AI1103: Assignment 7

Tanmay Garg CS20BTECH11063 EE20BTECH11048

Download all python codes from

https://github.com/tanmaygar/AI-Course/blob/main /Assignment7/Codes/CSIRUGC NET%20 EXAM (Dec%202016) Q51.py

and latex-tikz codes from

https://github.com/tanmaygar/AI-Course/blob/main /Assignment7/Assignment7.tex

PROBLEM CSIR UGC NET EXAM (Dec 2016), Q.51:

Suppose customers arrive in a shop according to a Poisson process with rate 4 per hour. The shop opens at 10:00 am. If it is given that the second customer arrives at 10:40 am, what is the probability that no customer arrived before 10:30 am?

1)
$$\frac{1}{4}$$

1)
$$\frac{1}{4}$$
 2) e^{-2} 3) $\frac{1}{2}$ 4) $e^{\frac{1}{2}}$

3)
$$\frac{1}{2}$$

SOLUTION:

Random Variable	Time at which people arrive
X_p	p = 10:00-10:30
X_q	q = 10:30-10:40
X_r	r = 10:00 - 10:40
Y	10:40

TABLE 4: Random Variables

We need to find

$$\Pr\left(X_p = 0 | Y = 2\right) \tag{0.0.1}$$

In the world where the 2^{nd} person arrives at 10:40am the (0.0.1) becomes:

$$= \frac{\Pr(X_p = 0, X_q = 1)}{\Pr(X_r = 1)}$$
(0.0.2)

$$= \frac{\Pr(X_p = 0) \times \Pr(X_q = 1)}{\Pr(X_r = 1)}$$
(0.0.3)

The Poisson function distribution for time interval t and rate λ for a random variable X:

$$f_X(x;t) = \frac{(\lambda t)^x \exp(-\lambda t)}{x!}$$

For the time interval p:

$$\lambda = 4, t = 0.5, x = 0$$
 (0.0.4)

$$\Pr(X_p = 0) = f_X(0; 0.5)$$
 (0.0.5)
= e^{-2} (0.0.6)

$$= e^{-2} (0.0.6)$$

For the time interval q:

$$\lambda = 4, t = \frac{1}{6}, x = 1$$
 (0.0.7)

$$\Pr(X_q = 1) = f_X(1; \frac{1}{6})$$
 (0.0.8)

$$=\frac{2}{3}e^{\frac{-2}{3}}\tag{0.0.9}$$

For the time interval *r*:

$$\lambda = 4, t = \frac{2}{3}, x = 1$$
 (0.0.10)

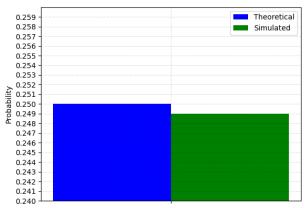
$$\Pr(X_r = 1) = f_X(1; \frac{2}{3}) \tag{0.0.11}$$

$$=\frac{8}{3}e^{\frac{-8}{3}}\tag{0.0.12}$$

Substituting (0.0.6) (0.0.9) (0.0.12) in (0.0.3):

$$\Pr(X_p = 0|Y = 2) = \frac{1}{4} \tag{0.0.13}$$

Hence, **Option 1** is correct



Theoretical v/s Simulated