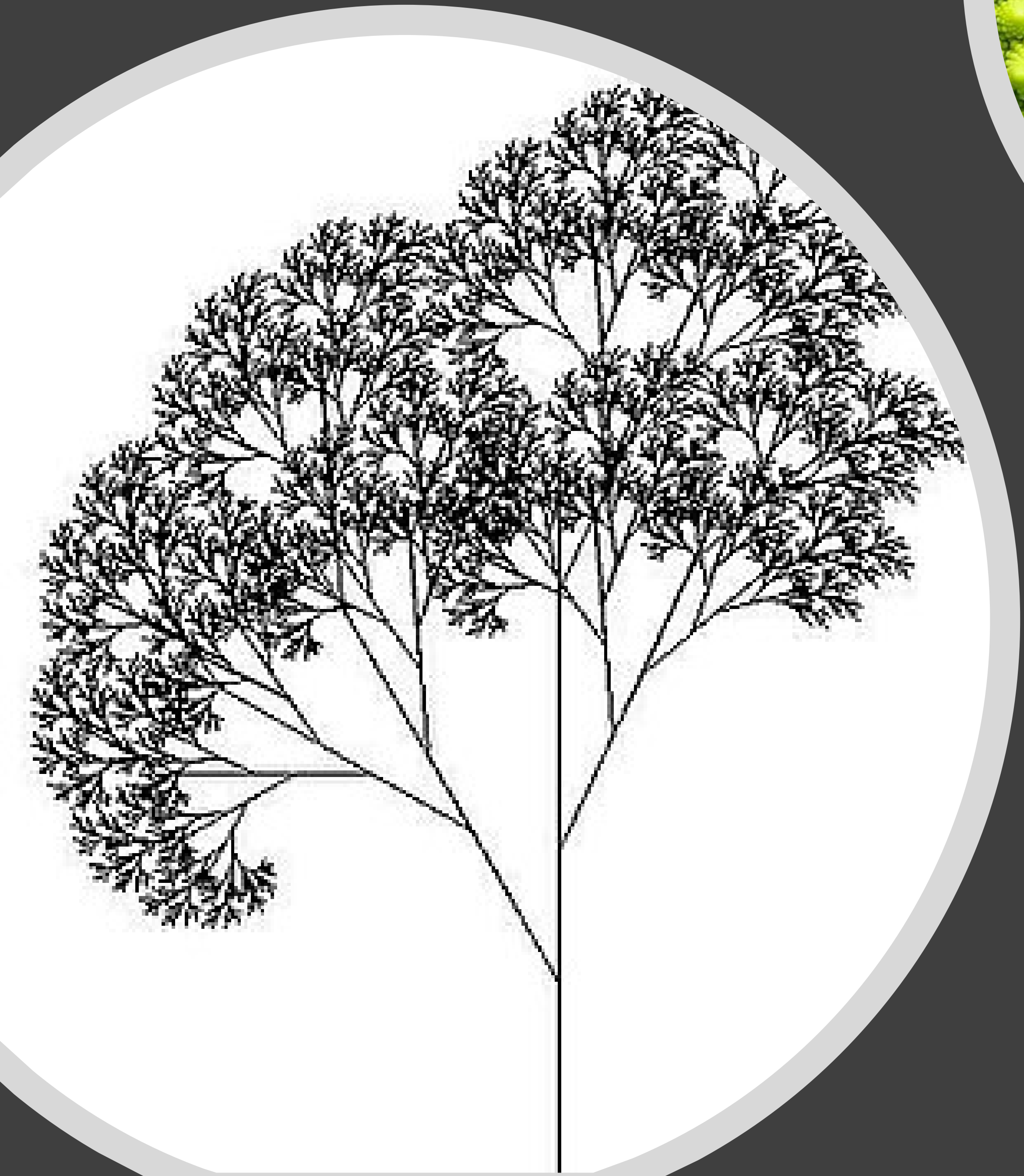
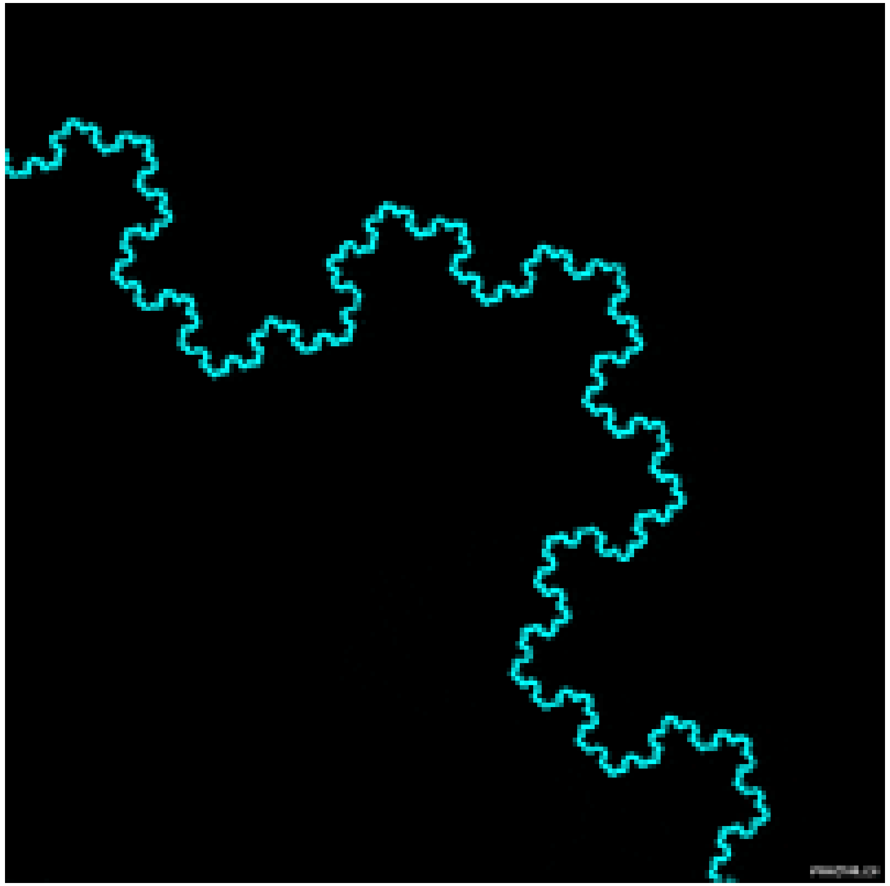


Recursion



Recursive Functions

A recursive function is a function that calls itself, reducing the problem a bit on each call:

```
void solveIt(the-Problem)
{
    . . .
    solveIt(the-Problem-a-bit-reduced);
}
```

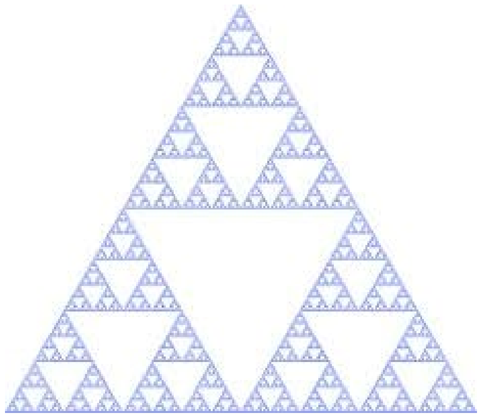
Of course, there's a lot/little more to it than this.

Recursion

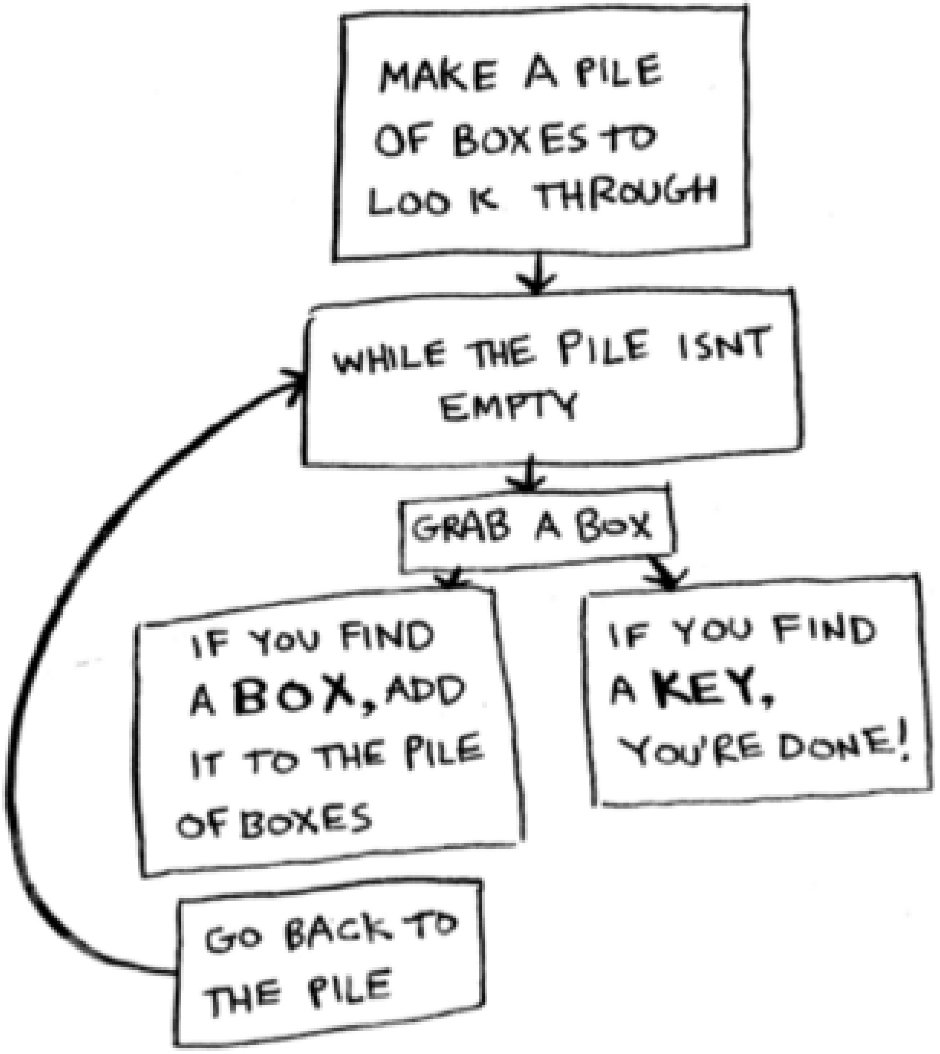
- *Recursion* is a powerful technique for breaking up complex computational problems into simpler ones,

  where the "simpler one" ***is*** the ***solution*** to the whole problem!

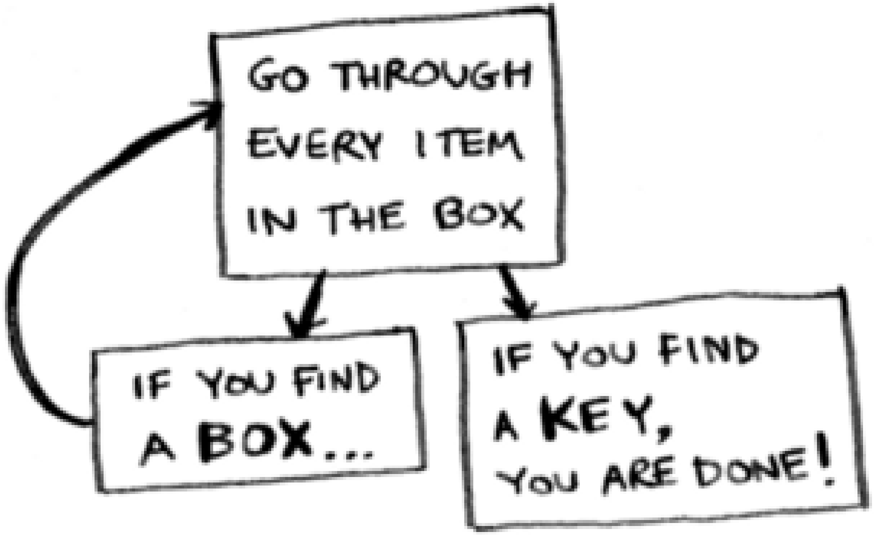
- Recursion is often the most natural way of thinking about a problem, and there are some computations that are difficult to perform without recursion.



Iterative Approach



Recursive Approach



How to think recursively? The Three Laws

The two keys requirements for a successful recursive function:

1. A recursive algorithm must have a **base case**
2. A recursive algorithm must change its state and move toward the base case
3. A recursive algorithm must call itself, recursively

Base case vs. recursive case

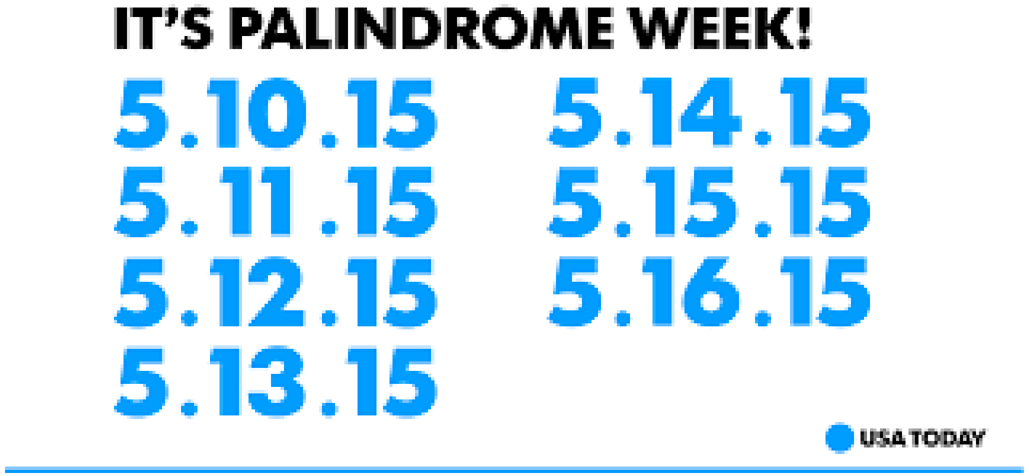
- Every recursive call must simplify the task in some way.

- There must be special cases to handle the simplest tasks directly so that the function will stop calling itself.
  - base case(s) – simplest task(s)
  - recursive case – break problem into smaller version of itself

Thinking Recursively – Palindromes

Palindrome: a string that is equal to itself when you reverse all characters

Example: Madam, I’m Adam



Thinking Recursively – Palindromes (1)

The problem: Write a function to test if a string is a palindrome.

```
bool is_palindrome(string s)
```

Thinking Recursively – Palindromes – an aside: iteratively

The problem: Write a function to test if a string is a palindrome.

```
bool is_palindrome(string s)
```

Thinking Recursively – Palindromes: recursively

The problem: Write a function to test if a string is a palindrome.

```
bool is_palindrome(string s)
```

Step 1: Break the input into parts that can themselves be inputs to the problem.

Focus on a particular input or set of inputs for the problem.

Think how you can simplify the inputs in such a way that the same function can be applied to the simpler input.

Thinking Recursively – Palindromes (2)

To get simpler inputs, how about :

- Remove the first character?
- Remove the last character?
- Remove a character from the middle?
- Cut the string into two halves?
- Remove both the first and the last character?

Thinking Recursively – Palindromes (3)

- Every palindrome’s first half is the same as its other half.
- In this problem, chopping in half seems to be a good way to reduce the problem. But:

`"rotor"`

(chop)

`"rot"` **`"or"`**

- Not sure how chopping in half gets us closer to a way to determine a palindromic situation.

Thinking Recursively – Palindromes (4)

- One character at a time seems not so good.
- How about chopping off BOTH ends at the same time?

`"rotor"`

(chop) (chop)

`"r"` **`"oto"`** **`"r"`**

- We can reduce the problem to the “middle” of the string for the recursive call.

Thinking Recursively – Palindromes (5)

- **Step 2:** Combine solutions with simpler inputs to a solution of the original problem.

`"rotor"`

(chop) (chop)

`"r"` **`"oto"`** **`"r"`**

- If the end letters are the same AND **is_palindrome(*the middle word*)** then the string is a palindrome!

Thinking Recursively – Palindromes (6)

Step 3: Find solutions to the simplest inputs.

- A recursive computation keeps simplifying its inputs.
- Eventually it arrives at very simple inputs. To make sure that the recursion comes to a stop, deal with the simplest inputs separately.
- That leaves us with two possible end situations, both of which are palindromes themselves:

string of length 0, and string of length 1

```
bool is_palindrome(string s)
{
   // Separate case for shortest strings
   if (s.length() <= 1 ){ return true; }
...
```

Thinking Recursively – Palindromes (7)

Step 4: Implement the solution by combining the simple cases and the reduction step.

```
...
// Get first and last character, converted to lowercase
   char first = tolower(s[0]);
   char last = tolower(s[s.length() - 1]);

   if (first == last)
   {
      string shorter = s.substr(1, s.length() - 2);
      return is_palindrome(shorter);
   }
   else
   {
      return false;
   }
}
```

Iteration vs. Recursion

- So is the iterative solution always faster than the recursive?
- Look at the iterative palindrome solution

```
bool is_palindrome(string s)
{
   int start = 0;
   int end = s.length() - 1;
   while (start < end)
   {
      if (s[start] != s[end]) { return false; }
      start++;
      end--;
   }
   return true;
}
```

Iteration vs. Recursion (2)

- If a palindrome has n characters,

    the iteration *executes* the loop **n/2** times.

    the recursive solution *calls itself* **n/2** times, because two characters are removed in each step.

The Fibonacci Sequence

- Recursion can lead to simpler solutions to problems, but recursive algorithms many perform poorly.
- The Fibonacci sequence is a sequence of numbers defined by the equations:

$$f_1 = 1$$
$$f_2 = 1$$
$$f_n = f_{n-1} + f_{n-2}$$

Each value in the sequence is the sum of the 2 preceding.

The first ten terms of the sequence:

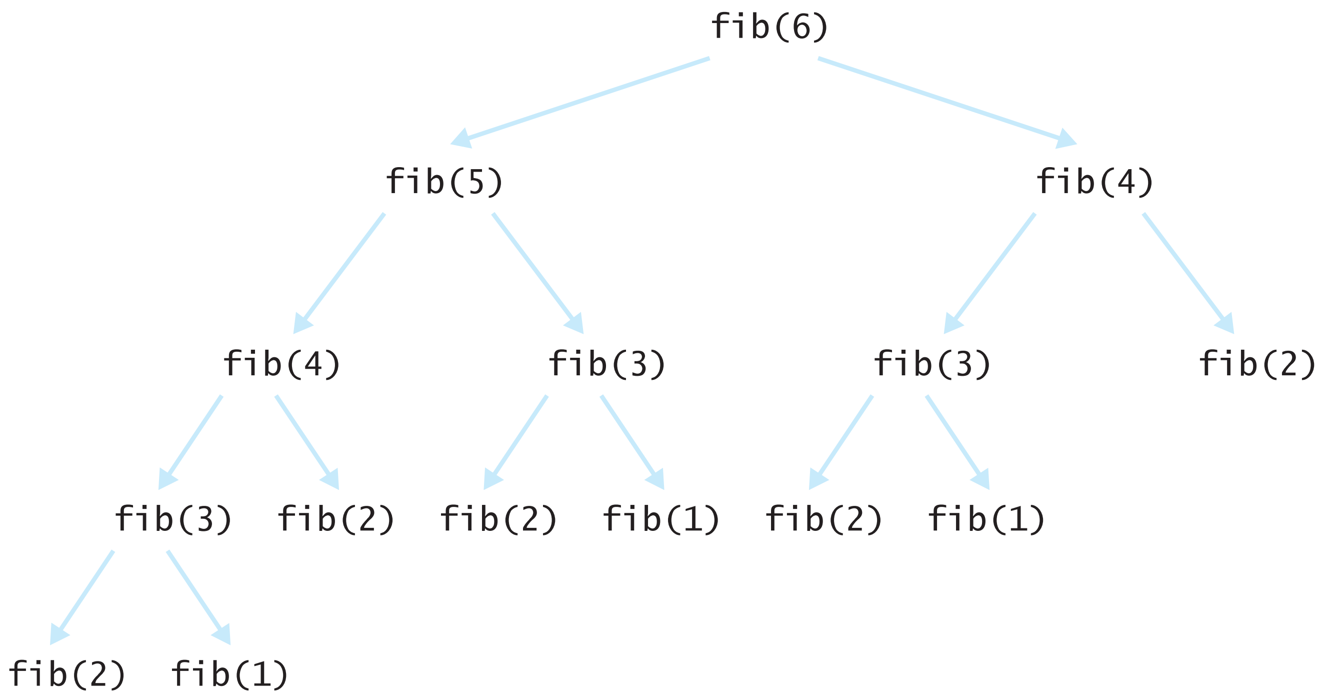
1, 1, 2, 3, 5, 8, 13, 21, 34, 55,…

Fibonacci Call Tree

Fibonacci Call Tree

This can be shown more clearly as a *call tree.*

Notice that the same values, for example, **fib(2)**, are computed over and over, **and each recursive call generates 2 more calls**



Fibonacci Call Tree