AI1103: Assignment 7

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Download all python codes from

https://github.com/tanmaygar/AI−Course/blob/main/Assignment7/Codes/CSIRUGC_NET%20EXAM_(Dec%202016)_Q51.py

and latex-tikz codes from

https://github.com/tanmaygar/AI−Course/blob/main/Assignment7/Assignment7.tex

Problem CSIR UGC NET EXAM (Dec 2016), Q.51:

Suppose customers arrive in a shop according to a Poisson process with rate 4 per hour. The shop opens at 10 : 00 am. If it is given that the second customer arrives at 10 : 40 am, what is the probability that no customer arrived before 10 : 30 am?

1) $\frac{1}{4}$ 2) e^{-2} 3) $\frac{1}{2}$ 4) $e^{\frac{1}{2}}$

Solution:

Random Variable	Time at which people arrive
X_p	$p = 10:00 - 10:30$
X_q	$q = 10:30 - 10:40$
X_r	$r = 10:00 - 10:40$
Y	10 : 40

TABLE 4: Random Variables

We need to find

$$\Pr\left(X_p = 0 | Y = 2\right) \tag{0.0.1}$$

In the world where the 2^{nd} person arrives at 10 : 40 am the (0.0.1) becomes:

$$= \frac{\Pr\left(X_p = 0, X_q = 1\right)}{\Pr\left(X_r = 1\right)} \tag{0.0.2}$$

$$= \frac{\Pr\left(X_p = 0\right) \times \Pr\left(X_q = 1\right)}{\Pr\left(X_r = 1\right)} \tag{0.0.3}$$

The Poisson function distribution for time interval t and rate λ for a random variable X:

$$f_X(x;t) = \frac{(\lambda t)^x \exp\left(-\lambda t\right)}{x!}$$

For the time interval p:

$$\lambda = 4, t = 0.5, x = 0 \tag{0.0.4}$$

$$\Pr\left(X_p = 0\right) = f_X\left(0; \frac{1}{2}\right) \tag{0.0.5}$$

$$= e^{-2} \tag{0.0.6}$$

$$\tag{0.0.7}$$

For the time interval q:

$$\lambda = 4, t = \frac{1}{6}, x = 1 \tag{0.0.8}$$

$$\Pr\left(X_q = 1\right) = f_X\left(1; \frac{1}{6}\right) \tag{0.0.9}$$

$$= \frac{2}{3} e^{\frac{-2}{3}} \tag{0.0.10}$$

For the time interval r:

$$\lambda = 4, t = \frac{2}{3}, x = 1 \tag{0.0.11}$$

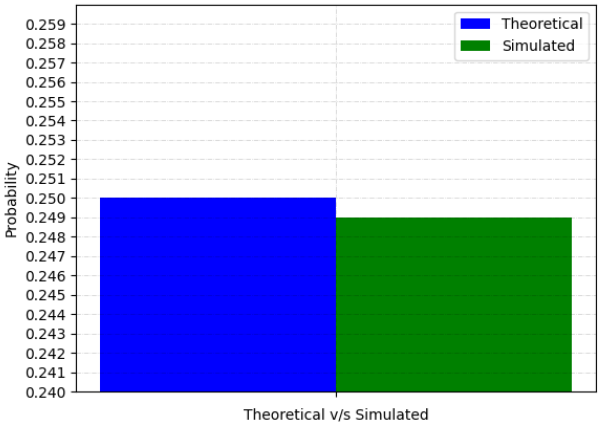
$$\Pr\left(X_r = 1\right) = f_X\left(1; \frac{2}{3}\right) \tag{0.0.12}$$

$$= \frac{8}{3} e^{\frac{-8}{3}} \tag{0.0.13}$$

Substituting (0.0.6) (0.0.10) (0.0.13) in (0.0.3):

$$\Pr\left(X_p = 0 | Y = 2\right) = \frac{1}{4} \tag{0.0.14}$$

Hence, **Option 1** is correct


Theoretical
Simulated
Probability
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0.258
0.257
0.256
0.255
0.254
0.253
0.252
0.251
0.250
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0.247
0.246
0.245
0.244
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0.242
0.241
0.240
Theoretical v/s Simulated