Assignment

Homework 4

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A MEGN544A Homework Assignment

COLORADO SCHOOL OF MINES

September 25, 2023

Problem 1

Derive (from the general solution provided in class and in the handouts) the simplified forward and inverse equations for the twist parameterization for the special case when the displacement $\mathbf{d}$ is parallel with the rotation axis $\hat{k}$.

Solution.

Q.1 We have forward kinematics equation for d.

Also, the case we are talking about is rotation along $\overline{\omega}$. where $v \parallel d \parallel \Omega$

$$d = \left((I - R)[\hat{k}]_\times + \Omega k^T \right) v$$

As $\hat{k} \times v = 0$

$$= \Omega v$$

$$= [\theta \cdot \hat{k}] v \qquad \text{As } \Omega = \theta \cdot \hat{k}$$

$$= \theta v$$

We have angle-axis inverse equation

$$v = \left(\frac{\sin\theta}{2(1-\cos\theta)} I + \frac{2(1-\cos\theta) - \theta\sin\theta}{2\theta(1-\cos\theta)} \hat{k}\hat{k}^T - \frac{1}{2}[\hat{k}]_\times \right) d$$

As $\hat{k} \times d = 0$ and $\hat{k}^T \times d = I$

$$= \frac{\sin\theta}{2(1-\cos\theta)} I \cdot d + \frac{2(1-\cos\theta) - \theta\sin\theta}{2\theta(1-\cos\theta)} d$$

$$= \left[\frac{\sin\theta}{2(1-\cos\theta)} I + \frac{2(1-\cos\theta) - \theta\sin\theta}{2\theta(1-\cos\theta)} \right] d$$

$$= \left[\frac{\theta\sin\theta + 2 - 2\cos\theta - \theta\sin\theta}{2\theta(1-\cos\theta)} \right] d$$

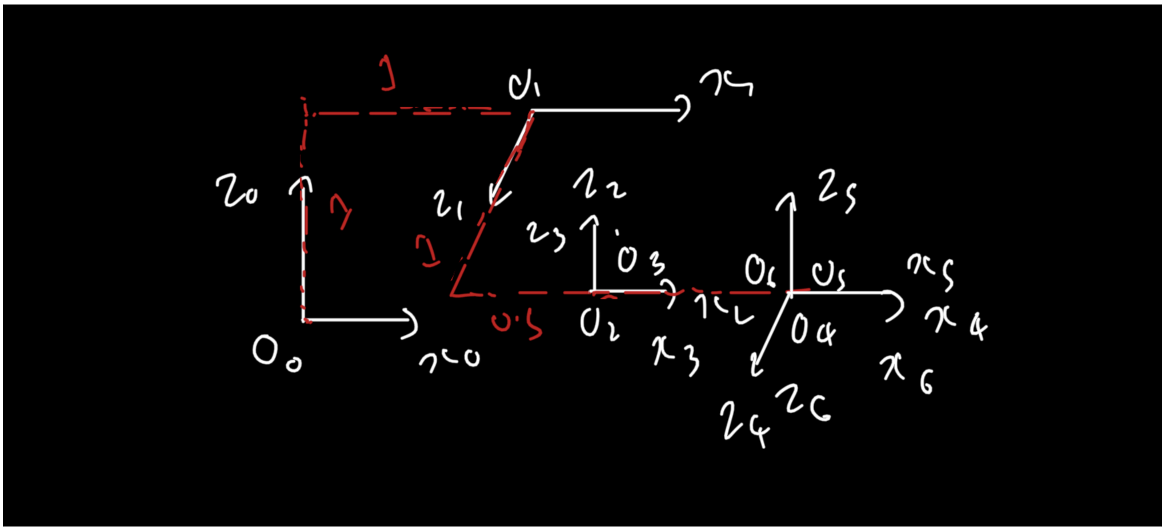
$$= \left[\frac{1}{\theta} \right] d$$

Problem 2

Sketch the Zero angle configuration for the following manipulator:

Link	a	d	α	θ
1	1	1	$\pi/2$	$*$
2	.5	1	$-\pi/2$	$*$
3	0	0	0	$*$
4	1	0	$\pi/2$	$*$
5	0	0	$-\pi/2$	$*$
6	0	0	$\pi/2$	$*$

Solution.

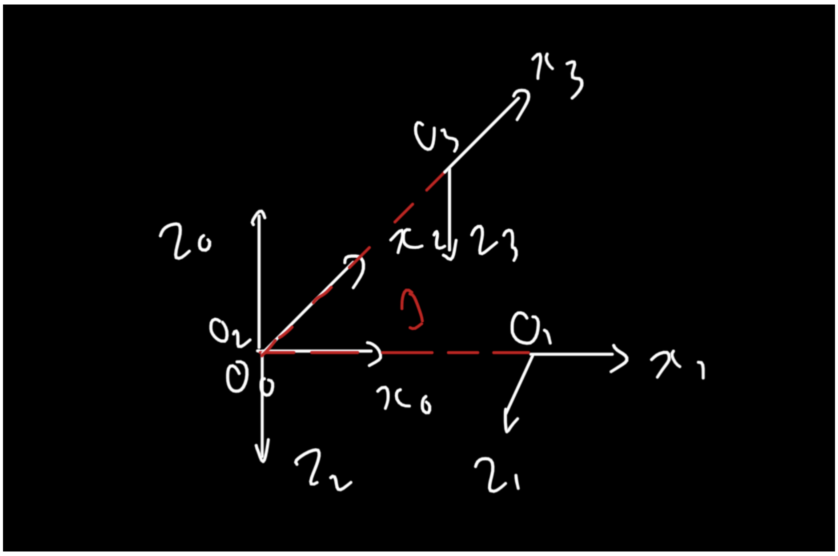


Problem 3

Sketch the Zero angle configuration for the following manipulator:

Link	a	α	d	θ
1	1	$\pi/2$	$*$	0
2	1	$\pi/2$	$*$	$\pi/2$
3	1	0	$*$	0

Solution.



Problem 4

For the manipulator of question 1.

This is the Matlab code I used for calculating transformation matrix.

```
  % Define DH parameters for a single joint
a = 0;          % X length
alpha = pi/2;     % Angle around X (in radians)
d = 0;          % Z length
theta = pi/6;    % Angle around Z (in radians)


A = [cos(theta), -sin(theta)*cos(alpha), sin(theta)*sin(alpha), a*
    cos(theta);
     sin(theta)0>, cos(theta)0>*cos(alpha)0>, -cos(theta)0>*sin(
    alpha)0>, a*sin(theta)0>;
     0, sin(alpha)0>, cos(alpha)0>, d;
     0, 0, 0, 1]; % Create the transformation matrix


disp('T:')0>;
disp(A)0>; % Display the transformation matrix
```

(a) What is the homogenous transformation ${}^0T_6$ if all joint angles are 0?

Solution. Here θ is 0 for all the links.

$$^{i-1}T_i = trans_Z(d)Rot_Z(\theta)trans_X(a)Rot_X(\alpha)$$

$$^0T_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 0.5 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^3T_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^4T_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } {}^5T_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By chaining we get, ${}^0T_6 = {}^0T_1 \times {}^1T_2 \times {}^2T_3 \times {}^3T_4 \times {}^4T_5 \times {}^5T_6$

$$^0T_6 = \begin{bmatrix} 1 & 0 & 0 & 2.5 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) What is the homogenous transformation ${}^{0}T_6$ if all joint angles are $\pi/6$?

Solution. Here θ is $\pi/6$ for all the links.

$$ {}^{i-1}T_i = trans_Z(d)Rot_Z(\theta)trans_X(a)Rot_X(\alpha) $$

$$ {}^{0}T_1 = \begin{bmatrix} 0.86 & 0 & 0.5 & 0.86 \\ 0.5 & 0 & -0.86 & 0.5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ {}^{1}T_2 = \begin{bmatrix} 0.86 & 0 & -0.5 & 0.43 \\ 0.5 & 0 & 0.86 & 0.25 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^{2}T_3 = \begin{bmatrix} 0.86 & -0.5 & 0 & 0 \\ 0.5 & 0.86 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ {}^{3}T_4 = \begin{bmatrix} 0.86 & 0 & 0.5 & 0.86 \\ 0.5 & 0 & -0.86 & 0.5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^{4}T_5 = \begin{bmatrix} 0.86 & 0 & -0.5 & 0 \\ 0.5 & 0 & 0.86 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ {}^{5}T_6 = \begin{bmatrix} 0.86 & 0 & 0.5 & 0 \\ 0.5 & 0 & -0.86 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

By chaining we get, ${}^{0}T_6 = {}^{0}T_1 \times {}^{1}T_2 \times {}^{2}T_3 \times {}^{3}T_4 \times {}^{4}T_5 \times {}^{5}T_6$

$$ {}^{0}T_6 = \begin{bmatrix} -0.6807 & -0.3460 & 0.6455 & 1.6829 \\ 0.6455 & -0.6997 & 0.3058 & 0.8169 \\ 0.3460 & 0.6250 & 0.6997 & 1.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Problem 5

For the manipulator of question 2.

This is the Matlab code I used for calculating transformation matrix.

```
   % Define DH parameters for a single joint
a = 0;          % X length
alpha = pi/2;     % Angle around X (in radians)
d = 0;          % Z length
theta = pi/6;    % Angle around Z (in radians)


A = [cos(theta), -sin(theta)*cos(alpha), sin(theta)*sin(alpha), a*
    cos(theta);
      sin(theta)0>, cos(theta)0>*cos(alpha)0>, -cos(theta)0>*sin(
    alpha)0>, a*sin(theta)0>;
      0, sin(alpha)0>, cos(alpha)0>, d;
      0, 0, 0, 1]; % Create the transformation matrix


disp('T:')0>;
disp(A)0>; % Display the transformation matrix
```

(a) What is ${}^0T_3$ when $d_1 = 1$ and all other joints are 0?

Solution.

Link	a	α	d	θ
1	1	$\pi/2$	1	0
2	1	$\pi/2$	0	$\pi/2$
3	1	0	0	0

$$^{i-1}T_i = trans_Z(d)Rot_Z(\theta)trans_X(a)Rot_X(\alpha)$$

$$^0T_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^1T_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } {}^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By chaining we get, ${}^0T_3 = {}^0T_1 \times {}^1T_2 \times {}^2T_3$

$$^0T_3 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) What is ${}^0T_3$ when $d_2 = 1$ and all other joints are 0?

Solution.

Link	a	α	d	θ
1	1	$\pi/2$	0	0
2	1	$\pi/2$	1	$\pi/2$
3	1	0	0	0

$$^{i-1}T_i = trans_Z(d)Rot_Z(\theta)trans_X(a)Rot_X(\alpha)$$

$$^0T_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ ^1T_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } ^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By chaining we get, $^0T_3 = {^0T_1} \times {^1T_2} \times {^2T_3}$

$$^0T_3 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) What is $^0T_3$ when $d_3 = 1$ and all other joints are 0?

Solution.

Link	a	α	d	θ
1	1	$\pi/2$	0	0
2	1	$\pi/2$	0	$\pi/2$
3	1	0	1	0

$$^{i-1}T_i = trans_Z(d)Rot_Z(\theta)trans_X(a)Rot_X(\alpha)$$

$$^0T_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ ^1T_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } ^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

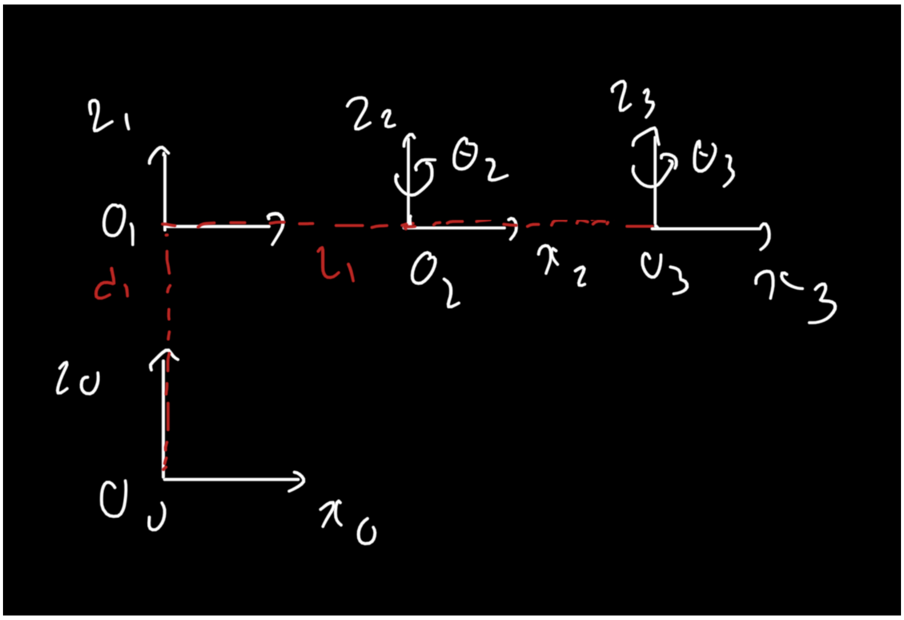
By chaining we get, $^0T_3 = {^0T_1} \times {^1T_2} \times {^2T_3}$

$$^0T_3 = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 6

Derive the DH parameters for the manipulator shown in Figure 3.40.

Solution. The zero angle configuration is as follows:



Here d_1, θ_2 and θ_3 are changing variables and L_1 and L_2 are fixed.

Link	a	α	d	θ
1	0	0	d_1	0
2	L_1	0	0	θ_2
3	L_2	0	0	θ_3