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Algo Quiz 1

1. (a) $T(n) = 5T(n/2) + n$

let $n = 2^k$ $\qquad k = \log_2 n$

$T(2^k) = 5T(2^{k-1}) + 2^k$

let $T(2^k) = f(k)$

$\therefore f(k) = 5f(k-1) + 2^k$

let $f(k) = 5^k g(k)$

$\therefore 5^k g(k) = 5 \cdot 5^{k-1} g(k-1) + 2^k$

$g(k) - g(k-1) = \left(\frac{2}{5}\right)^k$

$\vdots$

$g(1) - g(0) = \left(\frac{2}{5}\right)$

$g(k) - g(0) = \frac{2}{5} + \dots + \left(\frac{2}{5}\right)^k = \frac{2/5\left(1 - (2/5)^k\right)}{3/5} = \frac{2}{3}\left(1 - \left(\frac{2}{5}\right)^k\right)$

$\because T(1) = 1 \qquad \therefore f(0) = T(1) = 1 \qquad f(0) = 5^0 g(0) = T(1) = 1$

$g(k) = 1 + \frac{2}{5} + \dots + \left(\frac{2}{5}\right)^k = \frac{5}{3} - \frac{2}{3}\left(\frac{2}{5}\right)^k = \frac{5}{3}\left(1 - \left(\frac{2}{5}\right)^{k+1}\right)$

$\frac{f(k)}{5^k} = \frac{5}{3}\left(1 - \left(\frac{2}{5}\right)^{k+1}\right)$

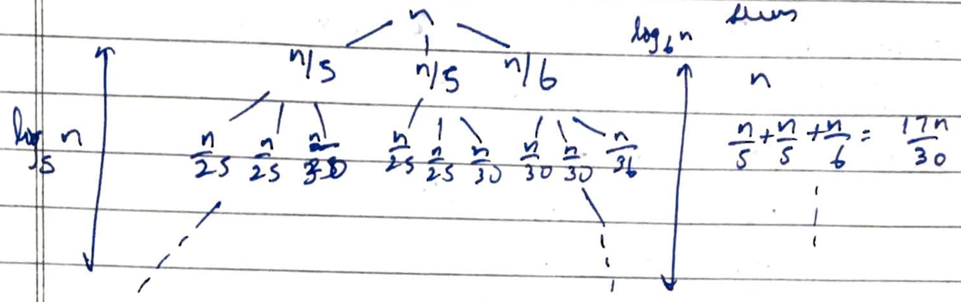
$T(2^k) = \frac{5}{3} 5^k - \frac{5}{3} \cdot \frac{2}{5}\left(\frac{2}{5}\right)^k \cdot 5^k$

$= \frac{5}{3} \cdot 5^k - \frac{2}{3} \cdot 2^k$

$T(n) = \frac{5}{3} \cdot 5^{\log_2 n} - \frac{2}{3} n = \frac{5}{3} \cdot n^{\log_2 5} - \frac{2}{3} n$

$\therefore T(n) = O\left(n^{\log_2 5}\right)$

(b) $T(n) = 2T\left(\frac{n}{5}\right) + T\left(\frac{n}{6}\right) + n$



using recursion tree method h of tree

$\log_6 n \le h \le \log_5 n$

$$T(n) = n + \frac{17n}{30} + \left(\frac{17}{30}\right)^2 n + \dots + \left(\frac{17}{30}\right)^h n$$

$$= n\left(1 + \frac{17}{30} + \left(\frac{17}{30}\right)^2 + \dots + \left(\frac{17}{30}\right)^h\right)$$

$$= n\,\frac{\left(1 - \left(\frac{17}{30}\right)^{h+1}\right)}{1 - 17/30} = \frac{30n}{13}\left(1 - \left(\frac{17}{30}\right)^{h+1}\right)$$

$$T(n) \le \frac{30n}{13}$$

$\therefore$ $T(n) = O(n)$

(c) $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

$$= 4T\left(\frac{n}{4}\right) + \frac{n}{\log n} + \frac{n}{\log n/2}$$

$$= \dots = 2^k T\left(\frac{n}{2^k}\right) + n \sum_{j=0}^{k-1} \frac{1}{\log n - j}$$

$\because$ base case $T(1) = 1$

$\therefore$ we can take $n = 2^k$

$$\therefore T(n) = n + n \underbrace{\sum_{j=0}^{k-1} \frac{1}{k-j}}_{\text{harmonic series sum}} \approx \log k \approx \log(\log n)$$

$\therefore T(n) \approx n + n \log(\log n)$

$\therefore T(n) = O(n \log(\log n))$

2. $A[1 \ldots n] = (2, 4, 3, 1, 6, 10, 9)$
1 2 3 4 5 6 7

$p = 5$

Partition ( ) :

swap $-$ $(2, 4, 3, 1, 9, 10, 6)$

$l = 0$

for $i = 1$ to $n-1 = 6$

$i = 1$ $A[1] < A[7] = 2 < 6$ true

$l = 1$

swap $A(l) \leftrightarrow A(i)$ $(2, 4, 3, 1, 9, 10, 6)$

$i = 2$ $A[2] < A[7] = 4 < 6$ true

$l = 2$

swap $A(2) \leftrightarrow A(2)$ $(2, 4, 3, 1, 9, 10, 6)$

$i = 3$ $A[3] < A[7] = 3 < 6$ True

$l = 3$

swap $A(3) \leftrightarrow A(3)$ $(2, 4, 3, 1, 9, 10, 6)$

$i = 4$ $A[4] < A[7] = 1 < 6$ true

$l = 4$

swap $(2, 4, 3, 1, 9, 10, 6)$

$i = 5$ $A[5] < A[7] = 9 < 6$ false

$l = 4$ $(2, 4, 3, 1, 9, 10, 6)$

$i=6$ $A[6] < A[7]$ $\equiv 10 < 6$ false

$l = 4$ $(2,4,3,1,9,10,6)$

swap $A[7] \leftrightarrow A[5]$ $(2,4,3,1,6,10,9)$

end.

~~8.~~ ~~let us take~~

3. The function XYZ is trying to put all the terms on one side & all -ve terms on the other side

loop invariant:

After each loop iteration the terms after $A[j] \geq 0$ & terms before $A[i] \leq 0$

$\therefore$ the loop invariant is that any term after $A[j] \geq 0$ & ~~$A[i]$~~ any terms before $A[i] \leq 0$ at a particular value of i & j during the iteration

~~Assumption: ① $A[1 \ldots i]$ is ≤ 0~~

~~② $A[j \ldots n]$ is ≥ 0~~

$\therefore$ Elements are swapped when $A[i] < 0$ & $A[j] \geq 0$ is false at a particular iteration

$\therefore$ If any of the above is true then no change occurs in the loop.

$\therefore$ After the function exits, all the -ve nos. are on left side & all the nos. are on right side of the array.