AI1103: Assignment 7

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Download all python codes from

https://github.com/tanmaygar/AI−Course/blob/main/Assignment7/Codes/CSIRUGC_NET%20EXAM_(Dec%202016)_Q51.py

and latex-tikz codes from

https://github.com/tanmaygar/AI−Course/blob/main/Assignment7/Assignment7.tex

Problem CSIR UGC NET EXAM (Dec 2016), Q.51: Suppose customers arrive in a shop according to a Poisson process with rate 4 per hour. The shop opens at 10 : 00 am. If it is given that the second customer arrives at 10 : 40 am, what is the probability that no customer arrived before 10 : 30 am?

1) $\frac{1}{4}$
2) e^{-2}
3) $\frac{1}{2}$
4) $e^{\frac{1}{2}}$

Solution:

Let the time interval be divided as:

$$t_1 = 10:00 - 10:30$$
$$t_2 = 10:30 - 10:40$$
$$t_3 = 10:00 - 10:40$$

We need to find

$$\Pr\left(0 \text{ in } t_1 | 2^{nd} \text{ arrives at 10:40}\right) \tag{0.0.1}$$

In the world where the 2^{nd} person arrives at 10 : 40 am the (0.0.1) becomes:

$$= \frac{\Pr(0 \text{ in } t_1, 1 \text{ in } t_2)}{\Pr(1 \text{ in } t_3)} \tag{0.0.2}$$
$$= \frac{\Pr(0 \text{ in } t_1) \cdot \Pr(1 \text{ in } t_2)}{\Pr(1 \text{ in } t_3)} \tag{0.0.3}$$

The Poisson function distribution for time interval t and rate λ for a random variable X:

$$f_X(x,t) = \frac{(\lambda \cdot t)^x \exp(-\lambda \cdot t)}{x!}$$

For the time interval t_1:

$$\lambda = 4, t = 0.5, x = 0 \tag{0.0.4}$$
$$\Pr(0 \text{ in } t_1) = f_X(0, 0.5) \tag{0.0.5}$$
$$= e^{-2} \tag{0.0.6}$$

For the time interval t_2:

$$\lambda = 4, t = \frac{1}{6}, x = 1 \tag{0.0.7}$$
$$\Pr(1 \text{ in } t_2) = f_X(1, \frac{1}{6}) \tag{0.0.8}$$
$$= \frac{2}{3}e^{\frac{-2}{3}} \tag{0.0.9}$$

For the time interval t_3:

$$\lambda = 4, t = \frac{2}{3}, x = 1 \tag{0.0.10}$$
$$\Pr(1 \text{ in } t_3) = f_X(1, \frac{2}{3}) \tag{0.0.11}$$
$$= \frac{8}{3}e^{\frac{-8}{3}} \tag{0.0.12}$$

Substituting (0.0.6) (0.0.9) (0.0.12) in (0.0.3):

$$\Pr\left(0 \text{ in } t_1 | 2^{nd} \text{ arrives at 10:40}\right) = \frac{1}{4} \tag{0.0.13}$$

Hence, **Option 1** is correct