

# CSIR UGC NET EXAM (June 2016), Q.118

Tanmay Garg - EE20BTECH11048

April 9, 2021

# CSIR UGC NET EXAM (June 2016), Q.118

## Question

Three types of components are used in electrical circuits 1, 2, 3 as shown below in the figure. Suppose that each of the three components fail with probability  $p$  and independently of each other. Let

$q_i = \Pr(\text{Circuit } i \text{ does not fail}); i = 1, 2, 3$  For  $0 < p < 1$ , we have

- 1  $q_3 > q_1$
- 2  $q_2 = q_1$
- 3  $q_2 > q_1$
- 4  $q_2 > q_3$

# Question

## Figure

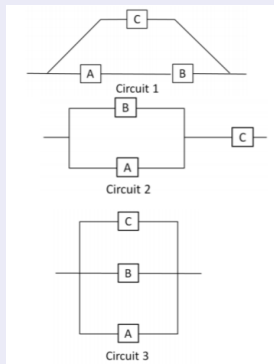


Figure: Circuits

# Boolean Algebra

## Boolean Expression for Series Circuit

Boolean Expression for series circuit:

$$AB$$

(1)

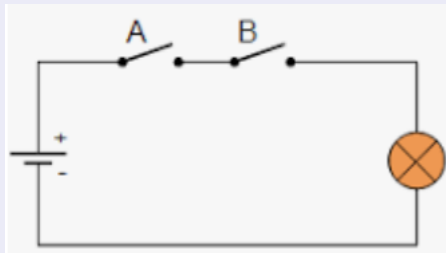


Figure: Series

# Boolean Algebra

## Boolean Expression for Parallel Circuit

Boolean Expression for parallel circuit:

$$A + B$$

(2)

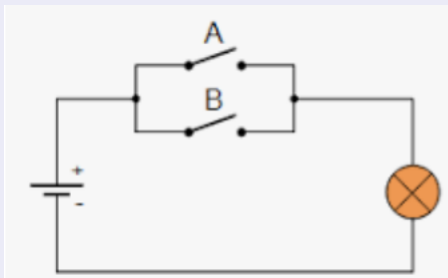


Figure: Parallel

# Circuit 1

## Boolean Expression

The Boolean Algebraic expression for this circuit is:

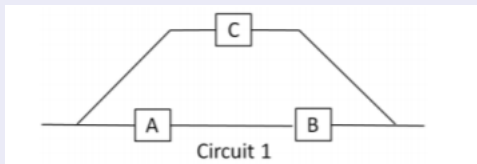


Figure: Circuit 1

We get:

$$AB + C$$

(3)

# Circuit 1

## Probabilities

For Circuit 1 to work the truth table will be:

A	B	C	$(AB) + C$	Probability
1	1	0	1	$p(1 - p)^2$
1	1	1	1	$(1 - p)^3$
0	1	1	1	$p(1 - p)^2$
0	0	1	1	$p^2(1 - p)$
1	0	1	1	$p(1 - p)^2$

Table: Circuit 1 working

Adding all we get  $\Pr(\text{Circuit 1 works})$  :

$$q_1 = p^3 - 2p^2 + 1 \quad (4)$$

## Circuit 2

### Boolean Expression

The Boolean Algebraic expression for this circuit is:

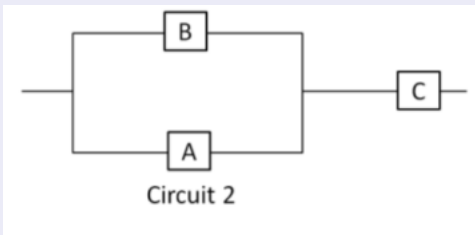


Figure: Circuit 2

We get:

$$(A + B)C$$

(5)



## Circuit 2

### Probabilities

For Circuit 2 to work the truth table will be:

A	B	C	$(A + B)C$	Probability
1	1	1	1	$(1 - p)^3$
1	0	1	1	$p(1 - p)^2$
0	1	1	1	$p(1 - p)^2$

Table: Circuit 2 working

Adding all we get Pr (Circuit 2 works):

$$q_2 = p^3 - p^2 - p + 1 \quad (6)$$

# Circuit 3

## Boolean Expression

The Boolean Algebraic expression for this circuit is:

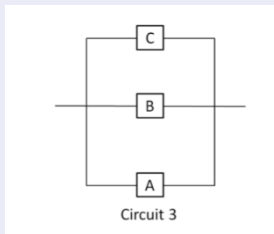


Figure: Circuit 3

We get:

$$A + B + C$$

(7)

# Circuit 3

## Probabilities

For Circuit 3 to work the truth table will be:

A	B	C	$A + B + C$	Probability
1	0	0	1	$p^2(1 - p)$
0	1	0	1	$p^2(1 - p)$
0	0	1	1	$p^2(1 - p)$
1	1	0	1	$p(1 - p)^2$
1	0	1	1	$p(1 - p)^2$
0	1	1	1	$p(1 - p)^2$
1	1	1	1	$(1 - p)^3$

Table: Circuit 3 working

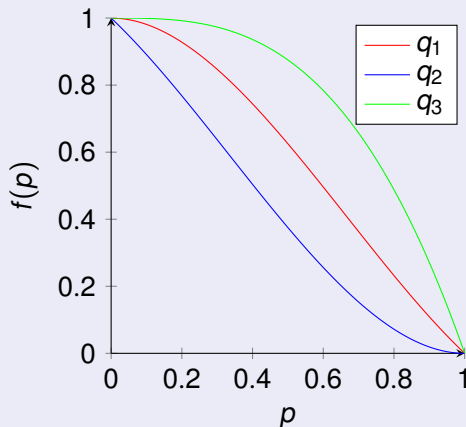
Adding all we get Pr (Circuit 3 works):

$$q_3 = 1 - p^3 \quad (8)$$

# Plotting the functions

## Graph

Plotting (4), (6) and (8)



# Answer

## Correct Answer

On comparing from the graph we can determine that:

$$\therefore q_3 > q_1 > q_2 \quad (9)$$

Hence **Option 1**:  $q_3 > q_1$  is correct