

CSIR UGC NET EXAM (June 2016), Q.118

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Question

Three types of components are used in electrical circuits 1, 2, 3 as shown below in the figure. Suppose that each of the three components fail with probability p and independently of each other. Let

$q_i = \Pr(\text{Circuit } i \text{ does not fail}); i = 1, 2, 3$ For $0 < p < 1$, we have

- 1 $q_3 > q_1$
- 2 $q_2 = q_1$
- 3 $q_2 > q_1$
- 4 $q_2 > q_3$

Question

Figure

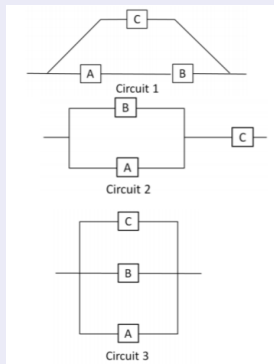


Figure: Circuits

Circuit 1

Boolean Expression

The Boolean Algebraic expression for this circuit is:

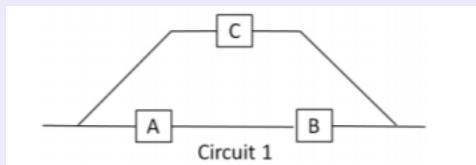


Figure: Circuit 1

We get:

$$AB + C$$

(1)

Circuit 1

Probabilities

For Circuit 1 to work the truth table will be:

A	B	C	$(AB) + C$	Probability
1	1	0	1	$p(1 - p)^2$
1	1	1	1	$(1 - p)^3$
0	1	1	1	$p(1 - p)^2$
0	0	1	1	$p^2(1 - p)$
1	0	1	1	$p(1 - p)^2$

Table: Circuit 1 working

Adding all we get $\Pr(\text{Circuit 1 works})$:

$$q_1 = p^3 - 2p^2 + 1 \quad (2)$$

Circuit 2

Boolean Expression

The Boolean Algebraic expression for this circuit is:

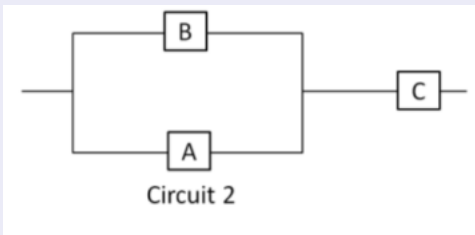


Figure: Circuit 2

We get:

$$(A + B)C$$

(3)

Circuit 2

Probabilities

For Circuit 2 to work the truth table will be:

A	B	C	$(A + B)C$	Probability
1	1	1	1	$(1 - p)^3$
1	0	1	1	$p(1 - p)^2$
0	1	1	1	$p(1 - p)^2$

Table: Circuit 2 working

Adding all we get $\Pr(\text{Circuit 2 works})$:

$$q_2 = p^3 - p^2 - p + 1 \quad (4)$$

Circuit 3

Boolean Expression

The Boolean Algebraic expression for this circuit is:

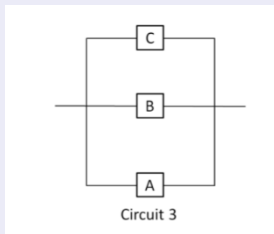


Figure: Circuit 3

We get:

$$A + B + C$$

(5)

Circuit 3

Probabilities

For Circuit 3 to work the truth table will be:

A	B	C	$A + B + C$	Probability
1	0	0	1	$p^2(1 - p)$
0	1	0	1	$p^2(1 - p)$
0	0	1	1	$p^2(1 - p)$
1	1	0	1	$p(1 - p)^2$
1	0	1	1	$p(1 - p)^2$
0	1	1	1	$p(1 - p)^2$
1	1	1	1	$(1 - p)^3$

Table: Circuit 3 working

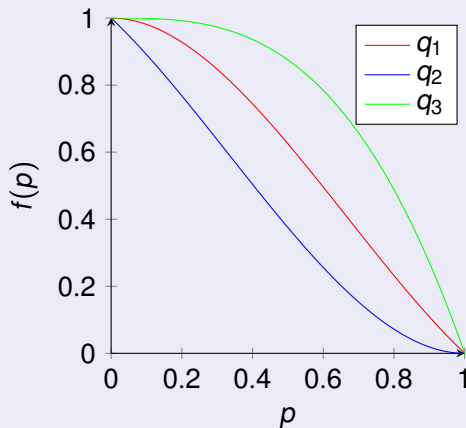
Adding all we get Pr (Circuit 3 works):

$$q_3 = 1 - p^3 \quad (6)$$

Plotting the functions

Graph

Plotting (2), (4) and (6)



Answer

Correct Answer

On comparing from the graph we can determine that:

$$\therefore q_3 > q_1 > q_2 \quad (7)$$

Hence **Option 1**: $q_3 > q_1$ is correct