TA # 2, EE 250 (Control System Analysis) - Spring 2025*

DEPARTMENT OF ELECTRICAL ENGINEERING, IIT KANPUR

1 Sketching of ABMPs

Sketch the asymptotic Bode magnitude plots of the following transfer functions (TFs): 1. $\dfrac{20}{s(s+10)}$ 2. $\dfrac{20}{s^2(s+10)}$ 3. $\dfrac{{\omega_n}^2}{s^2+2\zeta\omega_n+{\omega_n}^2}$ 4. $\dfrac{20(s+1)}{s(s+10)}$ 5. $\dfrac{20(s-1)}{s(s+10)}$ In each of the above cases, label the axes and slopes appropriately, and write the units along the axes.

2 Sketching ABPPs

Sketch the asymptotic Bode phase plots of the above TFs as combinations of elemental Bode phase plots.

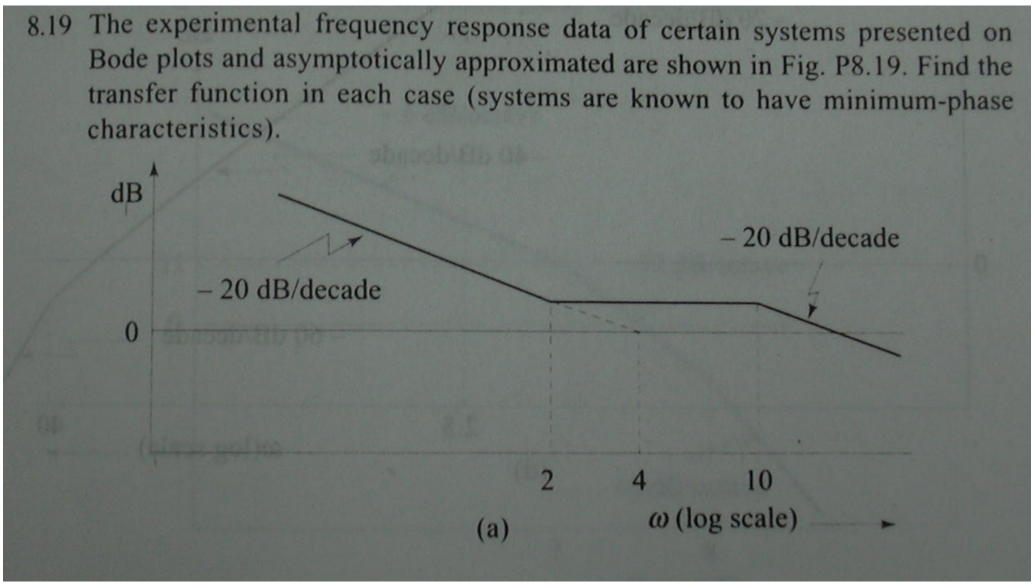
An elemental Bode plot is that of an elemental TF. The only elemental TFs that we may see in this course are

$$K,\ (s+\omega_0),\ \frac{{\omega_n}^2}{s^2+2\zeta\omega_n+{\omega_n}^2},\ e^{-t_d s}.$$

Here, K and ω_0 are real constants, ζ, t_d, and ω_n are non-negative real numbers, and n is a positive integer.

3 Transfer functions of minimum phase systems

This assignment is based on problems from [Gop93].


8.19 The experimental frequency response data of certain systems presented on Bode plots and asymptotically approximated are shown in Fig. P8.19. Find the transfer function in each case (systems are known to have minimum-phase characteristics).

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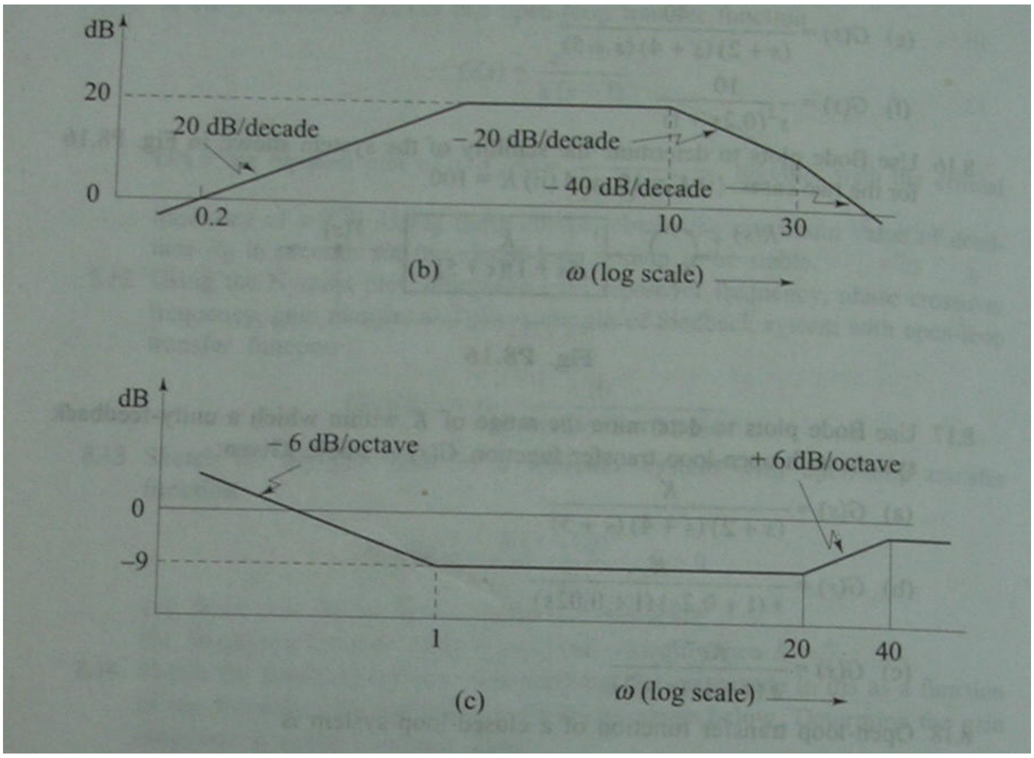
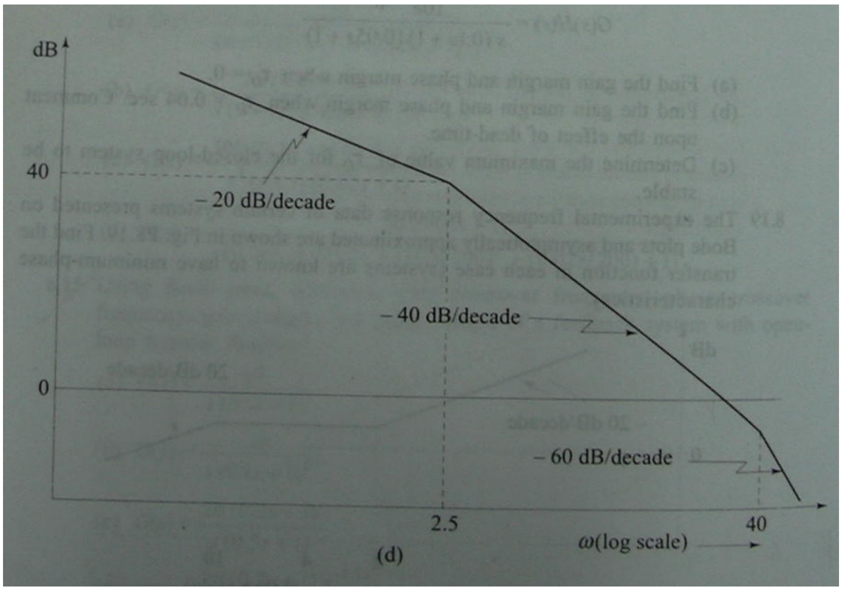

dB
20
20 dB/decade
− 20 dB/decade
− 40 dB/decade
0
0.2
10
30
(b)
ω (log scale)
dB
− 6 dB/octave
+ 6 dB/octave
0
−9
1
20
40
(c)
ω (log scale)


dB
40
− 20 dB/decade
− 40 dB/decade
0
− 60 dB/decade
2.5
40
ω(log scale)
(d)

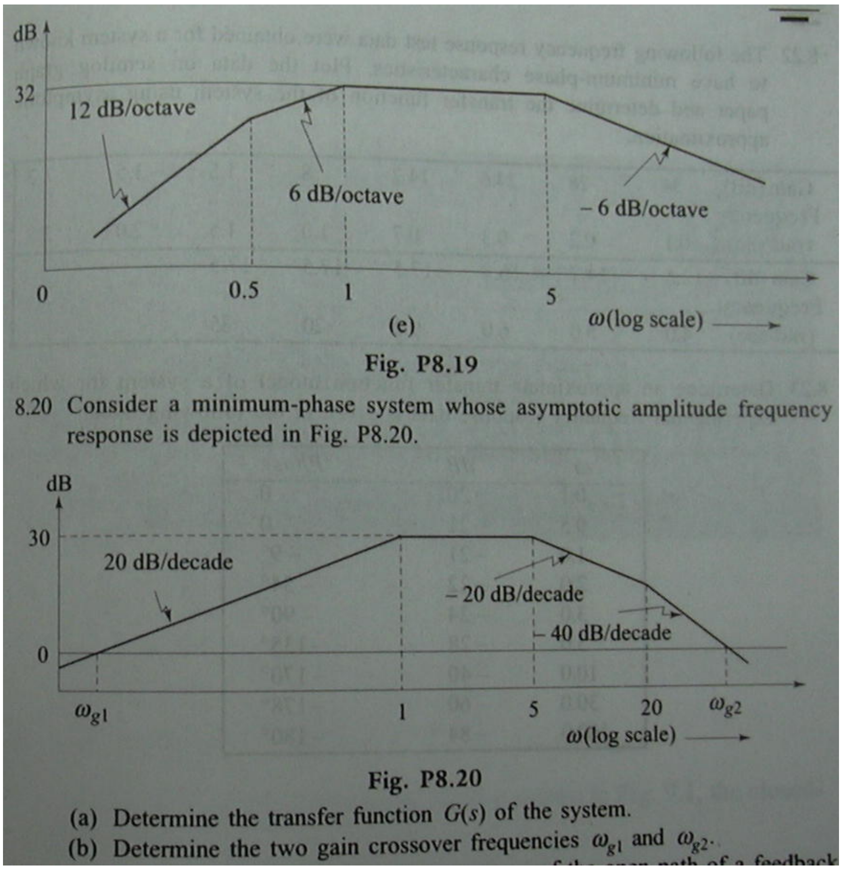


Fig. P8.19

8.20 Consider a minimum-phase system whose asymptotic amplitude frequency response is depicted in Fig. P8.20.

Fig. P8.20

(a) Determine the transfer function $G(s)$ of the system.

(b) Determine the two gain crossover frequencies ω_{g1} and ω_{g2}.

4 Non-Tutorial Problem: Sketching Polar Plots

Sketch the polar plots of the following transfer functions using Bode plots.

1. $\dfrac{K}{(s+1)^2}$
2. $\dfrac{K}{s(s+1)(\frac{s}{10}+1)}$
3. $\dfrac{K}{s(s+1)^2}$
4. $\dfrac{K}{s^2(s+1)^2}$
5. $\dfrac{K}{s^3(s+1)^2}$
6. $\dfrac{K(s+1)}{s(\frac{s}{10}-1)}$

5 Non-Tutorial Problems: MATLAB or GNU Octave

We do not discuss questions related to MATLAB or GNU Octave in the tutorials.

See if you can solve the following questions using MATLAB or GNU Octave.

1. Write a MATLAB code that uses the `bode` function of MATLAB to determine the angle of $G(j\omega) = \frac{1}{j\omega+10}$ at $\omega = 100$ rad/s.

2. Write a MATLAB code that declares the transfer function (TF) $G(s) = \frac{s+1}{(s+2)(s+3)}$ for further use by the MATLAB function `feedback`.

3. Write a MATLAB code that uses the MATLAB function `feedback` to evaluate the TF of the positive unity feedback closed-loop system corresponding to the open-loop TF $G(s) = \frac{10}{s^2+100}$.

4. Write a MATLAB code that uses the MATLAB function `tfdata` to extract the numerator and denominator of the closed-loop TF obtained in the Question 3.

5. Write a MATLAB code that uses the MATLAB function `roots` to extract the poles of the TF obtained in Question 3.

6. What does the command `logspace(2,3,100)` do?

7. What does the command `linspace(2,3,100)` do?

8. Write a MATLAB code to plot the step response of the TF $G(s) = 1/(s+10)$.

9. On the figure generated in Question 8, we wish to plot the step response of the TF $G(s) = 1/(s+20)$ without erasing the step response generated in Question 8. Write a MATLAB code to achieve this goal.

10. After you finished all the above questions, you feel that your computer's memory is choked with much data generated by all the above MATLAB code. Write a MATLAB code to clear the data and close all figure windows.

6 Solution to Section 1

Need to sketch the asymptotic Bode magnitude plots of the following TFs: $\frac{20}{s(s+10)}$, $\frac{20}{s^2(s+10)}$, $\frac{\omega_n^2}{s^2+2\zeta\omega_n+\omega_n^2}$, $\frac{20(s+1)}{s(s+10)}$, $\frac{20(s-1)}{s(s+10)}$

1. $\frac{20}{s(s+10)} \rightarrow \frac{20}{j\omega(j\omega+10)} \rightarrow \frac{2\cdot 0}{\omega\sqrt{\frac{\omega^2}{10^2}+1}} \rightarrow 20\log_{10}\frac{2/10}{\frac{\omega}{10}\sqrt{\frac{\omega^2}{10^2}+1}}$

TF given; sinusoidal TF; magnitude of the sine TF; dB-gain of the sine TF.

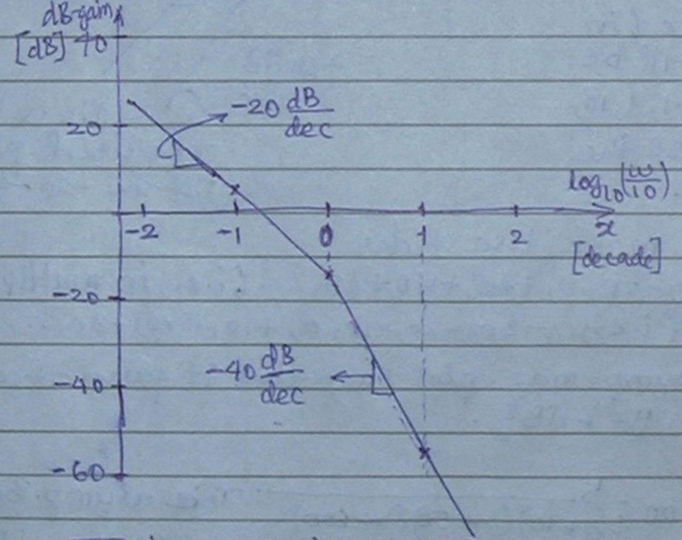
$$20\log_{10}0.2 - 20\log_{10}\frac{\omega}{10} - 20\log_{10}\sqrt{\frac{\omega^2}{10^2}+1}$$

$$\text{dB-gain} = \begin{cases} 20\log_{10}0.2 - 20\log_{10}\frac{\omega}{10}, & \frac{\omega}{10} \ll 1 \\ 20\log_{10}0.2 - 40\log_{10}\frac{\omega}{10}, & \frac{\omega}{10} \gg 1 \end{cases}$$

If we let $x = \log_{10}\frac{\omega}{10}$, then

$$\text{dB-gain} = \begin{cases} 20\log_{10}0.2 - 20x, & \frac{\omega}{10} \ll 1 \text{ — low frequency asymptote} \\ 20\log_{10}0.2 - 40x, & \frac{\omega}{10} \gg 1 \text{ — high frequency asymptote} \end{cases}$$

Let us now sketch the dB-gain against $x = \log_{10}\frac{\omega}{10}$:



$\left(\begin{array}{l}\log_{10}0.2 = -0.7 \\ 20\log_{10}0.2 = -14\end{array}\right)$

We use the following 2 points to define the low-freq asymp: $\{x=0, \text{dB-gain}=-14\}$, $\{x=-1, \text{dB-gain}=6\}$.

We use the following 2 points to define the high-freq. asymp: $\{x=0, \text{dB-gain}=-14\}$, $\{x=1, \text{dB-gain}=-54\}$

We see, ~~that~~ by comparing the TF with the asymptotic Bode magnitude plot, that $\frac{1}{s}$ term gives a slope of -20 dB/dec, and this changes to -40 dB/dec at $\omega=10$ rad/s, i.e., at $\log_{10}\frac{\omega}{10} = 0$ decade. So, we could have constructed this plot by sketching a line that has slopes of -20 dB/dec followed by -40 dB/dec and passing through, say, the point $\{\omega=100 \text{ rad/s}$ (i.e. $\log_{10}\frac{\omega}{10} = 1$ decade), dB-gain $= 20\log_{10}\frac{2}{1000} = -54$ dB$\}$.
Let us apply this technique in the remaining four problems.

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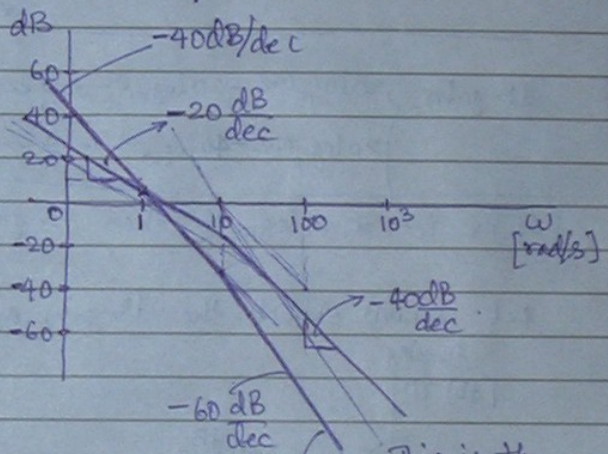
2. $\frac{20}{s^2(s+10)}$. The asymp. Bode mag. plot is a line that has a ~~slope~~ straight line segment of slope -40 dB/dec, followed by another straight line segment of slope -60 dB/dec beginning $\omega = 10$ rad/s. Finally, we need just one point define the exact location of the plot on the dB-gain vs. decade plain. This point is, e.g.,

$$\left\{ \omega = 1 \tfrac{\text{rad}}{s},\ \text{dB-gain} = 20\log_{10}\frac{20}{1\sqrt{101}} \approx 20\log_{10} 2 = 6\,\text{dB} \right\}.$$

The ~~20~~ -40 dB/dec section passes through this point.

Now, let us sketch the plot:

Here are the steps:



This is the required plot, (not the -20 -40 one).

Setting up the axes:

(1). Draw the x & y axes in decades and dBs respectively.

(2) Along the x-axis, we may mark the ticks either as ....., $-2, -1, 0, +1, +2$, etc (in which case the units will be decade), or as $0.01, 0.1, 1, 10, 10^2, 10^3$, etc (in which case the units will be in rad/s).

(3) Along the y-axis, we mark the ticks usually as ..., $-60, -40, -20, 0, +20, +40, +60, \ldots$ (i.e., in multiples of 20) or in multiples of 10 (i.e., ..., $-30, -20, -10, 0, +10, +20, +30, \ldots$).

(4) Then we sketch the asymp. mag. plot such that it passes through the point $\{\omega = 1$ rad/s, $y = 6$ dB$\}$.

Now, try this technique on $\frac{K}{s(s+10)(s+50)(s+600)}$. The asymp. Bode mag. plot of this TF has four straight line segments, respectively with the following slopes — -20 dB/dec, followed by -40 dB/dec beginning $\omega = 10$, followed by -60 dB/dec beginning 50 rad/s, followed by -80 dB/dec beginning $\omega = 600$ rad/s. The plot passes through the point $\left\{\omega = 1 \tfrac{\text{rad}}{s},\ \text{dB-gain} = 20\log_{10}\frac{K}{1(10)(50)(600)}\right\}$.

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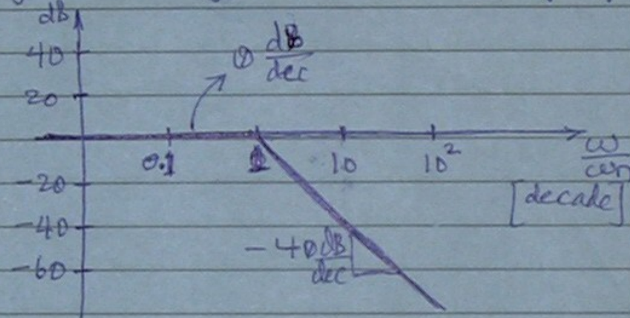
3. $\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \rightarrow \dfrac{\omega_n^2}{-\omega^2+j2\zeta\omega_n\omega+\omega_n^2} \rightarrow \dfrac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2+j\,2\zeta\frac{\omega}{\omega_n}}$

$$\text{dB-gain}=\begin{cases} 20\log_{10}(1)=0, & \frac{\omega}{\omega_n}\ll 1 \\ 20\log_{10}\dfrac{1}{\left(\frac{\omega}{\omega_n}\right)^2}, & \frac{\omega}{\omega_n}\gg 1 \end{cases} \leftarrow \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

So, the asymp. Bode mag. plot (let's abbreviate it ABMP) has a straight line segment with a slope of 0 dB/dec, followed by another straight line segment with a slope of −40 dB/dec beginning $\omega=\omega_n$.

The ABMP passes through the point $\left\{\frac{\omega}{\omega_n}=1, \text{dB-gain}=0\right\}$

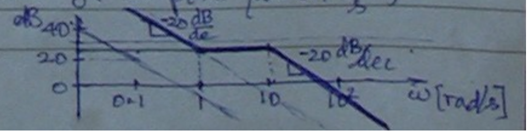


How will the ABMP of $\dfrac{\omega_n^2}{s^2+\omega_n^2}$ and $\dfrac{\omega_n^2}{(s+\omega_n)^2}$ look?

4. $\dfrac{20(s+1)}{s(s+10)}$. By now we have noticed that each pole gives the ABMP a slope of (−20 dB/dec)$*i$, where i is the multiplicity of the pole. Similarly, each zero gives the ABMP a slope of (+20 dB/dec)$*i$, where i is the multiplicity of the zero.

Thus, this TF's ABMP has a straight line segment of slope −20 $\frac{dB}{dec}$ (due to the $\frac{1}{s}$ term) until $\omega=1\ \frac{rad}{s}$, followed by a straight line segment of slope 0 dB/dec until $\omega=10\ \frac{rad}{s}$ (because of the cancellation of the −20 dB/dec slope by the zero's +20 dB/dec slope), followed by another straight line segment of slope −20 dB/dec (due to the $\frac{1}{s+10}$ term). The ABMP passes through the point $\left\{\omega=0.1\ \frac{rad}{s},\ 20\log_{10}\frac{20(1)}{(0.1)(10)}=20\log_{10}20=26\text{ dB}\right\}$.

The ABMP is shown



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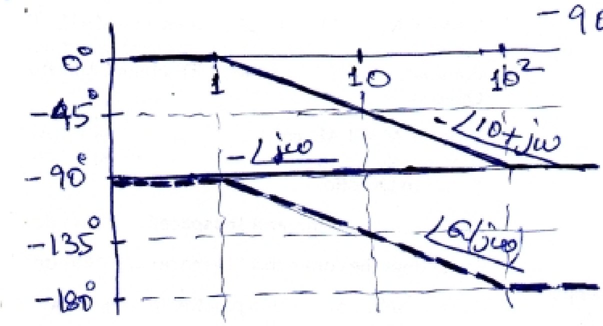
5. $\frac{20(s-1)}{s(s+10)}$. The ABMP of this TF coincides with that of $\frac{20(s+1)}{s(s+10)}$.

7 Solution to Section 2

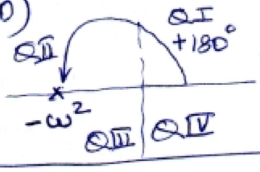
Solution to Sketching ABPPs

1. $\frac{20}{s(s+10)} \rightarrow \angle G(j\omega) = -\angle j\omega - \angle 10+j\omega = -90^\circ - \angle 10+j\omega$

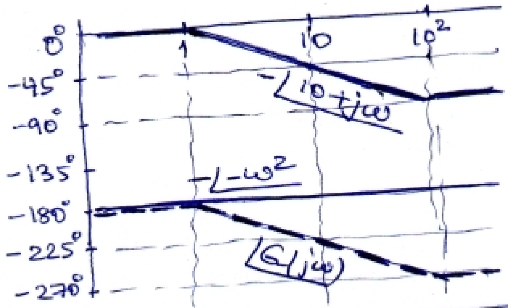
$10+j\omega$ is in the first quadrant (QI). So, $\angle 10+j\omega$ goes from 0° to $+90^\circ$. So, $\angle G(j\omega)$ is obtained by adding -90° to the ~~ABP~~ plot of $-\angle 10+j\omega$



2. $\frac{20}{s^2(s+10)} \rightarrow \angle G(j\omega) = -\angle (j\omega)^2 - \angle 10+j\omega = -\angle -\omega^2 - \angle 10+j\omega$

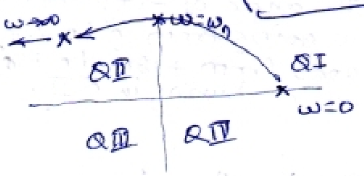


$\Longrightarrow \angle G(j\omega) = -180^\circ - \angle 10+j\omega$.

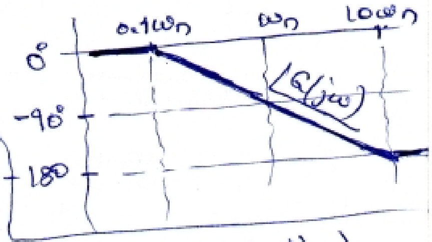


3. $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \rightarrow$

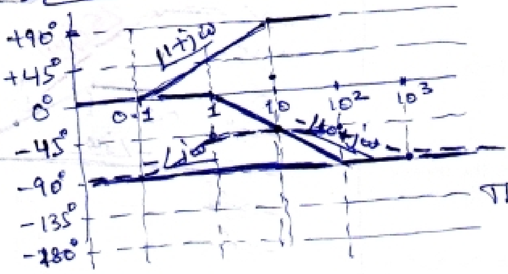
$\angle G(j\omega) = -\underbrace{\angle (\omega_n^2-\omega^2)+j2\zeta\omega_n\omega}_{\phi}$



From the adjacent figure, we see that ϕ goes from 0° to $+180^\circ$ through $+90^\circ$ as ω goes from 0 to ∞. So, $\angle G(j\omega)$ goes from 0° through -90° to -180° as shown below:



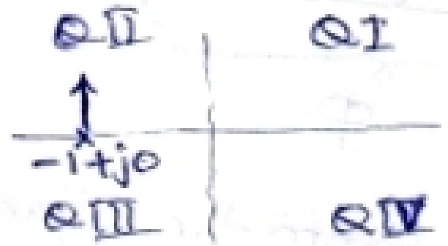
4. $\frac{20(s+1)}{s(s+10)} \rightarrow \angle G(j\omega) = \angle 1+j\omega - \angle j\omega - \angle 10+j\omega$.



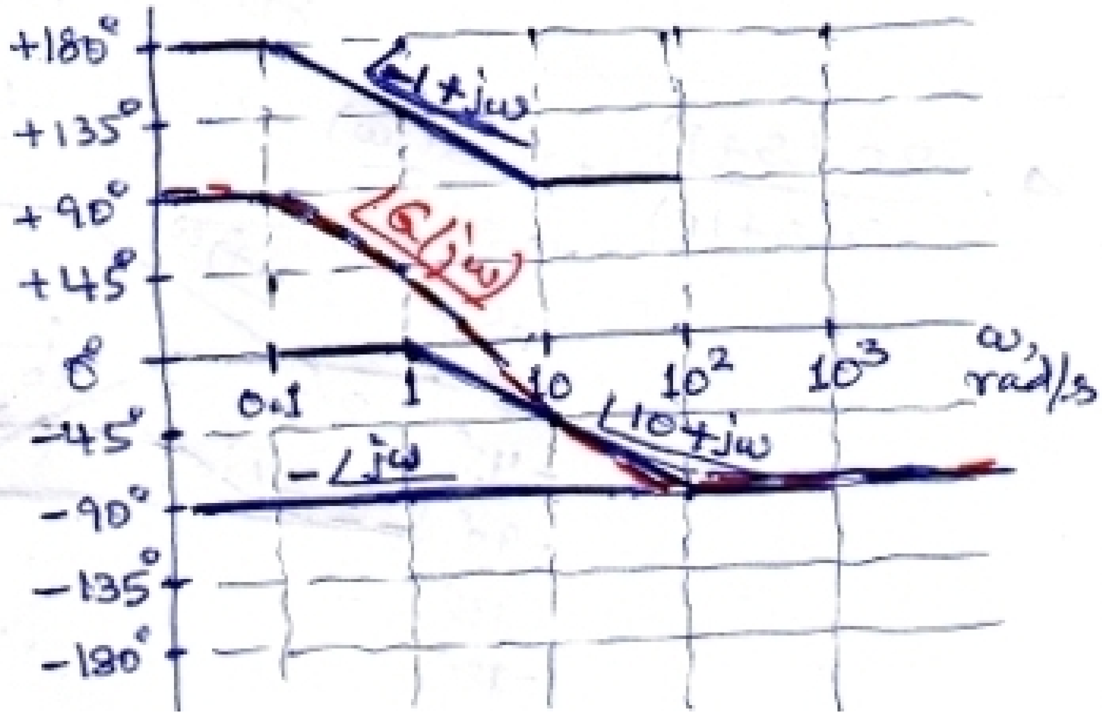
The dashed curve is $\angle G(j\omega)$

Solution to sketching ABPPs: (5.)

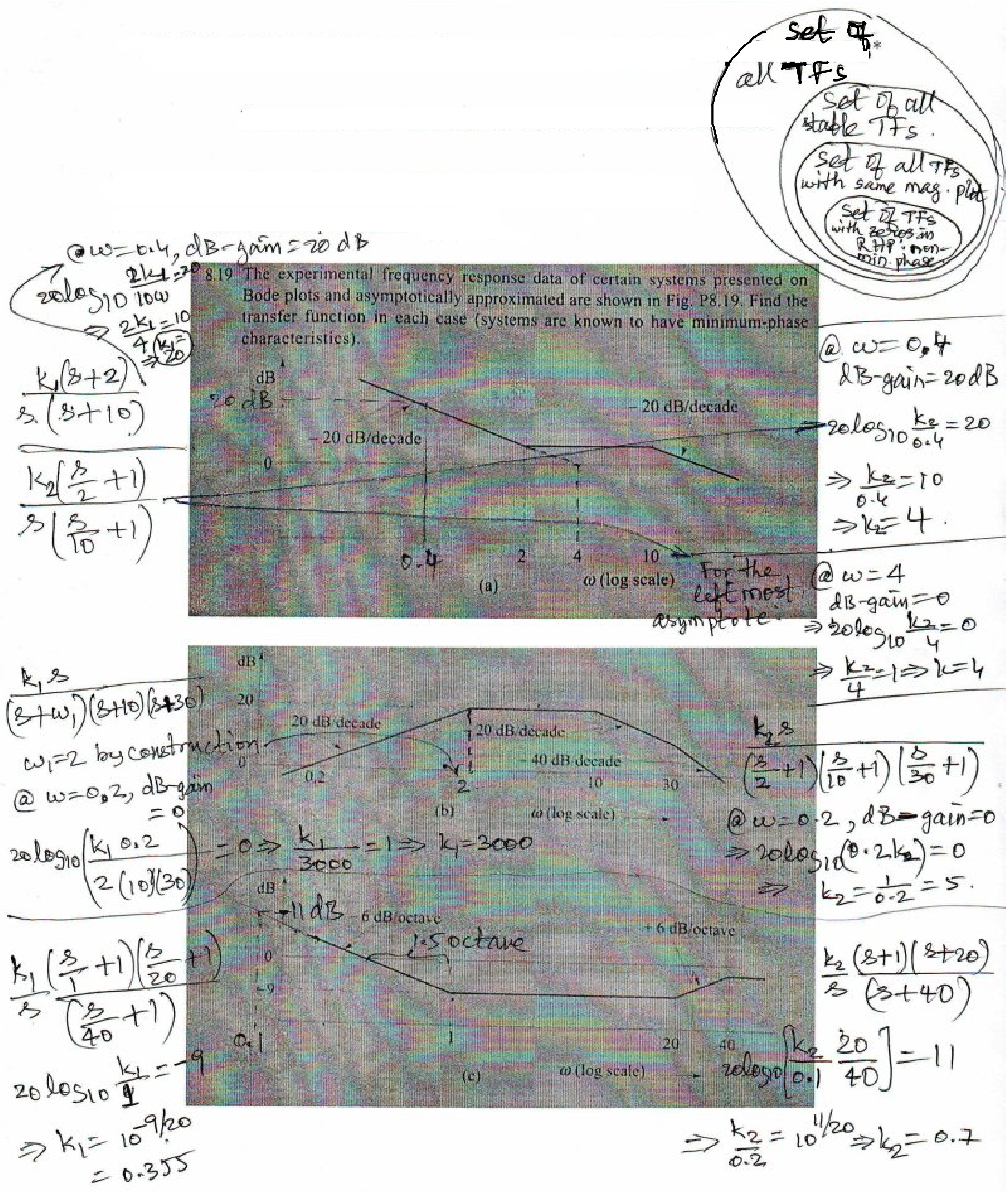
$$\frac{20(s-1)}{s(s+10)} \rightarrow \angle G(j\omega) = -90^\circ - \angle 10+j\omega + \angle -1+j\omega$$



The angle of $-1+j\omega$, with ω going from 0 to ∞, goes from $+180^\circ$ to $+90^\circ$, while staying in QII.



8 Solution to Section 3



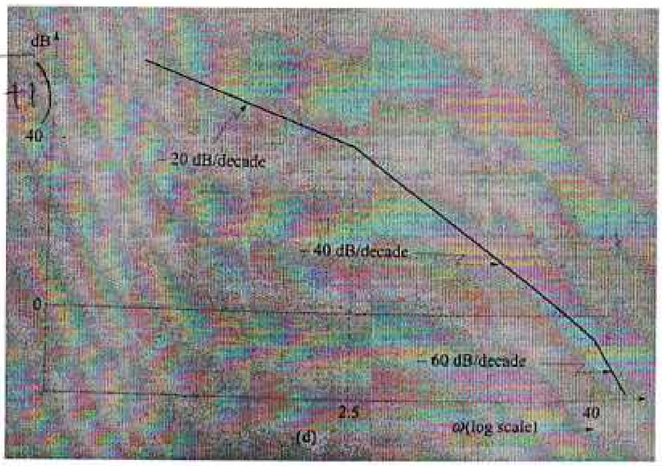
8.19 The experimental frequency response data of certain systems presented on Bode plots and asymptotically approximated are shown in Fig. P8.19. Find the transfer function in each case (systems are known to have minimum-phase characteristics).

$$\frac{k_1}{s\left(\frac{s}{2.5}+1\right)\left(\frac{s}{40}+1\right)}$$

@ $\omega = 2.5$,
dB-gain $= 40$

$\Rightarrow \frac{k_1}{2.5} = 100$

$\Rightarrow k_1 = 250$



$$\frac{k\, s^2}{\left(\frac{s}{0.5}+1\right)\left(\frac{s}{1}+1\right)\left(\frac{s}{5}+1\right)}$$

$\left(\frac{k\, 50^2}{100\,(50)(10)}\right) = 10^{12/20}$

$= \frac{k}{20} = 10^{12/20}$ $\Rightarrow k = 79.621$

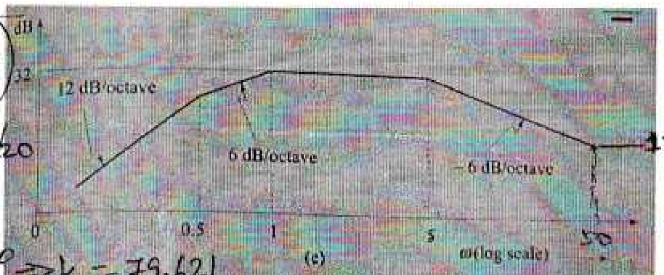


Fig. P8.19

8.20 Consider a minimum-phase system whose asymptotic amplitude frequency response is depicted in Fig. P8.20.

$$\frac{k s}{\left(\frac{s}{1}+1\right)\left(\frac{s}{5}+1\right)\left(\frac{s}{20}+1\right)}$$

$20\log_{10} k\cdot 1 = 30$

$\Rightarrow \log_{10} k = 1.5$

$\Rightarrow k = 10^{1.5} = 31.623$

$20\log_{10} k(\omega_{g1}) = 0$

$\Rightarrow \omega_{g1} = \frac{1}{k}$.

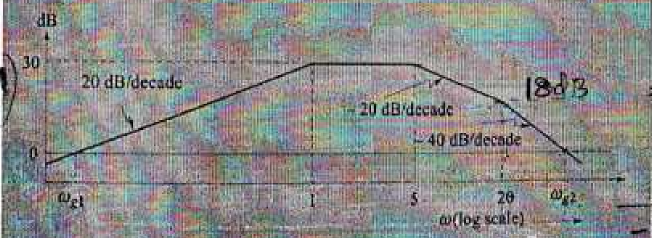


Fig. P8.20

(a) Determine the transfer function $G(s)$ of the system.
(b) Determine the two gain crossover frequencies ω_{g1} and ω_{g2}.

$\frac{18-0}{-\log_{10}\omega_{g2}+\log_{10}20} = -40$

$\Rightarrow -\frac{18}{40} = \log_{10}\frac{20}{\omega_{g2}}$

$\Rightarrow$

$10^{-18/40} = \frac{20}{\omega_{g2}}$

$\Rightarrow \omega_{g2} = \frac{20}{10^{-18/40}}$

$= 56.37\ \frac{rad}{s}$

References

[Gop93] Madan Gopal. *Modern Control System Theory*. New Age International (P) Ltd., New Delhi, India, second edition, 1993. 2003 Reprint.