CSIR UGC NET EXAM (June 2016), Q.118

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Question

Three types of components are used in electrical circuits 1, 2, 3 as shown below in the figure Suppose that each of the three components fail with probability p and independently of each other. Let
$q_i = \Pr(\text{Circuit } i \text{ does not fail});\ i = 1, 2, 3$ For $0 < p < 1$, we have

1. $q_3 > q_1$
2. $q_2 = q_1$
3. $q_2 > q_1$
4. $q_2 > q_3$

Question

Figure

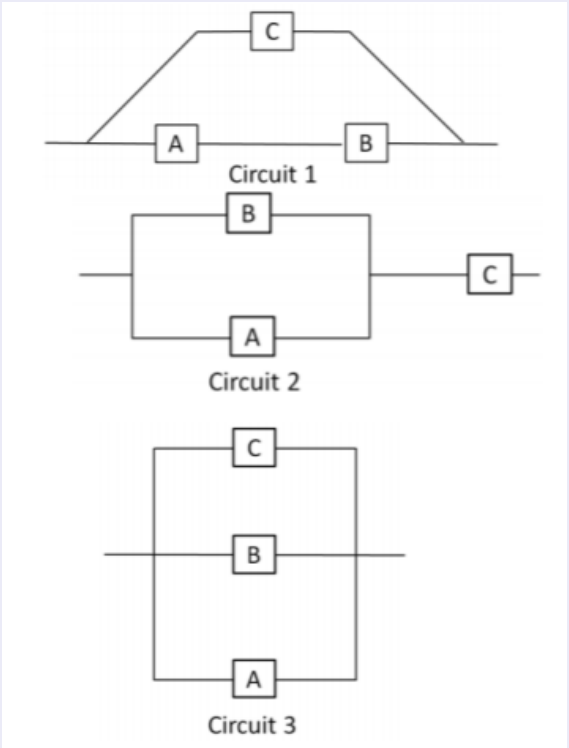


Figure: Circuits

Boolean Algebra

Boolean Expression for Series Circuit

Boolean Expression for series circuit:

$$AB \tag{1}$$

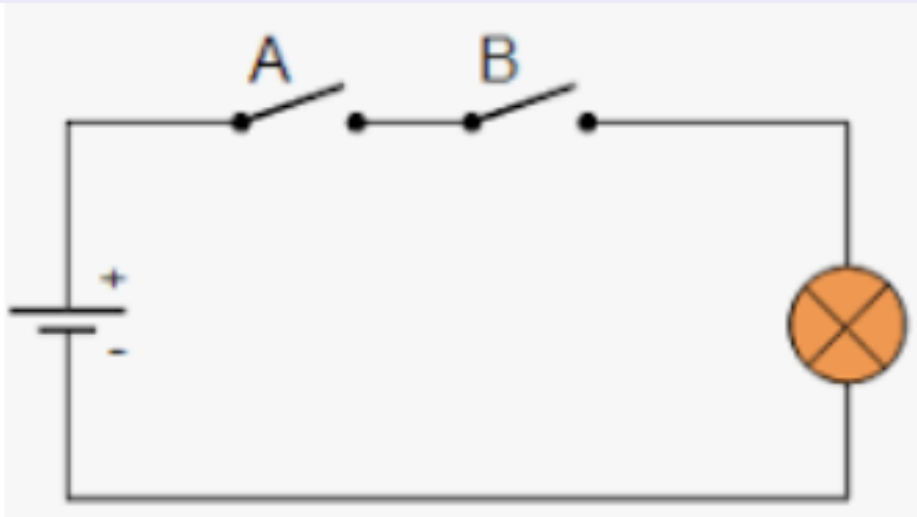


Figure: Series

Boolean Algebra

Boolean Expression for Parallel Circuit

Boolean Expression for parallel circuit:

$$A + B \tag{2}$$

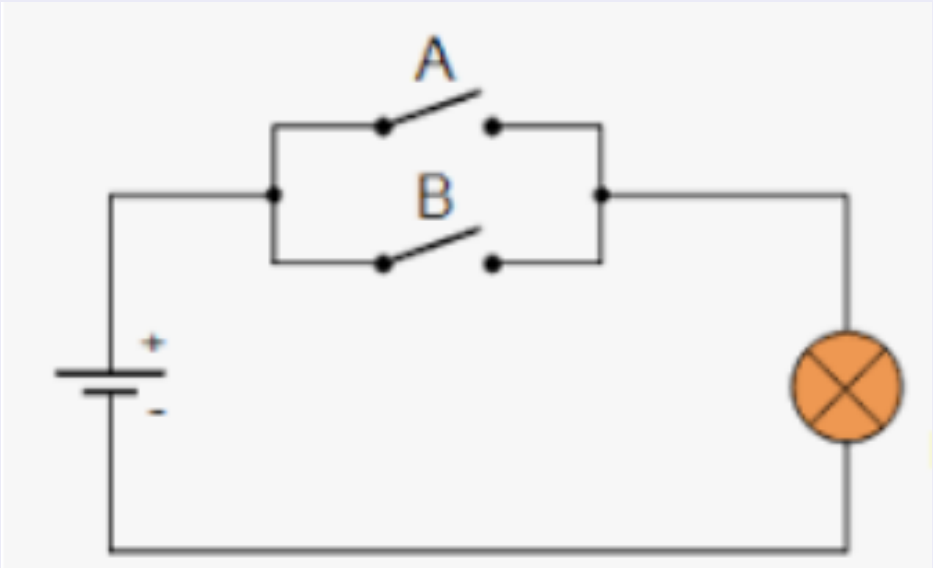


Figure: Parallel

Circuit 1

Boolean Expression

The Boolean Algebraic expression for this circuit is:

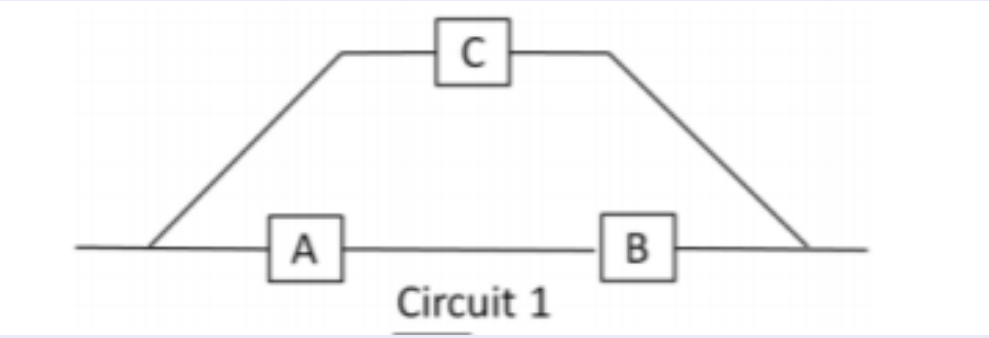


Figure: Circuit 1

We get:

$$AB + C \tag{3}$$

Circuit 1

Probabilities

For Circuit 1 to work the truth table will be:

A	B	C	$(AB)+C$	Probability
1	1	0	1	$p(1-p)^2$
1	1	1	1	$(1-p)^3$
0	1	1	1	$p(1-p)^2$
0	0	1	1	$p^2(1-p)$
1	0	1	1	$p(1-p)^2$

Table: Circuit 1 working

Adding all we get $\Pr(\text{Circuit 1 works})$:

$$q_1 = p^3 - 2p^2 + 1 \tag{4}$$

Circuit 2

Boolean Expression

The Boolean Algebraic expression for this circuit is:

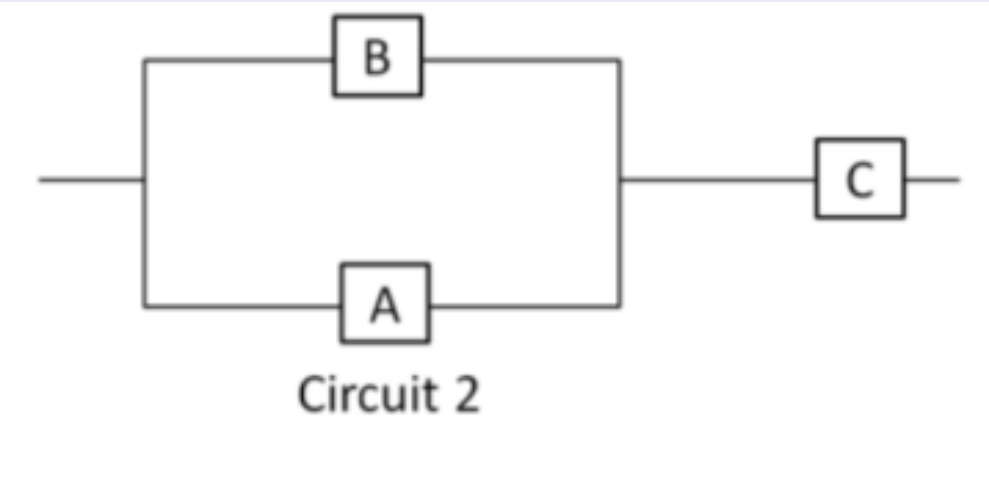


Figure: Circuit 2

We get:

$$(A + B)C \tag{5}$$

Circuit 2

Probabilities

For Circuit 2 to work the truth table will be:

A	B	C	$(A+B)C$	Probability
1	1	1	1	$(1-p)^3$
1	0	1	1	$p(1-p)^2$
0	1	1	1	$p(1-p)^2$

Table: Circuit 2 working

Adding all we get Pr (Circuit 2 works):

$$q_2 = p^3 - p^2 - p + 1 \tag{6}$$

Circuit 3

Boolean Expression

The Boolean Algebraic expression for this circuit is:

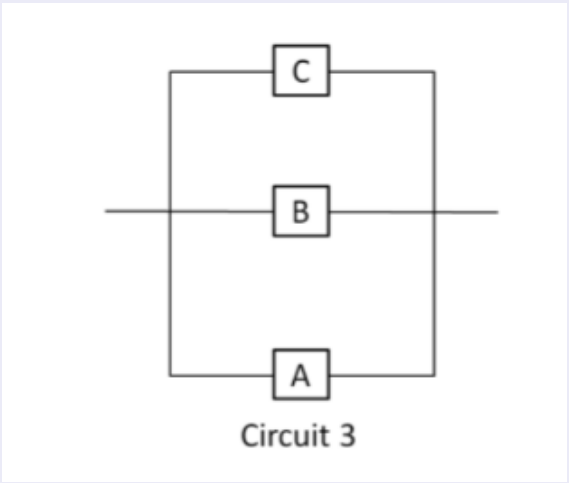


Figure: Circuit 3

We get:

$$A + B + C \tag{7}$$

Circuit 3

Probabilities

For Circuit 3 to work the truth table will be:

A	B	C	$A+B+C$	Probability
1	0	0	1	$p^2(1-p)$
0	1	0	1	$p^2(1-p)$
0	0	1	1	$p^2(1-p)$
1	1	0	1	$p(1-p)^2$
1	0	1	1	$p(1-p)^2$
0	1	1	1	$p(1-p)^2$
1	1	1	1	$(1-p)^3$

Table: Circuit 3 working

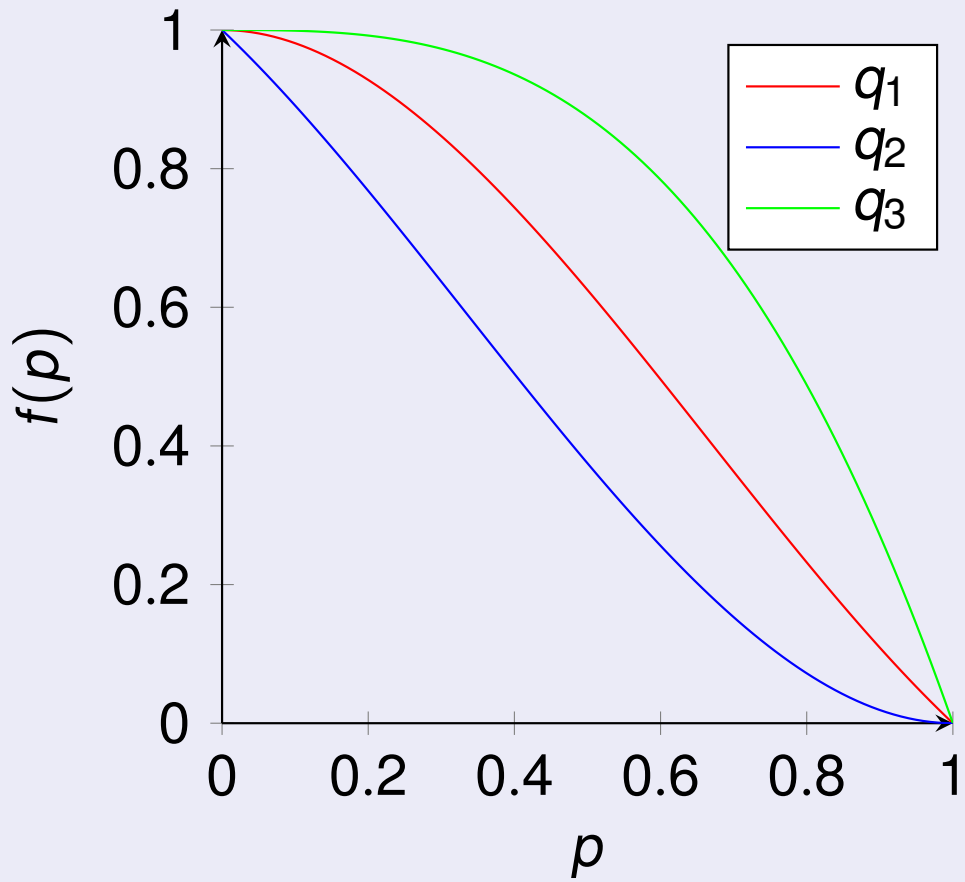
Adding all we get Pr (Circuit 3 works):

$$q_3 = 1 - p^3 \tag{8}$$

Plotting the functions

Graph

Plotting (4), (6) and (8)



Answer

Correct Answer

On comparing from the graph we can determine that:

$$\therefore q_3 > q_1 > q_2 \tag{9}$$

Hence **Option 1**: $q_3 > q_1$ is correct