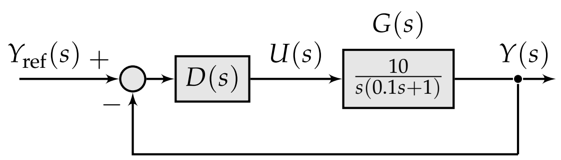
TA # 6, EE 250 (Control System Analysis) - Spring 2013*

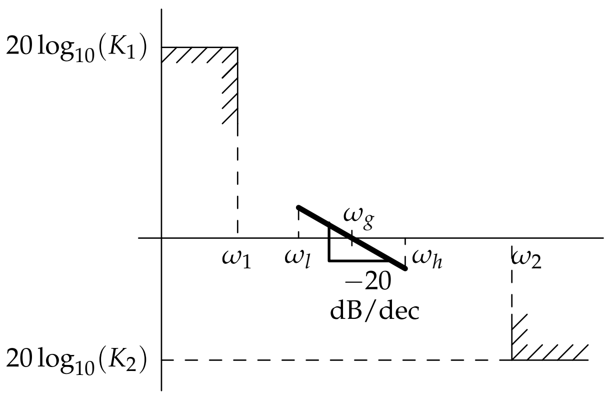
DEPARTMENT OF ELECTRICAL ENGINEERING, IIT KANPUR

Consider the following control system.



We wish to design a compensator $D(s)$ that satisfies the following design specifications: (a) $K_v = 100$. (b) PM $\approx 60°$. (c) Sinusoidal inputs of up to 1 rad/sec to be reproduced with $\leq 2\%$ error. (d) Sinusoidal inputs with a frequency of greater than 100 rad/sec to be attenuated at the output to $\leq 5\%$ of their input value.

For the following figure



1. Determine K_1 and K_2.

2. Write the numerical values of ω_1 and ω_2.

3. What is the decade distance needed between the corner frequencies ω_l and ω_h for the desired Bode plot?

4. Is this DD the distance on the BMP or on the ABMP or both?

5. On the semilog grid provided, draw the ABMPs of the desired $D(s)G(s)$ and of $G(s)$. Your figure must contain all the necessary labels.

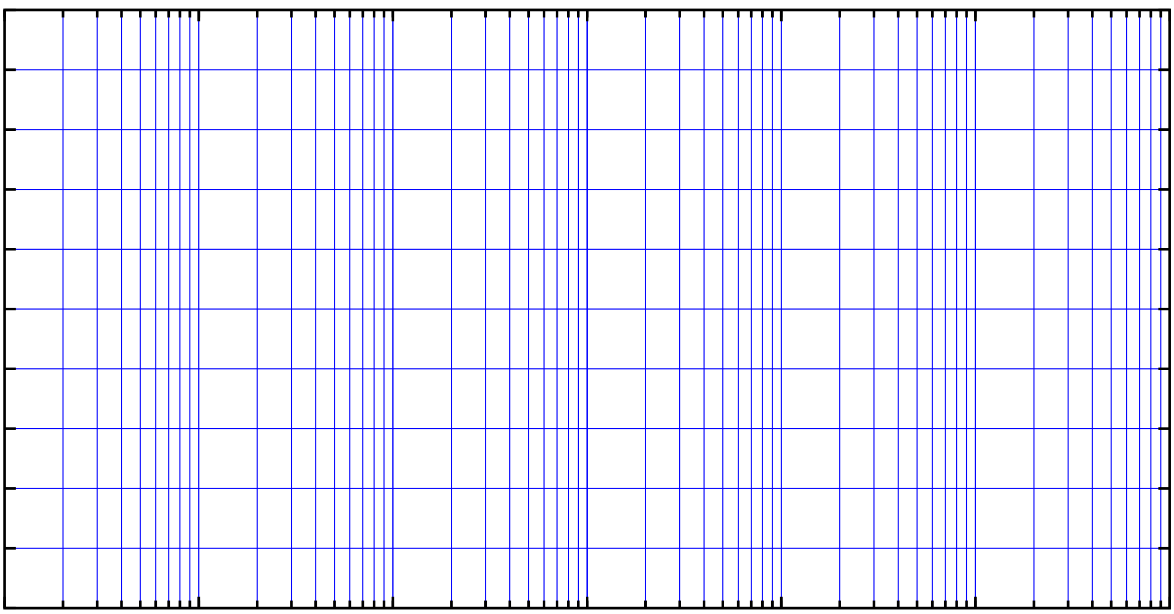
6. On the semilog grid provided, show the ABMP of the resulting $D(s)$. Write the TF of $D(s)$.

7. For the resulting CL system, given that $\omega_B \in [\omega_{\min}, \omega_{\max}]$, where ω_B is the bandwidth, what are the values of $\omega_{\min}$ and $\omega_{\max}$?

8. Convert the TF of $D(s)$ into a state-space model using the simulation diagram approach discussed in the lectures.

9. Discretize the state-space model using Euler's approximation. Assume the step size of numerical integration equals T.

10. Will this controller $D(s)$ work approximately as well as it works in the continuous-time case if $T = 2$ ms?



*Instructor: Ramprasad Potluri, E-mail: potluri@iitk.ac.in. Office: WL217A, Lab: WL217B. TA # 5 discussed mid-semester exam's solutions.

Solution to TA # 6, EE 250, Spring 2025*

DEPARTMENT OF ELECTRICAL ENGINEERING, IIT KANPUR

We first translate the specifications into actionable information.

(a) $K_v = 100$.

For a unity-feedback control system built around the minimum-phase TF $G_{\mathrm{OL}}(s)$ that contains a single integrator, K_v is the frequency where the left-most asymptote of the ABMP of $G_{\mathrm{OL}}(s)$ has a value of 0 dB. K_v has the units of rad/s.

In this problem, $K_v = 100$ implies that the left most asymptote of the BMP of $D(s)G(s)$ has a slope of -20 dB/decade and passes through the point

$\{\ \omega = 100$ rad/s, dB-gain $= 0$ dB $\}$.

(b) PM $\approx 60°$.

This specification can give us the decade distance (DD) between the ω_l and ω_h. Table 1 says that a DD of 1.144 decades between ω_l and ω_h with a -20 dB/decade decline between them implies PM $\approx 60°$.

This information can approximately also be read off the Figure 1; this approximate figure is adequate for practical purposes.

We have from this specification:

$$\log_{10}\omega_h - \log_{10}\omega_l = 1.144$$

(c) Sinusoidal inputs of up to 1 rad/sec to be reproduced with $\leq 2\%$ error.

$$G_e(s) \triangleq \frac{E(s)}{R(s)} = \frac{1}{1 + D(s)G(s)}$$
$$\Rightarrow G_e(j\omega) = \frac{1}{1 + D(j\omega)G(j\omega)}$$

Let us define $e(t) \triangleq r(t) - y(t)$. For an input sinusoid $r(t) = A\sin\omega t$, the steady-state value of $e(t)$ is $e_{ss}(t) = |G_e(j\omega)|A\sin(\omega t + \angle G_e(j\omega))$. It is given that $e_{ss}(t) \leq 2\%$ when $r(t)$ is upto 1 rad/s in frequency. Is this a specification for the magnitude and phase of $e_{ss}(t)$, or is it only for the phase of $e_{ss}(t)$? Note that, the frequency responses of even ideal filters are defined as having phases changing linearly with frequency. So, most likely, we are not being asked to constrain the phase as well as magnitude of a practical filter within 2%. This guess is verified by looking at specification (d). So, the 2% specification can only be on the magnitude of $e_{ss}(t)$. So, we shall constrain the magnitude of $e_{ss}(t)$ within 2%. This can be done as follows:

$$|G_e(j\omega)| = \frac{1}{|1 + D(j\omega)G(j\omega)|} \leq 2\%,$$
$$\forall\omega \leq 1 \text{ rad/s}.$$
$$\Rightarrow |D(j\omega)G(j\omega)| \gtrapprox 50, \quad \forall\omega \leq 1 \text{ rad/s}.$$

So, $K_1 \geq 50$ and $\omega_1 = 1$ rad/sec.

(d) Sinusoidal inputs with a frequency of greater than 100 rad/sec to be attenuated at the output to $\leq 5\%$ of their input value.

$$G_{\mathrm{CL}}(s) \triangleq \frac{Y(s)}{R(s)} = \frac{D(s)G(s)}{1 + D(s)G(s)}$$
$$\Rightarrow G_{\mathrm{CL}}(j\omega) = \frac{D(j\omega)G(j\omega)}{1 + D(j\omega)G(j\omega)}$$

Table 1: The required DD (Δ) between the corner frequencies of the lead (or lag) controller and the corresponding $\Phi_{\max}$ (or $\Phi_{\min}$) that the controller needs to provide for certain commonly occuring values of $\Phi_{\max}$ (or $\Phi_{\min}$). Note that this DD is the same on the BMP and the AMBP.

Required $\Phi_{\max}$ ($\Phi_{\min}$)	$90°$ ($-90°$)	$60°$ ($-60°$)	$45°$ ($-45°$)	$30°$ ($-30°$)
DD between corner frequencies of BP in decades	∞	1.1439 $\approx$ 1.144	0.76555 $\approx$ 0.766	0.47712 $\approx$ 0.477

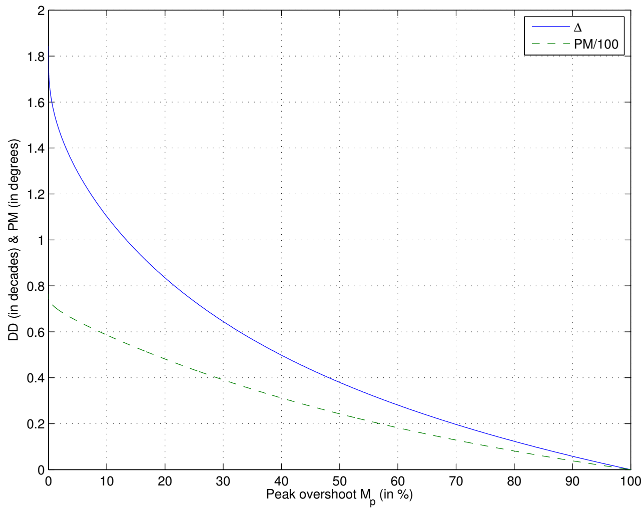


Figure 1: Plot of DD versus M_p and PM versus M_p.

*Instructor: Ramprasad Potluri, E-mail: potluri@iitk.ac.in. Office: WL217A, Lab: WL217B. TA # 5 discussed solutions to midsemester exam.

Want $|G_{\text{CL}}(j\omega)| \leq 5\%$ (wanting to keep $\angle G_{\text{CL}}(j\omega) \leq 5\%$ is unrealistic). This can be achieved thus:

$$|D(j\omega)G(j\omega)| \lessapprox 5\%, \quad \forall \omega \geq 100 \text{ rad/s}$$

So, $K_2 \lessapprox 0.05$ and $\omega_2 = 100$ rad/s.

Now that we have actionable information, we answer the questions.

1. Determine K_1 and K_2.

$$K_1 = 50, K_2 = 0.05$$

2. Write the numerical values of ω_1 and ω_2.

$$\omega_1 = 1 \text{ rad/sec}, \omega_2 = 100 \text{ rad/sec}$$

3. What is the DD needed between the corner frequencies ω_l and ω_h for the desired Bode plot?

1.144 decades.

4. Is this DD the distance on the BMP or on the ABMP or both?

Both.

5. On the semilog grid provided, draw the ABMPs of the desired $D(s)G(s)$ and of $G(s)$. Your figure must contain all the necessary labels.

See solution worked out on the semilog grid.

5.1. From the values of K_1, K_2 and ω_1, ω_2 we get the forbidden regions. The BMP of $D(s)G(s)$ cannot lie in this region.

5.2. Draw the left-most asymptote of the BMP of $D(s)G(s)$. For this, using $K_v = 100$, draw a straight line at a slope of -20 dB/decade and passing through 100 rad/sec at 0 dB.

5.3. As no data is given to fix ω_g, we choose ω_g to be approximately equidistant, on logarithmic scale, from ω_1 and ω_2. That is, $\omega_g \approx \sqrt{\omega_1 \omega_2}$. This gives $\omega_g = 10$ rad/sec.

5.4. We draw a 1.144 decade-wide -20 dB/decade straight line centered at ω_g.

5.5. We connect the left end of this -20 dB/decade section to the asymptote of step 5.2 via a straight line while being careful that this line does not intersect the LF forbidden region.

This line will need to have a slope that is integer multiple of -20 dB/decade. We get this slope as -60 dB /dec in our case.

5.6. From the right end of the -20 dB/decline (of step 5), we draw a straight line at a slope of -40 dB/dec (integral multiple of 20), that passes below the HF forbidden zone.

The ABMP of the desired $D(s)G(s)$ is shown on the semilog grid. Also shown on the same graph is the ABMP of $G(s)$.

6. On the semilog grid provided, show the ABMP of the resulting $D(s)$. Write the TF of $D(s)$.

See solution worked out on the semilog grid.

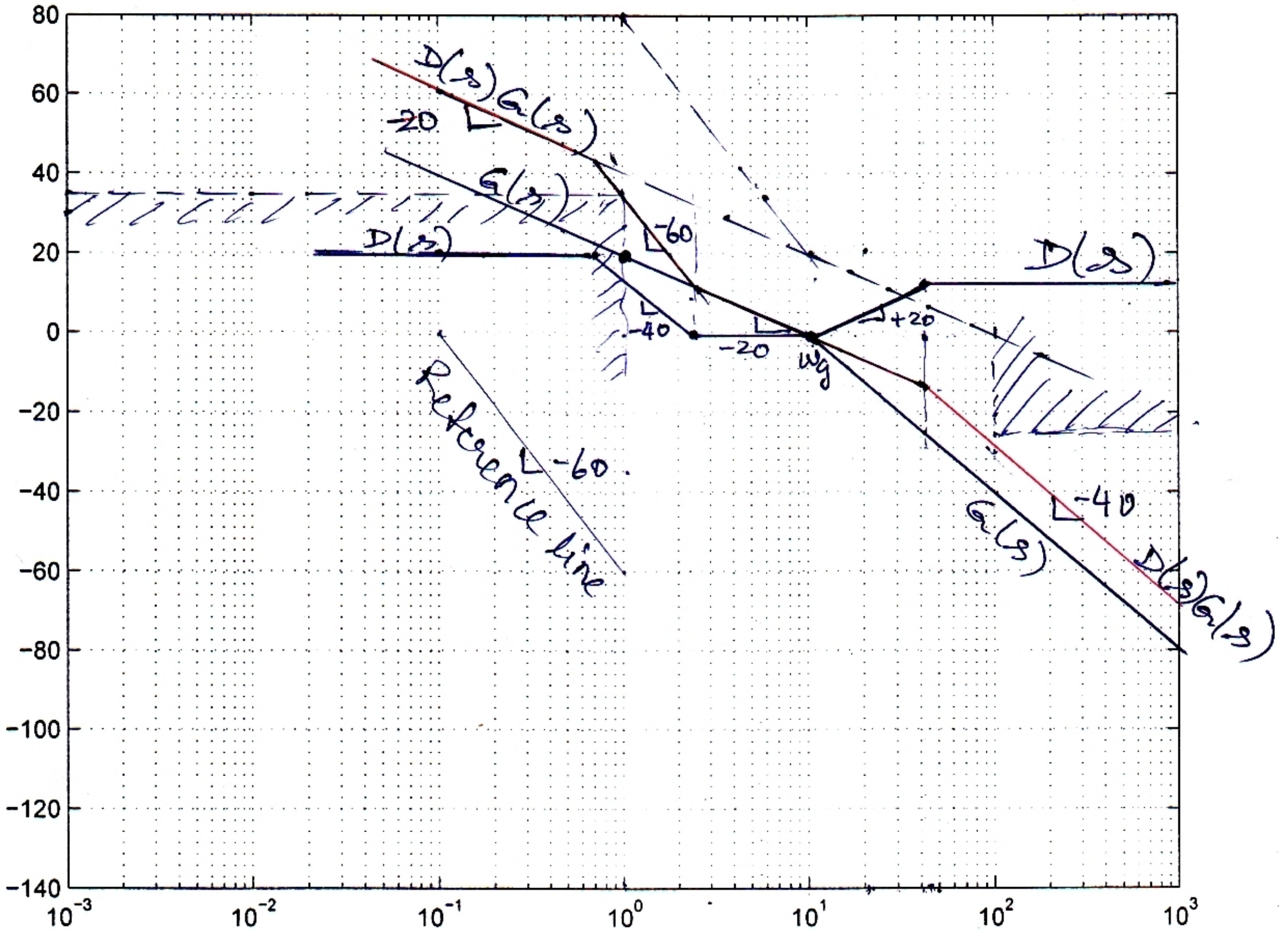
6.1. Subtract the ABMP of $G(s)$ from that of $D(s)G(s)$ to obtain the ABMP of $D(s)$ as shown on the graph.

6.2. The TF of D(s) is

$$D(s) = \frac{10\left(\frac{s}{2.5}+1\right)^2\left(\frac{s}{10}+1\right)}{\left(\frac{s}{0.7}+1\right)^2\left(\frac{s}{40}+1\right)}.$$

7. For the resulting CL system, given that $\omega_b \in [\omega_{\min}, \omega_{\max}]$, where ω_b is the bandwidth, what are the values of $\omega_{\min}$ and $\omega_{\max}$?

Since $\omega_g \leq \omega_b \leq 2\omega_g$, we have $\omega_{\min} = \omega_g = 10$ rad/s, and $\omega_{\max} = 2\omega_g = 20$ rad/s.


80
60
40
20
0
−20
−40
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10^{-3}
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10^{0}
10^{1}
10^{2}
10^{3}
D(s)G(s)
-20
G(s)
D(s)
-60
-40
-20
ω_g
+20
D(s)
Reference line
-60
-40
G(s)
D(s)G(s)