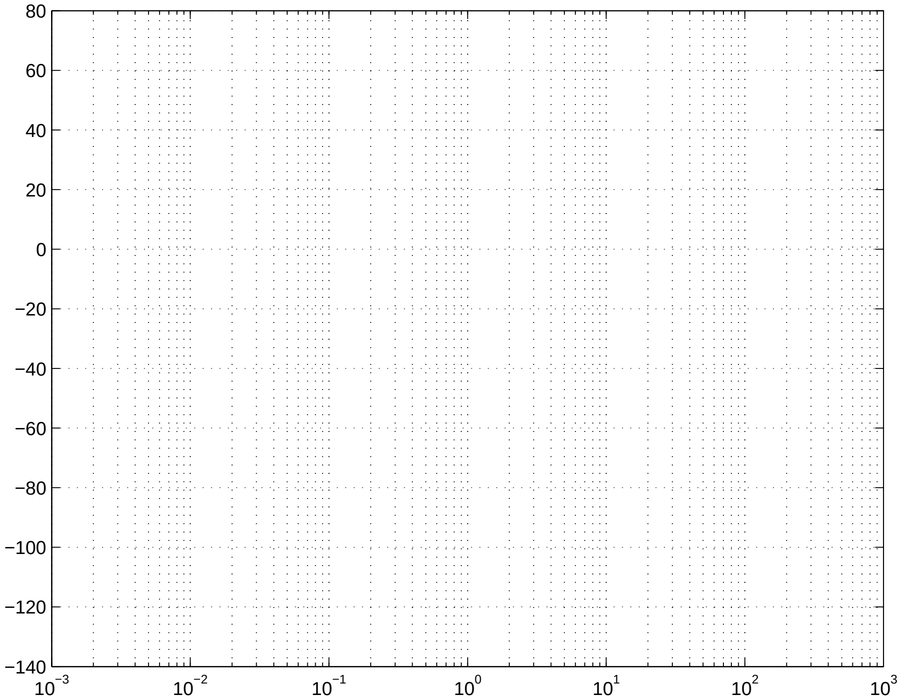
TA # 7, EE 250 (Control System Analysis) - Spring 2025*

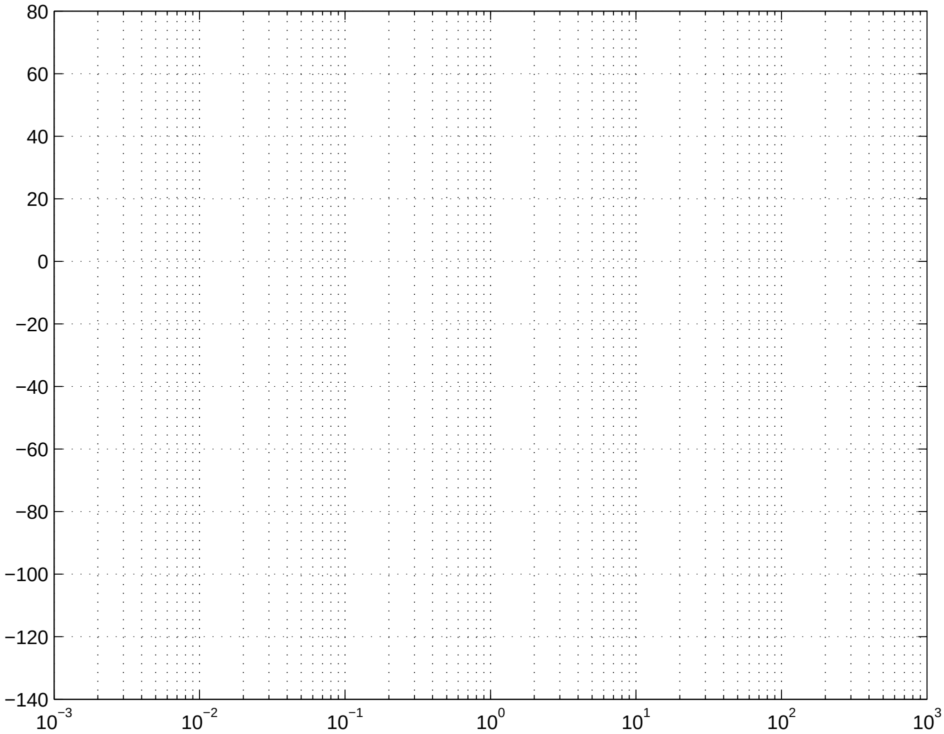
DEPARTMENT OF ELECTRICAL ENGINEERING, IIT KANPUR

This tutorial is on design of lead and lag compensators. The solutions are included. We will discuss these solutions in the tutorial. These problems are respectively Examples 10.1, 10.2, 10.4, 10.5 of [Gop93].

1. Consider a type-1 unity-feedback system with an OL TF $G(s) = \frac{K}{s(s+1)}$. We wish to have velocity error constant $K_v = 10$ and phase margin of at least 45°. Design a lead compensator for this purpose.

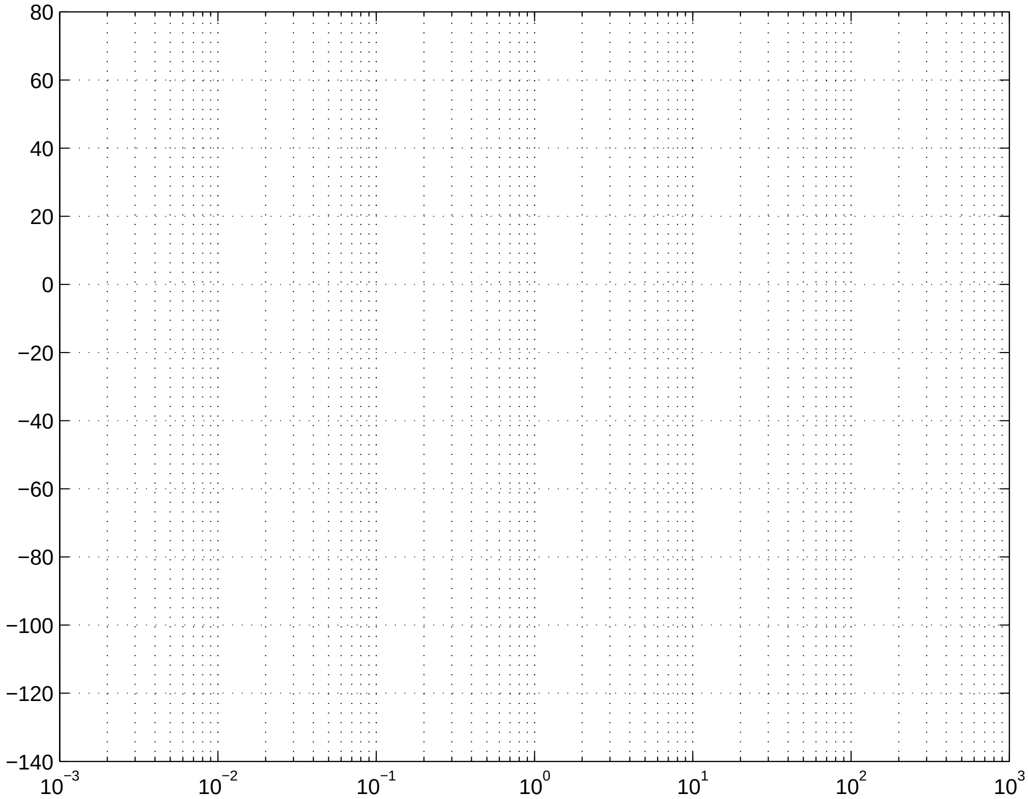


2. The uncompensated unity feedback CL TF of a system is $M(s) = \frac{1}{s^2+1}$. Design a compensator (decide between lead and lag) that will provide the CL system's step response a damping coefficient of $\zeta \geq 0.55$.

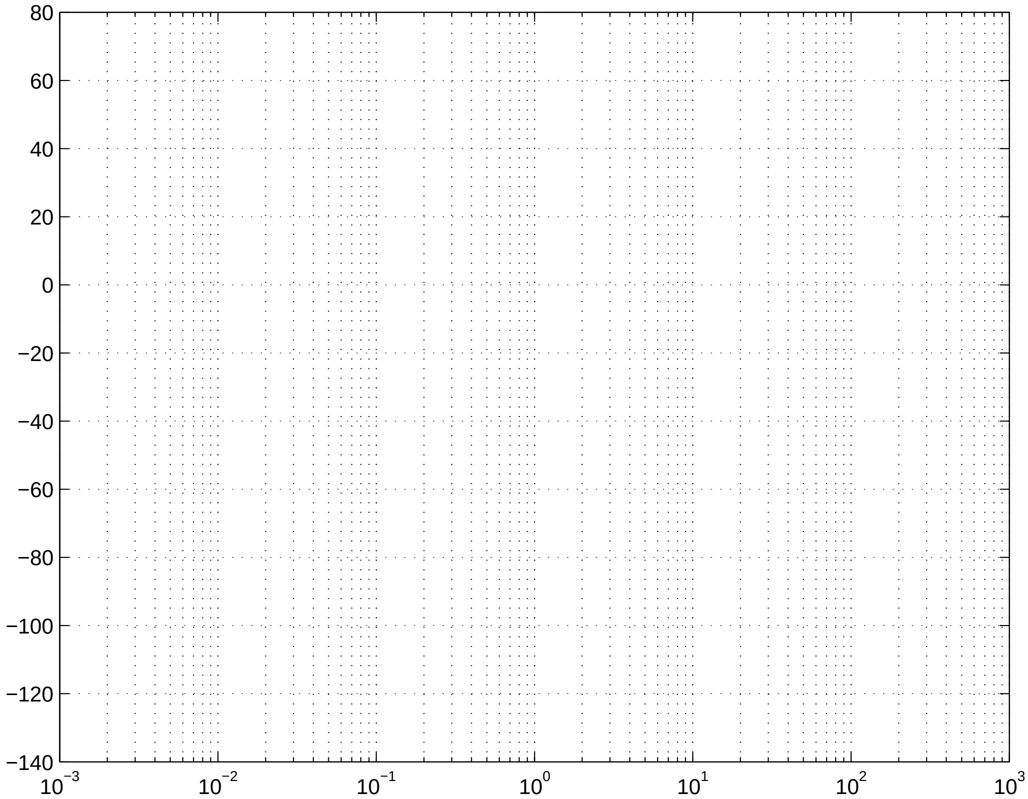


*Instructor: Ramprasad Potluri, E-mail: potluri@iitk.ac.in. Office: WL217A, Lab: WL217B, Phones: (0512) 259-6093, 259-7735

3. Solve Problem 1 using a lag compensator.



4. A unity-feedback control system has the OL TF $G(s) = \frac{K}{s(0.1s+1)(0.2s+1)}$. The system is required to satisfy the performance spec-s $K_v = 30$, PM $\geq 40°$, $\omega_b = 5$ rad/s. Design a lag compensator for this purpose.



References

[Gop93] Madan Gopal. *Modern Control System Theory*. New Age International (P) Ltd., New Delhi, India, second edition, 1993. 2003 Reprint.

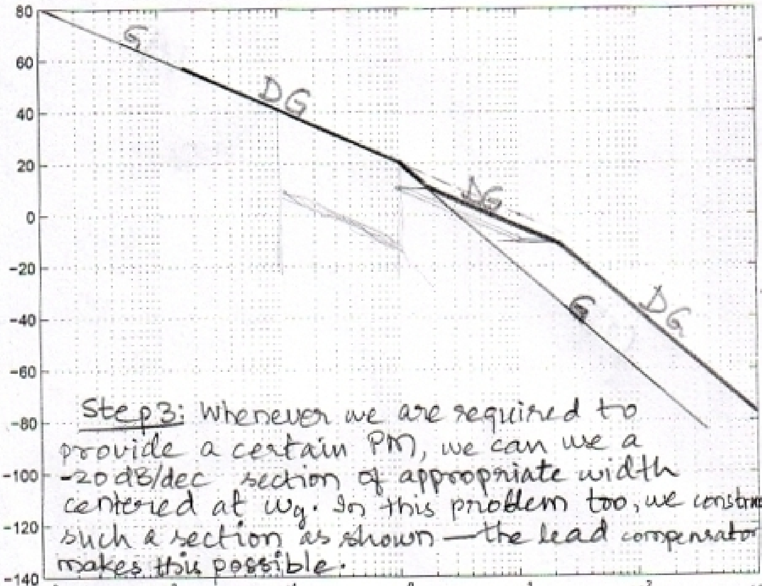
Solutions. Ramprasad P.

TA # 6 , EE 250 (Control System Analysis)

DEPARTMENT OF ELECTRICAL ENGINEERING, IIT KANPUR

This tutorial is on design of lead and lag compensators.

1. Consider a type-1 unity-feedback system with an OL TF $G(s) = \frac{K}{s(s+1)}$. We wish to have velocity error constant $K_v = 10$ and phase margin of at least 45°. Design a lead compensator for this purpose.



Step3: Whenever we are required to provide a certain PM, we can use a -20 dB/dec section of appropriate width centered at ω_g. In this problem too, we construct such a section as shown — the lead compensator makes this possible.

Step1: As $G(s)$ has one integrator in it, we have $K_v = \lim_{s\to 0} sG(s) = K$

$\therefore K = K_v = 10$. Draw ABMP of $G(s)$ with $K=10$.

Step2: For $\phi m \geq 45°$, $\Delta \geq 0.766$ decades. Say, $\Delta = 1$ decade for convenience

Step4: $DG = \frac{10\left(\frac{s}{2}+1\right)}{s\left(\frac{s}{1}+1\right)\left(\frac{s}{20}+1\right)}$

$\Rightarrow D(s) = \frac{\frac{s}{2}+1}{\frac{s}{20}+1}$; K=10 is included in $G(s)$.

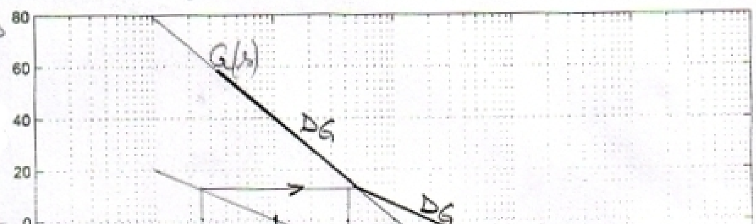
2. The uncompensated unity feedback CL TF of a system is $M(s) = \frac{1}{s^2+1}$. Design a compensator (decide between lead and lag) that will provide the CL system's step response a damping coefficient of $\zeta \geq 0.55$.

step1: The OL TF is $G(s) = \frac{1}{s^2}$

step2: The phase of $G(j\omega)$ is -180° across all ω.

A lag compensator adds negative phase. So, it cannot create a (+ve) PM for this $G(s)$

So, we choose a lead compensator.



Step3: $\zeta \geq 0.55 \Rightarrow PM \geq 55°$

$\Rightarrow$ Let's decided to provide a PM = 60°. For this we will create DG such that it has a -20 dB/decade section that is 1.2 dec. wide and is centered about ω_g. Also, since there are no other spec-s, we can retain the present value of K_a.

Step4: Construct a 22.8 mm-wide -20 dB/dec section symmetrically positioned about 0.1 rad/s as shown. Use this section to complete the ABMP of DG as shown.

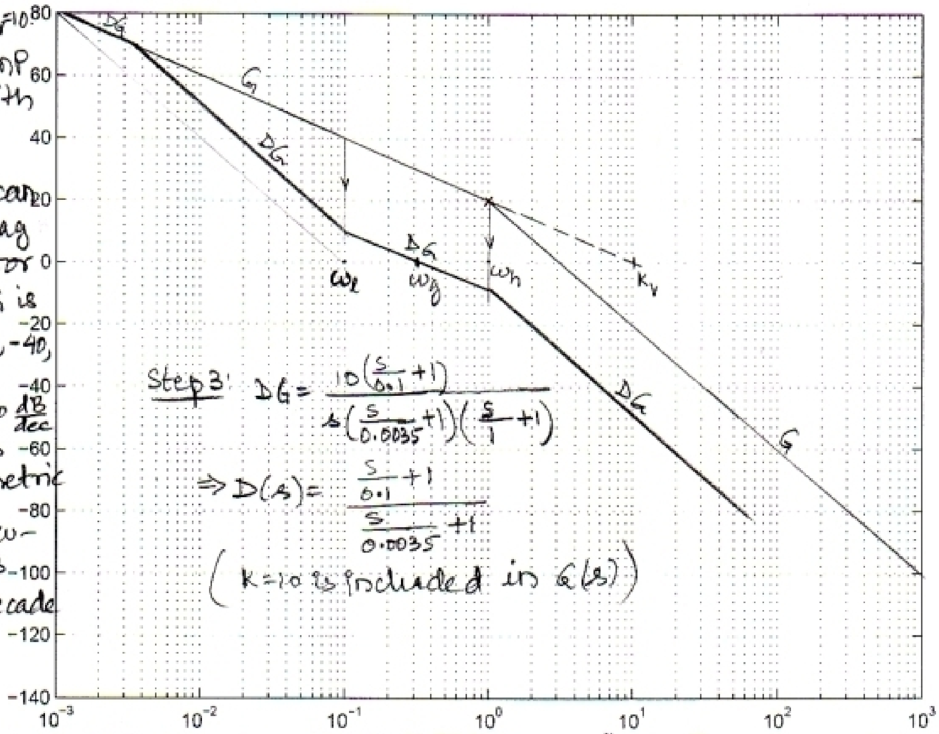
Instructor: Ramprasad Potluri, E-mail: potluri@iitk.ac.in. Office: WL217A, Lab: WL217B, Phones: (0512) 259-8837, 259-7735.
Assignment posted on March 20, 2009.

3. Solve Problem 1 using a lag compensator.

Step1: $K = K_v = 10$

Draw ABMP of $G(s)$ with $K = 10$.

Step2: We can use the lag compensator so that DG is $-20, -40, -20, -40$, where the second $-20\ \frac{dB}{dec}$ section is symmetric about the ω-axis and is about 1 decade-wide.



Step3: $DG = \dfrac{10\left(\frac{s}{0.1}+1\right)}{s\left(\frac{s}{0.0035}+1\right)\left(\frac{s}{1}+1\right)}$

$\Rightarrow D(s) = \dfrac{\frac{s}{0.1}+1}{\frac{s}{0.0035}+1}$

($K = 10$ is included in $G(s)$)

4. A unity-feedback control system has the OL TF $G(s) = \frac{K}{s(0.1s+1)(0.2s+1)}$. The system is required to satisfy the performance spec-s $K_v = 30$, PM $\geq 40°$, $\omega_b = 5$ rad/s. Design a lag compensator for this purpose.

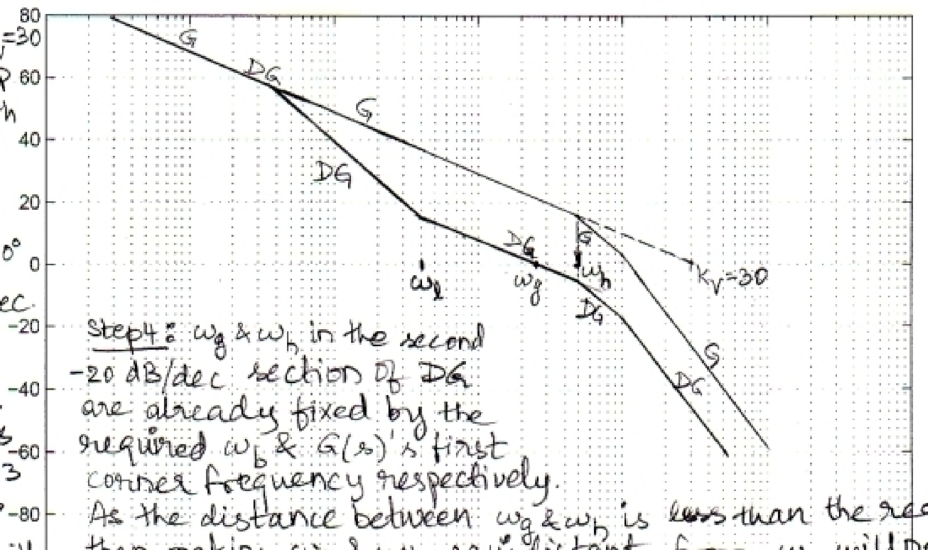
Step1: $K = K_v = 30$

Draw ABMP of $G(s)$ with $K = 10$.

Step2: PM $\geq 40°$ $\Rightarrow \Delta \geq 0.67$ dec.

$\omega_b = 5$ rad/s $\Rightarrow \omega_g \approx 2.5-3$ rad/s

Step3: We will use the lag compensator so that DG is $-20, -40, -20, -40, -60$.



Step4: ω_g & ω_h in the second -20 dB/dec section of DG are already fixed by the required ω_b & $G(s)$'s first corner frequency respectively.

As the distance between ω_g & ω_h is less than the required $\frac{0.67}{2}$ dec, then making ω_ℓ & ω_h equidistant from ω_g will not provide us the required PM. The solution is to make $\omega_\ell - \omega_g$ distance (in decades) larger than $\omega_g - \omega_h$ distance. How much larger? I tried two different values for ω_ℓ: 0.8 rad/s and 0.4 rad/s. I plotted the BP of DG in the two cases. PM $\approx 38°$ & PM $\approx 41°$ in the first and the second case respectively.

Step5: $DG(s) = \dfrac{30\left(\frac{s}{0.4}+1\right)}{s\left(\frac{s}{0.04}+1\right)\left(\frac{s}{5}+1\right)\left(\frac{s}{10}+1\right)}$

You must try this on MATLAB or Octave. We need such software for this problem.