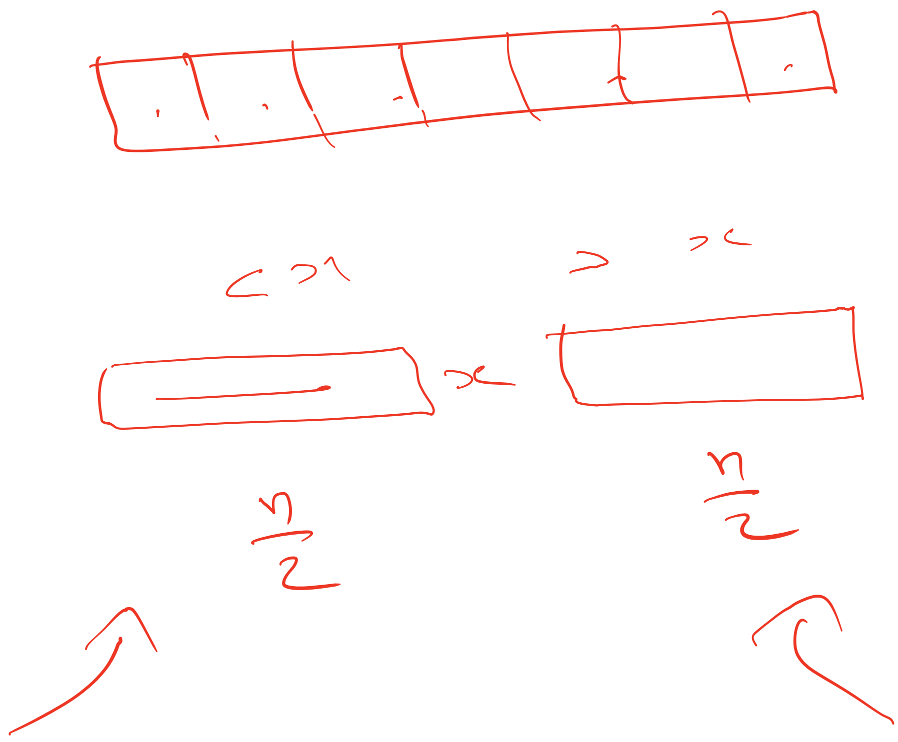
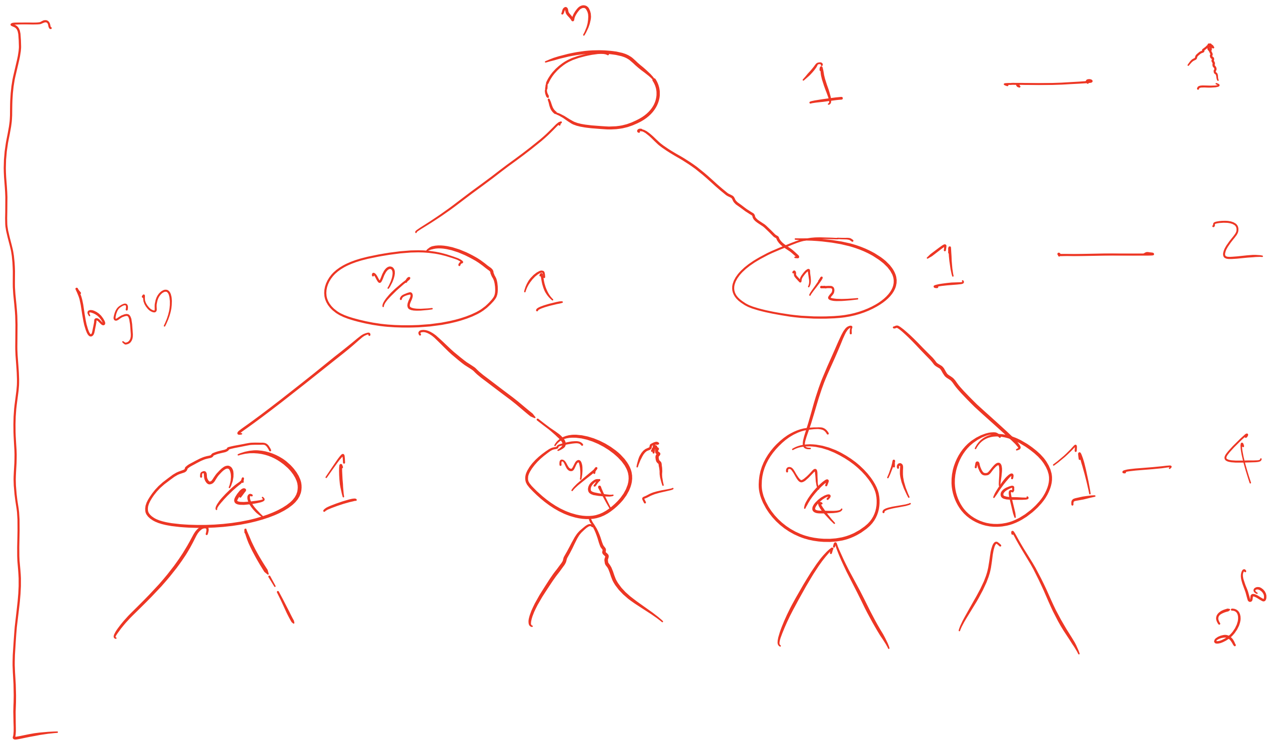
Question # 1

1. choose pivot by calling median finding algorithm.
2. Partition the number according to the pivot.
3. Recursively solve left and right parts.



Answer $O(n)$ ✓ — 1

Proof:



Total no. of calls:

$$1 + 2 + 2^2 + \cdots 2^{\log n}$$

$$= 2^{\log n}\left(1 + \frac{1}{2} + \frac{1}{4} + \cdots\right)$$

$$\leq 2^{\log n} \cdot \frac{1}{1 - 1/2}$$

$$= 2 \cdot 2^{\log n} = 2n = O(n)$$

$\square$

Question 2:

$$T(n) = n^{1/3} T(n^{1/3}) + 1$$

$$T(1) = T(2) = 1$$

substitute $n = 2^k$

$$T(2^k) = 2^{k/3} T(2^{k/3}) + 1$$

Divide by $2^{k/2}$

$$(1) - \frac{T(2^k)}{2^{k/2}} = \frac{T(2^{k/3})}{2^{k/6}} + \frac{1}{2^{k/2}}$$

Let $S(k) = \dfrac{T(2^k)}{2^{k/2}}$

$$S(0) = 1$$

$$S(k) = S(k/3) + 1/2^{k/2}$$

$$= \frac{1}{2^{k/2}} + \frac{1}{2^{k/6}} + S(k/9)$$

$$= \frac{1}{2^{k/2}} + \frac{1}{2^{k/6}} + \frac{1}{2^{k/18}} + \cdots$$

$$\leq 2$$

$$\leq \frac{1}{2^k} + \frac{1}{2^{k-1}} + \frac{1}{2^{k-2}} + \cdots$$

$$1 + \frac{1}{2} + \frac{1}{4} + \cdots$$

$$S(k) \leq 2$$

$$\frac{T(2^k)}{2^{k/2}} \leq 2$$

$$T(2^k) \leq 2 \cdot 2^{k/2}$$

$$\leq 2n^{1/2}$$

$$T(n) = T(2^k) = O(\sqrt{n})$$

Question 3:

A

$$T(n) = T\left(\frac{19}{26}n\right) + T\left(\frac{n}{13}\right) + O(n)$$

$$T(1) = T(2) = T(13) = \underline{\underline{O(1)}}$$

Question #4:

$A[1 \dots m]$, $B[1 \dots n]$.

$Edit(i,j)$ = min cost to Edit $A[1 \dots i]$ to $B[1 \dots j]$.

$$Edit(i,j) = \begin{cases} 0.75\, i & \text{if } j = 0 \\ 0.75\, j & \text{if } i = 0 \\ \min \begin{cases} 0.75 + Edit(i, j-1); \\ 0.75 + Edit(i-1, j); \\ Edit(i-1, j-1) + 0.5\, |A[i] - B[j]| \end{cases} & \text{o.w.} \end{cases}$$