

AI1103: Assignment 7

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Download all python codes from

[https://github.com/tanmaygar/AI-Course/blob/main/Assignment7/Codes/CSIRUGC_NET%20EXAM_\(Dec%202016\)_Q51.py](https://github.com/tanmaygar/AI-Course/blob/main/Assignment7/Codes/CSIRUGC_NET%20EXAM_(Dec%202016)_Q51.py)

and latex-tikz codes from

<https://github.com/tanmaygar/AI-Course/blob/main/Assignment7/Assignment7.tex>

PROBLEM CSIR UGC NET EXAM (DEC 2016), Q.51:

Suppose customers arrive in a shop according to a Poisson process with rate 4 per hour. The shop opens at 10 : 00 am. If it is given that the second customer arrives at 10 : 40 am, what is the probability that no customer arrived before 10 : 30 am?

- 1) $\frac{1}{4}$ 2) e^{-2} 3) $\frac{1}{2}$ 4) $e^{\frac{1}{2}}$

SOLUTION:

Random Variable	Time at which people arrive
X_p	$p = 10 : 00 - 10 : 30$
X_q	$q = 10 : 30 - 10 : 40$
X_r	$r = 10 : 00 - 10 : 40$
Y	10 : 40

TABLE 4: Random Variables

We need to find

$$\Pr(X_p = 0 | Y = 2) \quad (0.0.1)$$

In the world where the 2nd person arrives at 10 : 40 am the (0.0.1) becomes:

$$= \frac{\Pr(X_p = 0, X_q = 1)}{\Pr(X_r = 1)} \quad (0.0.2)$$

$$= \frac{\Pr(X_p = 0) \times \Pr(X_q = 1)}{\Pr(X_r = 1)} \quad (0.0.3)$$

The Poisson function distribution for time interval t and rate λ for a random variable X :

$$f_X(x; t) = \frac{(\lambda t)^x \exp(-\lambda t)}{x!}$$

For the time interval p :

$$\lambda = 4, t = 0.5, x = 0 \quad (0.0.4)$$

$$\Pr(X_p = 0) = f_X(0; 0.5) \quad (0.0.5)$$

$$= e^{-2} \quad (0.0.6)$$

For the time interval q :

$$\lambda = 4, t = \frac{1}{6}, x = 1 \quad (0.0.7)$$

$$\Pr(X_q = 1) = f_X(1; \frac{1}{6}) \quad (0.0.8)$$

$$= \frac{2}{3} e^{-\frac{2}{3}} \quad (0.0.9)$$

For the time interval r :

$$\lambda = 4, t = \frac{2}{3}, x = 1 \quad (0.0.10)$$

$$\Pr(X_r = 1) = f_X(1; \frac{2}{3}) \quad (0.0.11)$$

$$= \frac{8}{3} e^{-\frac{8}{3}} \quad (0.0.12)$$

Substituting (0.0.6) (0.0.9) (0.0.12) in (0.0.3):

$$\Pr(X_p = 0 | Y = 2) = \frac{1}{4} \quad (0.0.13)$$

Hence, **Option 1** is correct

