AI1103: Assignment 4

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PROBLEM GATE 2015(CS-SET 3), Q.37:

Suppose X_i for $i = 1, 2, 3$ are independent and identically distributed random variables whose probability mass functions are $\Pr(X_i = 0) = \Pr(X_i = 1) = \frac{1}{2}$ for $i = 1, 2, 3$. Define another random variable $Y = X_1 X_2 \oplus X_3$, where $\oplus$ denotes XOR. Then $\Pr(Y = 0 | X_3 = 0) =$

SOLUTION:

For

$$\because Y = (X_1 X_2) \oplus X_3 = 0 \qquad (0.0.1)$$

$$\implies X_1 X_2 = X_3 \qquad (0.0.2)$$

$$\Pr(Y = 0 | X_3 = 0) = \frac{\Pr(Y = 0, X_3 = 0)}{\Pr(X_3 = 0)} \qquad (0.0.3)$$

$$= \frac{\Pr(X_1 X_2 = X_3, X_3 = 0)}{\Pr(X_3 = 0)} \qquad (0.0.4)$$

$$\Pr(X_3 = 0) = \frac{1}{2} \qquad (0.0.5)$$

if $X_3 = 0$, from (0.0.2)

$$X_1 X_2 = 0 \qquad (0.0.6)$$

The random variables are independent of each other:

$\Pr(X_1 = 0, X_2 = 0)$	$\Pr(X_1 = 0) \cdot \Pr(X_2 = 0)$	0.25
$\Pr(X_1 = 1, X_2 = 0)$	$\Pr(X_1 = 1) \cdot \Pr(X_2 = 0)$	0.25
$\Pr(X_1 = 0, X_2 = 1)$	$\Pr(X_1 = 0) \cdot \Pr(X_2 = 1)$	0.25

TABLE 0: Probabilities

$$\Pr(X_1 X_2 = 0) = \Pr(X_1 = 0, X_2 = 0)$$
$$+ \Pr(X_1 = 0, X_2 = 1)$$
$$+ \Pr(X_1 = 1, X_2 = 0) \qquad (0.0.7)$$
$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4} \qquad (0.0.8)$$

$$\Pr(Y = 0, X_3 = 0) = \Pr(X_1 X_2 = X_3 = 0) \qquad (0.0.9)$$
$$= \Pr(X_1 X_2 = 0) \cdot \Pr(X_3 = 0) \qquad (0.0.10)$$
$$= \frac{3}{4} \cdot \frac{1}{2} \qquad (0.0.11)$$
$$= \frac{3}{8} \qquad (0.0.12)$$

Upon substituting (0.0.12) and (0.0.5) in (0.0.3)

$$\Pr(Y = 0 | X_3 = 0) = \frac{3}{4} = 0.75 \qquad (0.0.13)$$