AI1103: Challenge Problem Mixture

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Challenge Problem Mixed

Let $X \sim B\left(5, \frac{1}{2}\right)$ and $Y \sim U(0,1)$. The the value of:

$$\frac{\Pr(X+Y \le 2)}{\Pr(X+Y \ge 5)}$$

is equal to?

Solution:

It is given that X is random variable which follows binomial distribution with $n = 5$ and $p = \frac{1}{2}$. Y is a uniform distribution in the interval $(0,1)$.

$$\Pr(X = k) = \binom{5}{k}\left(\frac{1}{2}\right)^5 \tag{0.0.1}$$

$$F_Y(y) = \Pr(Y \le y) = \begin{cases} 0 & y < 0 \\ y & 0 \le y \le 1 \\ 1 & y > 1 \end{cases} \tag{0.0.2}$$

Calculating:

$$\begin{aligned} \Pr(X+Y \le 2) &= \Pr(X=0, Y \le 2) && (0.0.3)\\ &\quad + \Pr(X=1, Y \le 1) \\ &\quad + \Pr(X=2, Y \le 0) \\ &= 1 \cdot \binom{5}{0}\left(\frac{1}{2}\right)^5 + 1 \cdot \binom{5}{1}\left(\frac{1}{2}\right)^5 && (0.0.4)\\ &\quad + 0 \cdot \binom{5}{2}\left(\frac{1}{2}\right)^5 \\ &= \frac{3}{16} && (0.0.5) \end{aligned}$$

$$\begin{aligned} \Pr(X+Y \ge 5) &= \sum_{k=0}^{5} \Pr(X=k, Y \ge 5-k) && (0.0.6)\\ &= 0 + \Pr(X=5, Y \ge 0) && (0.0.7)\\ &= \binom{5}{5}\left(\frac{1}{2}\right)^5 && (0.0.8)\\ &= \frac{1}{32} && (0.0.9) \end{aligned}$$

Substituting (0.0.5) and (0.0.9)

$$\frac{\Pr(X+Y \le 2)}{\Pr(X+Y \ge 5)} = 6 \tag{0.0.10}$$

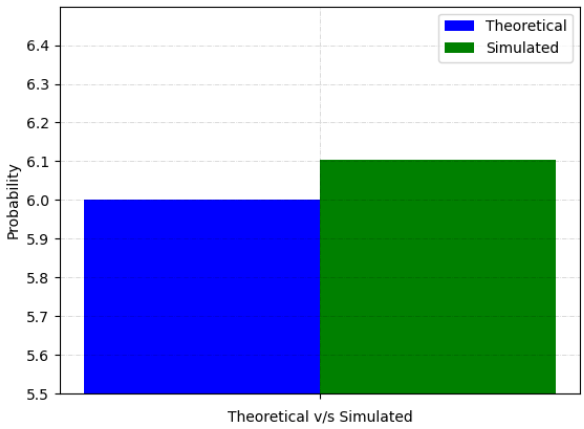

Fig. 0