TENSOR WORKSHEET

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Abstract

In this paper we will be convering most of the tensors and their various transformations. We will also take a look on the dirac notation formalizations

Contents

1 Initial Tensor And Their Transformations

Firstly we will be looking at the tensors which have only one index. Broadly as we all know there are two types of tensors

1.Contra-variant tensors

2.Co-variant tensors

In contra-variant tensors the components transfrom in a "contra" manner. If the components increase, their respective products will decrese(If still any confusion please refer the main notes). The various Transformations of contra-variant vectors are given as

$$X'^{i} = \frac{\partial X'^{i}}{\partial X^{j}} \cdot X^{j} \tag{1}$$

In co-variant tensors the components trnsform in a "co" manner. If the components increase or decrease, even the products increases or decreses. Their tansformation is

$$X_{i}^{'} = \frac{\partial X^{j}}{\partial X_{i}^{'}} \cdot X^{j} \tag{2}$$

These transformations can also be representes as follows

$$X^{,i} = \wedge^{ij} \cdot X^{j} \tag{3}$$

and

$$X_{i}^{'} = \wedge^{ji} \cdot X^{j} \tag{4}$$

here

$$\wedge^{\alpha\beta} = \frac{\partial X'^{\alpha}}{\partial X^{\beta}} \tag{5}$$

2 Transformations For Contra Tensors

For contra-variant tesnors a list of various transformations are

$$x^{'i} = \frac{\partial x^{'i}}{\partial x^j} \cdot x^j \text{ (or) } x^{'i} = \wedge^{ij} \cdot x^j \tag{6}$$

$$A^\alpha = \frac{\partial x^{'\alpha}}{\partial x^\beta} \cdot A^\beta \tag{7}$$

$$T^{\alpha\beta} = \frac{\partial x^{'\alpha}}{\partial x^j} \frac{\partial x^{'\beta}}{\partial x^j} \cdot T^{ij} \tag{8}$$

or

$$T^{\alpha\beta} = \wedge^{\alpha i} \wedge^{\beta j} \cdot T^{ij}$$

Proceeding

$$T^{\alpha\beta} = \frac{\partial y^\alpha}{\partial x^i} \frac{\partial y^\beta}{\partial x^j} \cdot T^{ij} \tag{9}$$

here

$$y^\alpha = x^{'}$$

Moving on, a mixed tensor is defined as

$$T^\alpha_\beta = \frac{\partial x^{'\alpha}}{\partial x^i} \frac{\partial x^j}{\partial x^{'\beta}} \cdot T^i_j \tag{10}$$

3 Transformations For Co-Tensors

For Co-variant tensors the list of various transformations are

$$x^{'}_i = \frac{\partial x^j}{\partial x^{'i}} \cdot x_j \tag{11}$$

$$A_i = \frac{\partial x^j}{\partial x^{'i}} \cdot A_j \tag{12}$$

$$T_{\alpha\beta} = \frac{\partial x^i}{\partial x^{'\alpha}} \frac{\partial x^j}{\partial x^{'\beta}} \cdot T_{ij} \tag{13}$$

$$\Gamma^\alpha_{\mu\nu} \tag{14}$$

$$\sum_i A_i \tag{15}$$