

AI1103: Challenge Problem Mixture

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CS20BTECH11063 EE20BTECH11048

Download all python codes from

https://github.com/tanmaygar/AI-Course/blob/main/challenge%20mixture/codes/challenge_mix.py

and latex-tikz codes from

https://github.com/tanmaygar/AI-Course/blob/main/challenge%20mixture/Challenge_Mixed.tex

CHALLENGE PROBLEM MIXED

Let $X \sim B\left(5, \frac{1}{2}\right)$ and $Y \sim U(0, 1)$. The the value of:

$$\frac{\Pr(X + Y \leq 2)}{\Pr(X + Y \geq 5)}$$

is equal to? (X and Y are independent)

SOLUTION:

It is given that X is random variable which follows binomial distribution with $n = 5$ and $p = \frac{1}{2}$. Y is a uniform distribution in the interval $(0, 1)$. Both X and Y are independent.

$$\Pr(X = k) = \binom{5}{k} \left(\frac{1}{2}\right)^5 \quad (0.0.1)$$

$$F_Y(y) = \Pr(Y \leq y) = \begin{cases} 0 & y < 0 \\ y & 0 \leq y \leq 1 \\ 1 & y > 1 \end{cases} \quad (0.0.2)$$

Calculating:

$$\Pr(X + Y \leq 2) = \Pr(X = 0, Y \leq 2) \quad (0.0.3)$$

$$+ \Pr(X = 1, Y \leq 1)$$

$$+ \Pr(X = 2, Y \leq 0)$$

$$= 1 \cdot \binom{5}{0} \left(\frac{1}{2}\right)^5 + 1 \cdot \binom{5}{1} \left(\frac{1}{2}\right)^5 \quad (0.0.4)$$

$$+ 0 \cdot \binom{5}{2} \left(\frac{1}{2}\right)^5$$

$$= \frac{3}{16} \quad (0.0.5)$$

$$\Pr(X + Y \geq 5) = \sum_{k=0}^5 \Pr(X = k, Y \geq 5 - k) \quad (0.0.6)$$

$$= \sum_{k=0}^5 \Pr(X = k) \Pr(Y \geq 5 - k) \quad (0.0.7)$$

$$= 0 + \Pr(X = 5) \Pr(Y \geq 0) \quad (0.0.8)$$

$$= \binom{5}{5} \left(\frac{1}{2}\right)^5 \quad (0.0.9)$$

$$= \frac{1}{32} \quad (0.0.10)$$

Substituting (0.0.5) and (0.0.10)

$$\frac{\Pr(X + Y \leq 2)}{\Pr(X + Y \geq 5)} = 6 \quad (0.0.11)$$

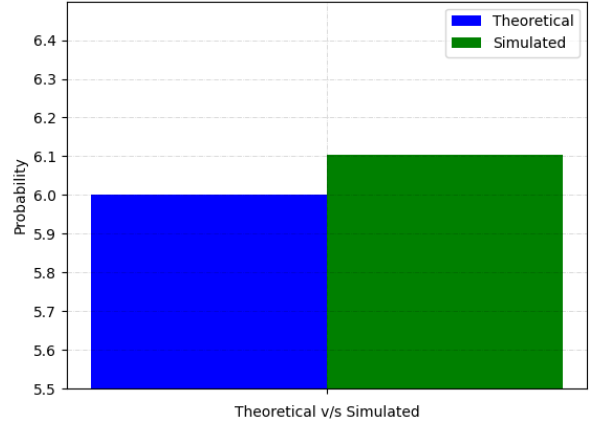


Fig. 0