TENSOR WORKSHEET

TANMAY AGARWAL

November 3, 2020

Abstract

In this paper we will be convering most of the tensors and their various transformations. We will also take a look on the dirac notation formalizations

Contents

1 Initial Tensor And Their Transformations

Firstly we will be looking at the tensors which have only one index. Broadly as we all know there are two types of tensors

1.Contra-variant tensors

2.Co-variant tensors

In contra-variant tensors the components transfrom in a "contra" manner. If the components increase, their respective products will decrese(If still any confusion please refer the main notes). The various Transformations of contra-variant vectors are given as

$$X^{'i} = \frac{\partial X^{'i}}{\partial X^j} \cdot X^j \tag{1}$$

In co-variant tensors the components trnsform in a "co" manner. If the components increase or decrease, even the products increases or decreses. Their tansformation is

$$X_i^{'} = \frac{\partial X^j}{\partial X_i^{'}} \cdot X^j \tag{2}$$

These transformations can also be representes as follows

$$X^{,i} = \wedge^{ij} \cdot X^j \tag{3}$$

and

$$X_i^{'} = \wedge^{ji} \cdot X^j \tag{4}$$

here

$$\wedge^{\alpha\beta} = \frac{\partial X^{'\alpha}}{\partial X^\beta} \tag{5}$$

2 Transformations For Contra Tensors

For contra-variant tesnors a list of various transformations are

$$x^{'i} = \frac{\partial x^{'i}}{\partial x^j} \cdot x^j \ (\text{or}) \ x^{'i} = \wedge^{ij} \cdot x^j \tag{6}$$

$$A^\alpha = \frac{\partial x^{'\alpha}}{\partial x^\beta} \cdot A^\beta \tag{7}$$

$$T^{\alpha\beta} = \frac{\partial x^{'\alpha}}{\partial x^j} \frac{\partial x^{'\beta}}{\partial x^j} \cdot T^{ij} \tag{8}$$

or

$$T^{\alpha\beta} = \wedge^{\alpha i} \wedge^{\beta j} \cdot T^{ij}$$

Proceeding

$$T^{\alpha\beta} = \frac{\partial y^\alpha}{\partial x^i} \frac{\partial y^\beta}{\partial x^j} \cdot T^{ij} \tag{9}$$

here

$$y^\alpha = x^{'}$$

Moving on, a mixed tensor is defined as

$$T_\beta^\alpha = \frac{\partial x^{'\alpha}}{\partial x^i} \frac{\partial x^j}{\partial x^{'\beta}} \cdot T_j^i \tag{10}$$

3 Transformations For Co-Tensors

For Co-variant tensors the list of various transformations are

$$x_i^{'} = \frac{\partial x^j}{\partial x^{'i}} \cdot x_j \tag{11}$$

$$A_i = \frac{\partial x^j}{\partial x^{'i}} \cdot A_j \tag{12}$$

$$T_{\alpha\beta} = \frac{\partial x^i}{\partial x^{'\alpha}} \frac{\partial x^j}{\partial x^{'\beta}} \cdot T_{ij} \tag{13}$$

2

$$\Gamma^{\alpha}_{\mu\nu} = \frac{\partial x^{\lambda}}{\partial x^{\alpha}} \cdot \frac{\partial^2 x^{\alpha}}{\partial x^{\mu} \partial x^{\nu}} \tag{14}$$

$$\sum_i A_i \tag{15}$$