Assignment

Homework 1

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A MEGN544A Homework Assignment

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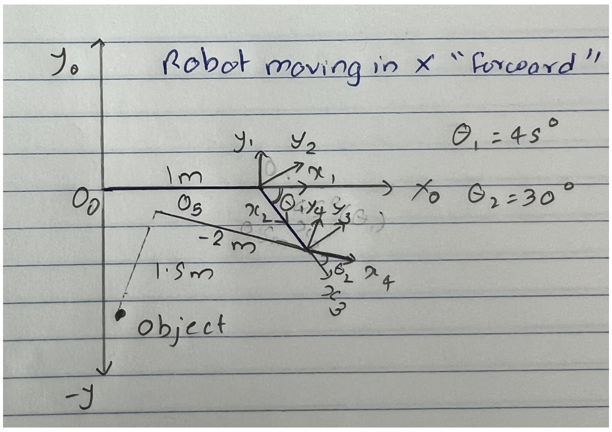
Problem 1

A mobile robot starts aligned with the World Coordinate system. It travels: forward 1 meter, turns clockwise 45°, travels forward 0.5 meter, turns counter clockwise 30º, travels backwards 2 meters. Once there it spots an object 1.5 meters to its right.

(a) Draw to scale the path taken by the robot and the location of the object.

Solution.



(b) Write out the series of translations and rotations required to reach the robot's final pose.

Solution.

$$^0d_{01} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; ^0r_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$^1d_{12} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; ^1r_2 = \begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix}$$

$$^2d_{23} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, \; ^2r_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$^3d_{34} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; ^3r_4 = \begin{bmatrix} 0.86 & 0.5 \\ -0.5 & 0.86 \end{bmatrix}$$

$$^4d_{45} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}, \; ^4r_5 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(c) Calculate the homogeneous transformation matrix that would move the world origin to the robot's the final pose, i.e. $^0T_{final}$.

Solution.

$$^{0}T_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, {}^{1}T_2 = \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}, {}^{2}T_3 = \begin{bmatrix} 1 & 0 & 0.5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$^{3}T_4 = \begin{bmatrix} 0.86 & 0.5 & 0 \\ -0.5 & 0.86 & 0 \\ 0 & 0 & 1 \end{bmatrix}, {}^{4}T_5 = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_{final} = {}^{0}T_1 \times {}^{1}T_2 \times {}^{2}T_3 \times {}^{3}T_4 \times {}^{4}T_5$$

$$^{0}T_{final} = \begin{bmatrix} 0.9615 & -0.2545 & -0.5695 \\ 0.2545 & 0.9615 & -0.1555 \\ 0 & 0 & 1 \end{bmatrix}$$

(d) Calculate the homogenous transformation matrix that would move the robot's final pose to the object, i.e. $^{final}T_{obj}$

Solution. Let's calculate the displacement from final position to the object.

$$^{final}d_{5obj} = \begin{bmatrix} 0 \\ -1.5 \end{bmatrix},$$

Now let's calculate the rotation.

$$^{final}r_{obj} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

So the transformation matrix $^{final}T_{obj}$ is

$$^{final}T_{obj} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1.5 \\ 0 & 0 & 1 \end{bmatrix}$$

(e) Calculate the homogenous transformation that would move the object to the world origin (assume the object is aligned with the robot), i.e. $^{obj}T_0$

Solution. For calculating $^{obj}T_0$ we need to first calculate the $^{0}T_{obj}$. In order to that let's do chaining from $^{0}T_{final}$ to $^{final}T_{obj}$.

$$^{0}T_{obj} = {}^{0}T_{final} \times {}^{final}T_{obj}$$

$$^{0}T_{obj} = \begin{bmatrix} 0.9615 & -0.2545 & -0.1878 \\ 0.2545 & 0.9615 & -1.5978 \\ 0 & 0 & 1 \end{bmatrix}$$

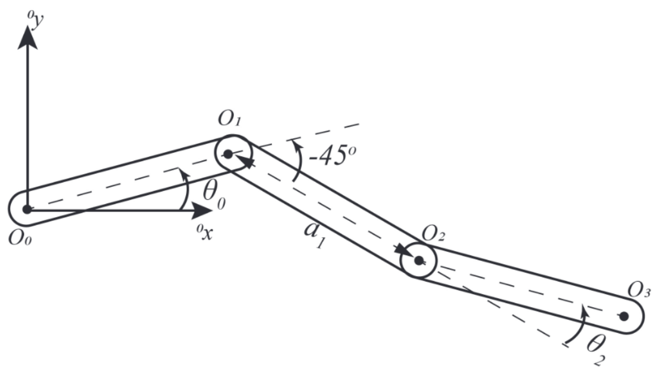
For getting the transformation matrix $^{obj}T_0$ we just use the standard equation on the above matrix.

$$^{obj}T_0 = \begin{bmatrix} ^{0}R_{obj}^{T} & -{}^{0}R_{obj}^{T}\,{}^{0}d_{0obj} \\ 0^T & 1 \end{bmatrix}$$

$$^{obj}T_0 = \begin{bmatrix} 0.9615 & 0.2545 & 0.5872 \\ -0.2545 & 0.9615 & 1.4885 \\ 0 & 0 & 1 \end{bmatrix}$$

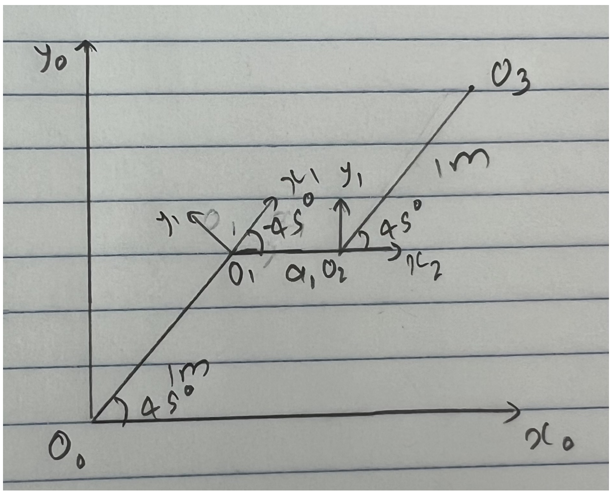
Problem 2

The figure below shows a 3 link manipulator. The first joint is at the origin ^0O and rotates about ^0z with an angle θ_0, it has a length of 1m. The second link is prismatic and extends and contracts with length a_1, it is attached at a 45 degree rotation clockwise about the ^1z direction. The third joint is again rotary and rotates relative to the second link by angle θ_2, it has a length of 1m.



(a) Draw the manipulator when $\theta_0 = \pi/4rad, a_1 = .5m, \theta_2 = \pi/4rad$, make sure you draw in and label the intermediate coordinate system directions.

Solution.



(b) Let θ_0 be the first joint's angle. What is the symbolic homogenous transformation from the world frame to the distal (far) end of link 1, i.e. $^0T_1$?

Solution. Let's calculate the displacement vector from O_0 to O_1, $\theta_0 = \pi/4$ rad and L1 = 1m

$$^0d_{01} = \begin{bmatrix} L1 \times \cos\theta_0 \\ L1 \times \sin\theta_0 \end{bmatrix}, {^0d_{01}} = \begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix}$$

Now lets calculate the rotation matrix $^0R_1$,

$$^0R_1 = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 \\ \sin\theta_0 & \cos\theta_0 \end{bmatrix}, {^0R_1} = \begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix}$$

Finally we calculate the transformation matrix $^0T_1$ using the displacement vector and rotation matrix above.

$$^0T_1 = \begin{bmatrix} ^0R_1 & ^0d_{01} \\ 0^T & 1 \end{bmatrix}, {^0T_1} = \begin{bmatrix} 0.707 & -0.707 & 0.707 \\ 0.707 & 0.707 & 0.707 \\ 0 & 0 & 1 \end{bmatrix}$$

(c) Let a_1 be the first joint's angle. What is the symbolic homogenous transformation from the distal end of link 1 to the distal end of link 2, i.e. $^1T_2$?

Solution. Let's calculate the displacement vector from O_1 to O_2, $\theta_1 = -\pi/4$ rad and $a_1 =$ 0.5m

$$^1d_{12} = \begin{bmatrix} a_1 \times \cos\theta_1 \\ a_1 \times \sin\theta_1 \end{bmatrix}, {^0d_{01}} = \begin{bmatrix} 0.3535 \\ -0.3535 \end{bmatrix}$$

Now lets calculate the rotation matrix $^0R_1$,

$$^1R_2 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix}, {^1R_2} = \begin{bmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{bmatrix}$$

Finally we calculate the transformation matrix $^0T_1$ using the displacement vector and rotation matrix above.

$$^1T_2 = \begin{bmatrix} ^1R_2 & ^1d_{12} \\ 0^T & 1 \end{bmatrix}, {^1T_2} = \begin{bmatrix} 0.707 & 0.707 & 0.3535 \\ -0.707 & 0.707 & -0.3535 \\ 0 & 0 & 1 \end{bmatrix}$$

(d) Let θ_2 be the first joint's angle. What is the symbolic homogenous transformation from the distal end of link 2 to the distal end of link 3,, i.e. $^2T_3$?

Solution. Let's calculate the displacement vector from O_2 to O_3, $\theta_2 = \pi/4$ rad and L1 = 1m

$$^2d_{23} = \begin{bmatrix} L2 \times \cos\theta_2 \\ L2 \times \sin\theta_2 \end{bmatrix}, {^2d_{23}} = \begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix}$$

Now lets calculate the rotation matrix $^2R_3$,

$$^2R_3 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix}, {^2R_3} = \begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix}$$

Finally we calculate the transformation matrix ${}^0T_1$ using the displacement vector and rotation matrix above.

$$ {}^2T_3 = \begin{bmatrix} {}^2R_3 & {}^2d_{23} \\ 0^T & 1 \end{bmatrix}, {}^2T_3 = \begin{bmatrix} 0.707 & -0.707 & 0.707 \\ 0.707 & 0.707 & 0.707 \\ 0 & 0 & 1 \end{bmatrix} $$

(e) What is the symbolic homogenous transformation from the world origin to the distal end of link 3, i.e. ${}^0T_3$?

Solution. In order to calculate the ${}^0T_3$ we can do chaining of ${}^0T_1$ ${}^1T_2$ ${}^2T_3$.

$$ {}^0T_3 = {}^0T_1 \times {}^1T_2 \times {}^2T_3 $$

$$ {}^0T_3 = \begin{bmatrix} 0.7068 & -0.7068 & 1.9136 \\ 0.7068 & 0.7068 & 1.4138 \\ 0 & 0 & 1 \end{bmatrix} $$

Problem 3

The figure below shows the path of a mobile robot. The displacements are given by: ${}^0d_{01} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, ${}^1d_{12} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, ${}^2d_{23} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, ${}^3d_{34} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, ${}^4d_{45} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

Solution.

(a) For calculating ${}^0T_1$ we need to get the rotation matrix ${}^0R_1$ and we have ${}^0d_{01}$. There is no rotation from O_0 to O_1.

$$ {}^0R_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, {}^0T_1 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} $$

(b) For calculating ${}^1T_2$ we need to get the rotation matrix ${}^1R_2$ and we have ${}^1d_{12}$. There is $\pi/4$ rotation from O_1 to O_2.

$$ {}^1R_2 = \begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix}, {}^1T_2 = \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

(c) For calculating ${}^2T_3$ we need to get the rotation matrix ${}^2R_3$ and we have ${}^2d_{23}$. There is no rotation from O_2 to O_3.

$$ {}^2R_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, {}^2T_3 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

(d) For calculating ${}^3T_4$ we need to get the rotation matrix ${}^3R_4$ and we have ${}^3d_{34}$. There is $3\pi/4$ rotation from O_3 to O_4.

$$ {}^3R_4 = \begin{bmatrix} -0.707 & -0.707 \\ 0.707 & -0.707 \end{bmatrix}, {}^3T_4 = \begin{bmatrix} -0.707 & -0.707 & 0 \\ 0.707 & -0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

(e) For calculating ${}^4T_5$ we need to get the rotation matrix ${}^4R_5$ and we have ${}^4d_{45}$. There is 0 rotation from O_4 to O_5.

$$ {}^4R_5 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, {}^4T_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} $$

(f) For calculating ${}^0T_5$ we need to chain the ${}^0T_1$, ${}^1T_2$, ${}^2T_3$, ${}^3T_4$ and ${}^4T_5$

$$ {}^0T_5 = {}^0T_1 \times {}^1T_2 \times {}^2T_3 \times {}^3T_4 \times {}^4T_5 $$

$$ {}^0T_5 = \begin{bmatrix} -0.9997 & 0 & 3.4140 \\ 0 & -0.9997 & -0.5851 \\ 0 & 0 & 1 \end{bmatrix} $$

(g) For calculating ${}^0d_{05}$ we can just use ${}^0T_5$,

$$ {}^0d_{05} = \begin{bmatrix} 3.4140 \\ -0.5851 \end{bmatrix} $$

(h) For calculating ${}^0R_5$ we can just use ${}^0T_5$,

$$ {}^0R_5 = \begin{bmatrix} -0.9997 & 0 \\ 0 & -0.9997 \end{bmatrix} $$

(i) What is the equation in terms of homogeneous transformations (T's) to find the displacement vector from origin 1 to origin 5 (d15) ex- pressed in the final (Origin 5) frame?

Answer:

The equations for calculating ${}^5d_{15}$ with homogeneous transformation are:

$$ {}^1T_5 = {}^1T_2 \times {}^2T_3 \times {}^3T_4 \times {}^4T_5 $$

$$ {}^5T_1 = {}^1T_5^{-1} $$

After getting the inverse the last column's first 2 elements will be the displacement vector.

(i) What is this numerical displacement? **Answer:**

$$ {}^1T_5 = \begin{bmatrix} -0.9997 & 0 & 1.4140 \\ 0 & -0.9997 & -1.5851 \end{bmatrix} $$

$$^{5}T_1 = \begin{bmatrix} -0.9997 & 0 & 1.4136 \\ 0 & -0.9997 & -1.5846 \end{bmatrix}$$

So the displacement $^{5}d_{15}$ is

$$^{5}d_{15} = \begin{bmatrix} 1.4136 \\ -1.5846 \end{bmatrix}$$

Problem 4

Determine if the following matrices are rotations and explain why or why not:

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Solution. In order to know if the matrix is a rotation matrix we have to check if it is orthonormal i.e. the rows and column elements will unit vectors and dot product of columns should be zero suggesting they are ortho.

Column 1: $\sqrt{0^2 + 1^2 + 0^2} = 1$, Column 2: $\sqrt{1^2 + 0^2 + 0^2} = 1$, Column 3: $\sqrt{0^2 + 0^2 + 1^2} = 1$

Column 1 ×Column 2: 0×1 + 1 ×0 + 0 ×0 = 0
Column 2 ×Column 3: 1 ×0 + 0 ×0 + 0 ×1 = 0
Column 3 ×Column 1: 1×0 + 0 ×1 + 1 ×0 = 0

Finally we calculate the determinant which should be equal to 1.
det = 0 ×(0 ×1 - 0 ×0) - 1 ×(1 ×1 - 0 ×0) + 0 ×(1 ×0 - 0 ×0) = -1

So as we see that the determinant is -1 and not +1 which suggests that the above matrix is not rotation matrix.

(b) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Solution. In order to know if the matrix is a rotation matrix we have to check if it is orthonormal i.e. the rows and column elements will unit vectors and dot product of columns should be zero suggesting they are orthogonal.

Column 1: $\sqrt{0^2 + 1^2 + 0^2} = 1$, Column 2: $\sqrt{1^2 + 0^2 + 0^2} = 1$,
Column 3: $\sqrt{0^2 + 0^2 + -1^2} = 1$

Column 1 ×Column 2: 0×1 + 1 ×0 + 0 ×0 = 0
Column 2 ×Column 3: 1×0 + 0 ×0 + 0 ×-1 = 0
Column 3 ×Column 1: 0×0 + 0 ×1 + (-1) ×0 = 0

Finally we calculate the determinant which should be equal to 1.
det = 0 ×(0 ×-1 - 0 ×0) - 1 ×(1 ×-1 - 0 ×0) + 0 ×(1 ×0 - 0 ×0) = 1

So as we see that the determinant is +1 which suggests that the above matrix is a rotation matrix.

(c) $\begin{bmatrix} -0.254 & 0.087 & -0.963 \\ 0.967 & 0.0169 & -0.253 \\ -0.00572 & -0.996 & -0.0884 \end{bmatrix}$

Solution. In order to know if the matrix is a rotation matrix we have to check if it is orthonormal i.e. the rows and column elements will unit vectors and dot product of columns should be zero suggesting they are orthogonal.

$$Column1 : \sqrt{(-0.254)^2 + (0.967)^2 + (-0.00572)^2} = 0.99 \approx 1$$

$$Column2 : \sqrt{(0.087)^2 + (0.0169)^2 + (-0.996)^2} = 0.99 \approx 1$$

$$Column3 : \sqrt{(-0.963)^2 + (-0.253)^2 + (-0.0884)^2} = 0.99 \approx 1$$

$$Column1 \times Column2 : (-0.254) \times (0.087) + (0.967) \times (0.0169) + (-0.00572) \times (-0.996) \approx 0$$

$$Column2 \times Column3 : (0.087) \times (-0.963) + (0.0169) \times (-0.253) + (-0.996) \times (-0.0884) \approx 0$$

$$Column3 \times Column1 : (-0.963) \times (-0.254) + (-0.253) \times (0.967) + (-0.0884) \times (-0.00572) \approx 0$$

Finally we calculate the determinant which should be equal to 1.

$$Det = (-0.254)\times((0.0169)\times(-0.0884)-(-0.253)\times(-0.996))-(-0.087)\times((0.967)\times(-0.0884)-(-0.253)\times(-0.00572))$$
$$+(-0.963) \times ((0.967) \times (-0.996) - (0.0169) \times (-0.00572)) \approx 1$$

So as we see that the determinant is approximately +1 which suggests that the above matrix is a rotation matrix.

(d) $\begin{bmatrix} 0.546 & 0.0719 & -1.47 \\ 0.814 & -0.696 & -0.0502 \\ -1.23 & -0.813 & 0.474 \end{bmatrix}$

Solution. In order to know if the matrix is a rotation matrix we have to check if it is orthonormal i.e. the rows and column elements will unit vectors and dot product of columns should be zero suggesting they are orthogonal.

$$Column1 : \sqrt{(0.546)^2 + (0.814)^2 + (-1.23)^2} = 1.57$$

$$Column2 : \sqrt{(0.0719)^2 + (-0.696)^2 + (-0.813)^2} = 1.07$$

$$Column3 : \sqrt{(-1.47)^2 + (-0.0502)^2 + (0.474)^2} = 1.54$$

As the columns are not unit vectors we can say this matrix is not a rotation matrix.

Problem 5

Complete the below rotation matrices. How do you know its correct?

(a) $\begin{bmatrix} 0 & 0 & ? \\ 0 & 1 & ? \\ 1 & 0 & ? \end{bmatrix}$

Solution. So in order find the missing values we can just calculate values considering these are rotation matrices and the rows are unit vectors.

$$Row1 : \sqrt{(0)^2 + (0)^2 + (?)^2} = 1$$

$$0 + 0 + ?^2 = 1$$

which gives

$$? = \pm 1$$

$$Row2 : \sqrt{(0)^2 + (1)^2 + (?)^2} = 1$$

$$0 + 1 + ?^2 = 1$$

which gives

$$? = 0$$

$$Row3 : \sqrt{(1)^2 + (0)^2 + (?)^2} = 1$$

$$1 + 0 + ?^2 = 1$$

which gives

$$? = 0$$

Final matrices are $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, Or $\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

We need to take determinant of both and check if its +1 in order to make sure it is a rotation matrix and not a reflection. The final rotation matrix is $\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, gives us +1 determinant so it is the correct rotation matrix.

(b) $\begin{bmatrix} -1 & ? & ? \\ ? & 1 & ? \\ ? & ? & ? \end{bmatrix}$.

Solution. So in order find the missing values we can just calculate values considering these are rotation matrices and the column are unit vectors. I can assume that the missing values are equal as the row or column vectors are already unit from given problem.

$$Column1 : \sqrt{(-1)^2 + (?)^2 + (?)^2} = 1$$

$$1 + ?^2 + ?^2 = 1$$

which gives

$$? = 0$$

$$Column2 : \sqrt{(?)^2 + (1)^2 + (?)^2} = 1$$

$$?^2 + 1 + ?^2 = 1$$

which gives

$$? = 0$$

Now that we have the first two columns we can get the last column elements using the rows as unit vectors.

$$Row1 : \sqrt{(-1)^2 + (0)^2 + (?)^2} = 1$$

$$1 + 0 + ?^2 = 1$$

which gives

$$? = 0$$

$$Row2 : \sqrt{(0)^2 + (1)^2 + (?)^2} = 1$$

$$0 + 1 + ?^2 = 1$$

which gives

$$? = 0$$

$$Row3 : \sqrt{(0)^2 + (0)^2 + (?)^2} = 1$$

$$0 + 0 + ?^2 = 1$$

which gives

$$? = \pm 1$$

Final matrices are $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, or $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ We need to take determinant of both and check if its +1 in order to make sure it is a rotation matrix and reflection. The final rotation matrix is $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ gives us +1 determinant so it is the correct rotation matrix.

(c) $\begin{bmatrix} 0.3698 & ? & 0.8665 \\ -0.9276 & ? & 0.324 \\ -0.05236 & ? & 0.3797 \end{bmatrix}$.

Solution. So in order find the missing values we can just calculate values considering these are rotation matrices and the rows are unit vectors.

$$Row1 : \sqrt{(0.3698)^2 + (?)^2 + (0.8665)^2} = 1$$

which gives

$$? = \pm 0.3353$$

$$Row2 : \sqrt{(-0.9276)^2 + (?)^2 + (0.324)^2} = 1$$

which gives

$$? = \pm 0.1860$$

$$Row3 : \sqrt{(-0.05236)^2 + (?)^2 + (0.3797)^2} = 1$$

which gives

$$? = \pm 0.9236$$

Matrix that we get $\begin{bmatrix} 0.3698 & \pm 0.3353 & 0.8665 \\ -0.9276 & \pm 0.1860 & 0.324 \\ -0.05236 & \pm 0.9236 & 0.3797 \end{bmatrix}$.

I am using Matlab for this problem to determine the determinant as we have many combinations. Determinant +1 makes sure that the matrix is not a reflection.

The final matrix that has determinant +1 is $\begin{bmatrix} 0.3698 & 0.3353 & 0.8665 \\ -0.9276 & 0.1860 & 0.324 \\ -0.05236 & -0.9236 & 0.3797 \end{bmatrix}$.

(d) $\begin{bmatrix} ? & 0.1166 & ? \\ -0.2039 & ? & 0.2153 \\ ? & ? & -0.3672 \end{bmatrix}$.

Solution. So in order find the missing values we can just calculate values considering these are rotation matrices and the columns are unit vectors.

$$Column3 : \sqrt{(?)^2 + (0.2153)^2 + (-0.3672)^2} = 1$$

which gives

$$? = \pm 0.9049$$

We now have all values for column three so we can use the constraint that all rows are unit vectors.

$$Row1 : \sqrt{(?)^2 + (0.1166)^2 + (\pm 0.9049)^2} = 1$$

which gives

$$? = \pm 0.4093$$

$$Row2 : \sqrt{(-0.2039)^2 + (?)^2 + (\pm 0.2153)^2} = 1$$

which gives

$$? = \pm 0.9550$$

$$Column2 : \sqrt{(0.1166)^2 + (\pm 0.9550)^2 + (?)^2} = 1$$

which gives

$$? = \pm 0.2727$$

$$Column1 : \sqrt{(\pm 0.4093)^2 + (-0.2039)^2 + (?)^2} = 1$$

which gives

$$? = \pm 0.8893$$

Matrix that we get $\begin{bmatrix} \pm 0.4093 & 0.1166 & \pm 0.9049 \\ -0.2039 & \pm 0.9550 & 0.2153 \\ \pm 0.8893 & \pm 0.2727 & -0.3672 \end{bmatrix}$.

I am using Matlab for this problem to determine the determinant as we have many combinations.Determinant +1 makes sure that the matrix is not a reflection.

The final matrix that has determinant +1 is $\begin{bmatrix} -0.4093 & 0.1166 & -0.9049 \\ -0.2039 & 0.9550 & 0.2153 \\ 0.8893 & 0.2727 & -0.3672 \end{bmatrix}$.