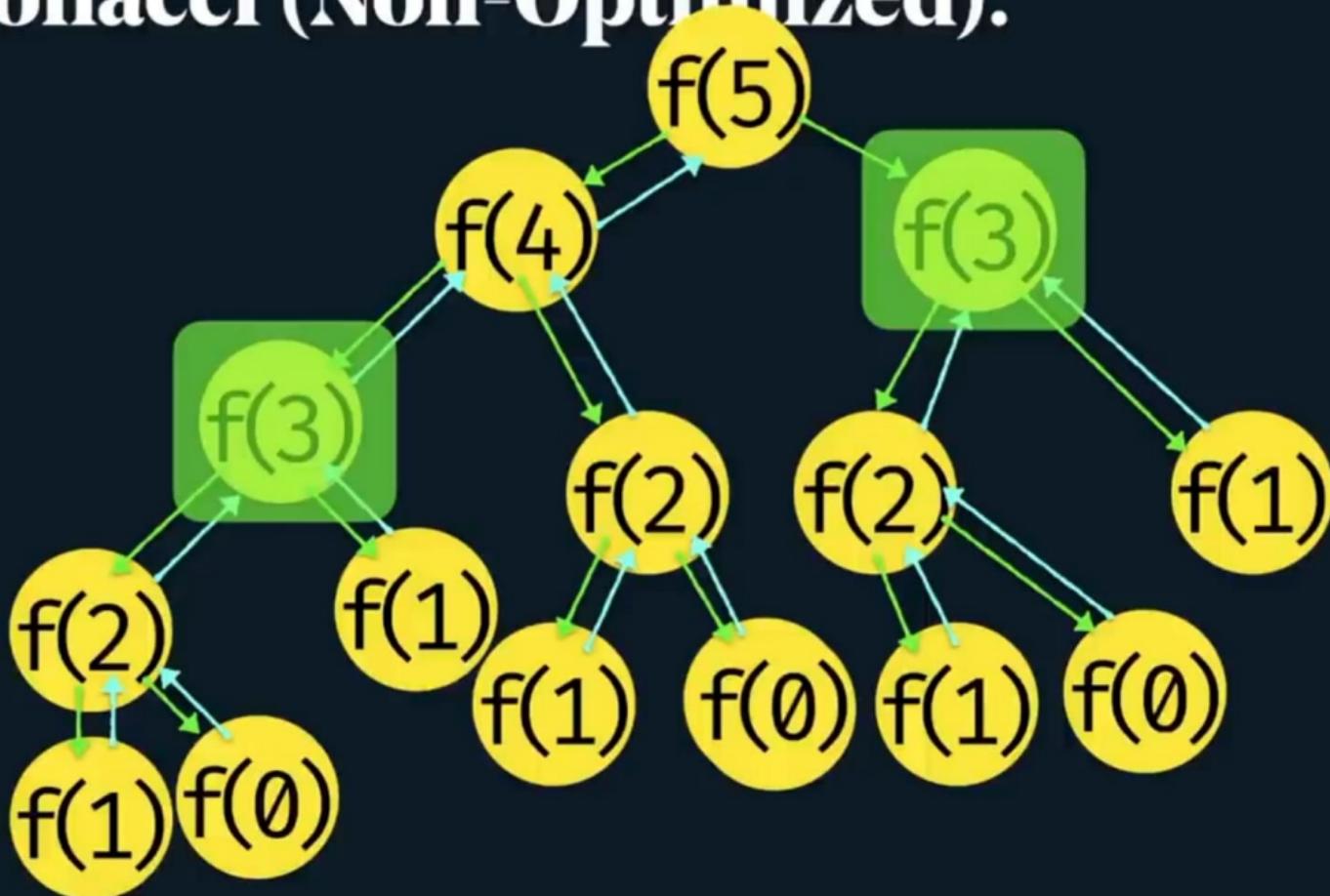
$$F_0 = 0 \quad F_1 = 1$$

$$\{F_n\}_{n=1}^{\infty}$$

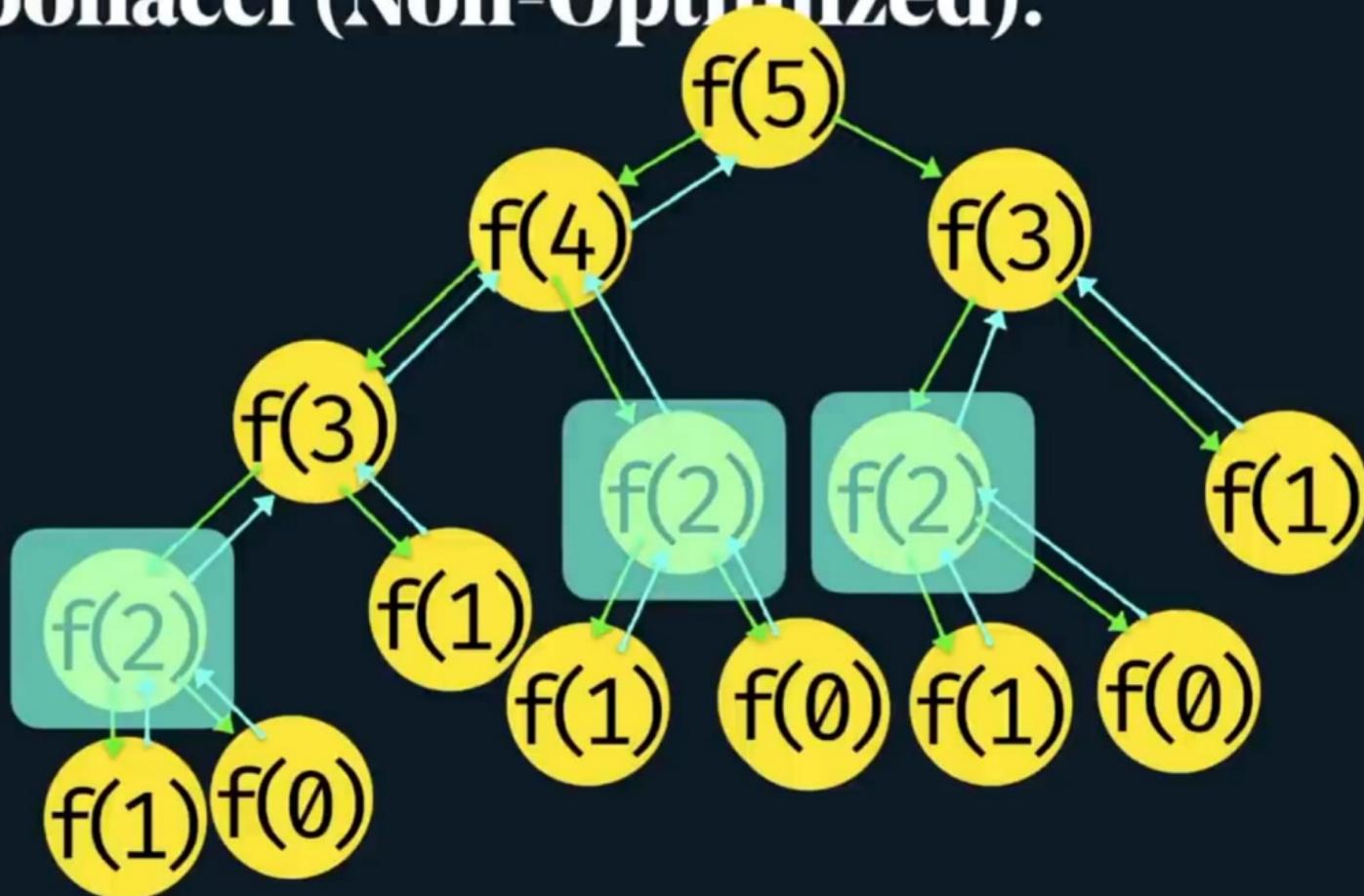
Fibonacci (Non-Optimized).



Fibonacci (Non-Optimized).



Fibonacci (Non-Optimized).



RUN AND DEBUG ... SumOfNaturalNumbers.java MergeSort.java 4 ●

RUN Run and Debug To customize Run and Debug create a launch.json file. Show all automatic debug configurations.

BREAKPOINTS Uncaught Exceptions Caught Exceptions DecimalToBinary.java DecimalToBinary.java DecimalToBinary.java SumOfNaturalNumbers.java SumOfNaturalNumbers.java SumOfNaturalNumbers.java SumOfNaturalNumbers.java

```
Run | Debug
public static void main(String[] args) {
}

public static void mergeSort(int[] data, int start, int end) {
    if (start < end) {
        int mid = (start + end) / 2;
        mergeSort(data, start, mid);
        mergeSort(data, mid + 1, end);
        merge(data, start, mid, end);
    }
}

public static void merge(int[] data, int start, int mid, int end) {
    // build temp array to avoid modifying the original contents
    int[] temp = new int[end - start + 1];
    int i = start, j = mid + 1, k = 0;
```

PROBLEMS 5 OUTPUT TERMINAL DEBUG CONSOLE Java Debug Console + ^ x

```
in/java -agentlib:jdwp=transport=dt_socket,server=n,suspend=y,address=localhost:63939 -Dfile.encoding=UTF-8 -cp "/Users/schachte/Library/Application Support/Code/User/workspaceStorage/f2cd88f30f4bda116e292ab7f651b4fd/redhat.java/jdt_ws/LiveCode_5 bfbfa06/bin" DecimalToBinary
> cd /Users/schachte/Desktop/LiveCode ; /usr/bin/env /Library/Java/JavaVirtualMachines/adoptopenjdk-11.jdk/Contents/Home/bin/java -agentlib:jdwp=transport=dt_socket,server=n,suspend=y,address=localhost:64366 -Dfile.encoding=UTF-8 -cp "/Users/schachte/Library/Application Support/Code/User/workspaceStorage/f2cd88f30f4bda116e292ab7f651b4fd/redhat.java/jdt_ws/LiveCode_5 bfbfa06/bin" SumOfNaturalNumbers
15
16
```

~Desktop/LiveCode 32m 17s anaconda3

Ln 19, Col 42 Spaces: 4 UTF-8 LF Java JavaSE-11

RUN AND DEBUG ... SumOfNaturalNumbers.java MergeSort.java 1 ●

RUN

Run and Debug

To customize Run and Debug create a [launch.json file](#).

Show all automatic debug configurations.

BREAKPOINTS

- Uncaught Exceptions
- Caught Exceptions
- DecimalToBinary.java
- DecimalToBinary.java
- DecimalToBinary.java
- SumOfNaturalNumbers.java
- SumOfNaturalNumbers.java
- SumOfNaturalNumbers.java
- SumOfNaturalNumbers.java

PROBLEMS 2 OUTPUT TERMINAL DEBUG CONSOLE

Java Debug Console + ^ X

```
18 int i = start, j = mid + 1, k = 0;
19
20 // While both sub-array have values, then try and merge them in sorted order
21 while (i <= mid && j <= end) {
22     if (data[i] <= data[j]) {
23         temp[k++] = data[i++];
24     } else {
25         temp[k++] = data[j++];
26     }
27 }
28
29
30 // Add the rest of the values from the left sub-array into the result
31 while (i <= mid) {
32     temp[k] = data[i];
33     k++; i++;
34 }
35
36 while ()
```

37 }

```
in/java -agentlib:jdwp=transport=dt_socket,server=n,suspend=y,address=localhost:63939 -Dfile.encoding=UTF-8 -cp "/Users/schachte/Library/Application Support/Code/User/workspaceStorage/f2cd88f30f4bda116e292ab7f651b4fd/redhat.java/jdt_ws/LiveCode_5bfba06/bin" DecimalToBinary
> cd /Users/schachte/Desktop/LiveCode ; /usr/bin/env /Library/Java/JavaVirtualMachines/adoptopenjdk-11.jdk/Contents/Home/bin/java -agentlib:jdwp=transport=dt_socket,server=n,suspend=y,address=localhost:64366 -Dfile.encoding=UTF-8 -cp "/Users/schachte/Library/Application Support/Code/User/workspaceStorage/f2cd88f30f4bda116e292ab7f651b4fd/redhat.java/jdt_ws/LiveCode_5bfba06/bin" SumOfNaturalNumbers
15
16 ~Desktop/LiveCode 32m 17s anaconda3
```

Ln 36, Col 16 Spaces: 4 UTF-8 LF Java ⚡ JavaSE-11 ⚡

RUN AND DEBUG

... SumOfNaturalNumbers.java MergeSort.java •

RUN

Run and Debug

To customize Run and Debug create a [launch.json file](#).

Show all automatic debug configurations.

```
33     k++; i++;
34 }
35
36     // Add the rest of the values from the right sub-array into the result
37     while (j <= end) {
38         temp[k] = data[j];
39         k++; j++;
40     }
41
42     for (i = start; i <= end; i++) {
43         data[i] = temp[i - start];
44     }
45 }
46
47 }
```

BREAKPOINTS

- Uncaught Exceptions
- Caught Exceptions
- DecimalToBinary.java
- DecimalToBinary.java
- DecimalToBinary.java
- SumOfNaturalNumbers.java
- SumOfNaturalNumbers.java
- SumOfNaturalNumbers.java
- SumOfNaturalNumbers.java

PROBLEMS 1 OUTPUT TERMINAL DEBUG CONSOLE

Java Debug Console + ^ X

```
3     in/java -agentlib:jdwp=transport=dt_socket,server=n,suspend=y,address=localhost:63939 -Dfile.encoding=UTF-8 -cp "/Users/schachte/Library/Application Support/Code/User/workspaceStorage/f2cd88f30f4bda116e292ab7f651b4fd/redhat.java/jdt_ws/LiveCode_5
4         bfbfa06/bin" DecimalToBinary
5         > cd /Users/schachte/Desktop/LiveCode ; /usr/bin/env /Library/Java/JavaVirtualMachines/adoptopenjdk-11.jdk/Contents/Home/b
6         in/java -agentlib:jdwp=transport=dt_socket,server=n,suspend=y,address=localhost:64366 -Dfile.encoding=UTF-8 -cp "/Users/schachte/Library/Application Support/Code/User/workspaceStorage/f2cd88f30f4bda116e292ab7f651b4fd/redhat.java/jdt_ws/LiveCode_5
7         bfbfa06/bin" SumOfNaturalNumbers
8         15
9         16 ~Desktop/LiveCode 32m 17s anaconda3
```

RUN AND DEBUG ...

RUN

Run and Debug

To customize Run and Debug create a launch.json file.

Show all automatic debug configurations.

BREAKPOINTS

- Uncaught Exceptions
- Caught Exceptions
- DecimalToBinary.java
- DecimalToBinary.java
- DecimalToBinary.java
- MergeSort.java
- SumOfNaturalNumbers.java
- SumOfNaturalNumbers.java
- SumOfNaturalNumbers.java

MergeSort.java > MergeSort > main(String[])

```
1 public class MergeSort {  
2     public static void main(String[] args) {  
3         int[] data = new int[]{-5, 20, 10, 3, 2, 0};  
4         mergeSort(data, 0, data.length - 1);  
5         System.out.println("stop");  
6     }  
7  
8     public static void mergeSort(int[] data, int start, int end) {  
9         if (start < end) {  
10             int mid = (start + end) / 2;  
11             mergeSort(data, start, mid);  
12             mergeSort(data, mid + 1, end);  
13             merge(data, start, mid, end);  
14         }  
15     }  
16 }  
17  
18 public static void merge(int[] data, int start, int mid, int end) {  
19     // build temp array to avoid modifying the original contents
```

PROBLEMS 1 OUTPUT TERMINAL DEBUG CONSOLE

Java Debug Console + ▾ ^ ×

```
in/java -agentlib:jdwp=transport=dt_socket,server=n,suspend=y,address=localhost:64366 -Dfile.encoding=UTF-8 -cp "/Users/schachte/Library/Application Support/Code/User/workspaceStorage/f2cd88f30f4bda116e292ab7f651b4fd/redhat.java/jdt_ws/LiveCode_5bfba06/bin" SumOfNaturalNumbers  
15  
cd /Users/schachte/Desktop/LiveCode ; /usr/bin/env /Library/Java/JavaVirtualMachines/adoptopenjdk-11.jdk/Contents/Home/bin/java -agentlib:jdwp=transport=dt_socket,server=n,suspend=y,address=localhost:49535 -Dfile.encoding=UTF-8 -cp "/Users/schachte/Library/Application Support/Code/User/workspaceStorage/f2cd88f30f4bda116e292ab7f651b4fd/redhat.java/jdt_ws/LiveCode_5bfba06/bin" MergeSort
```

Ln 5, Col 36 Spaces: 4 LF Java ⌂ JavaSE-11 ⌂ ⌂

LinkedList Reversal.



```
1 public static ListNode reverseList(ListNode head) {  
2     if (head == null || head.next == null) return head;  
3     ListNode p = reverseList(head.next);  
4     head.next.next = head;  
5     head.next = null;  
6     return p;  
7 }
```



LinkedList Reversal.



```
1 public static ListNode reverseList(ListNode head) {  
2     if (head == null || head.next == null) return head;  
3     ListNode p = reverseList(head.next);  
4     head.next.next = head;  
5     head.next = null;  
6     return p;  
7 }
```



Live Code & Debug

Merge Two Sorted Linked Lists.



```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```

Merge Two Sorted Linked Lists.



```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```



sortedMerge(1, 4)

Merge Two Sorted Linked Lists.



```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```



sortedMerge(8, 4)

sortedMerge(1, 4)

Merge Two Sorted Linked Lists.

```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```



sortedMerge(8, 11)

sortedMerge(8, 4)

sortedMerge(1, 4)

Merge Two Sorted Linked Lists.

```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```



sortedMerge(22, 11)

sortedMerge(8, 11)

sortedMerge(8, 4)

sortedMerge(1, 4)

Merge Two Sorted Linked Lists.

```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```



sortedMerge(22, 16)

sortedMerge(22, 11)

sortedMerge(8, 11)

sortedMerge(8, 4)

sortedMerge(1, 4)

Merge Two Sorted Linked Lists.

```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```



sortedMerge(22, 20)

sortedMerge(22, 16)

sortedMerge(22, 11)

sortedMerge(8, 11)

sortedMerge(8, 4)

sortedMerge(1, 4)

Merge Two Sorted Linked Lists.

```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```



sortedMerge(22, null)

sortedMerge(22, 20)

sortedMerge(22, 16)

sortedMerge(22, 11)

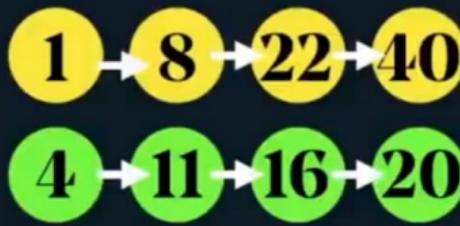
sortedMerge(8, 11)

sortedMerge(8, 4)

sortedMerge(1, 4)

Merge Two Sorted Linked Lists.

```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```



return 22

sortedMerge(22, null)

sortedMerge(22, 20)

sortedMerge(22, 16)

sortedMerge(22, 11)

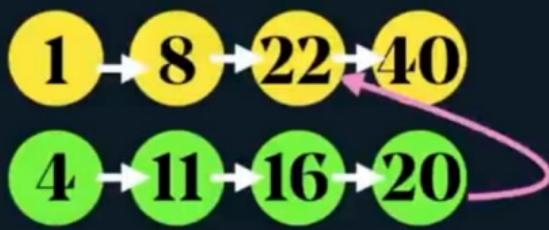
sortedMerge(8, 11)

sortedMerge(8, 4)

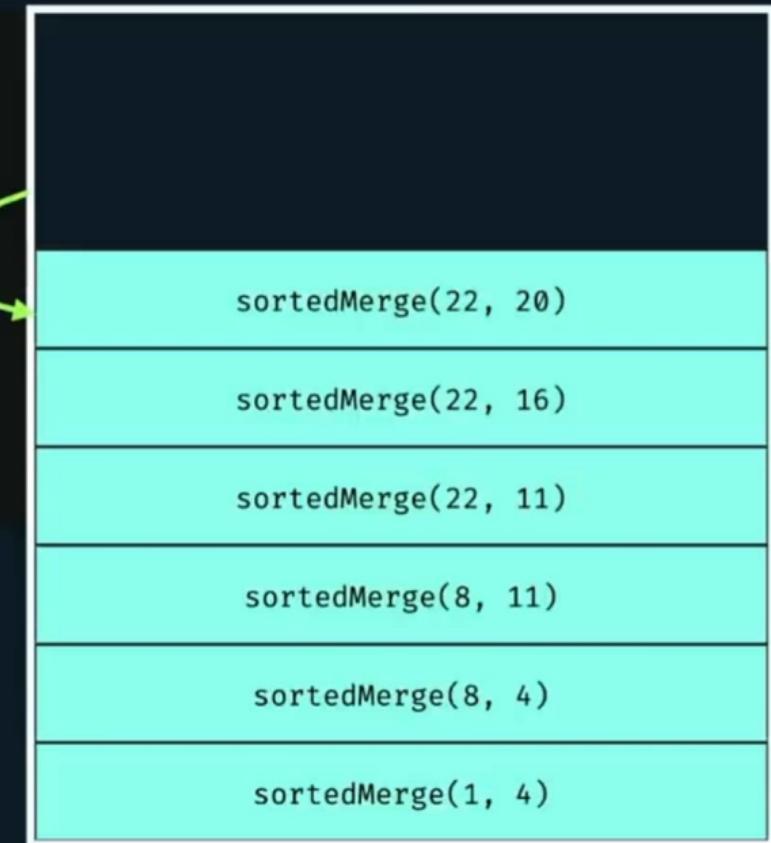
sortedMerge(1, 4)

Merge Two Sorted Linked Lists.

```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```



return 22

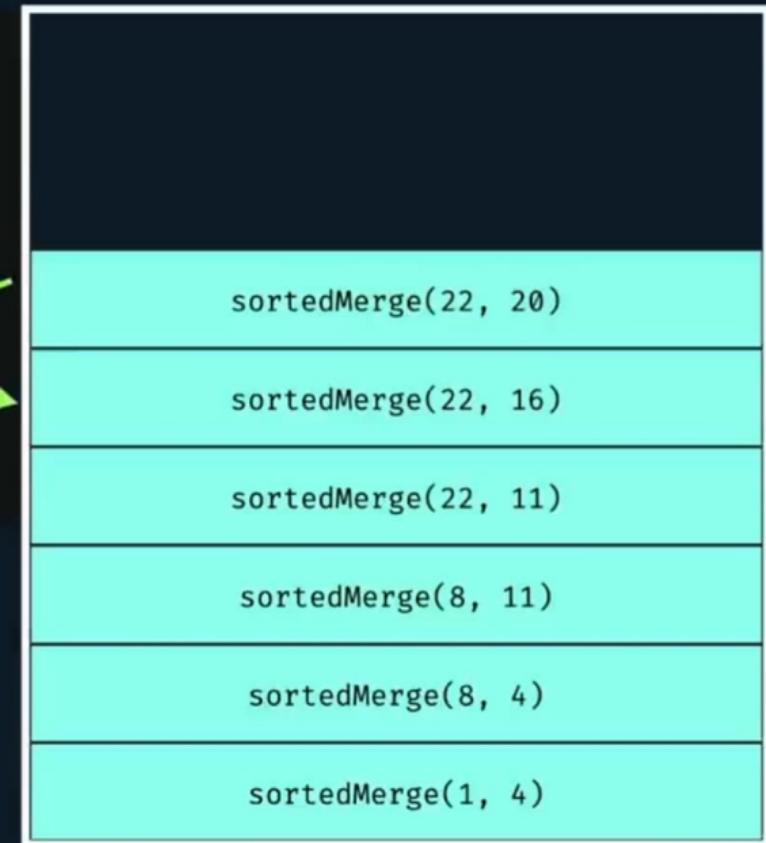


Merge Two Sorted Linked Lists.

```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```



return 20

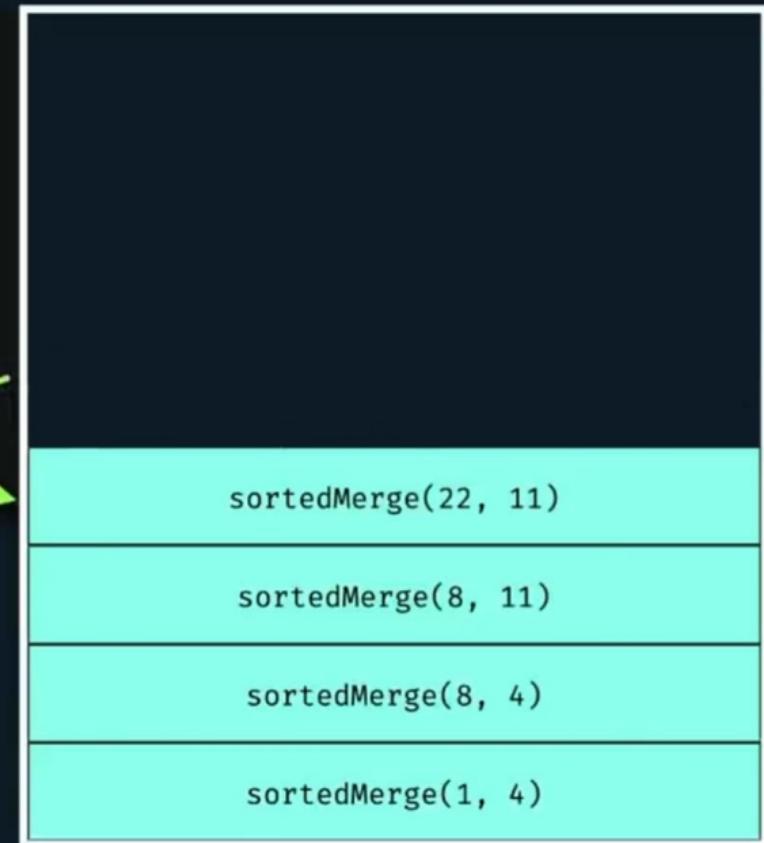


Merge Two Sorted Linked Lists.

```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```



return 16



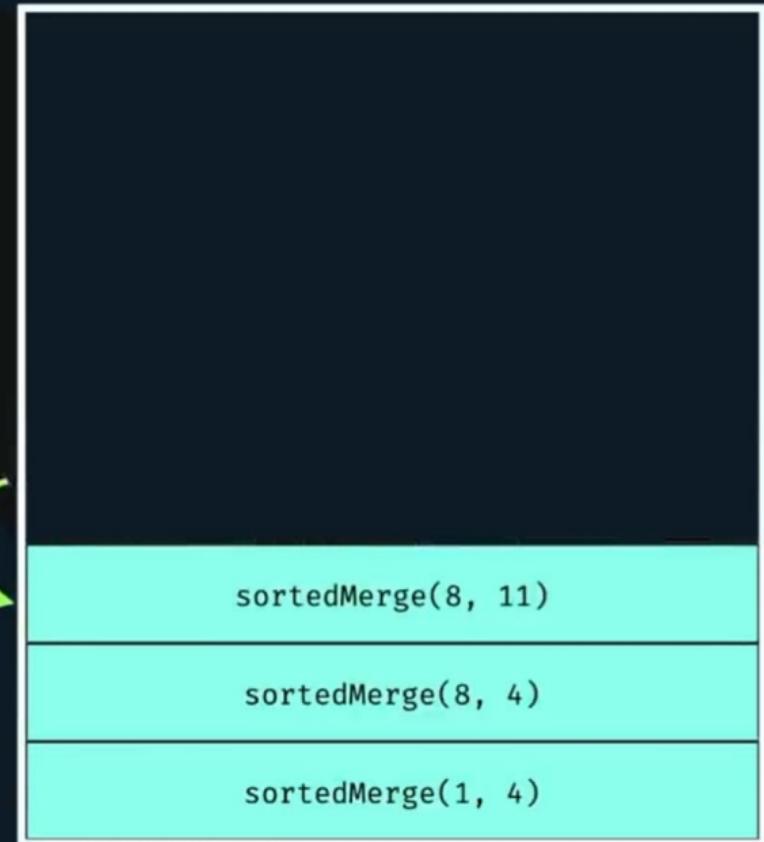
Merge Two Sorted Linked Lists.



```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```



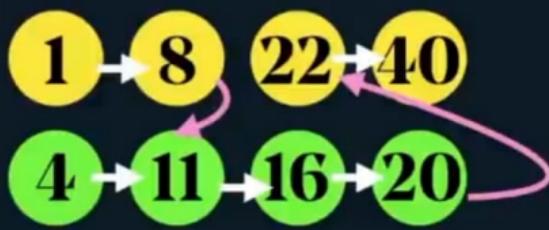
return 11



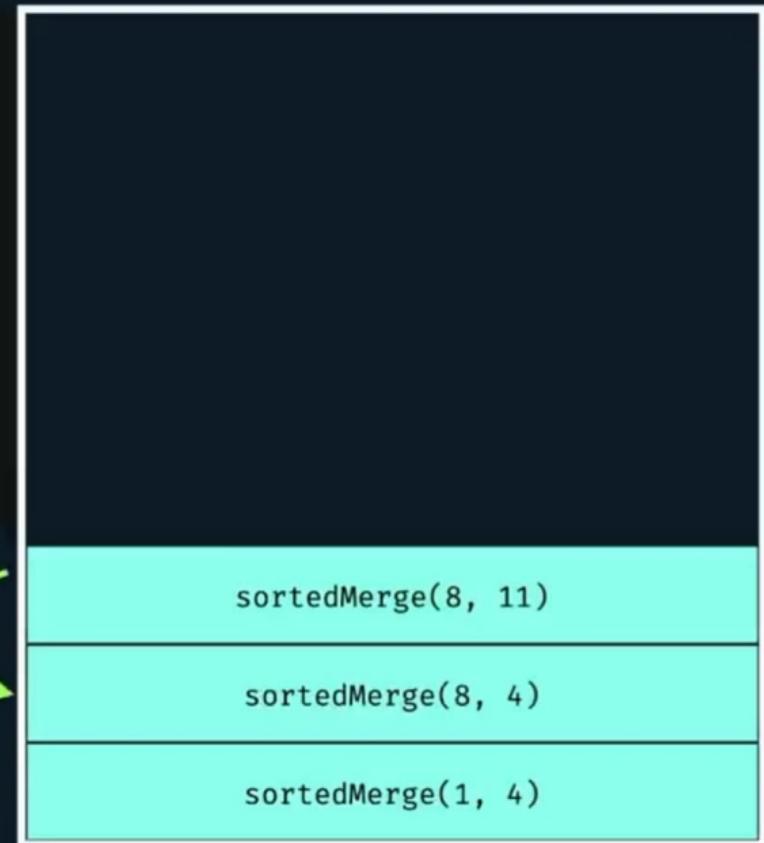
Merge Two Sorted Linked Lists.



```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```



return 8



Merge Two Sorted Linked Lists.



```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```



return 4

sortedMerge(8, 4)

sortedMerge(1, 4)

Merge Two Sorted Linked Lists.

```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```



return 1

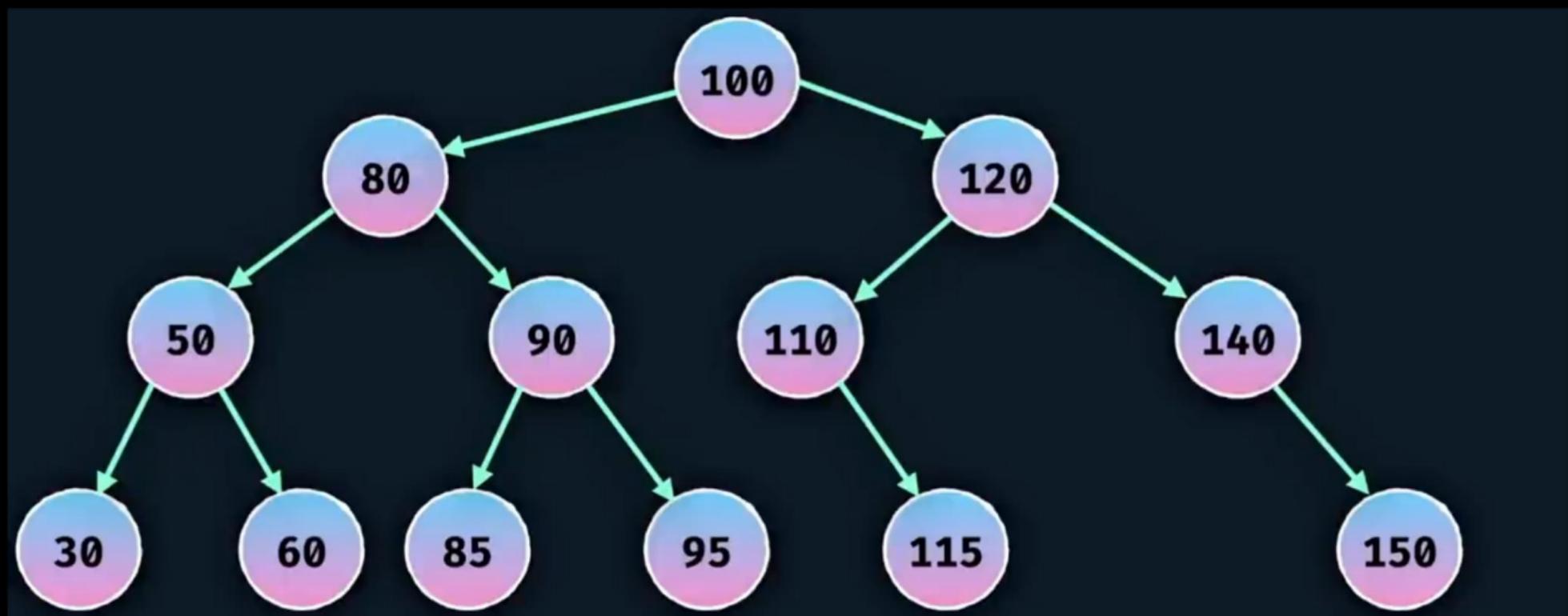
sortedMerge(1, 4)

Merge Two Sorted Linked Lists.

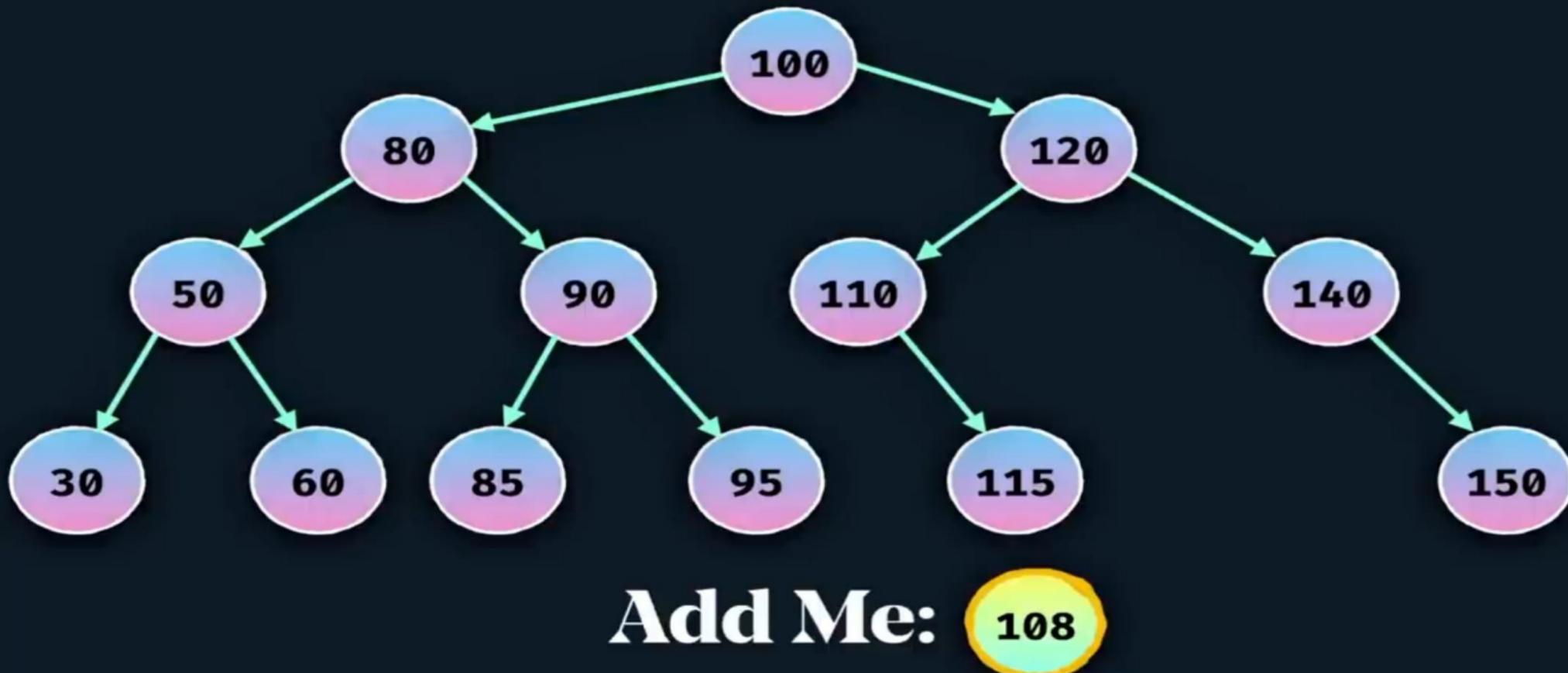
```
1 public Node SortedMerge(Node A, Node B) {  
2     if(A == null) return B;  
3     if(B == null) return A;  
4  
5     if(A.data < B.data) {  
6         A.next = SortedMerge(A.next, B);  
7         return A;  
8     } else {  
9         B.next = SortedMerge(A, B.next);  
10        return B;  
11    }  
12 }
```



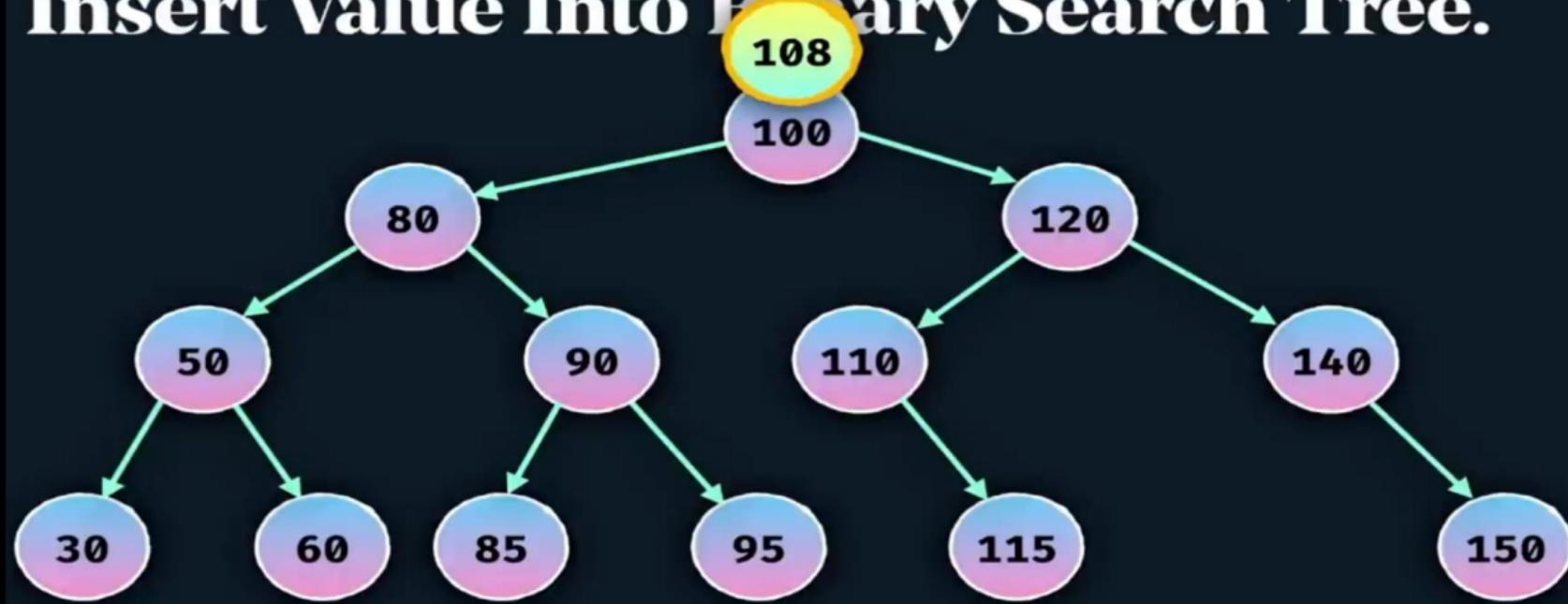
Live Code & Debug



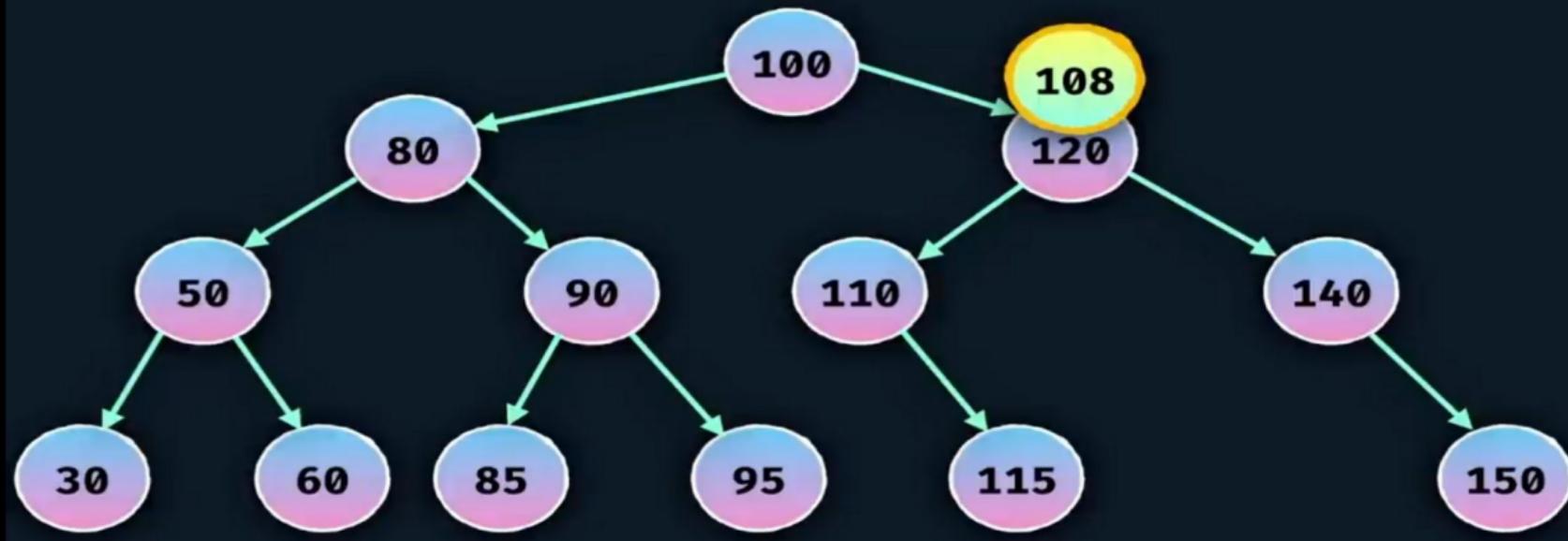
Insert Value Into Binary Search Tree.



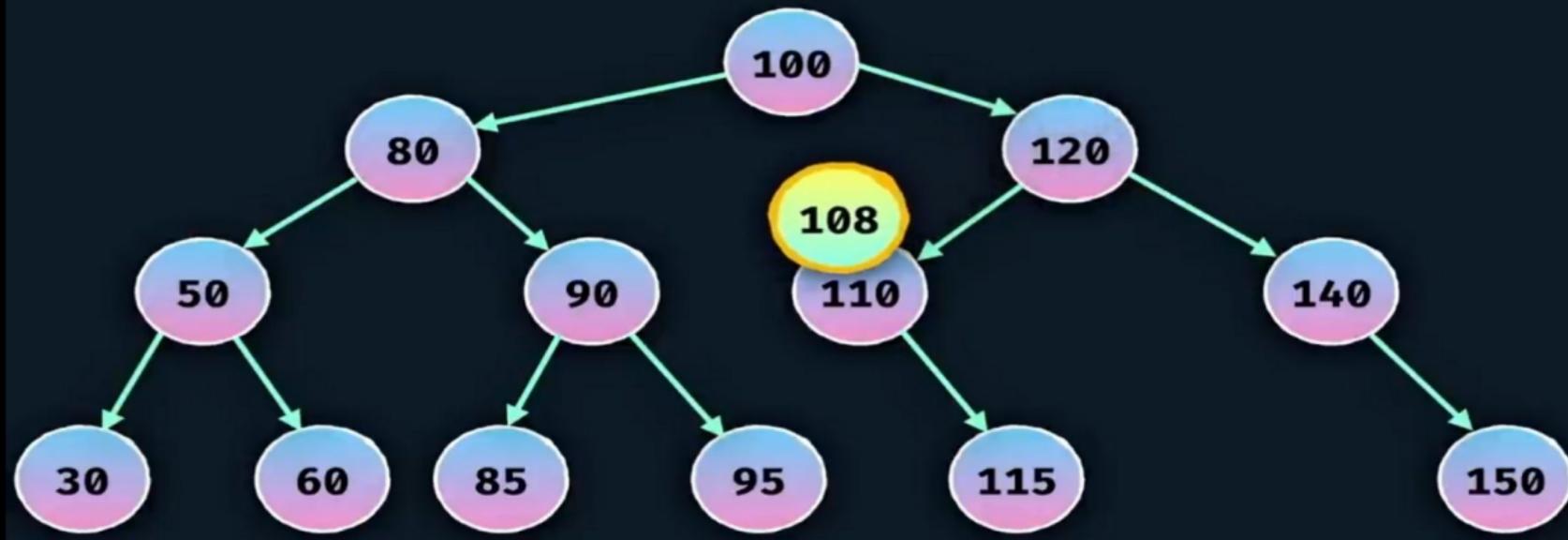
Insert Value Into Binary Search Tree.



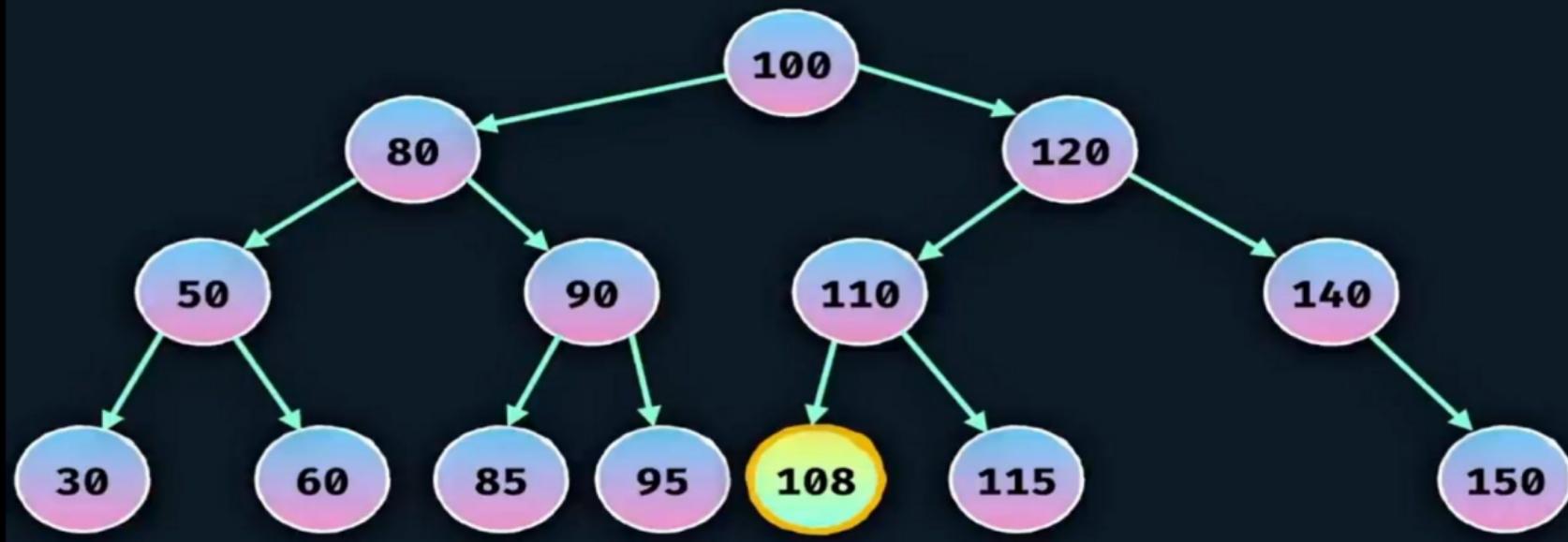
Insert Value Into Binary Search Tree.



Insert Value Into Binary Search Tree.



Insert Value Into Binary Search Tree.



Insert Value Into Binary Search Tree.

```
1 public Node insertNode(Node head, int data) {  
2     if(head == null){  
3         head = new Node();  
4         head.data = data;  
5         return head;  
6     }  
7     if(head.data < data) {  
8         head.right = insertNode(head.right,data);  
9     } else {  
10        head.left = insertNode(head.left, data);  
11    }  
12    return head;  
13 }
```



Insert Value Into Binary Search Tree.



```
1 public Node insertNode(Node head, int data) {  
2     if(head == null){  
3         head = new Node();  
4         head.data = data;  
5         return head;  
6     }  
7     if(head.data < data) {  
8         head.right = insertNode(head.right,data);  
9     } else {  
10        head.left = insertNode(head.left, data);  
11    }  
12    return head;  
13 }
```

If I hit `null`, then I've recursed to my end goal according to the BST property.

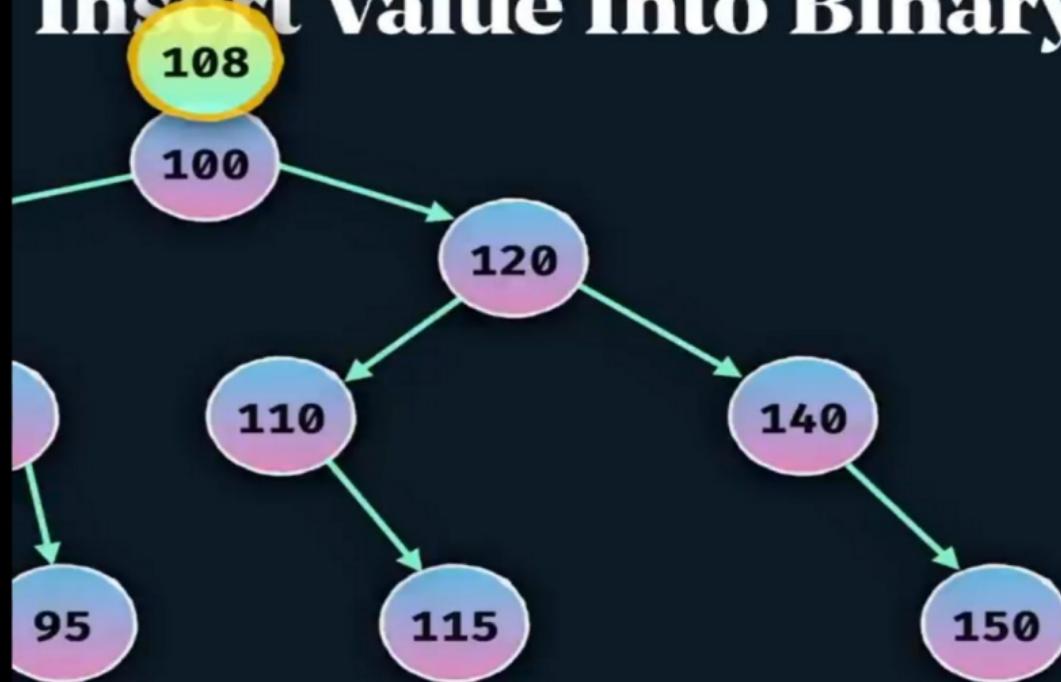
If [data] is greater than the node, then I need to recurse **right**.

If [data] is less than the node, then I need to recurse **left**.

Return the original root node of the tree

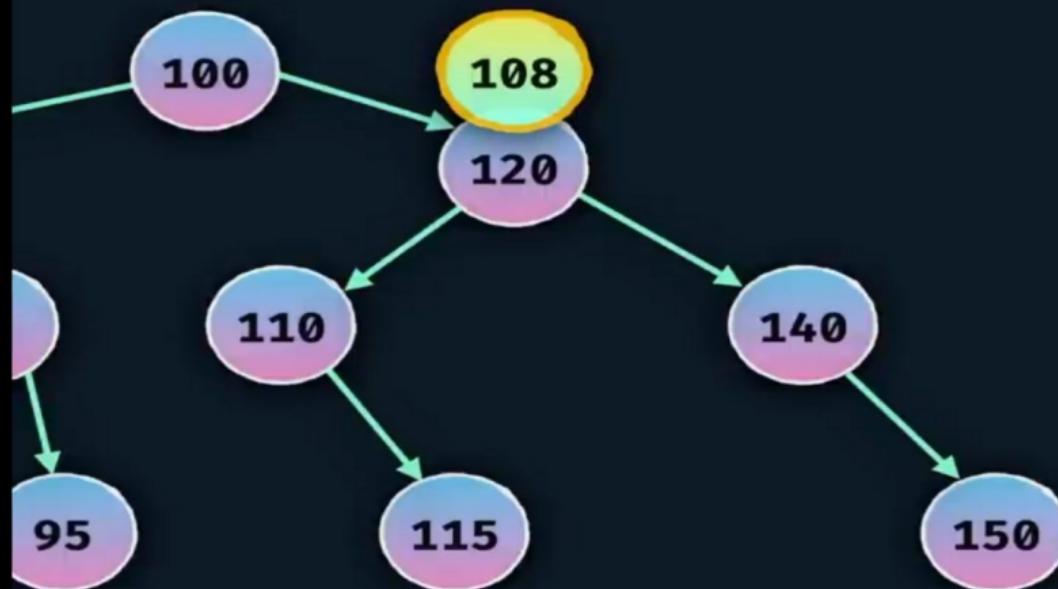


Insert Value Into Binary Search Tree.



```
insertNode(100, 108)
```

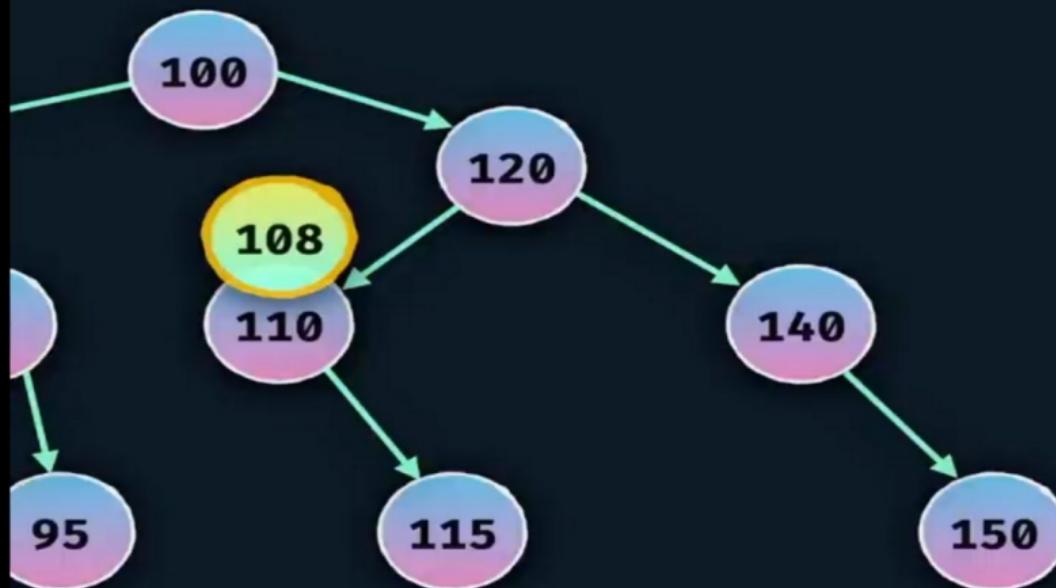
Insert Value Into Binary Search Tree.



```
insertNode(120, 108)
```

```
insertNode(100, 108)
```

Insert Value Into Binary Search Tree.

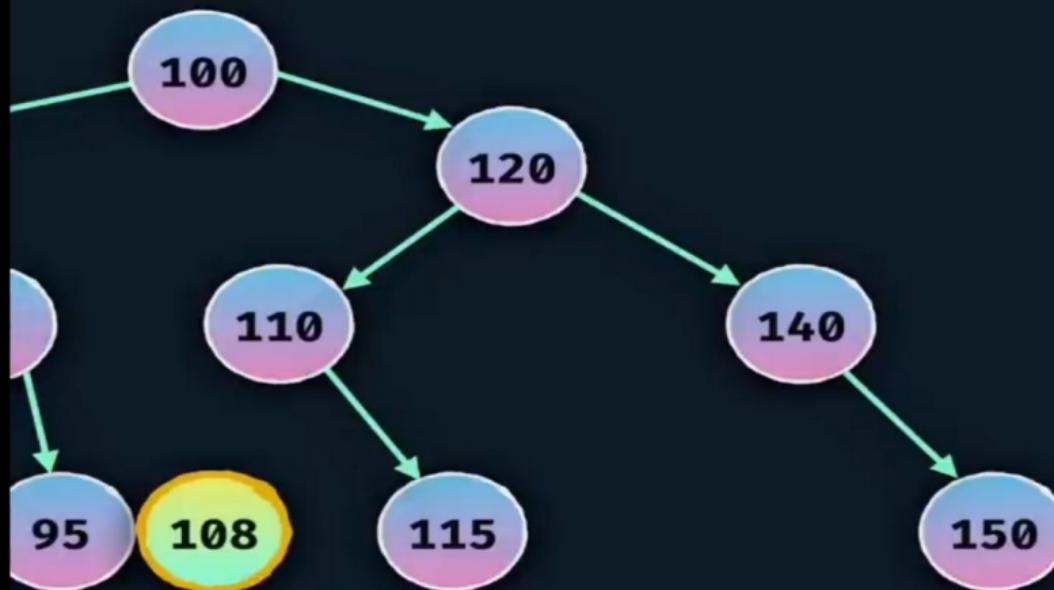


```
insertNode(110, 108)
```

```
insertNode(120, 108)
```

```
insertNode(100, 108)
```

Insert Value Into Binary Search Tree.



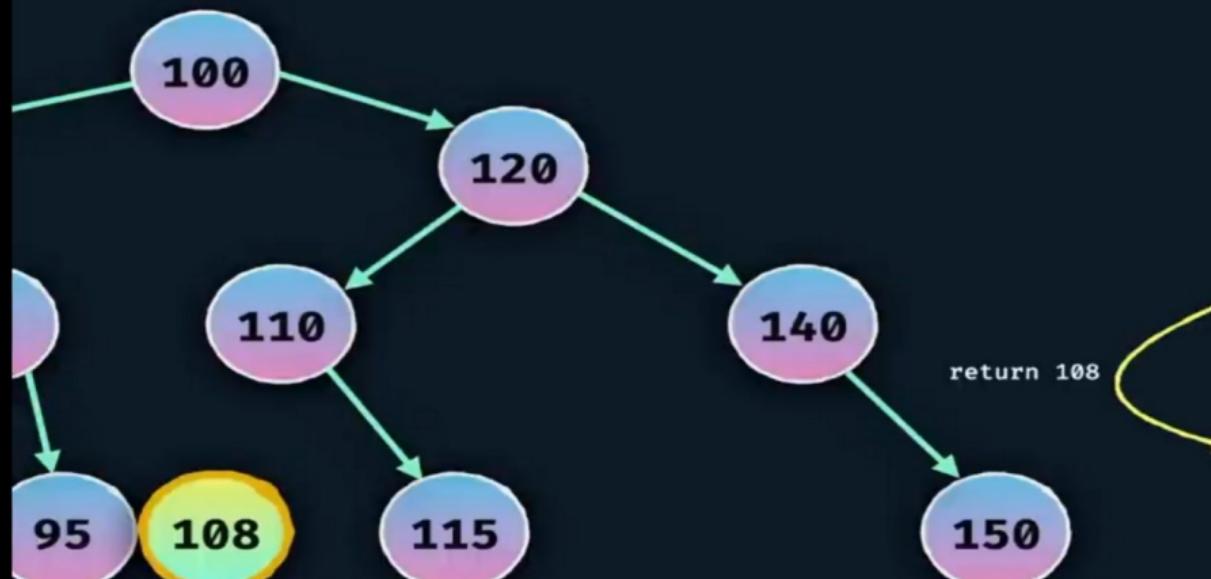
```
insertNode(null, 108)
```

```
insertNode(110, 108)
```

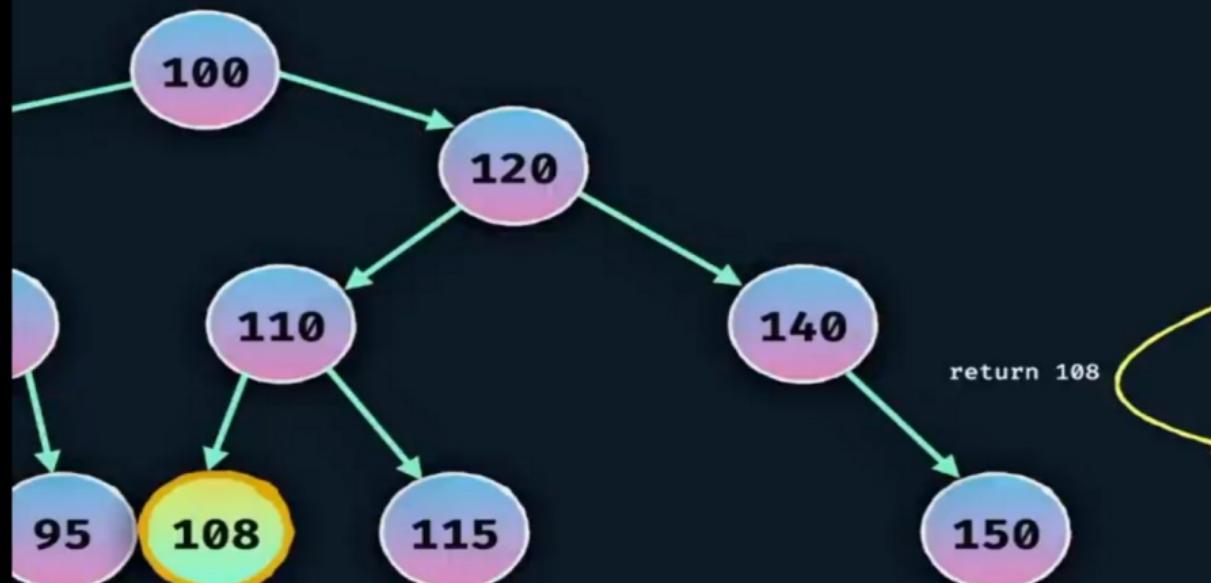
```
insertNode(120, 108)
```

```
insertNode(100, 108)
```

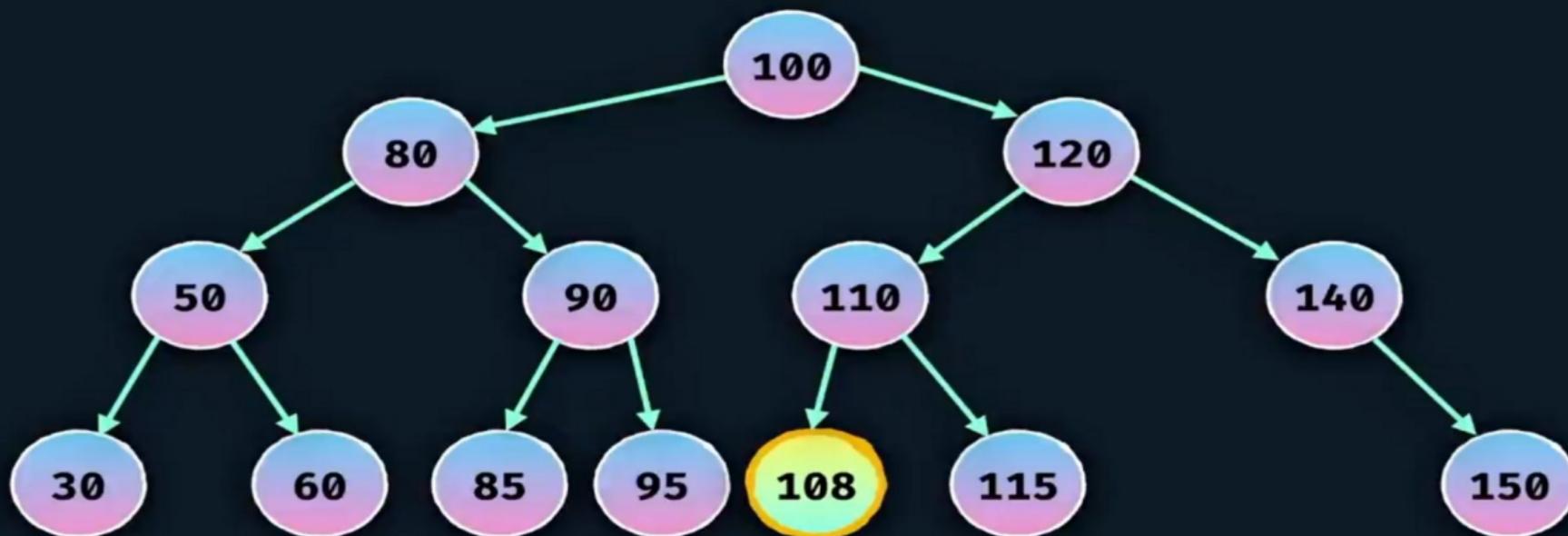
Insert Value Into Binary Search Tree.



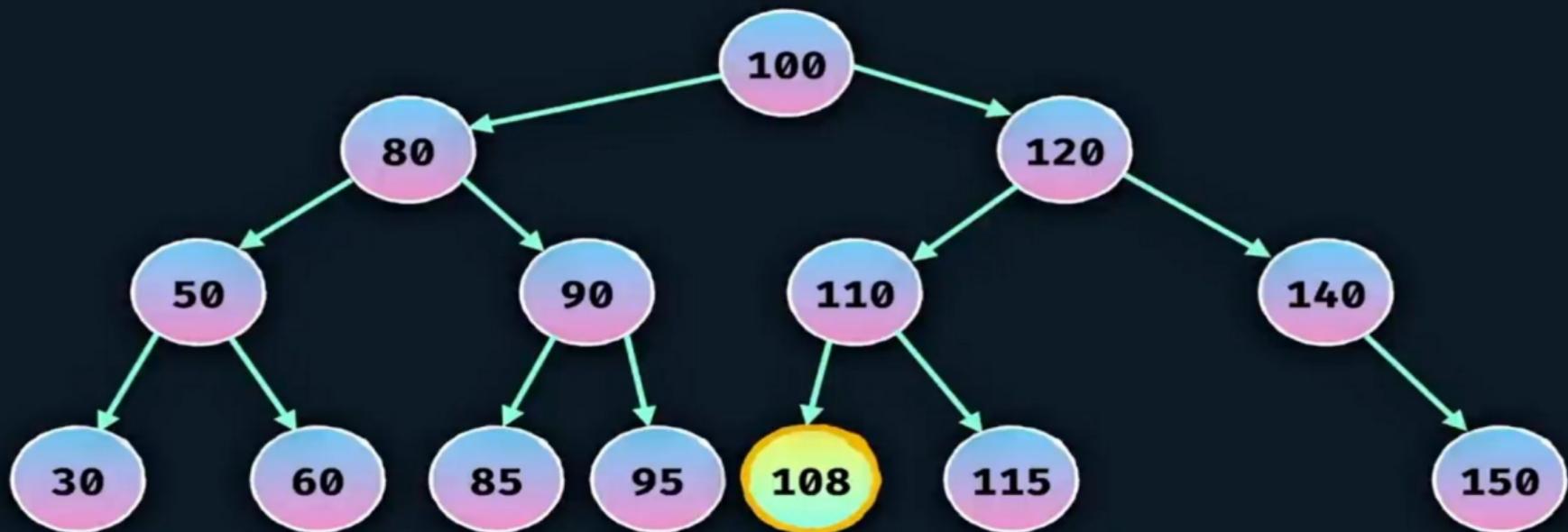
Insert Value Into Binary Search Tree.



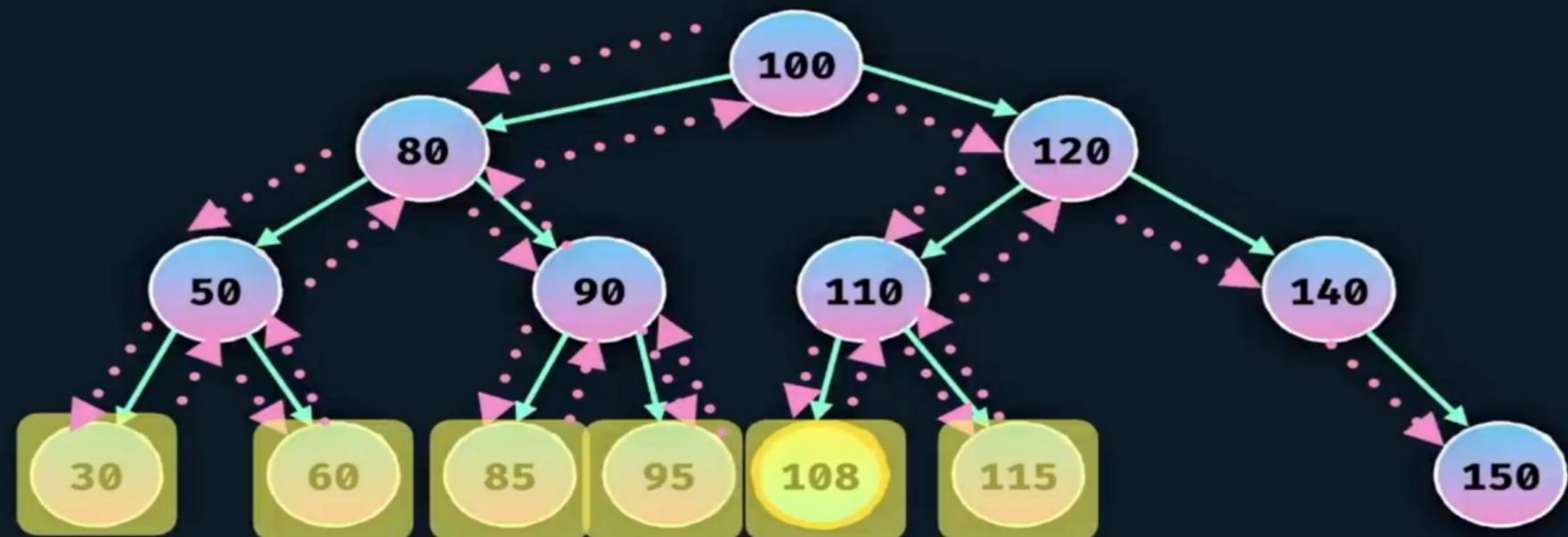
Print All Leaf Nodes.



Print All Leaf Nodes.



Print All Leaf Nodes.



Print All Leaf Nodes.

```
● ● ●  
1 public static void printLeaves(Node root) {  
2     if (root == null) return;  
3  
4     if (root.left == null && root.right == null) {  
5         System.out.print(root.val + ", ");  
6         return;  
7     }  
8     if (root.left != null)  
9         printLeaves(root.left);  
10    if (root.right != null)  
11        printLeaves(root.right);  
12 }
```

Let's analyze the call stack in real-time

Depth-First Search.



```
1  boolean depthFirstSearch(Node node, Set<Node> visited, int goal) {  
2      if (node == null) return false;  
3  
4      if (node.val == goal) {  
5          return true;  
6      }  
7  
8      for (Node neighbor : node.getNeighbors()) {  
9          if (visited.contains(neighbor)) continue;  
10         visited.add(neighbor);  
11         boolean isFound = depthFirstSearch(neighbor, visited, goal);  
12  
13         if (isFound) return true;  
14     }  
15     return false;  
16 }
```

(b)

(i)

Memoization & Caching.

How can we speed up our program by “caching” things we’ve already done?

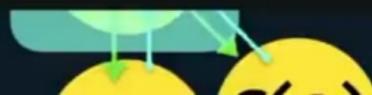


Memoization & Caching.



done?

```
1 Map<Integer, Integer> memoizedCache = new HashMap<>();
2 memoizedCache.put(1,1);
3 memoizedCache.put(0, 1);
4
5 public static long fib(long n) {
6     if (memoizedCache.containsKey(n)) {
7         return memoizedCache.get(n);
8     }
9     int result = fib(n - 1) + fib(n - 2);
10    memoizedCache.put(n, result);
11    return result;
12 }
```



Tail-Call Recursion Optimization.

Compiler optimization in certain languages to reduce stack overflows.

- This works by ensuring the last function call is a recursive one

```
1 int factorial(int x) {  
2     if (x > 0) {  
3         return x * factorial(x - 1);  
4     }  
5     return 1;  
6 }
```

```
1 int factorial(int x) {  
2     return tailfactorial(x, 1);  
3 }  
4  
5 int tailfactorial(int x, int multiplier) {  
6     if (x > 0) {  
7         return tailfactorial(x - 1, x * multiplier);  
8     }  
9     return multiplier;  
10 }
```

Credit: Viliam Búr - <https://cs.stackexchange.com/questions/6230/what-is-tail-recursion>

Tail-Call Recursion Optimization.

Compiler optimization in certain languages to reduce stack overflows.

- This works by ensuring the last function call is a recursive one
- Rule of thumb: Make the recursive call the last instruction
- Supported by mostly all functional languages.
Not supported by Python, Java. Supported for JS in Safari.